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## FIXED POINT THEOREM IN FUZZY METRIC SPACE USING SEMI-COMPATIBLE MAPPINGS

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**Abstract:** The present paper deals with common fixed point theorem in fuzzy metric space by employing the notion of semi-compatible and occasionally weakly compatible mappings. Our result generalizes the recent result of Singh et. al. [17].

**Keywords:** Common fixed points, fuzzy metric space, compatible maps, occasionally weakly compatible mappings and semi-compatible mappings.

**AMS Subject Classification:** Primary 47H10, Secondary 54H25.

### 1. Introduction

In 1965, Zadeh [18] introduced the concept of Fuzzy set as a new way to represent vagueness in our everyday life. However, when the uncertainty is due to fuzziness rather than randomness, as sometimes in the measurement of an ordinary length, it seems that the concept of a fuzzy metric space

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is more suitable. We can divide them into following two groups: The first group involves those results in which a fuzzy metric on a set  $X$  is treated as a map where  $X$  represents the totality of all fuzzy points of a set and satisfy some axioms which are analogous to the ordinary metric axioms. Thus, in such an approach numerical distances are set up between fuzzy objects. On the other hand in second group, we keep those results in which the distance between objects is fuzzy and the objects themselves may or may not be fuzzy. In this paper we deal with the Fuzzy metric space defined by Kramosil and Michalek [10] and modified by George and Veeramani [3]. Recently, Grabiec [4] has proved fixed point results for Fuzzy metric space. In the sequel, Singh and Chauhan [13] introduced the concept of compatible mappings in Fuzzy metric space and proved the common fixed point theorem. Jungck et. al. [7] introduced the concept of compatible maps of type (A) in metric space and proved fixed point theorems. Singh et.al. [16] proved various fixed point theorems using the concepts of semi-compatibility, compatibility and implicit relations in fuzzy metric space. In 2011, using the concept of compatible maps of type (A) and type ( $\beta$ ), Singh et. al. [14, 15] proved fixed point theorems in a fuzzy metric space. Recently in 2012, Jain et. al. [5, 6] and Sharma et. al. [12] proved various fixed point theorems using the concepts of semi-compatible mappings, property (E.A.) and absorbing mappings. The concept of occasionally weakly compatible mappings in metric spaces is introduced by Al-Thagafi and Shahzad [1] which is most general among all the commutativity concepts. Recently, Khan and Sumitra [8] extended the notion of occasionally weakly compatible maps to fuzzy metric space.

In this paper, a fixed point theorem for four self maps has been established using the concept of semi-compatible and occasionally weak compatible maps which generalizes the result of Singh et. al. [17].

For the sake of completeness, we recall some definitions and known results in Fuzzy metric space.

## 2. Preliminaries

**Definition 2.1.** [11] A binary operation  $*$  :  $[0, 1] \times [0, 1] \rightarrow [0, 1]$  is called a *t-norm* if  $([0, 1], *)$  is an abelian topological monoid with unit 1 such that  $a * b \leq c * d$  whenever  $a \leq c$  and  $b \leq d$  for  $a, b, c, d \in [0, 1]$ .

Examples of t-norms are  $a * b = ab$  and  $a * b = \min\{a, b\}$ .

**Definition 2.2.** [11] The 3-tuple  $(X, M, *)$  is said to be a *Fuzzy metric space* if  $X$  is an arbitrary set,  $*$  is a continuous t-norm and  $M$  is a Fuzzy set in  $X^2 \times [0, \infty)$  satisfying the following conditions :

for all  $x, y, z \in X$  and  $s, t > 0$ .

(FM-1)  $M(x, y, 0) = 0,$

(FM-2)  $M(x, y, t) = 1$  for all  $t > 0$  if and only if  $x = y,$

(FM-3)  $M(x, y, t) = M(y, x, t),$

(FM-4)  $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s),$

(FM-5)  $M(x, y, \cdot) : [0, \infty) \rightarrow [0, 1]$  is left continuous,

(FM-6)  $\lim_{t \rightarrow \infty} M(x, y, t) = 1.$

Note that  $M(x, y, t)$  can be considered as the degree of nearness between  $x$  and  $y$  with respect to  $t$ . We identify  $x = y$  with  $M(x, y, t) = 1$  for all  $t > 0$ . The following example shows that every metric space induces a Fuzzy metric space.

**Example 2.1.** [11] Let  $(X, d)$  be a metric space. Define  $a * b = \min\{a, b\}$  and

$M(x, y, t) = \frac{t}{t + d(x, y)}$  for all  $x, y \in X$  and all  $t > 0$ . Then  $(X, M, *)$  is a Fuzzy metric space. It is

called the Fuzzy metric space induced by  $d$ .

**Definition 2.3.** [4] Let  $(X, M, *)$  be a fuzzy metric space. Then

(a) a sequence  $\{x_n\}$  in  $X$  is said to

(i) be a Cauchy sequence if

$$\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, t) = 1 \text{ for all } t > 0 \text{ and } n, p \in \mathbb{N},$$

(ii) be convergent to a point  $x \in X$  if

$$\lim_{n \rightarrow \infty} M(x_n, x, t) = 1 \text{ for all } t > 0.$$

(b)  $X$  is said to be *complete* if every Cauchy sequence in it converges to a point in it.

**Definition 2.4.** [13] Self mappings  $A$  and  $S$  of a Fuzzy metric space  $(X, M, *)$  are said to be *compatible* if and only if  $M(ASx_n, SAsx_n, t) \rightarrow 1$  for all  $t > 0$ , whenever  $\{x_n\}$  is a sequence in  $X$  such that  $Sx_n, Ax_n \rightarrow p$  for some  $p$  in  $X$  as  $n \rightarrow \infty$ .

**Definition 2.5.** [16] Suppose  $A$  and  $S$  be two maps from a Fuzzy metric space  $(X, M, *)$  into itself. Then they are said to be semi-compatible if  $\lim_{n \rightarrow \infty} ASx_n = Sx$ , whenever  $\{x_n\}$  is a sequence such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = x \in X.$$

**Definition 2.6.** [13] Two self maps  $A$  and  $B$  of a fuzzy metric space  $(X, M, *)$  are said to be weak compatible if they commute at their coincidence points, i.e.  $Ax = Bx$  implies  $ABx = BAx$ .

**Definition 2.7.** [8] Self maps  $A$  and  $S$  of a Fuzzy metric space  $(X, M, *)$  are said to be occasionally weakly compatible (owc) if and only if there is a point  $x$  in  $X$  which is coincidence point of  $A$  and  $S$  at which  $A$  and  $S$  commute.

**Proposition 2.1.** [15] In a fuzzy metric space  $(X, M, *)$  limit of a sequence is unique.

**Proposition 2.2.** [14] Let  $S$  and  $T$  be compatible self maps of a Fuzzy metric space  $(X, M, *)$  and let  $\{x_n\}$  be a sequence in  $X$  such that  $Sx_n, Tx_n \rightarrow u$  for some  $u$  in  $X$ . Then  $STx_n \rightarrow Tu$  provided  $T$  is continuous.

**Proposition 2.3.** [14] Let  $S$  and  $T$  be compatible self maps of a Fuzzy metric space  $(X, M, *)$  and  $Su = Tu$  for some  $u$  in  $X$  then  $STu = TSu = SSu = TTu$ .

**Lemma 2.1.** [3] Let  $(X, M, *)$  be a fuzzy metric space. Then for all  $x, y \in X$ ,  $M(x, y, \cdot)$  is a non-decreasing function.

**Lemma 2.2.** [2] Let  $(X, M, *)$  be a fuzzy metric space. If there exists  $k \in (0, 1)$  such that for all  $x, y \in X$ ,  $M(x, y, kt) \geq M(x, y, t) \forall t > 0$ , then  $x = y$ .

**Lemma 2.3.** [9] *The only  $t$ -norm  $*$  satisfying  $r * r \geq r$  for all  $r \in [0, 1]$  is the minimum  $t$ -norm, that is  $a * b = \min \{a, b\}$  for all  $a, b \in [0, 1]$ .*

### 3. Main Result

Now, we are in a position to give the main results.

**Theorem 3.1.** *Let  $A, B, S$  and  $T$  be self mappings of a complete fuzzy metric space  $(X, M, *)$ . Suppose that they satisfy the following conditions :*

$$(3.1.1) \quad A(X) \subseteq T(X), B(X) \subseteq S(X);$$

(3.1.2) *the pair  $(A, S)$  is semi-compatible and  $(B, T)$  is occasionally weakly compatible;*

(3.1.3) *there exists  $k \in (0, 1)$  such that  $\forall x, y \in X$  and  $t > 0$ ,*

$$M(Ax, By, kt) \geq \text{Min} \{M(By, Ty, t), M(Sx, Ty, t), M(Ax, Sx, t)\}.$$

*Then  $A, B, S$  and  $T$  have a unique common fixed point in  $X$ .*

**Proof.** Let  $x_0 \in X$  be an arbitrary point. As  $A(X) \subseteq T(X)$  and  $B(X) \subseteq S(X)$  then there exists  $x_1, x_2 \in X$  such that  $Ax_0 = Tx_1, Bx_1 = Sx_2$ . Inductively, we can construct sequence  $\{y_n\}$  and  $\{x_n\}$  in  $X$  such that

$$y_{2n+1} = Ax_{2n} = Tx_{2n+1}, y_{2n+2} = Bx_{2n+1} = Sx_{2n+2} \text{ for } n=0, 1, 2 \dots$$

We first show that  $\{y_n\}$  is a Cauchy sequence in  $X$ .

Now, by (3.1.3) with  $x = x_{2n}, y = x_{2n+1}$ , we obtain that

$$\begin{aligned} M(Ax_{2n}, Bx_{2n+1}, kt) &= M(y_{2n+1}, y_{2n+2}, kt) \\ &\geq \text{Min} \{M(Bx_{2n+1}, Tx_{2n+2}, t), M(Sx_{2n}, Tx_{2n+1}, t), M(Ax_{2n}, Sx_{2n}, t)\} \\ &\geq \text{Min} \{M(y_{2n+1}, y_{2n+2}, t), M(y_{2n}, y_{2n+1}, t), M(y_{2n}, y_{2n+1}, t)\} \\ &\geq \text{Min} \{M(y_{2n+1}, y_{2n+2}, t), M(y_{2n}, y_{2n+1}, t)\}. \end{aligned} \tag{i}$$

Thus we have,

$$M(y_{2n+1}, y_{2n+2}, t) \geq \text{Min} \{M(y_{2n+1}, y_{2n+2}, t/k), M(y_{2n}, y_{2n+1}, t/k)\}. \tag{ii}$$

By putting (ii) in (i), we have,

$$M(y_{2n+1}, y_{2n+2}, kt) \geq \text{Min} \{M(y_{2n+1}, y_{2n+2}, t/k), M(y_{2n}, y_{2n+1}, t/k), M(y_{2n}, y_{2n+1}, t)\}$$

$$\begin{aligned}
&= \text{Min} \{M(y_{2n+1}, y_{2n+2}, t/k), M(y_{2n}, y_{2n+1}, t)\} \\
&\geq \text{Min} \{M(y_{2n+1}, y_{2n+2}, t/k^2), M(y_{2n}, y_{2n+1}, t/k^2), M(y_{2n}, y_{2n+1}, t)\} \\
&= \text{Min} \{M(y_{2n+1}, y_{2n+2}, t/k^2), M(y_{2n}, y_{2n+1}, t)\} \\
&\geq \dots \\
&\geq \text{Min} \{M(y_{2n+1}, y_{2n+2}, t/k^m), M(y_{2n}, y_{2n+1}, t)\}.
\end{aligned}$$

Taking limit as  $m \rightarrow \infty$ , we have

$$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t), \quad \forall t > 0.$$

Similarly, we also have

$$M(y_{2n+2}, y_{2n+3}, kt) \geq M(y_{2n+1}, y_{2n+2}, t), \quad \forall t > 0.$$

Thus, for all  $n$ , and  $t > 0$

$$M(y_n, y_{n+1}, kt) \geq M(y_n, y_{n-1}, t).$$

Therefore,

$$M(y_n, y_{n+1}, t) \geq M(y_{n-1}, y_n, t/k) \geq M(y_{n-2}, y_{n-1}, t/k^2) \geq \dots \geq M(y_0, y_1, t/k^n).$$

Hence,  $\lim_{n \rightarrow \infty} M(y_n, y_{n+1}, t) = 1 \quad \forall t > 0$ . Now, for any integer  $p$ , we have

$$M(y_n, y_{n+p}, t) \geq M(y_n, y_{n+1}, t/p) * M(y_{n+1}, y_{n+2}, t/p) * \dots * M(y_{n+p-1}, y_{n+p}, t/p)$$

Therefore,  $\lim_{n \rightarrow \infty} M(y_n, y_{n+p}, t) = 1 * 1 * 1 * \dots * 1 = 1$

$$\lim_{n \rightarrow \infty} M(y_n, y_{n+p}, t) = 1.$$

This shows that  $\{y_n\}$  is a Cauchy sequence in  $X$ , which is complete. Therefore,  $\{y_n\}$  converges to  $z \in X$ . We have the following subsequences

$$\{Ax_{2n}\} \rightarrow z, \quad \{Bx_{2n+1}\} \rightarrow z \tag{1}$$

$$\{Sx_{2n}\} \rightarrow z, \quad \{Tx_{2n+1}\} \rightarrow z \tag{2}$$

Since  $A(X) \subseteq T(X)$  then there exists  $p \in X$  such that  $p = T^{-1}z$  i.e.  $Tp = z$ .

By (3.1.3) we have (at  $x = x_{2n}, y = p$ )

$$M(Ax_{2n}, Bp, kt) \geq \text{Min} \{M(Bp, Tp, t), M(Sx_{2n}, Tp, t), M(Ax_{2n}, Sx_{2n}, t)\}$$

$$M(Ax_{2n}, Bp, kt) \geq \text{Min} \{M(Bp, z, t), M(Sx_{2n}, z, t), M(Ax_{2n}, Sx_{2n}, t)\}$$

Taking the limit  $n \rightarrow \infty$  and using (i) and (ii), we have

$$M(z, Bp, kt) \geq \text{Min} \{M(Bp, z, t), M(z, z, t), M(z, z, t)\}$$

$$M(z, Bp, kt) \geq M(Bp, z, t).$$

Therefore by lemma 2.2, we have  $z = Bp$ . Since  $z = Tp$ , therefore  $z = Bp = Tp$ .

i. e.  $p$  is a coincidence point of  $B$  and  $T$ . Similarly, since  $B(X) \subseteq S(X)$  then there exists  $q \in X$  such that  $q = S^{-1}z$  i.e.  $Sq = z$ . By (3.1.3) we have (at  $x = q, y = x_{2n+1}$ )

$$M(Aq, Bx_{2n+1}, kt) \geq \text{Min} \{M(Bx_{2n+1}, Tx_{2n+1}, t), M(Sq, Tx_{2n+1}, t), M(Aq, Sq, t)\}.$$

$$M(Aq, Bx_{2n+1}, kt) \geq \text{Min} \{M(Bx_{2n+1}, Tx_{2n+1}, t), M(z, Tx_{2n+1}, t), M(Aq, z, t)\}.$$

Taking the limit  $n \rightarrow \infty$  and using (i) and (ii), we have

$$M(Aq, z, kt) \geq \text{Min} \{M(Bz, Tz, t), M(z, z, t), M(Aq, z, t)\}.$$

$$M(Aq, z, kt) \geq M(Aq, z, t).$$

Therefore by lemma 2.2, we have  $Aq = z$ . Since  $Sq = z$ , therefore,  $z = Aq = Sq$ , i.e.  $q$  is a coincidence point of  $A$  and  $S$ . Since  $A$  and  $S$  are semi-compatible, so  $\lim_{n \rightarrow \infty} ASx_{2n} = Sz$ . Also,  $\lim_{n \rightarrow \infty} ASx_{2n} = Az$ .

Since the limit in a Fuzzy metric space is unique, we get  $Az = Sz$ . Since  $\{B, T\}$  is occasionally weakly compatible, therefore, we have  $BTp = TBp$  or  $Bz = Tz$ . Now at  $x = z, y = x_{2n+1}$ , we have by (3.1.3)

$$M(Az, Bx_{2n+1}, kt) \geq \text{Min} \{M(Bx_{2n+1}, Tx_{2n+1}, t), M(Sz, Tx_{2n+1}, t), M(Az, Sz, t)\}.$$

$$M(Az, Bx_{2n+1}, kt) \geq \text{Min} \{M(Bx_{2n+1}, Tx_{2n+1}, t), M(Az, Tx_{2n+1}, t), M(Az, Sz, t)\}.$$

Taking the limit  $n \rightarrow \infty$ , we have

$$M(Az, z, kt) \geq \text{Min} \{M(z, z, t), M(Az, z, t), 1\}.$$

$$M(Az, z, kt) \geq M(Az, z, t)$$

Therefore by lemma 2.2, we have  $Az = z$ . Since  $Az = Sz$ , therefore  $z = Az = Sz$ . Again by (3.1.3) we have (at  $x = x_{2n}, y = z$ )

$$M(Ax_{2n}, Bz, kt) \geq \text{Min} \{M(Bz, Tz, t), M(Sx_{2n}, Tz, t), M(Ax_{2n}, Sx_{2n}, t)\}.$$

$$M(Ax_{2n}, Bz, kt) \geq \text{Min} \{M(Bz, Bz, t), M(Sx_{2n}, Bz, t), M(Ax_{2n}, Sx_{2n}, t)\}.$$

Taking the limit  $n \rightarrow \infty$ , we have

$$M(z, Bz, kt) \geq \text{Min} \{1, M(z, Bz, t), M(z, z, t)\}$$

$$M(z, Bz, kt) \geq \text{Min} \{1, M(z, Bz, t), 1\}$$

$$M(z, Bz, kt) \geq M(z, Bz, t)$$

Therefore by lemma 2.2, we have  $z = Bz$ . Since  $Bz = Tz$ , therefore  $z = Bz = Tz$ .

Thus, we have  $z = Az = Sz = Bz = Tz$ . Hence  $z$  is a common fixed point of  $A, B, S$  and  $T$ .

**Uniqueness** - Let  $z$  and  $z'$  be two common fixed points of the maps  $A, B, S$  and  $T$ .

Then

$$z = Az = Sz = Bz = Tz = \text{and } z' = Az' = Sz' = Bz' = Tz'.$$

Now by (3.1.3) we have (at  $x = z, y = z'$ )

$$M(Az, Bz', kt) \geq \text{Min} \{M(Bz', Tz', t), M(Sz, Tz', t), M(Az, Sz, t)\}$$

$$M(z, z', kt) \geq \text{Min} \{M(z', z', t), M(z, z', t), M(z, z, t)\}$$

$$M(z, z', kt) \geq \text{Min} \{1, M(z, z', t), 1\}$$

$$M(z, z', kt) \geq M(z, z', t)$$

Therefore, by lemma 2.2, we have,  $z = z'$ . Hence,  $z$  is the unique common fixed point of the four self maps  $A, B, S$  and  $T$ . This completes the proof.

If we take  $B = A$  in theorem 3.1, we get the following corollary for three self maps.

**Corollary 3.2.** *Let  $A, S$  and  $T$  be self mappings of a complete fuzzy metric space  $(X, M, *)$  satisfying :*

$$(3.2.1) A(X) \subseteq S(X) \cap T(X),$$

(3.2.2) *Pair  $(A, S)$  is semi-compatible and  $(A, T)$  is occasionally weak compatible,*

$$(3.2.3) M(Ax, Ay, kt) \geq \text{Min} \{M(Ay, Ty, t) M(Sx, Ty, t), M(Ax, Sx, t)\}$$

*for all  $x, y \in X, t > 0$  and  $0 < k < 1$ .*

*Then,  $A, S$  and  $T$  have a unique common fixed point in  $X$ .*

**Proof.** The proof is similar to the proof of theorem 3.1.



If we take  $S = T = I$ , the identity maps on  $X$  in corollary 3.2, then the conditions (3.2.1) and (3.2.2) are trivially satisfied.

#### 4. An Application

**Theorem 4.1.** *Let  $A$  be a self map on a complete fuzzy metric space  $(X, M, *)$  such that for some  $k \in (0, 1)$ .*

$$M(Ax, Ay, kt) \geq M(x, y, t) \text{ for all } x, y \in X, t > 0.$$

*Then  $A$  has a unique common fixed point in  $X$ .*

**Proof.** On taking only one factor in R.H.S. of the contraction (3.2.3), we obtain the desired result, which is Grabeic's [4] Banach contraction principle in fuzzy metric space.

**Conclusion.** Theorem 3.1 is a generalization of the result of Singh et. al. [17] in the sense that condition of compatibility and weak compatibility of the pairs of self maps has been restricted to semi-compatible and occasionally weakly compatible self maps and the requirement of continuity is completely removed.

#### Conflict of Interests

The authors declare that there is no conflict of interests.

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