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GENERALIZED FIXED POINT THEOREMS IN DISLOCATED QUASI - METRIC SPACES

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Abstract: In this paper, the concept of generalized contraction mappings has been used in proving fixed point theorems. We establish some new common fixed point theorems in complete dislocated quasi metric spaces using contraction mappings. The presented results extend and complement some known existence results from the literature.

Keywords: Dislocated quasi-metric space; fixed point; dq-Cauchy sequence.

2000 AMS Subject Classification: 47H17, 47H05, 47H09

1. Introduction

P.Hitzler et al., introduced the notation of dislocated metric spaces in which self distance of a point need not be equal to zero. They also generalized the famous Banach contraction principle in this space, and satisfying certain contractive conditions has been at the center of vigorous research activity. Dislocated metric space plays very important role in topology, logical programming and in electronics engineering. Zeyada et al. (4) initiated the concept of dislocated quasi-metric space and generalized the result of Hitzler and Seda in dislocated qusi-metric spaces. Results on fixed points in dislocated and dislocated quasi-metric spaces followed by Isufati (1) and Aage and Salunke (3), and recently by Shrivastava, Ansari and Sharma(7). Our result generalizes some results of fixed points.

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2. Preliminaries

Definition 2.1: Let X be a nonempty set, let d: X xX $\rightarrow$ $[0,\infty)$ be a function satisfying following conditions.

(i) d (x , y) = d (y , x) = 0 implies x = y .

(ii) d (x , y) $\leq$ d (x , z) + d (z , y) for all x, y , z $\in$ X .

Then d is called a dislocated quasi metric spaces or dq - metric on X.

Definition 2.2 : A sequence { x_n } in dq-metric space (X , d) is said to be a Cauchy sequence if for given $\in > 0$, there exists $n_0 \in N$ such that for all m , n $\geq n_0$, implies $d(x_m, x_n) < \in$.

Definition 2.3: A sequence { x_n } in dq-metric space (X , d) is said to be a convergent to x if $\lim_{n\to\infty} d(x_n, x) = 0$.

Definition 2. 4: A dq-metric space (X , d) is said to be a Complete if every Cauchy sequence in convergent in X .

Definition 2. 5: Let (X,d) be a dq-metric space . A mapping f : X $\rightarrow$ X is called contraction if there exists $0 \leq \lambda < 1$ *such that* $d[f(x), f(y)] \leq \lambda d(x,y)$ *for all* $x, y \in X$.

Lemma 2.6: dq-limits in a dq-metric space are unique.

Theorem 2.7: Let (X,d) be complete dq-metric space and let f : X$\rightarrow$ X be a continuous contraction function then f has a unique fixed point.

3. Main Results

Theorem 3.1: Let (X , d) be a complete dislocated quasi metric space .Let T be a continuous mapping from X to X satisfying the following condition

$$d(Tx,Ty)) \leq \alpha \frac{d(y,Ty)[1+d(x,Tx)]}{1+d(x,y)} + \beta d(x,y) + \gamma \left(\frac{d(y,Tx)+d(x,Ty)}{d(x,y)+d(y,Ty)} \right) d(y,Ty)$$
$$+ \delta[d(x,Tx)+d(y,Ty)] + \mu[d(x,Ty)+d(y,Tx)] \quad .....(1)$$

for all x,y $\in$ X , $\alpha,\beta,\gamma,\delta,\mu > 0$ *and* $\alpha+\beta+\gamma+2\delta+2\mu<1$. Then T has a unique fixed point.

Proof: Let { x_n} be a sequence in X , defined as follows

Let $x_0 \in$ X, $Tx_0 = x_1$, $Tx_1 = x_2$,............... , $Tx_n = x_{n+1}$.

Consider

$$d(x_n,x_{n+1}) = d(Tx_{n-1},Tx_n)$$
$$\leq \frac{\alpha\, d(x_n,Tx_n)\left[1+d(x_{n-1},Tx_{n-1})\right]}{1+d(x_{n-1},x_n)} + \beta\, d(x_{n-1},x_n)$$

$$+\gamma\left(\frac{d(x_n,Tx_{n-1})+d(x_{n-1},Tx_n)}{d(x_{n-1},x_n)+d(x_n,Tx_n)}\right)d(x_n,Tx_n)$$

$$+\delta\left[d(x_{n-1},Tx_{n-1})+d(x_n,Tx_n)\right]+\mu\left[d(x_{n-1},Tx_n)+d(x_n,Tx_{n-1})\right].$$

$$\leq \frac{\alpha d(x_n,x_{n+1})\left[1+d(x_{n-1},x_n)\right]}{1+d(x_{n-1},x_n)} + \beta\, d(x_{n-1},x_n)$$

$$+\gamma\left(\frac{d(x_n,x_n)+d(x_{n-1},x_{n+1})}{d(x_{n-1},x_n)+d(x_n,x_{n+1})}\right)d(x_n,x_{n+1})$$

$$+\delta\left[d(x_{n-1},x_n)+d(x_n,x_{n+1})\right]+\mu\left[d(x_{n-1},x_{n+1})+d(x_n,x_n)\right]$$

$$d(x_n,x_{n+1}) \leq \left(\frac{\beta+\delta+\mu}{1-\alpha-\gamma-\delta-\mu}\right)d(x_{n-1},x_n)$$

$$d(x_n,x_{n+1}) \leq h\ d(x_{n-1},x_n) \text{........}(2) \qquad \textit{where} \quad h=\left(\frac{\beta+\delta+\mu}{1-\alpha-\gamma-\delta-\mu}\right)<1$$

In the same way, wehave $d(x_{n-1},x_n) \leq h\ d(x_{n-2},x_{n-1})$.

By (2), we get $d(x_n,x_{n+1}) \leq h^2\ d(x_{n-2},x_{n-1})$ Continue this process , we get in general $d(x_n,x_{n+1}) \leq h^n\ d(x_1,x_0)$. *Since* $0 \leq h < 1$ *as* $n \to \infty$, $h^n \to 0$. Hence {x_n } is a dq–

cauchy sequence in X . Thus { x_n} dislocated quasi converges to some u in X . Since T is continuous we have T(u) = lim T(x_n) = lim x_{n+1} = u .Thus u is a fixed point T.

Uniqueness: Let $x \in X$ is a fixed point. Then by (1),

$$d(x,x) \le \frac{\alpha\, d(x,x)\left[1+d(x,x)\right]}{1+d(x,x)} + \beta\, d(x,x) + \gamma\left(\frac{d(x,x)+d(x,x)}{d(x,x)+d(x,x)}\right)d(x,x)$$
$$+\delta\left[d(x,x)+d(x,x)\right]+\mu\left[d(x,x)+d(x,x)\right]$$

$$d(x,x) \le \left(\alpha+\beta+\gamma+2\delta+2\mu\right)d(x,x)$$

Which is true only if d (x , x) = 0, since $0 \le \left(\alpha+\beta+\gamma+2\delta+2\mu\right) < 1$ *and* $d(x,x) \ge 0$.

Thus d(x, x) =0 if x is fixed point of T.

Let x, y be fixed point, (i.e.) Tx = x , Ty = y,

Then by condition (1), we have,

$$d(x,y) = d\left(Tx,Ty\right) \le \frac{\alpha\, d(y,y)\left[1+d\left(x,x\right)\right]}{\left[1+d\left(x,y\right)\right]} + \beta d\left(x,y\right)$$
$$+\gamma\left[\frac{d(y,x)+d(x,y)}{d(x,y)+d(y,y)}\right]d(y,y)$$
$$+\delta\left[d(x,x)+d\left(y,y\right)\right]+\mu\left[d(x,y)+d(y,x)\right]$$
$$d(x,y) \le \beta\ d(x,y)+\mu\left[d(x,y)+d(y,x)\right]$$

similarly

$$d(y,x) \le \beta\ d(y,x)+\mu\left[d(y,x)+d(x,y)\right]$$

$$\textit{Hence}\quad \left|d(x,y)-d(y,x)\right| \le \beta\left|d(x,y)-d(y,x)\right|,$$
$$\textit{which implies } d(x,y) = d(y,x).$$

Since $0 \le \beta < 1$.

Again by condition (1) , we have,

$d(x,y) \le \left(\beta+2\mu\right) d(x,y)$ *which gives* , $d\left(x,y\right)=0$. Since $0 \le \beta+2\mu < 1$, *further* $d(x,y) = d(y,x) = 0 \Rightarrow x = y$.

This proves the uniqueness. This completes the proof.

Corollary 3.2: Let (X , d) be a complete dislocated quasi metric space .Let T be a continuous mapping from X to X satisfying the following condition

$$d(Tx,Ty)) \leq \frac{\alpha d(y,Ty)\left[1+d(x,Tx)\right]}{1+d(x,y)} + \beta d(x,y) + \gamma\left(\frac{d\left(y,Tx\right)+d\left(x,Ty\right)}{d(x,y)+d\left(y,Ty\right)}\right)d\left(y,Ty\right) \text{.......(1)}$$

for all x,y $\in$ X , $\alpha,\beta,\gamma > 0$ *and* $\alpha+\beta+\gamma < 1$. Then T has a unique fixed point.

Proof: Put $\delta = \mu = 0$ in the above theorem 3.1, it can be easily proved.

Corollary 3.3: Let (X , d) be a complete dislocated quasi metric space .Let T be a continuous mapping from X to X satisfying the following condition

$$d(T(x),T(y)) \leq \frac{\alpha d(y,T(y))\left[1+d(x,Tx)\right]}{1+d(x,y)} + \beta d(x,y)$$

for all x,y $\in$ X , $\alpha,\beta > 0$ *and* $\alpha+\beta < 1$. Then T has a unique fixed point.

Proof: Put $\gamma = \delta = \mu = 0$ in the above theorem 3.1, it can be easily proved.

Theorem 3.4: Let (X , d) be a complete dislocated quasi metric space .Let T be a continuous mapping from X to X satisfying the following condition

$$d(Tx,Ty)) \leq \alpha\frac{d(y,Ty)\left[1+d(x,Tx)\right]}{1+d(x,y)} + \beta d(x,y) + \gamma\left(\frac{d\left(x,Ty\right)\ d\left(y,Ty\right)}{d(x,y)+d\left(y,Ty\right)}\right)$$
$$+\delta\left[d(x,Tx)+d\left(y,Ty\right)\right] + \mu\left[d(x,Ty)+d(y,Tx)\right] \text{.....(1)}$$

for all x,y $\in$ X , $\alpha,\beta,\gamma,\delta,\mu > 0$ *and* $\alpha+\beta+\gamma+2\delta+2\mu < 1$. Then T has a unique fixed point.

Proof: Let { x_n} be a sequence in X , defined as follows

Let $x_0 \varepsilon$ X , $Tx_0 = x_1$, $Tx_1 = x_2$,............... , $Tx_n = x_{n+1}$.

Consider

$$d\left(x_n,x_{n+1}\right)= d\left(Tx_{n-1},Tx_n\right)$$
$$\leq \frac{\alpha d\left(x_n,Tx_n\right)\left[1+d\left(x_{n-1},Tx_{n-1}\right)\right]}{1+d\left(x_{n-1},x_n\right)}+\beta\, d\left(x_{n-1},x_n\right)$$
$$+\gamma\left(\frac{d\left(x_{n-1},Tx_n\right) d\left(x_n,Tx_n\right)}{d(x_{n-1},x_n)+d\left(x_n,Tx_n\right)}\right)$$
$$+\delta\left[d(x_{n-1},Tx_{n-1})+d\left(x_n,Tx_n\right)\right]+\mu\left[d(x_{n-1},Tx_n)+d(x_n,Tx_{n-1})\right]$$
$$\leq \frac{\alpha\, d\left(x_n,x_{n+1}\right)\left[1+d\left(x_{n-1},x_n\right)\right]}{1+d\left(x_{n-1},x_n\right)}+\beta\, d\left(x_{n-1},x_n\right)$$
$$+\gamma\left(\frac{d\left(x_{n-1},x_{n+1}\right) d\left(x_n,x_{n+1}\right)}{d(x_{n-1},x_n)+d\left(x_n,x_{n+1}\right)}\right)$$
$$+\delta\left[d(x_{n-1},x_n)+d\left(x_n,x_{n+1}\right)\right]+\mu\left[d(x_{n-1},x_{n+1})+d(x_n,x_n)\right]$$
$$d\left(x_n,x_{n+1}\right)\leq\left(\frac{\beta+\delta+\mu}{1-\alpha-\gamma-\delta-\mu}\right)d\left(x_{n-1},x_n\right)$$
$$d\left(x_n,x_{n+1}\right)\leq h\, d\left(x_{n-1},x_n\right)\text{.......}(2)\qquad where\quad h=\left(\frac{\beta+\delta+\mu}{1-\alpha-\gamma-\delta-\mu}\right)<1$$

In the same way, we have $d\left(x_{n-1},x_n\right)\leq h\, d\left(x_{n-2},x_{n-1}\right)$.

By (2), we get $d\left(x_n,x_{n+1}\right)\leq h^2\, d\left(x_{n-2},x_{n-1}\right)$ Continue this process , we get in general $d\left(x_n,x_{n+1}\right)\leq h^n\, d\left(x_1,x_0\right)$. *Since* $0\leq h<1$ *as* $n\to\infty$, $h^n\to 0$. Hence { x_n } is a dq–Cauchy sequence in X . Thus { x_n} dislocated quasi converges to some u in X . Since T is continuous we have

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Uniqueness: Let $x\in X$ is a fixed point. Then by (1),

$$d(x,x)\leq\frac{\alpha\, d(x,x)\left[1+d(x,x)\right]}{1+d(x,x)}+\beta\, d(x,x)+\gamma\left(\frac{d(x,x)d(x,x)}{d(x,x)+d(x,x)}\right)$$
$$+\delta\left[d(x,x)+.d\left(x,x\right)\right]+\mu\left[d(x,x)+d(x,x)\right].$$

$$d(x,x) \le \left(\alpha+\beta+\frac{\gamma}{2}+2\delta+2\mu\right) d(x,x)$$

Which is true only if d (x , x) = 0, since $0 \le \left(\alpha+\beta+\frac{\gamma}{2}+2\delta+2\mu\right) < 1$ *and* $d(x,x) \ge 0$.

Thus d(x, x) =0 if x is fixed point of T.

Let x,y be fixed point , (i.e.) Tx = x , Ty = y ,

Then by condition (1), we have,

$$\begin{aligned} d(x,y) = d\,(Tx,Ty) \le & \frac{\alpha\, d(y,y)\left[1+d(x,x)\right]}{\left[1+d(x,y)\right]} + \beta\; d(x,y) \\ & + \gamma\left[\frac{d(x,y)\,d(y,y)}{d(x,y)+d(y,y)}\right] \\ & + \delta\left[d(x,x)+d(y,y)\right] + \mu\left[d(x,y)+d\;(y,x)\right] \end{aligned}$$

$$d(x,y) \le \beta\; d(x,y) + \mu\left[d(x,y)+d(y,x)\right]$$

similarly

$$d(y,x) \le \beta\; d(y,x) + \mu\left[d(y,x)+d(x,y)\right]$$

Hence $|d(x,y) - d(y,x)| \le \beta |d(x,y) - d(y,x)|$,
which implies $d(x,y) = d(y,x)$.

Since $0 \le \beta < 1$.

Again by condition (1) , we have,

$d(x,y) \le (\beta+2\mu)\, d(x,y)$ *which gives* , $d(x,y) = 0$. Since $0 \le \beta+2\mu < 1$,

further $d(x,y) = d(y,x) = 0 \Rightarrow x = y$.

This proves the uniqueness. This completes the proof.

Corollary 3.5: Let (X , d) be a complete dislocated quasi metric space .Let T be a continuous mapping from X to X satisfying the following condition

$$d(Tx,Ty)) \leq \frac{\alpha d(y,Ty)\left[1+d(x,Tx)\right]}{1+d(x,y)} + \beta d(x,y) + \gamma\left(\frac{d\left(y,Tx\right)d\left(y,Ty\right)}{d(x,y)+d\left(y,Ty\right)}\right) \ldots\ldots.(1)$$

for all x,y $\in$ X , $\alpha,\beta,\gamma > 0$ *and* $\alpha+\beta+\gamma < 1$. Then T has a unique fixed point.

Proof: Put $\delta = \mu = 0$ in the above theorem 3.1, it can be easily proved.

Corollary 3.6: Let (X , d) be a complete dislocated quasi metric space .Let T be a continuous mapping from X to X satisfying the following condition

$$d(T(x),T(y)) \leq \frac{\alpha d(y,T(y))\left[1+d(x,Tx)\right]}{1+d(x,y)} + \beta d(x,y)$$

for all x,y $\in$ X , $\alpha,\beta > 0$ *and* $\alpha+\beta < 1$. Then T has a unique fixed point.

Proof: Put $\gamma = \delta = \mu = 0$ in the above theorem 3.1, it can be easily proved.

Theorem 3.7: Let (X , d) be a complete dislocated quasi metric space .Let T be a continuous mapping from X to X satisfying the following condition

$$d(Tx,Ty)) \leq \frac{\alpha\, d(y,Ty)\, d(x,Tx)}{d(x,y)} + \beta\, d(x,y) + \gamma\left(\frac{d\left(y,Tx\right)+d\left(x,Ty\right)}{d(x,y)+d\left(y,Ty\right)}\right) d\left(y,Ty\right)$$
$$+\delta\left[d(x,Tx)+d\left(y,Ty\right)\right] + \mu\left[d(x,Ty)+d(y,Tx)\right] \ldots..(1)$$

for all x,y $\in$ X , $\alpha,\beta,\gamma,\delta,\mu > 0$ *and* $\alpha+\beta+\gamma+2\delta+2\mu < 1$. Then T has a unique fixed point.

Proof: Let { x_n} be a sequence in X , defined as follows

Let $x_0 \in$ X , $Tx_0 = x_1$, $Tx_1 = x_2$,............... , $Tx_n = x_{n+1}$.

Consider

$$d\left(x_n, x_{n+1}\right) = d\left(Tx_{n-1}, Tx_n\right)$$
$$\leq \frac{\alpha\, d\left(x_n, Tx_n\right) d\left(x_{n-1}, Tx_{n-1}\right)}{d\left(x_{n-1}, x_n\right)} + \beta\, d\left(x_{n-1}, x_n\right)$$

$$+\gamma\left(\frac{d\left(x_n, Tx_{n-1}\right)+d\left(x_{n-1}, Tx_n\right)}{d(x_{n-1}, x_n)+d\left(x_n, Tx_n\right)}\right) d\left(x_n, Tx_n\right)$$

$$+\delta\left[d(x_{n-1},Tx_{n-1})+d\left(x_n,Tx_n\right)\right]+\mu\left[d(x_{n-1},Tx_n)+d(x_n,Tx_{n-1})\right].$$

$$\leq \frac{\alpha\ d\left(x_n,x_{n+1}\right)d\left(x_{n-1},x_n\right)}{d\left(x_{n-1},x_n\right)}+\beta\, d\left(x_{n-1},x_n\right)$$

$$+\gamma\left(\frac{d\left(x_n,x_n\right)+d\left(x_{n-1},x_{n+1}\right)}{d\left(x_{n-1},x_n\right)+d\left(x_n,x_{n+1}\right)}\right)d\left(x_n,x_{n+1}\right)$$

$$+\delta\left[d\left(x_{n-1},x_n\right)+d\left(x_n,x_{n+1}\right)\right]+\mu\left[d(x_{n-1},x_{n+1})+d\left(x_n,x_n\right)\right]$$

$$\left(1-\alpha-\gamma-\delta-\mu\right)d\left(x_n,x_{n+1}\right)\leq\left(\beta+\delta+\mu\right)\ d\left(x_{n-1},x_n\right)$$

$$d\left(x_n,x_{n+1}\right)\leq\left(\frac{\beta+\delta+\mu}{1-\alpha-\gamma-\delta-\mu}\right)d\left(x_{n-1},x_n\right)$$

$$d\left(x_n,x_{n+1}\right)\leq h\ d\left(x_{n-1},x_n\right)\text{.......}(2)\qquad where\quad h=\left(\frac{\beta+\delta+\mu}{1-\alpha-\gamma-\delta-\mu}\right)<1$$

In the same way, we have $d\left(x_{n-1},x_n\right)\leq h\ d\left(x_{n-2},x_{n-1}\right)$.

By (2), we get $d\left(x_n,x_{n+1}\right)\leq h^2\ d\left(x_{n-2},x_{n-1}\right)$ Continue this process, we get in general $d\left(x_n,x_{n+1}\right)\leq h^n\ d\left(x_1,x_0\right)$. *Since* $0\leq h<1$ *as* $n\to\infty$, $h^n\to 0$. Hence { x_n } is a dq–cauchy sequence in X . Thus{ x_n} dislocated quasi converges to some u in X . Since T is continuous we have T(u) = lim T(x_n) = lim x_{n+1} = u .Thus u is a fixed point T.

Uniqueness: Let $x\in X$ is a fixed point. Then by (1),

$$d(x,x)\leq\frac{\alpha\, d(x,x)\, d(x,x)}{d(x,x)}+\beta\, d(x,x)+\gamma\left(\frac{d(x,x))+d(x,x))}{d(x,x)+d(x,x)}\right)d(x,x)$$
$$+\delta\left[d(x,x)+d\left(x,x\right)\right]+\mu\left[d(x,x)+d(x,x)\right].$$

$$d(x,x)\leq\left(\alpha+\beta+\gamma+2\delta+2\mu\right)d(x,x)$$

Which is true only if d (x , x) = 0, since $0\leq\left(\alpha+\beta+\gamma+2\delta+2\mu\right)<1$ *and* $d(x,x)\geq 0$.

Thus d(x, x) =0 if x is fixed point of T.

Let x,y be fixed point , (i.e.) Tx = x , Ty = y ,

Then by condition (1), we have,

$$d(x,y) = d\left(Tx,Ty\right) \le \frac{\alpha\, d(y,y)d\left(x,x\right)}{d\left(x,y\right)} + \beta d\left(x,y\right) + \gamma\left[\frac{d\,(y,x)+d\,(x,y)}{d\,(x,y)+d\,(y,y)}\right]d\,(y,y)$$
$$+\delta\left[d\,(x,x)+d\left(y,y\right)\right]+\mu\left[d\,(x,y)+d\,\,(y,x)\right]$$
$$d(x,y)\ \le\beta\ \ d(x,y)+\mu\left[d(x,y)+d(y,x)\right]$$

similarly

$$d(y,x)\ \le\beta\ \ d(y,x)+\mu\left[d(y,x)+d(x,y)\right]$$

Hence $\left|d(x,y) - d(y,x)\right| \le \beta\left|d(x,y) - d(y,x)\right|$, *which implies* $d(x,y) = d(y,x)$.

Since $0\ \le\beta<1$.

Again by condition (1), we have,

$d(x,y)\ \le\left(\beta+2\mu\right) d(x,y)$ *which gives* , $d\left(x,y\right)=0$. Since $0\ \le\beta+2\mu<1$,

further $d(x,y) = d(y,x)=0\ \Rightarrow x=y$.

This proves the uniqueness. This completes the proof.

Corollary 3.8: Let (X , d) be a complete dislocated quasi metric space .Let T be a continuous mapping from X to X satisfying the following condition

$$d(Tx,Ty))\le\alpha\frac{d(y,Ty)\ \ d(x,Tx)}{d(x,y)}+\beta d(x,y)+\gamma\left(\frac{d\left(y,Tx\right)+d\left(x,Ty\right)}{d(x,y)+d\left(y,Ty\right)}\right)d\left(y,Ty\right).......(1)$$

for all x,y $\in$ X , $\alpha,\beta,\gamma>0$ *and* $\alpha+\beta+\gamma\ <1$. Then T has a unique fixed point.

Proof: Put $\delta=\mu=0$ in the above theorem 3.1, it can be easily proved.

Corollary 3.9: Let (X , d) be a complete dislocated quasi metric space .Let T be a continuous mapping from X to X satisfying the following condition

$$d(T(x),T(y))\le\frac{\alpha d(y,T(y))\,d(x,Tx)}{d(x,y)}+\beta d(x,y)$$

for all x,y $\in$ X , $\alpha, \beta \in [0,1]$ *and* $\alpha + \beta < 1$. Then T has a unique fixed point.

Proof: Put $\gamma = \delta = \mu = 0$ in the above theorem 3.1, it can be easily proved.

Theorem 3.10: Let (X , d) be a complete dislocated quasi metric space .Let T be a continuous mapping from X to X satisfying the following condition

$$d(Tx,Ty)) \le \alpha \frac{d(y,Ty)\ d(x,Tx)}{d(x,y)} + \beta d(x,y) + \gamma\left(\frac{d(x,Ty)\ d(y,Ty)}{d(x,y)+d(y,Ty)}\right)$$
$$+\delta\left[d(x,Tx)+.d(y,Ty)\right] + \mu\left[d(x,Ty)+d(y,Tx)\right].....(1)$$

for all x,y $\in$ X , $\alpha, \beta, \gamma, \delta, \mu > 0$ *and* $\alpha + \beta + \gamma + 2\delta + 2\mu < 1$. Then T has a unique fixed point.

Proof: Let { x_n} be a sequence in X , defined as follows

Let $x_0 \,\varepsilon$ X , $Tx_0 = x_1$, $Tx_1 = x_2$,............... , $Tx_n = x_{n+1.}$

Consider

$$d(x_n, x_{n+1}) = d(Tx_{n-1}, Tx_n)$$
$$\le \frac{\alpha d(x_n, Tx_n) d(x_{n-1}, Tx_{n-1})}{d(x_{n-1}, x_n)} + \beta d(x_{n-1}, x_n)$$
$$+\gamma\left(\frac{d(x_{n-1}, Tx_n)\ d(x_n, Tx_n)}{d(x_{n-1}, x_n) + d(x_n, Tx_n)}\right)$$
$$+\delta\left[d(x_{n-1}, Tx_{n-1}) + .d(x_n, Tx_n)\right] + \mu\left[d(x_{n-1}, Tx_n) + d(x_n, Tx_{n-1})\right].$$

$$\le \frac{\alpha d(x_n, x_{n+1}) d(x_{n-1}, x_n)}{d(x_{n-1}, x_n)} + \beta d(x_{n-1}, x_n)$$
$$+\gamma\left(\frac{d(x_{n-1}, x_{n+1})\ d(x_n, x_{n+1})}{d(x_{n-1}, x_n) + d(x_n, x_{n+1})}\right)$$
$$+\delta\left[d(x_{n-1}, x_n) + d(x_n, x_{n+1})\right] + \mu\left[d(x_{n-1}, x_{n+1}) + d(x_n, x_n)\right]$$
$$d(x_n, x_{n+1}) \le \left(\frac{\beta+\delta+\mu}{1-\alpha-\gamma-\delta-\mu}\right) d(x_{n-1}, x_n)$$
$$d(x_n, x_{n+1}) \le h\ d(x_{n-1}, x_n)........(2) \qquad where \quad h = \left(\frac{\beta+\delta+\mu}{1-\alpha-\gamma-\delta-\mu}\right) < 1$$

In the same way, we have $d(x_{n-1}, x_n) \le h\ d(x_{n-2}, x_{n-1})$.

By (2), we get $d\left(x_n, x_{n+1}\right) \le h^2\ d\left(x_{n-2}, x_{n-1}\right)$ Continue this process, we get in general $d\left(x_n, x_{n+1}\right) \le h^n\ d\left(x_1, x_0\right)$. *Since* $0 \le h < 1$ *as* $n \to \infty$, $h^n \to 0$. Hence { x_n } is a dq–cauchy sequence in X . Thus { x_n} dislocated quasi converges to some u in X . Since T is continuous we have T(u) = lim T(x $_n$) = lim x_{n+1} = u .Thus u is a fixed point T.

Uniqueness: Let $x \in X$ is a fixed point. Then by (1),

$$d(x,x) \le \frac{\alpha\, d(x,x)\, d(x,x)}{d(x,x)} + \beta\, d(x,x) + \gamma \left(\frac{d(x,x) d(x,x)}{d(x,x) + d(x,x)} \right)$$
$$+ \delta \left[d(x,x) + .d\left(x,x\right) \right] + \mu \left[d(x,x) + d(x,x) \right].$$

$$d(x,x) \le \left(\alpha + \beta + \frac{\gamma}{2} + 2\delta + 2\mu \right) d(x,x)$$

Which is true only if d (x , x) = 0, since $0 \le \left(\alpha + \beta + \frac{\gamma}{2} + 2\delta + 2\mu \right) < 1$ *and* $d(x,x) \ge 0$.

Thus d(x, x) =0 if x is fixed point of T.

Let x,y be fixed point , (i.e.) Tx = x , Ty = y ,

Then by condition (1), we have,

$$d(x,y) = d\ \left(Tx,Ty\right) \le \frac{\alpha\, d(y,y)\, d\left(x,x\right)}{d\left(x,y\right)} + \beta d\left(x,y\right) + \gamma \left[\frac{d(x,y) d(y,y)}{d(x,y) + d(y,y)} \right]$$
$$+ \delta \left[d(x,x) + d\left(y,y\right) \right] + \mu \left[d(x,y) + d\ (y,x) \right]$$
$$d(x,y)\ \le \beta\ \ d(x,y) + \mu \left[d(x,y) + d(y,x) \right]$$

similarly

$$d(y,x)\ \le \beta\ \ d(y,x) + \mu \left[d(y,x) + d(x,y) \right]$$

Hence $\left| d(x,y) - d(y,x) \right| \le \beta \left| d(x,y) - d(y,x) \right|$, *which implies* $d(x,y) = d(y,x)$.

Since $0\ \le \beta < 1$.

Again by condition (1), we have,

$d(x,y)\ \le \left(\beta + 2\mu\right)\ d(x,y)$ *which gives* , $d\left(x,y\right) = 0$. Since $0\ \le \beta + 2\mu < 1$, *further* $d(x,y)=d(y,x)=0 \Rightarrow x=y$.

This proves the uniqueness. This completes the proof.

Corollary 3.11: Let (X , d) be a complete dislocated quasi metric space .Let T be a continuous mapping from X to X satisfying the following condition

$$d(Tx,Ty)) \leq \frac{\alpha d(y,Ty)\, d(x,Tx)}{d(x,y)} + \beta d(x,y) + \gamma\left(\frac{d(y,Tx)\, d(y,Ty)}{d(x,y)+d(y,Ty)}\right) \text{.......(1)}$$

for all x,y $\in$ X , $\alpha,\beta,\gamma > 0$ *and* $\alpha+\beta+\gamma < 1$. Then T has a unique fixed point.

Proof: Put $\delta = \mu = 0$ in the above theorem 3.1, it can be easily proved.

Corollary 3.12: Let (X , d) be a complete dislocated quasi metric space .Let T be a continuous mapping from X to X satisfying the following condition

$$d(T(x),T(y)) \leq \frac{\alpha d(y,T(y))\, d(x,Tx)}{d(x,y)} + \beta d(x,y)$$

for all x,y $\in$ X , $\alpha,\beta \in [0,1]$ *and* $\alpha+\beta < 1$. Then T has a unique fixed point.

Proof: Put $\gamma = \delta = \mu = 0$ in the above theorem 3.1, it can be easily proved.

Conflict of Interests

The author declares that there is no conflict of interests.

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