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COMMON COUPLED FIXED POINT THEOREM FOR CONTRACTIVE TYPE MAPPINGS IN CLOSED BALL OF COMPLEX VALUED METRIC SPACES

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Abstract. In this paper, we extend and improve the condition of contraction of results of Azam et al. for two single-valued mappings on a closed ball in complex valued metric spaces.

Key words: complex valued metric space, closed ball, common fixed point.

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1. Introduction

Azam et al. [1] introduced the concept of complex-valued metric spaces and obtained sufficient conditions for the existence of common fixed points of a pair of contractive type mappings involving rational expressions. Subsequently, several authors have studied the existence and uniqueness of the fixed points and common fixed points of self-mappings in view of contrasting contractive conditions.

In [2], Bhaskar and Lakshmikantham introduced the concept of coupled fixed points for a given partially ordered set X. Recently Samet et al. [3, 4] proved that most of the coupled fixed point theorems (on ordered metric spaces) are in fact immediate consequences of well-known fixed point theorems in the literature. In this paper, we deal with the corresponding definition of coupled fixed point for mappings on a complex-valued metric space along with

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generalized contraction involving rational expressions. Our results extend and improve several fixed point theorems in closed ball.

2. Preliminaries

Let ℂ the set of complex numbers and $z_1, z_2 \in \mathbb{C}$. We define a partial order $\preccurlyeq$ on ℂ as follows:

$z_1 \preccurlyeq z_2$ if and only if $\mathrm{Re}(z_1) \le \mathrm{Re}(z_2)$ and $\mathrm{Im}(z_1) \le \mathrm{Im}(z_2)$

that is $z_1 \preccurlyeq z_2$ if one of the following holds

C1: $\mathrm{Re}(z_1) = \mathrm{Re}(z_2)$ and $\mathrm{Im}(z_1) = \mathrm{Im}(z_2)$ C2: $\mathrm{Re}(z_1) < \mathrm{Re}(z_2)$ and $\mathrm{Im}(z_1) = \mathrm{Im}(z_2)$

C3: $\mathrm{Re}(z_1) = \mathrm{Re}(z_2)$ and $\mathrm{Im}(z_1) < \mathrm{Im}(z_2)$ C4: $\mathrm{Re}(z_1) < \mathrm{Re}(z_2)$ and $\mathrm{Im}(z_1) < \mathrm{Im}(z_2)$

In particular, we will write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of (C2), (C3), and (C4) is satisfied and we will write $z_1 \prec z_2$ if only (C4) is satisfied.

Definition 2: Let X be a non empty set. A mapping $d: X \times X \to C$ is called a complex valued matrix on X if the following conditions are satisfied:

(CM1) $0 \preccurlyeq d(x, y)$ for all $x, y \in X$ and $d(x, y) = 0$ if and only if $x = y$;

(CM2) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(CM3) $d(x, y) \preccurlyeq d(x, z) + d(z, y)$, for all $x, y, z \in X$.

Then d is called a complex valued metric space.

Definition 3: Let (X, d) be a complex valued metric space.

1. A point $x \in X$ is called interior point of set $A \subseteq X$ whenever there exist $0 \prec r \in \mathbb{C}$ such that $B(x, r) := \{\gamma \in X \mid d(x, y) \prec r\} \subseteq A$, Where B(x, r) is an open Ball.

Then $\overline{B(x,r)} = \{y \in X \mid d(x, y) \preccurlyeq r\}$ is a closed ball.

2. A point $x \in X$ is called a limit of A whenever for every $0 \prec r \in \mathbb{C}$,

We have $B(x, r) \cap (A \setminus \{x\}) \neq \emptyset$.

3. A subset $A \subseteq X$ is called open whenever each element A is an interior point of A.

4. A sub set $B \subseteq X$ is called closed whenever each limit point of B belongs to B.

(v) A sub-basis for a Hausdorff topology τ on X is a family $F = \{B(x, r) \mid x \in X \text{ and } 0 \prec r\}$.

Definition 4: Let (x, d) be a complex valued metric space, $\{x_n\}$ be a sequence in X and $x \in X$.

(i) If for every $c \in C$, with $0 \prec c$ there is $N \in \mathbb{N}$ such that for all $n > N$, $d(x_n, x) \prec c$, then $\{x_n\}$ is said to be convergent, $\{x_n\}$ converges to x and x is the limit point of $\{x_n\}$, we denote this by $\lim_{n\to\infty} x_n = x$ (or) $\{x_n\} \to x$ as $n \to \infty$.

(ii) If for every $c \in C$, with $0 \prec c$ there is $N \in \mathbb{N}$ such that for all $n > N$, $d(x_n, x_{n+m}) \prec c$, where $m \in \mathbb{N}$, then $\{x_n\}$ is said to be Cauchy sequence.

(iii) If for every Cauchy sequence in X is convergent, then (x, d) is said to be a complete complex valued metric space.

Lemma 5: [1] Let (x, d) be a complex valued metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $|d(x_n, x)| \to 0$, as $n\to \infty$.

Lemma 6: [1] Let (x, d) be a complex valued metric space and, let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $d(x_n, x_{n+m})| \to 0$, as $n\to \infty$, where $m \in \mathbb{N}$.

Remark 7: We obtain the following statements hold.

(i) If $z_1 \preccurlyeq z_2$ and $z_2 \preccurlyeq z_3$ then $z_1 \preccurlyeq z_3$. (ii) If $z \in \mathbb{C}$, a, b$\in \mathbb{R}$, and $a \leq b$, then $az \preccurlyeq bz$.

(iii) If $0 \preccurlyeq z_1 \preccurlyeq z_2$, then $|z_1| \preccurlyeq |z_2|$.

Definition 8: [2]Let (X, d) be a complex valued metric space. Then an element $(x, y) \in X \times X$ is said to be a common coupled fixed point of $S, T: X \times X \to X$ if $= S(x, y) = T(x, y)$, $y = S(y, x) = T(y, x)$.

Example 9: Let $X = R$ and $S, T: X \times X \to X$ defined as $S(x, y) = x\left(\frac{y-1}{2}\right)$ $and\ T(x, y) = x\left(\frac{y}{3}\right), for\ all\ x, y \in X$. Then (0, 0) and (1, 3) are common coupled fixed point of S and T

3. Main Results

In this section, we discuss the existence of common coupled fixed-point theorems for the generalized contractive mappings on the closed ball in complex valued metric spaces.

Theorem 10: Let (X, d) be a complete complex vale metric space, and let the mappings $S, T: X \times X \to X$ satisfying the following condition

$$d\big(S(x, y), T(u, v)\big) \preccurlyeq Ad(x, u) + \frac{Bd\big(x, S(x, y)\big)d\big(u, T(u, v)\big)}{1 + d(x, u)} + \frac{Cd\big(u, S(x, y)\big)d\big(x, T(u, v)}{1 + d(x, u)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

for all $x, y, u, v \in \overline{B(x_0, r)}$where A, B, C are nonnegative with A+B+C< 1.

$|d(x_0, S(x_0, y_0) + d(x_0, T(y_0, x_0)| \preccurlyeq (1 - \lambda)|r|$ where $\lambda = \frac{A}{[1-B]}$. Then S and T have a unique common coupled fixed point.

Proof: Let x_0 and y_0 be arbitrary in $\overline{B(x_0, r)}$.

Define $x_{2k+1} = S(x_{2k}, y_{2k})$, $y_{2k+1} = S(y_{2k}, x_{2k})$

$x_{2k+2} = T(x_{2k+1}, y_{2k+1})$, $y_{2k+2} = T(y_{2k+1}, x_{2k+1})$, for all $k \geq 0$.

We will prove that $x_n, y_n \in \overline{B(x_0, r)}$ for all $n \in N$, by the mathematical induction.

Using inequality (2) and the fact that, where $\lambda = \frac{A}{[1-B]} < 1$, we have $|d(x_0, S(x_0, y_0) + d(x_0, T(y_0, x_0)| \preccurlyeq |r|$.

It implies that $x_1, y_1 \in \overline{\mathrm{B}(x_0, \mathrm{r})}$, $Let\ x_{2,}\ x_{3,} \dots x_j \in \overline{\mathrm{B}(x_0, \mathrm{r})} and\ , let\ y_{2,}\ y_{3,} \dots y_j \in \overline{\mathrm{B}(x_0, \mathrm{r})}$,

for some $j \in N$. If j = 2k + 1, where $k = 0, 1, 2, \dots \dots \dots \frac{j-1}{2}$ $or\ j = 2k + 2\ where$

$k = 0, 1, 2, \dots \frac{j-2}{2}$, we obtain by using inequality (1)

$$d(x_{2k+1}, x_{2k+2}) = d(S(x_{2k}, y_{2k}), T(x_{2k+1}, y_{2k+1}))$$

$$\preccurlyeq Ad(x_{2k}, x_{2k+1}) + \frac{Bd(x_{2k}, S(x_{2k}, y_{2k}))d\big(x_{2k+1}, T(x_{2k+1}, y_{2k+1})\big)}{1 + d(x_{2k}, x_{2k+1})} + \frac{Cd(x_{2k+1}, S(x_{2k}, y_{2k}))d\big(x_{2k}, T(x_{2k+1}, y_{2k+1})\big)}{1 + d(x_{2k}, x_{2k+1})}$$

$$d(x_{2k+1}, x_{2k+2}) \preccurlyeq Ad(x_{2k}, x_{2k+1}) + \frac{Bd(x_{2k}, x_{2k+1})d(x_{2k+1}, x_{2k+2})}{1 + d(x_{2k}, x_{2k+1})} + \frac{Cd(x_{2k+1}, x_{2k+1})d(x_{2k}, x_{2k+2})}{1 + d(x_{2k}, x_{2k+1})}$$

$$d(x_{2k+1}, x_{2k+2}) \preccurlyeq Ad(x_{2k}, x_{2k+1}) + \frac{Bd(x_{2k}, x_{2k+1})d(x_{2k+1}, x_{2k+2})}{1 + d(x_{2k}, x_{2k+1})} \dots \dots \dots \dots \dots \dots \dots (2)$$

$$|d(x_{2k+1}, x_{2k+2})| \le A|d(x_{2k}, x_{2k+1})| + \frac{B|d(x_{2k}, x_{2k+1})||d(x_{2k+1}, x_{2k+2})|}{|1 + d(x_{2k}, x_{2k+1})|} \dots \dots \dots \dots (3)$$

$$= A|d(x_{2k}, x_{2k+1})| + B|d(x_{2k+1}, x_{2k+2})| \left[\frac{|d(x_{2k}, x_{2k+1})|}{|1 + d(x_{2k}, x_{2k+1})|}\right]$$

$$|d(x_{2k+1}, x_{2k+2})| \preccurlyeq A|d(x_{2k}, x_{2k+1})| + B|d(x_{2k+1}, x_{2k+2})|$$

$$|d(x_{2k+1}, x_{2k+2})[1 - B] \preccurlyeq Ad(x_{2k}, x_{2k+1})|$$

it follows that $|d(x_{2k+1}, x_{2k+2})| \preccurlyeq \frac{A}{[1-B]}|d(x_{2k}, x_{2k+1})| \dots \dots \dots \dots (4)$

Similarly, $|d(y_{2k+1}, y_{2k+2})| \preccurlyeq \frac{A}{[1-B]}|d(y_{2k}, y_{2k+1})| \dots \dots \dots \dots (5)$

$$d(x_{2k+2}, x_{2k+3}) = d(T(x_{2k+1}, y_{2k+1}), S(x_{2k+2}, y_{2k+2}))$$
$$= d(S(x_{2k+2}, y_{2k+2}), T(x_{2k+1}, y_{2k+1}))$$

$$\preccurlyeq Ad(x_{2k+2}, x_{2k+1}) + \frac{Bd(x_{2k+2},\ S(x_{2k+2}, y_{2k+2}))\ d(x_{2k+1},\ T(x_{2k+1}, y_{2k+1}))}{1 + d(x_{2k+2},\ x_{2k+1})} + \frac{Cd(x_{2k+1},\ S(x_{2k+2}, y_{2k+2}))\ d(x_{2k+2},\ T(x_{2k+1}, y_{2k+1}))}{1 + d(x_{2k+2},\ x_{2k+1})}$$

$$\preccurlyeq Ad(x_{2k+2}, x_{2k+1}) + \frac{Bd(x_{2k+2},\ x_{2k+3})\ d(x_{2k+1},\ x_{2k+2})}{1 + d(x_{2k+2},\ x_{2k+1})} + \frac{Cd(x_{2k+1},\ x_{2k+3})\ d(x_{2k+2},\ x_{2k+2})}{1 + d(x_{2k+2},\ x_{2k+1}) \dots \dots \dots \dots (6)}$$

$$|d(x_{2k+2}, x_{2k+3})| \leq A|d(x_{2k+2}, x_{2k+1})| + \frac{B|d(x_{2k+2}, \ x_{2k+3})||d(x_{2k+1}, x_{2k+2})|}{|1 + d(x_{2k+2}, \ x_{2k+1})|} \ldots (7)$$

$$= A|d(x_{2k+2}, x_{2k+1})| + B|d(x_{2k+2}, x_{2k+3})| \left[\frac{|d(x_{2k+1}, \ x_{2k+2})|}{|1 + d(x_{2k+2}, \ x_{2k+1})|}\right]$$

$|d(x_{2k+2}, x_{2k+3})| \preccurlyeq A|d(x_{2k+2}, x_{2k+1})| + B|d(x_{2k+2}, x_{2k+3})|$

$|d(x_{2k+2}, x_{2k+3})[1 - B] \preccurlyeq Ad(x_{2k+2}, x_{2k+1})|$

it follows that $|d(x_{2k+2}, x_{2k+3})| \preccurlyeq \frac{A}{[1-B]}|d(x_{2k+2}, x_{2k+1})| \ldots \ldots \ldots \ldots (8)$

Similarly $|d(y_{2k+2}, y_{2k+3})| \preccurlyeq \frac{A}{[1-B]}|d(y_{2k+2}, y_{2k+1})| \ldots \ldots \ldots \ldots (9)$

Adding (4)-(9), we get

$$|d(x_{2k+1}, x_{2k+2})| + |d(y_{2k+1}, y_{2k+2})| \preccurlyeq \frac{A}{[1 - B]}|d(x_{2k}, x_{2k+1})| + \frac{A}{[1 - B]}|d(y_{2k}, y_{2k+1})|$$

$$|d(x_{2k+2}, x_{2k+3})| + |d(y_{2k+2}, y_{2k+3})|$$

$$\preccurlyeq \frac{A}{[1 - B]}|d(x_{2k+2}, x_{2k+1})| + \frac{A}{[1 - B]}|d(y_{2k+2}, y_{2k+1})| \ldots \ldots \ldots \ldots (10)$$

If $\lambda = \frac{A}{[1-B]} < 1$, then from (10), we get

$$\left|d\left(x_{n,} \, x_{n+1}\right)\right| + \left|d\left(y_{n,} \, y_{n+1}\right)\right| \leq \lambda\left(\left|d\left(x_{n-1,} \, x_n\right)\right| + \left|d\left(y_{n-1,} \, y_n\right)\right|\right)$$

$$\leq \cdots \leq \lambda^n\left(\left|d\left(x_{0,} \, x_1\right)\right| + \left|d\left(y_{0,} \, y_1\right)\right|\right), \text{ for all } n \in N$$

Now if $\left|d\left(x_{n,} \, x_{n+1}\right)\right| + \left|d\left(y_{n,} \, y_{n+1}\right)\right| = \theta_n, then$

$\theta_n \leq \lambda\theta_{n-1} \leq \cdots \leq \lambda^n\theta_0 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (11)$

Now $\left|d\left(x_{0,} \, x_{n+1}\right)\right| + \left|d\left(y_{0,} \, y_{n+1}\right)\right|$

$$\leq \left(\left|d\left(x_{0,} \, x_1\right)\right| + \left|d\left(y_{0,} \, y_1\right)\right|\right) + \cdots + \left(\left|d\left(x_{n,} \, x_{n+1}\right)\right| + \left|d\left(y_{n,} \, y_{n+1}\right)\right|\right)$$

$$\leq \left(\left|d\left(x_{0,} \, x_1\right)\right| + \left|d\left(y_{0,} \, y_1\right)\right|\right) + \cdots + \lambda^n\left(\left|d\left(x_{0,} \, x_1\right)\right| + \left|d\left(y_{0,} \, y_1\right)\right|\right), \text{ for all } n \in N$$

$= (\left|d\left(x_{0,} \, x_1\right)\right| + \left|d\left(y_{0,} \, y_1\right)\right|)[1 + \cdots + \lambda^{n-1} + \lambda^n] \leq (1 - \lambda)|r|\frac{(1-\lambda^{n-1})}{1-\lambda} \leq |r|$

gives $x_{n+1} \in \overline{B(x_0, r)}$. Hence $x_n \in \overline{B(x_0, r)}$ for all $n \in N$ and

$\left|d\left(x_{n,} \, x_{n+1}\right)\right| + \left|d\left(y_{n,} \, y_{n+1}\right)\right| \leq \lambda^n\left(\left|d\left(x_{0,} \, x_1\right)\right| + \left|d\left(y_{0,} \, y_1\right)\right|\right)$

for all $n \in N$. Without loss of generality, we take m > n, then

$$\left|d\left(x_{n,} \, x_m\right)\right| + \left|d\left(y_{n,} \, y_m\right)\right| \leq \left(\left|d\left(x_{n,} x_{n+1}\right)\right| + \left|d\left(y_{n,} \, y_{n+1}\right)\right|\right) + \cdots$$

$$+\left(\left|d\left(x_{m-1,} \, x_m\right)\right| + \left|d\left(y_{m-1,} y_m\right)\right|\right)$$

$\leq [\lambda^n\theta_0 + \lambda^{n+1}\theta_0 + \cdots + \leq \lambda^{m-1}\theta_0] \leq [\lambda^n + \lambda^{n+1} + \cdots + \lambda^{m-1}]\theta_0$

$\sum_{i=n}^{m-1} \lambda^i\theta_0 \to 0, as \; m, n \to \infty.$

This implies that the sequence $\{x_n\}$ and $\{y_n\}$ are Cauchy in $\overline{\mathrm{B}(x_0,\mathrm{r})}$. Since $\overline{\mathrm{B}(x_0,\mathrm{r})}$ is complete, there exists x, y ∈ $\overline{\mathrm{B}(x_0,\mathrm{r})}$ such that $x_n \to x \text{ and } y_n \to y \text{ as } n \to +\infty$.

We now show that $x = S(x,y) and\ y = S(y,x)$. We suppose on the contrary that $x \neq S(x,y)$ and $y \neq S(y,x)$, so that $0 < d\big(x,S(x,y)\big) = l_1 and\ 0 < d\big(y,S(y,x)\big) = l_2$

$l_1 = d\big(x,S(x,y)\big) \leq d(x,x_{2k+2}) + d\big(x_{2k+2},S(x,y)\big)$

$\leq d(x,x_{2k+2}) + d\big(T(x_{2k+1},y_{2k+1}),S(x,y)\big)$

$$\leq d(x,x_{2k+2}) + Ad(x_{2k+1},x) + \frac{Bd(x_{2k+1},T(x_{2k+1},y_{2k+1}))d(x,S(x,y))}{1+d(x_{2k+1},x)} + \frac{Cd(x,T(x_{2k+1},y_{2k+1}))d(x_{2k+1},S(x,y))}{1+d(x_{2k+1},x)}$$

$$= d(x,x_{2k+2}) + Ad(x_{2k+1},x) + \frac{Bd(x_{2k+1},\ x_{2k+1})d(x,S(x,y))}{1+d(x_{2k+1},x)} + \frac{Cd(x,\ x_{2k+2})d(x_{2k+1},S(x,y))}{1+d(x_{2k+1},x)}$$

$$So\ that\ |l_1| \leq |d(x,x_{2k+2})| + A|d(x_{2k+1},x)| + \frac{B|d(x_{2k+1},\ x_{2k+1})||d(x,S(x,y))|}{|1+d(x_{2k+1},x)|} + \frac{C|d(x,\ x_{2k+2})||d(x_{2k+1},S(x,y))|}{|1+d(x_{2k+1},x)|}$$

By taking $k \to +\infty$, we get $|d(x,S(x,y))| = 0$ which is contradiction so that $x = S(x,y)$. Similarly one can prove that $y = S(x,y)$. It follows that similarly that $x = T(x,y)$ and $y = T(x,y)$. So we have prove that (x, y) is a common fixed point of S and T. We now show that S and T have a unique common coupled fixed point.

For this, assume that $(x^*,y^*) \in \overline{\mathrm{B}(x_0,\mathrm{r})}$ is a second common coupled fixed point of S and T. Then

$$d(x,x^*) = d(S(x,y),T(x^*,y^*)) \preccurlyeq Ad(x,x^*) + \frac{Bd(x,S(x,y))d(x^*,T(x^*,y^*))}{1+d(x,x^*)} + \frac{Cd(x,T(x^*,y^*))d(x^*,S(x,y))}{1+d(x,x^*)}$$

$$d(x,x^*) \preccurlyeq Ad(x,x^*) + \frac{Bd(x,\ \ x)\,d(x^*,\ \ x^*)}{1+d(x,\ \ x^*)} + \frac{Cd(x,\ \ x^*)\,d(x^*,\ \ x\,))}{1+d(x,\ \ x^*)}$$

$$|d(x,x^*)| \preccurlyeq A|d(x,x^*)| + C|d(x,\ x^*)|\left|\frac{d(x^*,\ \ x\,))}{1+d(x,\ \ x^*)}\right| \preccurlyeq A|d(x,x^*)| + C|d(x,\ x^*)|$$

$|d(x,x^*)| \preccurlyeq [A+C]|d(x,x^*)|$, which is a contradiction because A+B+C<1. Thus we get $x^* = x \text{ and } y^* = y$, which is prove the uniqueness of common coupled fixed point of S and T.

Corollary 11: Let (X, d) be a complete complex vale metric space, and let the mapping $T: X \times X \rightarrow X$ satisfying the following condition

$$d\big(T(x,y),T(u,v)\big) \precsim Ad(x,u) + \frac{Bd\big(x,T(x,y)\big)d\big(u,T(u,v)\big)}{1+d(x,u)} + \frac{Cd\big(u,T(x,y)\big)d(x,T(u,v)}{1+d(x,u)}$$

for all $x,y,u,v \in \overline{\mathrm{B}(x_0,\mathrm{r})}$ where A, B, C are nonnegative with A+B+C< 1.

$|d(x_0,S(x_0,y_0)+d(x_0,T(y_0,x_0)| \precsim (1-\lambda)|r|$, where $\lambda=\frac{A}{[1-B]}$. Then T has a unique coupled fixed point.

Corollary 12: Let (X, d) be a complete complex vale metric space, and let the mapping $T: X \times X \rightarrow X$ satisfying the following condition

$$d\big(T^n(x,y),T^n(u,v)\big) \precsim Ad(x,u) + \frac{Bd\big(x,\ T^n(x,y)\big)d\big(u,T^n(u,v)\big)}{1+d(x,u)} + \frac{Cd\big(u,T^n(x,y)\big)d(x,T^n(u,v)}{1+d(x,u)}$$

for all $x,y,u,v \in \overline{\mathrm{B}(x_0,\mathrm{r})}$ where A, B, C are nonnegative with A+B+C< 1.

$|d(x_0,S(x_0,y_0)+d(x_0,T(y_0,x_0)| \precsim (1-\lambda)|r|$, Where $\lambda=\frac{A}{[1-B]}$

Then T has a unique coupled fixed point.

Conflict of Interests

The authors declare that there is no conflict of interests.

REFERENCES

[1] Azam, B. Fisher, and M. Khan, "common fixed point theorems in complex valued metric spaces" Numerical Functional Analysis and Optimization, 32(2011), 243-253.

[2] T. Gnana Bhaskar and V. Lakshmikantham, "Fixed point theorems in partially ordered metric spaces and applications,"Nonlinear Analysis: Theory,Methods & Applications, 65(2006), 1379–1393.

[3] Samet, E. Karapinar, H. Aydi, and V. Cojbasic, "Discussionon some coupled fixed point theorems," Fixed Point Theory and Applications, 2013(2013), Article ID 50.

[4] Samet and C.Vetro, "Coupled fixed point theorems for multivalued nonlinear contraction mappings in partially orderedmetric spaces," Nonlinear Analysis:Theory, Methods & Applications, 74(2011), 4260–4268.

[5] Jamshaid Ahmad,Akbar Azam and Satit Saejung," Common fixed point results for contractive mappings in complex valued metric spaces" Fixed Point Theory and Applications, 2014 (2014), Article ID 67.