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COMMON FIXED POINT THEOREMS FOR WEAKLY COMPATIBLE MAPS IN COMPLEX VALUED B-METRIC SPACES

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Abstract. In this paper, we prove some common fixed point theorems for weakly compatible mappings in complex valued b-metric spaces. Examples are provided to support the results.

Keywords: Complex valued b-metric spaces; Fixed point; Weakly compatible mapping; (E.A) property, (CLR) property

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1. Introduction

Complex valued metric spaces were introduced by Azam et.al.[1] in which they obtained fixed point theorems for mappings satisfying various contractive conditions. Many researchers Bhatt et al. [3], Sintunavarat and Kumam [10] and Mohanta et al. [11] have proved results in these spaces including those for rational expressions which are meaningless in the context of

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cone metric spaces. The notion of b-metric spaces which are more general than metric spaces was introduced by Czerwik [4] in 1998.

In 2013, Rao et al. [9] combined these concepts and introduced generalizations of these spaces called Complex-valued b-metric spaces. They proved a theorem for four weakly compatible maps in a complete complex valued b-metric space. Later A.A.Mukeimar [2] obtained the results of Azam et al. [1], S.Bhatt et al. [3] in the setting of the complex valued b-metric space. Also Singh et al. [6], Dubey et al. [7], have obtained fixed point theorems for mapping satisfying contractive conditions in complex valued b-metric spaces.

In this paper, we prove common fixed point theorems for four weakly compatible mappings which satisfy a rational contractive condition in the complex valued b-metric space. Also we obtain these results using the (E.A) property and the Common limit range (CLR) property in complex valued *b*-metric spaces.

2. Preliminaries

Let us denote the set of complex numbers by $\mathbb{C}$. Let$z_1, z_2 \in \mathbb{C}$. We define a partial order $\precsim$ on $\mathbb{C}$ as follows:

$z_1 \precsim z_2$, iff $Re(z_1) \leq Re(z_2)$ and $Im(z_1) \leq Im(z_2)$. Thus we can say that $z_1 \precsim z_2$ if one of the following holds:

(i) $Re(z_1) = Re(z_2)$ and $Im(z_1) = Im(z_2)$;
(ii) $Re(z_1) < Re(z_2)$ and $Im(z_1) = Im(z_2)$;
(iii) $Re(z_1) = Re(z_2)$ and $Im(z_1) < Im(z_2)$;
(iv) $Re(z_1) < Re(z_2)$ and $Im(z_1) < Im(z_2)$.

In particular we write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of (ii), (iii), (iv) holds. Also we write $z_1 \prec z_2$ if only (iv) is satisfied.

Note that the following statements hold:

(i) $0 \precsim z_1 \precnsim z_2$ implies that $|z_1| < |z_2|$;
(ii) $z_1 \precsim z_2$ and $z_2 \prec z_3$ implies that $z_1 \prec z_3$;
(iii) $0 \precsim z_1 \precsim z_2$ implies that $|z_1| \leq |z_2|$;

(iv) $a, b \in \mathbb{R}$ and $a \leq b$ implies that $az \precsim bz$, for all $z \in \mathbb{C}$.

Definition 2.1. [1] Let X be a nonempty set. If the mapping $d : X \times X \to \mathbb{C}$ satisfies the conditions:

(i) $0 \precsim d(x,y)$, for all $x, y \in X$ and $d(x,y) = 0$ if and only if $x = y$;

(ii) $d(x,y) = d(y,x)$, for all $x, y \in X$;

(iii) $d(x,y) \precsim d(x,z) + d(z,y)$, for all $x, y, z \in X$.

Then d is called a complex valued metric on X and (X,d) is called a complex valued metric space.

Definition 2.2. [9] Let X be a nonempty set and $s \geq 1$, a given real number. A function $d : X \times X \to \mathbb{C}$ satisfies the following conditions:

(i) $0 \precsim d(x,y)$, for all $x, y \in X$ and $d(x,y) = 0$ if and only if $x = y$;

(ii) $d(x,y) = d(y,x)$, for all $x, y \in X$;

(iii) $d(x,y) \precsim s[d(x,z) + d(z,y)]$, for all $x, y, z \in X$.

Then d is called a complex valued b-metric on X and (X,d) is called a complex valued b-metric space.

Example 2.3. [9] Let $X = [0,1]$, define the mapping $d : X \times X \to \mathbb{C}$ by $d(x,y) = |x-y|^2 + i|x-y|^2$ for all $x, y \in X$. Then (X,d) is a complex valued b-metric space with $s = 2$.

Definition 2.4.[9] Let (X,d) be a complex valued b-metric space.

(i) A point $x \in A$ is called a interior point of a set $A \subseteq X$, whenever there exists $0 \prec r \in \mathbb{C}$ such that $B(x,r) = \{y \in X : d(x,y) \prec r\} \subseteq A$.

(ii) A point $x \in X$ is called a limit point of a set $A \subseteq X$ whenever for every $0 \prec r \in \mathbb{C}$ such that $B(x,r) \cap (X - A) \neq \phi$.

(iii) A subset $B \subseteq X$ is called open whenever each limit point of B is an interior point of B.

(iv) A subset $B \subseteq X$ is called closed whenever each limit point of B belongs to B.

(v) The family $F = \{B(x,r) : x \in X \text{ and } 0 \prec r\}$ is a subbasis for a topology on X. We denote this complex topology by τ_c. Indeed, the topology τ_c is Hausdroff.

Definition 2.5. [9] Let (X,d) be a complex valued b-metric space and let $\{x_n\}$ be a sequence in X and $x \in X$.

(i) If for every $c \in C$,with $0 \prec c$, there exists $n_0 \in \mathbb{N}$, such that $d(x_n, x) \prec c$ for all $n > n_0$, then $\{x_n\}$ is said to converge to x and x is a limit point of of $\{x_n\}$. We denote this by $x_n \to x$ as $n \to \infty$ or $\lim_{x\to\infty} x_n = x$.

(ii) If for every $c \in \mathbb{C}$, with $0 \prec c$, there exists $n_0 \in \mathbb{N}$ such that for all $n > n_0, d(x_n, x_{n+m}) \prec c$, where $m \in \mathbb{N}$, then $\{x_n\}$ is said to be Cauchy sequence.

(iii) If every Cauchy sequence is convergent in (X,d), then (X,d) is called a complete complex valued b-metric space.

Lemma 2.6. [9] Let (X,d) be a complex valued b-metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $|d(x_n, x)| \to 0$ as $n \to \infty$.

Lemma 2.7. [9] Let (X,d) be a complex valued b-metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $|d(x_n, x_{n+m})| \to 0$ as $n \to \infty$ where $m \in \mathbb{N}$.

Definition 2.8. Let S and f be two self maps of a nonempty set X. If $Sx = fx = y$ for some $x \in X$, then x is called the coincidence point of S and f and y is called the point of coincidence of S and f.

Definition 2.9. Two self mappings S and f are said to be weakly compatible if they commute at their coincidence points, i.e. $Sx = fx$ implies that $Sfx = fSx$.

3. A common fixed point theorem

In this section we prove common fixed point theorems for four weakly compatible maps in complex valued b-metric spaces.

Theorem 3.1. Let S, T, f and g be four self mappings of a complete complex valued b-metric space (X,d) which satisfy the following,

$$\begin{aligned} d(Sx,Ty) \precsim\, & Ad(fx,gy) + B\frac{d(fx,Sx)d(Ty,gy)}{1+d(fx,gy)} + C\frac{d(fx,Ty)d(Sx,gy)}{1+d(fx,gy)} \\ & + D\frac{d(fx,Sx)d(fx,Ty)+d(gy,Ty)d(gy,Sx)}{1+d(fx,Ty)+d(gy,Sx)} \qquad (1) \end{aligned}$$

for all $x, y \in X$ where A, B, C and D are nonnegative real numbers such that $s(A+C+D)+B < 1$. Here if,

(i) $S(X) \subseteq g(X)$ and $T(X) \subseteq f(X)$,

(ii) the pairs (S,f) and (T,g) are weakly compatible,

(iii) the subspace $f(X)$ or $g(X)$ is closed,

then S,T,f and g have a unique common fixed point.

Proof: We construct a sequence $\{y_k\}$ in X such that,

$$y_{2k}=Sx_{2k}=gx_{2k+1} \text{ and } y_{2k+1}=Tx_{2k+1}=fx_{2k+2}, k\geq 0,$$

where $\{x_{2k}\}$ is another sequence in X.

We show that $\{y_k\}$ is a Cauchy sequence in X.

Consider,

$$\begin{aligned}
d(y_{2k},y_{2k+1}) &= d(Sx_{2k},Tx_{2k+1})\\
&\precsim Ad(fx_{2k},gx_{2k+1})+B\frac{d(fx_{2k},Sx_{2k})d(Tx_{2k+1},gx_{2k+1})}{1+d(fx_{2k},gx_{2k+1})}\\
&+C\frac{d(fx_{2k},Tx_{2k+1})d(Sx_{2k},gx_{2k+1})}{1+d(fx_{2k},gx_{2k+1})}\\
&+D\frac{d(fx_{2k},Sx_{2k})d(fx_{2k},Tx_{2k+1})+d(gx_{2k+1},Tx_{2k+1})d(gx_{2k+1},Sx_{2k})}{1+d(fx_{2k},Tx_{2k+1})+d(gx_{2k+1},Sx_{2k})}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
d(y_{2k},y_{2k+1}) \precsim Ad(y_{2k-1},y_{2k})+B\frac{d(y_{2k-1},y_{2k})d(y_{2k+1},y_{2k})}{1+d(y_{2k-1},y_{2k})}+C\frac{d(y_{2k-1},y_{2k+1})d(y_{2k},y_{2k})}{1+d(y_{2k-1},y_{2k})}\\
+D\frac{d(y_{2k-1},y_{2k})d(y_{2k-1},y_{2k+1})+d(y_{2k},y_{2k+1})d(y_{2k},y_{2k})}{1+d(y_{2k-1},y_{2k+1})+d(y_{2k},y_{2k})},
\end{aligned}$$

and,

$|d(y_{2k},y_{2k+1})|\leq A|d(y_{2k-1},y_{2k})|+B|d(y_{2k+1},y_{2k})|\left|\frac{d(y_{2k-1},y_{2k})}{1+d(y_{2k-1},y_{2k})}\right|+D|d(y_{2k-1},y_{2k})|\left|\frac{d(y_{2k-1},y_{2k+1})}{1+d(y_{2k-1},y_{2k+1})}\right|$.
Since $\left|\frac{d(y_{2k-1},y_{2k})}{1+d(y_{2k-1},y_{2k})}\right|<1, \left|\frac{d(y_{2k-1},y_{2k+1})}{1+d(y_{2k-1},y_{2k+1})}\right|<1$,

$$|d(y_{2k},y_{2k+1})|\leq A|d(y_{2k-1},y_{2k})|+B|d(y_{2k+1},y_{2k})|+D|d(y_{2k-1},y_{2k})|.$$

Hence,

$$|d(y_{2k},y_{2k+1})|\leq \tfrac{A+D}{1-B}|d(y_{2k-1},y_{2k})|.$$

Let $\lambda=\frac{A+D}{1-B}$, then since $s(A+C+D)+B<1$ and $s\geq 1$, we have $\lambda<1$.

Therefore,

$$|d(y_{2k},y_{2k+1})|\leq \lambda|d(y_{2k-1},y_{2k})|$$

Similarly we obtain,

$$|d(y_{2k-1},y_{2k})| \leq \lambda |d(y_{2k-2},y_{2k-1})|.$$

Consequently we conclude that,

$$|d(y_{2k},y_{2k+1})| \leq \lambda |d(y_{2k-1},y_{2k})| \leq \lambda^2 |d(y_{2k-2},y_{2k-1})| \leq \lambda^3 |d(y_{2k-3},y_{2k-2})| \leq ... \leq \lambda^{2k} |d(y_0,y_1)|.$$

Finally, we have,

$$|d(y_k,y_{k+1})| \leq \lambda^k |d(y_0,y_1)| \tag{2}$$

For all $m > n, m, n \in \mathbb{N}$, since $s(A+C+D)+B < 1$, we have, $s\lambda = s\frac{(A+D)}{1-B} < 1$, thus using the triangular inequality,

$$\begin{aligned}
|d(y_n,y_m)| &\leq s|d(y_n,y_{n+1})| + s|d(y_{n+1},y_m)| \\
&\leq s|d(y_n,y_{n+1})| + s^2|d(y_{n+1},y_{n+2})| + s^2|d(y_{n+2},y_m)| \\
&\leq s|d(y_n,y_{n+1})| + s^2|d(y_{n+1},y_{n+2})| + s^3|d(y_{n+2},y_{n+3})| + s^3|d(y_{n+3},y_m)| \\
&\vdots \\
&\leq s|d(y_n,y_{n+1})| + s^2|d(y_{n+1},y_{n+2})| + s^3|d(y_{n+2},y_{n+3})| + ... + \\
&s^{m-n-1}|d(y_{m-2},y_{m-1})| + s^{m-n}|d(y_{m-1},y_m)|.
\end{aligned}$$

By using (2), we have,

$$|d(y_n,y_m)| \leq s\lambda^n |d(y_0,y_1| + s^2\lambda^{n+1}|d(y_0,y_1)| + s^3\lambda^{n+2}|d(y_0,y_1)| + ... + \\
s^{m-n-1}\lambda^{m-2}|d(y_0,y_1)| + s^{m-n}\lambda^{m-1}|d(y_0,y_1)|$$

Hence,

$$\begin{aligned}
|d(y_n,y_m)| &\leq (s\lambda^n + s^2\lambda^{n+1} + s^3\lambda^{n+2} + ... + s^{m-n-1}\lambda^{m-2} + s^{m-n}\lambda^{m-1})|d(y_0,y_1)| \\
&= \sum_{i=1}^{m-n} s^i \lambda^{i+n-1} |d(y_0,y_1)|.
\end{aligned}$$

Therefore,

$$
\begin{aligned}
|d(y_n,y_m)| &\leq \sum_{i=1}^{m-n} s^{i+n-1}\lambda^{i+n-1}|d(y_0,y_1)| \\
&= \sum_{t=n}^{m-1} s^t\lambda^t|d(y_0,y_1)| \\
&\leq \sum_{t=n}^{\infty}(s\lambda)^t|d(y_0,y_1)| \\
&\leq \frac{(s\lambda)^n}{1-s\lambda}|d(y_0,y_1)|
\end{aligned}
$$

Since $s\lambda < 1, |d(y_n,y_m)| \leq \frac{(s\lambda)^n}{1-s\lambda}|d(y_0,y_1)| \to 0$ as $n \to \infty$, for any $m \in \mathbb{N}$.

Thus $\{y_n\}$ is a Cauchy sequence in X. Since X is complete, there exists a point $z \in X$ such that,

$$\lim_{k\to\infty} Sx_{2k} = \lim_{k\to\infty} gx_{2k+1} = \lim_{k\to\infty} Tx_{2k+1} = \lim_{k\to\infty} fx_{2k+2} = z.$$

Assuming $f(X)$ is a closed subspace of X, $z \in f(X)$ and $z = fu$ for some $u \in X$. Now we show that $Su = fu$.

We have, using the triangular inequality,

$$d(Su,z) \precsim s[d(Su,Tx_{2k+1}) + d(Tx_{2k+1},z)].$$

Hence, by (1),

$$
\begin{aligned}
\frac{1}{s}d(Su,z) \precsim{}& Ad(fu,gx_{2k+1}) + B\frac{d(fu,Su)d(Tx_{2k+1},gx_{2k+1})}{1+d(fu,gx_{2k+1})} + C\frac{d(fu,Tx_{2k+1})d(Su,gx_{2k+1})}{1+d(fu,gx_{2k+1})} \\
&+ D\frac{d(fu,Su)d(fu,Tx_{2k+1}) + d(gx_{2k+1},Tx_{2k+1})d(gx_{2k+1},fu)}{1+d(fu,Tx_{2k+1})+d(gx_{2k+1},Su)} + sd(Tx_{2k+1},z).
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\frac{1}{s}d(Su,z) \precsim{}& Ad(z,gx_{2k+1}) + B\frac{d(z,Su)d(Tx_{2k+1},gx_{2k+1})}{1+d(z,gx_{2k+1})} + C\frac{d(z,Tx_{2k+1})d(Su,gx_{2k+1})}{1+d(z,gx_{2k+1})} \\
&+ D\frac{d(z,Su)d(z,Tx_{2k+1}) + d(gx_{2k+1},Tx_{2k+1})d(gx_{2k+1},Su)}{1+d(z,Tx_{2k+1})+d(gx_{2k+1},Su)} + sd(Tx_{2k+1},z).
\end{aligned}
$$

Therefore, as $k \to \infty$, we have,

$$\frac{1}{s}d(Su,z) \precsim 0$$

which implies that $|d(Su,z)| = 0$ and that $Su = z$. Thus $Su = fu = z$ and u is a coincidence point of f and S.

Since $S(X) \subseteq g(X)$ and $z \in g(X)$, there exists point $v \in X$, such that $Su = gv$. Thus $Su = fu = gv = z$.

Consider, by the triangular inequality,

$$d(z,Tv) \precsim s[d(z,Sx_{2k}) + d(Sx_{2k},Tv)]$$

i.e.

$$\frac{1}{s}d(z,Tv) \precsim d(z,Sx_{2k}) + d(Sx_{2k},Tv).$$

Using (1), we have,

$$\frac{1}{s}d(z,Tv) \precsim d(z,Sx_{2k}) + Ad(fx_{2k},gv) + B\frac{d(fx_{2k},Sx_{2k})d(Tv,gv)}{1+d(fx_{2k},gv)} + C\frac{d(fx_{2k},Tv)d(Sx_{2k},gv)}{1+d(fx_{2k},gv)}$$
$$+D\frac{d(fx_{2k},Sx_{2k})d(fx_{2k},Tv) + d(gv,Tv)d(gv,Sx_{2k})}{1+d(fx_{2k},Tv)+d(gv,Sx_{2k})}.$$

Thus,

$$\frac{1}{s}d(z,Tv) \precsim d(z,Sx_{2k}) + Ad(fx_{2k},z) + B\frac{d(fx_{2k},Sx_{2k})d(Tv,z)}{1+d(fx_{2k},z)} + C\frac{d(fx_{2k},Tv)d(Sx_{2k},z)}{1+d(fx_{2k},z)}$$
$$+D\frac{d(fx_{2k},Sx_{2k})d(fx_{2k},Tv) + d(z,Tv)d(z,Sx_{2k})}{1+d(fx_{2k},Tv)+d(z,Sx_{2k})}.$$

As $k \to \infty$, we get,

$$\frac{1}{s}d(z,Tv) \precsim 0.$$

Hence it implies that $|d(z,Tv)| = 0$ and $Tv = z$. Thus $Su = fu = gv = Tv = z$.

Since S and f are weakly compatible, $Sfu = fSu$ and $Sz = fz$.

We prove that z is a fixed point of S i.e. $Sz = z$, suppose not, $Sz \neq z$, then again by the triangular inequality,

$$d(Sz,z) \precsim s[d(Sz,Tx_{2k+1}) + d(Tx_{2k+1},z)].$$

By (1) we get,

$$\frac{1}{s}d(Sz,z) \precsim Ad(fz,gx_{2k+1}) + B\frac{d(fz,Sz)d(Tx_{2k+1},gx_{2k+1})}{1+d(fz,gx_{2k+1})} + C\frac{d(fz,Tx_{2k+1})d(Sz,gx_{2k+1})}{1+d(fz,gx_{2k+1})}$$
$$+D\frac{d(fz,Sz)d(fz,Tx_{2k+1}) + d(gx_{2k+1},Tx_{2k+1})d(gx_{2k+1},Sz)}{1+d(fz,Tx_{2k+1})+d(gx_{2k+1},Sz)} + d(Tx_{2k+1},z).$$

As $k \to \infty$,

$$\frac{1}{s}d(Sz,z) \precsim Ad(Sz,z) + B\frac{d(Sz,Sz)d(z,z)}{1+d(Sz,z)} + C\frac{d(Sz,z)d(Sz,z)}{1+d(Sz,z)} + D\frac{d(Sz,Sz)d(Sz,z)+d(z,z)d(z,Sz)}{1+d(Sz,z)+d(z,Sz)} + d(z,z).$$

Hence,

$$\frac{1}{s}|d(Sz,z)| \leq A|d(Sz,z)| + C|d(Sz,z)|.$$

Thus,

$$|d(Sz,z)| \leq s(A+C)|d(Sz,z)|.$$

Since $s(A+C) < 1$, it implies that $|d(Sz,z)| = 0$ and $Sz = z$. Hence $fz = Sz = z$.
Since T and g are weakly compatible, $Tgv = gTv$ i.e. $Tz = gz$. We can prove in a similar way that $Tz = z$. Hence $Sz = fz = Tz = gz = z$ and z is a common fixed point of S, T, f and g.
Similar argument holds if we assume that $g(X)$ is a closed subspace of X.
To prove uniqueness of the common fixed point, suppose there is another point $z_1 \in X$ such that,

$$Sz_1 = Tz_1 = fz_1 = gz_1 = z_1.$$

Again, using (1),

$$d(z,z_1) = d(Sz,Tz_1) \precsim Ad(fz,gz_1) + B\frac{d(fz,Sz)d(Tz_1,gz_1)}{1+d(fz,gz_1)} + C\frac{d(fz,Tz_1)d(Sz,gz_1)}{1+d(fz,gz_1)} + D\frac{d(fz,Sz)d(fz,Tz_1)+d(gz_1,Tz_1)d(gz_1,Sz)}{1+d(fz,Tz_1)+d(gz_1,Sz)}.$$

Hence,

$$d(z,z_1) \precsim Ad(z,z_1) + B\frac{d(z,z)d(z,z_1)}{1+d(z,z_1)} + C\frac{d(z,z_1)d(z,z_1)}{1+d(z,z_1)} + D\frac{d(z,z)d(z,z_1)+d(z_1,z_1)d(z_1,z)}{1+d(z,z_1)+d(z,z_1)}.$$

Thus,

$$|d(z,z_1)| \leq A|d(z,z_1)| + C|d(z,z_1)|\left|\frac{d(z,z_1)}{1+d(z,z_1)}\right|,$$

which implies that,

$$|d(z,z_1)| \leq (A+C)|d(z,z_1)|.$$

As $s(A+C+D)+B<1$ and $s\geq 1$, $A+C<1$ and $|d(z,z_1)|=0$, which implies that $z=z_1$ and S,T,f and g have a unique common fixed point in X. □

Corollary 3.2. Let S and f be two self mappings of a complete complex valued b-metric space (X,d) which satisfy the following,

$$d(Sx,Sy) \precsim Ad(fx,fy)+B\frac{d(fx,Sx)d(Sy,fy)}{1+d(fx,fy)}+C\frac{d(fx,Sy)d(Sx,fy)}{1+d(fx,fy)}$$
$$+D\frac{d(fx,Sx)d(fx,Sy)+d(fy,Sy)d(fy,Sx)}{1+d(fx,Sy)+d(fy,Sx)}$$

for all $x,y\in X$ where A,B,C and D are nonnegative real numbers such that $s(A+C+D)+B<1$. Here if,

(i) $S(X)\subseteq f(X)$,

(ii) the pair (S,f) is weakly compatible,

(iii) the subspace $f(X)$ is closed,

then S and f have a unique common fixed point.

Proof: Taking $T=S$ and $g=f$ in Theorem 3.1, we get the proof. □

Corollary 3.3. Let S,T,f and g be four self mappings of a complete complex valued b-metric space (X,d) which satisfy the following

$$d(Sx,Ty) \precsim Ad(fx,gy)+B\frac{d(fx,Sx)d(Ty,gy)}{1+d(fx,gy)}+C\frac{d(fx,Ty)d(Sx,gy)}{1+d(fx,gy)}$$

for all $x,y\in X$ where A,B and C are nonnegative real numbers such that $s(A+C)+B<1$. Here if,

(i) $S(X)\subseteq g(X)$ and $T(X)\subseteq f(X)$,

(ii) the pairs (S,f) and (T,g) are weakly compatible,

(iii) the subspace $f(X)$ or $g(X)$ is closed,

then S,T,f and g have a unique common fixed point.

Proof: The result is obtained by putting D = 0 in Theorem 3.1 which is similar to Theorem 3.1 of [6]. □

Corollary 3.4. Let S,T,f and g be four self mappings of a complete complex valued b-metric space (X,d) which satisfy the following

$$d(Sx,Ty) \precsim Ad(fx,gy)$$

for all $x,y \in X$ where A is nonnegative real number such that $sA < 1$. Here if,

(i) $S(X) \subseteq g(X)$ and $T(X) \subseteq f(X)$,
(ii) the pairs (S,f) and (T,g) are weakly compatible,
(iii) the subspace $f(X)$ or $g(X)$ is closed,

then S,T,f and g have a unique common fixed point.

Proof: Putting $B = C = D = 0$, in Theorem 3.1 we get the required result. □

Example 3.5. Let (X,d) be a complex valued b-metric space, where $X = [0,1]$ and $d : X \times X \to \mathbb{C}$ is defined by $d(x,y) = |x-y|^2 + i|x-y|^2$. To show that (X,d) is a complex valued b-metric space with $s = 2$ let us verify the triangular inequality.

$$\begin{aligned}
d(x,y) &= |x-y|^2 + i|x-y|^2 \\
&\precsim |(x-z)+(z-y)|^2 + i|(x-z)+(z-y)|^2 \\
&\precsim [|x-z|^2 + |z-y|^2 + 2|x-z||z-y|] + i[|x-z|^2 + |z-y|^2 + 2|x-z||z-y| \\
&\precsim [|x-z|^2 + |z-y|^2 + |x-z|^2 + |z-y|^2] + i[|x-z|^2 + |z-y|^2 + |x-z|^2 + |z-y|^2] \\
&\precsim [|x-z|^2 + i|x-z|^2] + [|z-y|^2 + i|z-y|^2] \\
&\precsim 2[d(x,z) + d(z,y)].
\end{aligned}$$

Here $s = 2$. We define mappings S,T,f and g as $Sx = \frac{x}{6}$,$Tx = \frac{x^2}{9}$, $fx = \frac{x}{2}$ and $gx = \frac{x^2}{3}$.

$$\begin{aligned}
d(Sx,Ty) &= |Sx-Ty|^2 + i|Sx-Ty|^2 \\
&= \left|\frac{x}{6} - \frac{y^2}{9}\right|^2 + i\left|\frac{x}{6} - \frac{y^2}{9}\right|^2 \\
&= \frac{1}{9}\left[\left|\frac{x}{2} - \frac{y^2}{3}\right|^2 + i\left|\frac{x}{2} - \frac{y^2}{3}\right|^2\right].
\end{aligned}$$

$$\begin{aligned} d(fx,gy) &= |fx-gy|^2+i|fx-gy|^2 \\ &= \left|\frac{x}{2}-\frac{y^2}{3}\right|^2+i\left|\frac{x}{2}-\frac{y^2}{3}\right|^2. \end{aligned}$$

Hence,

$$d(Sx,Ty)=\frac{1}{9}d(fx,gy).$$

$$d(Sx,Ty)\precsim\frac{1}{10}d(fx,gy).$$

We have all the conditions of Corollary 3.4 with $A=\frac{1}{10}$ and $sA=2.\frac{1}{10}=\frac{1}{5}<1$. Hence $0\in X$ is the unique common fixed point of S,T,f and g. □

4. Fixed point results for mappings satisfying the (E.A) and (CLR) properties

Common fixed point theorems for mappings satisfying the $(E.A)$ property and the (CLR) property in complex valued metric spaces are proved by S.Chandok and D.Kumar [5], Manoj Kumar et.al.[8] and S.Shukla and S.Pagey [12]. We aim to extend these concepts in the complex valued b-metric spaces by proving Theorem 3.1 using the (E.A) and common limit range (CLR) properties. These properties relax the assumption of completeness of X or completeness of any of the range subspaces.

Following Verma et.al.[13], the definition of mappings satisfying property (E.A) in context of complex valued b-metric space is as follows:

Definition 4.1. [13] Let $S,T:X\to X$ be two selfmappings of a complex valued b-metric space (X,d). The pair is said to satisfy (E.A.) property if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty}Sx_n=\lim_{n\to\infty}Tx_n=t$, for some $t\in X$.
The (E.A.) property and weak compatibility are shown to be independent in [13].

Definition 4.2. [14] The self mappings $S,T:X\to X$ are said to satisfy the common limit in the range of S property (CLR_S) property if $\lim_{n\to\infty}Sx_n=\lim_{n\to\infty}Tx_n=Sx$, for some $x\in X$.
We now prove a theorem in the framework of complex valued b-metric spaces in which the mappings satisfy the property (E.A).

Theorem 4.3. Let (X,d) be a complex valued b-metric space and S,T,f and g be self mappings of X which satisfy the following:

$$d(Sx,Ty) \precsim Ad(fx,gy)+B\frac{d(fx,Sx)d(Ty,gy)}{1+d(fx,gy)}+C\frac{d(fx,Ty)d(Sx,gy)}{1+d(fx,gy)}$$
$$+D\frac{d(fx,Sx)d(fx,Ty)+d(gy,Ty)d(gy,Sx)}{1+d(fx,Ty)+d(gy,Sx)} \qquad (3)$$

for all $x,y\in X$ and A,B,C and D are nonnegative numbers such that $A+C<1$. Also if,

(i) $S(X)\subseteq g(X)$ and $T(X)\subseteq f(X)$,

(ii) the pairs (S,f) and (T,g) are weakly compatible,

(iii) one of the pairs (S,f) or (T,g) satisfy the property (E.A),

(iv) the subspace $f(X)$ or $g(X)$ is closed,

then S,T,f and g have a unique common fixed point.

Proof: Suppose the pair (T,g) satisfies the (E.A) property, then there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty}Tx_n=\lim_{n\to\infty}gx_n=t$, for some $t\in X$. Since $T(X)\subseteq f(X)$, there exists a sequence $\{y_n\}$ in X such that $Tx_n=fy_n$, hence $\lim_{n\to\infty}fy_n=t$.

We claim that $\lim_{n\to\infty}Sy_n=t$.

Suppose that $\lim_{n\to\infty}Sy_n=t^*\neq t$, then from (3) we get,

$$d(Sy_n,Tx_n) \precsim Ad(fy_n,gx_n)+B\frac{d(fx_n,Sy_n)d(Tx_n,gx_n)}{1+d(fy_n,gx_n)}+C\frac{d(fy_n,Tx_n)d(Sy_n,gx_n)}{1+d(fy_n,gx_n)}+$$
$$D\frac{d(fy_n,Sy_n)d(fy_n,Tx_n)+d(gx_n,Tx_n)d(gx_n,Sy_n)}{1+d(fy_n,Tx_n)+d(gx_n,Sy_n)}.$$

As $n\to\infty$,

$$d(t^*,t) \precsim Ad(t,t)+B\frac{d(t,t^*)d(t,t)}{1+d(t,t)}+C\frac{d(t,t)d(t^*,t)}{1+d(t,t)}+D\frac{d(t,t^*)d(t,t)+d(t,t)d(t,t^*)}{1+d(t,t)+d(t,t^*)}.$$

Thus, $d(t^*,t)\precsim 0$, i.e. $|d(t^*,t)|=0$ and $t^*=t$.

Hence, $\lim_{n\to\infty}Sy_n=\lim_{n\to\infty}Tx_n=t$. Suppose that $f(X)$ is a closed subspace of X, then $t=fu$, for some $u\in X$.

Thus, $\lim_{n\to\infty}Sy_n=\lim_{n\to\infty}Tx_n=\lim_{n\to\infty}fy_n=\lim_{n\to\infty}gx_n=t=fu$.

Now we claim that $Su=fu$.

From (3), we get,

$$d(Su,Tx_n) \precsim Ad(fu,gx_n)+B\frac{d(fu,Su)d(Tx_n,gx_n)}{1+d(fu,gx_n)}+C\frac{d(fu,Tx_n)d(Su,gx_n)}{1+d(fu,gx_n)}+$$
$$D\frac{d(fu,Su)d(fu,Tx_n)+d(gx_n,Tx_n)d(gx_n,Su)}{1+d(fu,Tx_n)+d(gx_n,Su)}.$$

As $n\to\infty$, we have,

$$d(Su,t) \precsim Ad(t,t)+B\frac{d(t,Su)d(t,t)}{1+d(t,t)}+C\frac{d(t,t)d(Su,t)}{1+d(t,t)}+D\frac{d(t,Su)d(t,t)+d(t,t)d(t,Su)}{1+d(t,t)+d(t,Su)}.$$

Hence $|d(Su,t)|=0$ which implies that $Su=t$, thus $fu=Su=t$ and u is a coincidence point of f and S.

Since the pair (S,f) is weakly compatible, we have $Sfu=fSu$ and $St=ft$. Now since $S(X)\subseteq g(X)$, there exists v in X such that $Su=gv$. Thus $Su=fu=gv=t$.

We claim that v is a coincidence point of T and g i.e. $gv=Tv=t$.

From (3), we get,

$$d(Su,Tv) \precsim Ad(fu,gv)+B\frac{d(fu,Su)d(Tv,gv)}{1+d(fu,gv)}+C\frac{d(fu,Tv)d(Su,gv)}{1+d(fu,gv)}+$$
$$D\frac{d(fu,Su)d(fu,Tv)+d(gv,Tv)d(gv,Su)}{1+d(fu,Tv)+d(gv,Su)}.$$

Thus,

$$d(t,Tv) \precsim Ad(t,t)+B\frac{d(t,t)d(Tv,t)}{1+d(t,t)}+C\frac{d(t,Tv)d(t,t)}{1+d(t,t)}+D\frac{d(t,t)d(t,Tv)+d(t,Tv)d(t,t)}{1+d(t,Tv)+d(t,t)}.$$

Thus $|d(t,Tv)|=0$ which implies that $Tv=t$. Hence $gv=Tv=t$ and v is a coincidence point of g and T.

Since T and g are weakly compatible we have, $gTv=Tgv$ i.e. $gt=Tt$.

We claim that $Tt=t$. From (3) we have,

$$d(t,Tt)=d(Su,Tt) \precsim Ad(fu,gt)+B\frac{d(fu,Su)d(Tt,gt)}{1+d(fu,gt)}+C\frac{d(fu,Tt)d(Su,gt)}{1+d(fu,gt)}+$$
$$D\frac{d(fu,Su)d(Su,Tt)+d(gt,Tt)d(gt,Su)}{1+d(fu,Tt)+d(gt,Su)}.$$

i.e.

$$d(t,Tt) \precsim Ad(t,Tt)+B\frac{d(t,t)d(Tt,Tt)}{1+d(t,Tt)}+C\frac{d(t,Tt)d(t,Tt)}{1+d(t,Tt)}+D\frac{d(t,t)d(t,Tt)+d(Tt,Tt)d(Tt,t)}{1+d(t,Tt)+d(Tt,t)}.$$

Therefore,

$$|d(t,Tt)| \leq A|d(t,Tt)|+C|d(t,Tt)|\left|\frac{d(t,Tt)}{1+d(t,Tt)}\right|.$$

Since $\left|\frac{d(t,Tt)}{1+d(t,Tt)}\right| < 1$,
we get,

$$|d(t,Tt)| \leq (A+C)|d(t,Tt)|.$$

As $A+C<1$, $|d(t,Tt)| = 0$ and $t = Tt$ i.e. $gt = Tt = t$. We can show in a similar way that $St = t$ i.e. $ft = St = t$.
So $St = Tt = ft = gt = t$ and t is a common fixed point of S, T, f and g.
To prove uniqueness of the fixed point, let us assume that there in another point w such that $Sw = Tw = fw = gw = w$.
From (3), we have,

$$d(w,t) = d(Sw,Tt) \precsim Ad(fw,gt)+B\frac{d(fw,Sw)d(Tt,gt)}{1+d(fw,gt)}+C\frac{d(fw,Tt)d(Sw,gt)}{1+d(fw,gt)} \\ +D\frac{d(fw,Sw)d(Sw,Tt)+d(gt,Tt)d(gt,Sw)}{1+d(fw,Tt)+d(gt,Sw)}.$$

Hence,

$$d(w,t) \precsim Ad(w,t)+B\frac{d(w,w)d(t,t)}{1+d(w,t)}+C\frac{d(w,t)d(w,t)}{1+d(w,t)}+D\frac{d(w,w)d(w,t)+d(t,t)d(t,w)}{1+d(w,t)+d(t,w)}.$$

Thus,

$$|d(w,t)| \leq A|d(w,t)|+C|d(w,t)|\left|\frac{d(w,t)}{1+d(w,t)}\right|.$$

Since $\left|\frac{d(w,t)}{1+d(w,t)}\right| < 1$, we have,

$$|d(w,t)| \leq (A+C)|d(w,t)|.$$

As $A+C<1$, it implies that $|d(w,t)|=0$ i.e. $w=t$ and the mappings S,T,f and g have a unique common fixed point in X.

The proof is similar assuming that $g(X)$ is a closed subspace of X. Also similar result can be obtained if the pair (S,f) satisfies the property $(E.A)$. □

Corollary 4.4. Let (X,d) be a complex valued b-metric space and S,T,f and g be self mappings of X which satisfy the following:

$$d(Sx,Ty) \precsim Ad(fx,gy)$$

for all $x,y\in X$ and A is a nonnegative number such that $A<1$. Also if,

(i) $S(X)\subseteq g(X)$ and $T(X)\subseteq f(X)$,

(ii) the pairs (S,f) and (T,g) are weakly compatible,

(iii) one of the pairs (S,f) or (T,g) satisfy the property (E.A),

(iv) the subspace $f(X)$ or $g(X)$ is closed,

then S,T,f and g have a unique common fixed point.

Proof. By taking $B=C=D=0$ in Theorem 4.3, we get the result. □

Example 4.5. Let (X,d) be a complex valued b-metric space, where $X=[0,1]$ and $d:X\times X\to\mathbb{C}$ is defined by $d(x,y)=|x-y|^2+i|x-y|^2$. In Example 3.5, we have shown that (X,d) is a complex valued b-metric space with s = 2.

We define mappings S,T,f and g as $Sx=\frac{x}{6}$,$Tx=\frac{x^2}{9}$, $fx=\frac{x}{2}$ and $gx=\frac{x^2}{3}$. Here $SX=\left[0,\frac{1}{6}\right]\subseteq\left[0,\frac{1}{3}\right]=g(X)$ and $TX=\left[0,\frac{1}{9}\right]\subseteq\left[0,\frac{1}{2}\right]=f(X)$. Also the pairs (T,g) and (S,f) are weakly compatible pairs.

Consider the sequence $\{x_n\}$ = $\{\frac{1}{n}\}$,$n\in\mathbb{N}$.

Clearly $\lim_{n\to\infty}Sx_n=\lim_{n\to\infty}fx_n=0$, and $0\in X$. Hence S and f satisfy the property (E.A).

Also $\lim_{n\to\infty}Tx_n=\lim_{n\to\infty}gx_n=0$, $0\in X$, and the mappings T and g satisfy the property (E.A).

Hence as discussed in Example 3.5, here $d(Sx,Ty)=\frac{1}{9}d(fx,gy)$

i.e. $d(Sx,Ty)\precsim\frac{1}{10}d(fx,gy)$.

Hence all the conditions of Corollary 4.4 are fulfilled for $A=\frac{1}{10}$ and $0\in X$ is unique common fixed point of the mappings S,T,f and g. □

We now prove a theorem in which the selfmappings of X satisfy the common limit range (CLR) property.

Theorem 4.6. Let (X,d) be a complex valued b-metric space and S,T,f and g be self mappings of X which satisfy the following:

$$\begin{aligned} d(Sx,Ty) \precsim\ & Ad(fx,gy)+B\frac{d(fx,Sx)d(Ty,gy)}{1+d(fx,gy)}+C\frac{d(fx,Ty)d(Sx,gy)}{1+d(fx,gy)} \\ & +D\frac{d(fx,Sx)d(fx,Ty)+d(gy,Ty)d(gy,Sx)}{1+d(fx,Ty)+d(gy,Sx)} \end{aligned} \tag{4}$$

for all $x,y\in X$ and A,B,C and D are nonnegative numbers such that $A+C<1$. Also if,

(i) $S(X)\subseteq g(X)$ and $T(X)\subseteq f(X)$,
(ii) the pairs (S,f) and (T,g) are weakly compatible,
(iii) the pair (S,f) satisfies the (CLR_S) property or (T,g) satisfies the (CLR_T) property,

then S,T,f and g have a unique common fixed point.

Proof. Suppose the pair T and g satisfies the (CLR_T) property, then there exists sequence $\{x_n\}$ in X such that,

$$\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} gx_n = Tx$$

for some $x\in X$.
Since $T(X)\subseteq f(X)$, we have $Tx=fu$ for some $u\in X$.
We claim that $Su=fu=t$,say.
From (4), we have,

$$\begin{aligned} d(Su,Tx_n) \precsim\ & Ad(fu,gx_n)+B\frac{d(fu,Su)d(Tx_n,gx_n)}{1+d(fu,gx_n)}+C\frac{d(fu,Tx_n)d(Su,gx_n)}{1+d(fu,gx_n)} \\ & +D\frac{d(fu,Su)d(fu,Tx_n)+d(gx_n,Tx_n)d(gx_n,Su)}{1+d(fu,Tx_n)+d(gx_n,Su)}. \end{aligned}$$

As $n\to\infty$, we have,

$$\begin{aligned} d(Su,Tx) \precsim\ & Ad(fu,Tx)+B\frac{d(fu,Su)d(Tx,Tx)}{1+d(fu,Tx)}+C\frac{d(fu,Tx)d(Su,Tx)}{1+d(fu,Tx)} \\ & +D\frac{d(fu,Su)d(fu,Tx)+d(Tx,Tx)d(Tx,Su)}{1+d(fu,Tx)+d(Tx,Su)}. \end{aligned}$$

Since $fu = Tx$,

$$d(Su,Tx) \precsim Ad(Tx,Tx) + B\frac{d(Tx,Su)d(Tx,Tx)}{1+d(Tx,Tx)} + C\frac{d(Tx,Tx)d(Su,Tx)}{1+d(Tx,Tx)} + D\frac{d(Tx,Su)d(Tx,Tx)+d(Tx,Tx)d(Tx,Su)}{1+d(Tx,Tx)+d(Tx,Su)}.$$

Thus,

$$d(Su,Tx) \precsim 0.$$

Hence, $|d(Su,Tx)| = 0$ which implies that $Su = Tx$ i.e. $Su = Tx = fu = t$.

Since S and f are weakly compatible, $Sfu = fSu$ i.e. $St = ft$.

Also $S(X) \subseteq g(X)$, so there exists v in X such that $Su = gv$. Therefore $Su = fu = gv = t$.

Further we show that v is a coincidence point of T and g i.e. $gv = Tv = t$.

Now from (4), we get,

$$d(Su,Tv) \precsim Ad(fu,gv) + B\frac{d(fu,Su)d(Tv,gv)}{1+d(fu,gv)} + C\frac{d(fu,Tv)d(Su,gv)}{1+d(fu,gv)} + D\frac{d(fu,Su)d(fu,Tv)+d(gv,Tv)d(gv,Su)}{1+d(fu,Tv)+d(gv,Su)}.$$

Hence,

$$d(t,Tv) \precsim Ad(t,t) + B\frac{d(t,t)d(Tv,t)}{1+d(t,t)} + C\frac{d(t,Tv)d(t,t)}{1+d(t,t)} + D\frac{d(t,t)d(t,Tv)+d(t,Tv)d(t,t)}{1+d(t,Tv)+d(t,t)}.$$

Therefore $|d(t,Tv)| = 0$ which implies that $Tv = t$ i.e. $Tv = gv = t$ and v is a coincidence point of T and g.

Further since (T,g) is a weakly compatible pair, we have $gTv = Tgv$ and hence $gt = Tt$. We claim that $Tt = t$. From (4), we get,

$$d(Su,Tt) \precsim Ad(fu,gt) + B\frac{d(fu,Su)d(Tt,gt)}{1+d(fu,gt)} + C\frac{d(fu,gt)d(Su,gt)}{1+d(fu,gt)} + D\frac{d(fu,Su)d(fu,Tt)+d(gt,Tt)d(gt,Su)}{1+d(fu,Tt)+d(gt,Su)}.$$

Hence,

$$\begin{aligned} d(t,Tt) \precsim Ad(t,Tt) + B\frac{d(t,t)d(Tt,Tt)}{1+d(t,Tt)} + C\frac{d(t,Tt)d(t,Tt)}{1+d(t,Tt)} \\ +D\frac{d(t,t)d(t,Tt)+d(Tt,Tt)d(Tt,t)}{1+d(t,Tt)+d(Tt,t)}, \end{aligned}$$

which implies that,

$$|d(t,Tt)| \leq (A+C)|d(t,Tt)|.$$

Hence as $A+C<1$, $|d(t,Tt)|=0$ i.e. $Tt=t$. Thus $Tt=gt=t$.

In a similar way we can show that $St=t$ i.e. $St=ft=t$.

Hence $Tt=St=ft=gt=t$ and t is a common fixed point of S,T,f and g.

To show that the fixed point is unique, let us assume that there exists another point w such that $Sw=Tw=fw=gw=w$. Then by (4),

$$\begin{aligned} d(t,w) = d(St,Tw) \precsim Ad(ft,gw) + B\frac{d(ft,St)d(Tw,gw)}{1+d(ft,gw)} + C\frac{d(ft,gw)d(St,gw)}{1+d(ft,gw)} \\ +D\frac{d(ft,St)d(ft,Tw)+d(gw,Tw)d(gw,St)}{1+d(ft,Tw)+d(gw,St)}, \end{aligned}$$

i.e.

$$\begin{aligned} d(t,w) \precsim Ad(t,w) + B\frac{d(t,t)d(w,w)}{1+d(t,w)} + C\frac{d(t,w)d(t,w)}{1+d(t,w)} \\ +D\frac{d(t,t)d(t,w)+d(w,w)d(w,t)}{1+d(t,w)+d(w,t)}. \end{aligned}$$

Hence,

$$|d(t,w)| \leq (A+C)|d(t,w)|.$$

Since $A+C<1$, $|d(t,w)|=0$ and $t=w$ which proves the uniqueness of the fixed point.

In a similar manner, we can prove that if the pair (S,f) satisfies the (CLR_S) property, then the mappings S,T,f and g have a unique common fixed point.

Conflict of Interests

The authors declare that there is no conflict of interests.

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