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SOME COMMON FIXED POINT THEOREMS FOR VARIOUS TYPES OF COMPATIBLE MAPPINGS IN INTUITIONISTIC FUZZY METRIC SPACES

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Abstract. The aim of this paper is to obtain some common fixed point theorems for compatible mapping of type (A) and type (P) in intuitionistic fuzzy metric spaces, which extends and generalizes the results of Aage and Salunke [1], Badshah et al.[5] Yadav and Thakur [15]and other existing, known fixed point results in metric, fuzzy metric spaces to intuitionistic fuzzy metric spaces.

Keywords: common fixed points; intuitionistic fuzzy metric space; compatible mapping; compatible mapping of type (A); compatible mapping of type (P).

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1. Introduction.

The concept of Fuzzy set was initially investigated by Zadeh [17] as a new way to represent vagueness in everyday life. Subsequently, it was developed by many authors and used in various fields. To use this concept in Topology and Analysis, several researchers have defined Fuzzy metric space in various ways. The notion of Fuzzy metric spaces due to George and Veeramani [6]. Sing and Chouhan [13] introduced the concept of compatible mappings in Fuzzy metric space and proved some common fixed point theorems. Jungck [7] introduced the concept of compatible maps, Jungck et al. [8] introduced the concept of compatible maps of type (A) in metric space and proved the fixed point theorems. The concept of compatible maps of type (P) introduced by Pathak et. al [10] and proved some common fixed point theorems.

Atanassov [4] introduced and studies the concept of intuitionistic Fuzzy sets, further, using the idea of intuitionistic fuzzy metric set, Park [9] introduced intuitionistic fuzzy metric space with

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the help of continuous t- norm and continuous t- conorm, as a generalization of fuzzy metric space due to George and Veeramani [6]. Alaca et al.[3] defined the notion of intuitionistic fuzzy metric spaces and proved common fixed point theorems. Turkoglu et al. [14] first formulate the definition of weakly commuting
and *R*-weakly commuting mappings in intuitionistic fuzzy metric spaces.
Turkoglu et al. [15] introduced the concept of compatible maps and compatible maps of types(α) and (β) in intuitionistic fuzzy metric spaces and gave some relations between the concepts of compatible maps and compatible maps of types (α) and (β).Alaca et al.[2] proved some common fixed point theorem for four mappings under the condition of compatible mappings of type (I) and of type (II) in complete intuitionistic fuzzy metric spaces.
The purpose of this paper is to prove some common fixed point theorems for compatible maps of type (A) and compatible maps of type (P) in intuitionistic fuzzy metric spaces. It is also worth mentioning that the concept of compatible maps of type (A) and compatible maps of type (P) in intuitionistic fuzzy metric spaces are same as compatible maps of types (α) and (β) in Turkoglu et al. [15] respectively.

2. Basic Definitions and Preliminaries.

Definition 2.1.[11] A binary operation $* : [0,1] \times [0,1] \rightarrow [0,1]$ is a continuous *t-norm* if it satisfies the following conditions:

i. $*$ is commutative and associative;
ii. $*$ is continuous,
iii. $a * 1 = a$ for all $a \in [0, 1]$;
iv. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0,1]$.

Examples of *t-norm* $a * b = ab$ and $a * b = \min\{a, b\}$.

Definition 2.2.[3] A binary operation $\Diamond : [0,1] \times [0,1] \rightarrow [0,1]$ is continuous *t-conorm* if it satisfies the following conditions:

i. $\Diamond$ is associative and commutative;
ii. $\Diamond$ is continuous;
iii. $a \Diamond 0 = a$ for all $a \in [0,1]$;
iv. $a \Diamond b \leq c \Diamond d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0,1]$.

Examples of *t-conorm* $a \Diamond b = \min\{a+b , 1\}$ and $a \Diamond b = \max\{a, b\}$.

Definition 2.3. [3] A 5- tuple (X, M, N, *, ◊) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous *t-norm,* ◊ continuous *t-conorm* and M, N are fuzzy sets on $X^2 \times [0,\infty)$ satisfying the following conditions:

For all x, y, z ∈ X and t, s > 0

(IFM-1) $M(x, y, t) + N(x, y, t) \leq 1$;

(IFM-2) $M(x, y, 0) = 0$;

(IFM-3) $M(x, y, t) = 1$ if and only if x=y;

(IFM-4) $M(x, y, t) = M(y, x, t)$;

(IFM-5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;

(IFM-6) $M(x, y, .): [0, \infty) \to [0,1]$ is left continuous;

(IFM-7) $\lim_{t\to\infty} M(x, y, t) = 1$;

(IFM-8) $N(x, y, 0) = 1$;

(IFM-9) $N(x, y, t) = 0$, if x = y;

(IFM-10) $N(x, y, t) = N(y, x, t)$;

(IFM-11) $N(x, y, t) \lozenge N(y, z, s) \geq N(x, z, t + s)$;

(IFM-12) $N(x, y, .): [0, \infty) \to [0,1]$ is right continuous,

(IFM-13) $\lim_{t\to\infty} N(x, y, t) = 0$.

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and degree of non nearness between x and y with respect to t, respectively.

Remark 2.1.[2] In Intuitionistic Fuzzy Metric space X, M(x, y, .) is non decreasing and N(x, y, .) is non increasing for all x, y ∈ X.

Example 2.1. Let (X, d) be a metric space. Define a * b = ab and a ◊ b = min{1, a+b }, for all a, b ∈ [0, 1] and let M and N be fuzzy sets on $X^2 \times [0, \infty)$ defined as follows:

$M(x, y, t) = \frac{t}{t+d(x,y)}$ and $N(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$ for all x, y ∈ X and all t > 0.

then (M, N) is called an intuitionistic fuzzy metric space on X. We call this intuitionistic fuzzy metric induced by a metric d, the standard intuitionistic fuzzy metric.

Remark 2.2. Note that the above examples holds even with the t- norm a * b = min{a, b} and t-conorm a ◊ b= max{a, b}and hence (M, N) is an intuitionistic fuzzy metric with respect to any continuous t – norm and continuous t – conorm.

Definition 2.4[3]. Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space. Then

(i) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all t > 0, p > 0,

$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1 \text{ and } \lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0.$$

(ii) a sequence $\{x_n\}$ in X is said to be convergent to a point x ∈ X if, for all t > 0,

$$\lim_{n\to\infty} M(x_n, x, t) = 1, \text{ and } \lim_{n\to\infty} N(x_n, x, t) = 0.$$

An Intuitionistic Fuzzy metric space (X, M, N, *, ◊) is said to be complete if and only if every Cauchy sequence in X is convergent.

Throughout in this section, (X, d) denote a metric space. We recall the following definitions and propositions in metric spaces.

The first ever attempt to relax the commutativity of mappings to a smaller subset of the domain of mappings was initiated by Sessa [12] who in 1982 gave the notion of weak commutativity.

Definition 2.5.[12] Self mappings S and T of a metric space (X, d) are said to be weakly commuting pair if, for all x ∈ X, d(STx, TSx) ≤ d(Sx, Tx).

Definition 2.6.[7] Let S , T: (X, d)→ (X, d) be mappings. The mappings S and T are said to be compatible if $\lim_{n\to\infty} d(STx_n, TSx_n) = 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some t in X.

Definition 2.7.[8] Let S , T: (X, d)→ (X, d) be mappings. The mappings S and T are are said to be compatible of type (A) if $\lim_{n\to\infty} d(TSx_n, SSx_n) = 0$ and $\lim_{n\to\infty} d(STx_n, TTx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$, for some t in X.

Definition 2.8.[10] Let S , T: (X, d)→ (X, d) be mappings. The mappings S and T are said to be compatible of type (P) if $\lim_{n\to\infty} d(SSx_n, TTx_n) = 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some t in X.

The following propositions of Pathak et al.[10], shows that definitions 2.6 and 2.7are equivalent under some conditions:

Proposition 2.1. Let **S,** T: (X, d) → (X, d) be continuous mappings. If S and T are compatible, then they are compatible of type (A).

Proposition 2.2. Let **S,** T: (X, d) → (X, d) be compatible mappings of type (A). If one of S and T is continuous, then S and T are compatible.

The following is a direct consequence of above Proposition 2.1 and 2.2:

Proposition 2.3. Let S**,** T: (X, d) →(X, d) be continuous mappings. Then S and T are compatible if and only if they are compatible of type (A).

In [8], it is shown that the above Proposition 2.3**.** is not true if S and T are not continuous on a metric space.

Also Pathak et al. [10], prove the following proposition:

Proposition 2.4. Let **S,** T: (X, d) →(X,d) be continuous mappings. Then S and T are compatible if and only if they are compatible of type (P).

Proposition 2.5. Let **S,** T: (X, d) → (X, d) be compatible mappings of type(A). If one of S and T is continuous, then S and T are compatible of type (P).

As a direct consequence of above Propositions2.3-2.5, Pathak et al. [10], prove the following:

Proposition 2.6. Let **S,** T: (X, d) → (X,d) be continuous mappings. Then

(1) S and T are compatible if and only if they are compatible of type (P).

(2) S and T are compatible of type (A) if and only if they are compatible of type (P).

Proposition2.7. Let **S,** T: (X, d) → (X,d) be mappings. If S and T are compatible of type (P) and Sz = Tz for some z ∈X, then SSz =STz = TSz = TTz.

Proposition 2.8. Let **S,** T: (X, d) → (X,d) be mappings. Let Sand T are compatible of type (P) and let Sxn , Txn → z as n → ∞ for some z ∈X.Then we have the following:

(i) $\lim_{n\to\infty}$ TTx_n =Sz if S is continuous at z.

(ii) $\lim_{n\to\infty}$ SSxn = Tz if T is continuous at z.

(iii) STz=TSz and Sz=Tz if S and T are continuous at z.

For the proof of the above propositions one can see Pathak et al. [10].

We now gave the above definitions and propositions in intuitionistic fuzzy metric spaces.

Definition 2.9. [14] A pair (A, S) of self mappings of an intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be commuting if $M(ASx, SAx, t) = 1$ *and* $N(ASx, SAx, t) = 0$ *for all* $x \in X$.

Definition 2.10. [14] A pair (A, S) of self mappings of an intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be weakly commuting if $M(ASx, SAx, t) \geq M(Ax, Sx, t)$ *and* $N(ASx, SAx, t) \leq N(Ax, Sx, t)$

$$for\ all\ x \in X and\ t > 0.$$

Definition 2.11. [15] A pair (A, S) of self mapping of an intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be compatible if $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$ *and* $\lim_{n\to\infty} N(ASx_n, SAx_n, t) =$

$0\ for\ all\ t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = u\ for\ some\ u \in X.$

We now define compatible mappings of type (A) (or type (α) in Turkoglu et al.[15]) and type (P) (or type(β) Turkoglu et al.[15]) in intuitionistic fuzzy metric space as follows:

Definition 2.12. Two self mappings S and T of an intuitionistic fuzzy metric space (X, M, N, *, ◊) are said to be compatible mappings of type (A) if,

$$\lim_{n\to\infty} M(STx_n, TTx_n, \mathrm{t}) = 1 \text{ and } \lim_{n\to\infty} N(STx_n, TTx_n, \mathrm{t}) = 0$$

$\lim_{n\to\infty} M(TSx_n, SSx_n, \mathrm{t}) = 1$ and $\lim_{n\to\infty} N(TSx_n, SSx_n, \mathrm{t}) = 0$ for all t > 0,

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = u\ for\ some\ u \in X.$

Definition 2.13. Two self mappings S and T of an intuitionistic fuzzy metric space (X, M, N, *, ◊) are said to be compatible mappings of type (P) if,

$\lim_{n\to\infty} M(SSx_n, TTx_n, \mathrm{t}) = 1$ and $\lim_{n\to\infty} N(SSx_n, TTx_n, \mathrm{t}) = 0$ for all t > 0.

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = u\ for\ some\ u \in X.$.

Proposition 2.9. Let S, T be continuous mappings from an intuitionistic fuzzy metric space X into itself. If S and T are compatible, then they are compatible of type (A).

Proposition 2.10. Let S, T be compatible mappings of type (A) in an intuitionistic fuzzy metric space X into itself. If one of S and T is continuous, then S and T are compatible.

Proposition 2.11. Let S, T be continuous mappings from an intuitionistic fuzzy metric space X into itself. Then S and T are compatible if and only if they are compatible mappings of type (A).

Proposition 2.12. Let S , T be continuous mappings from an intuitionistic fuzzy metric space X into itself. Then S and T are compatible if and only if they are compatible mappings of type (P).

Proposition 2.13. If S and T are compatible mappings of type (A) in an intuitionistic fuzzy metric space X into itself. If one of S and T are continuous then S and T are compatible mappings of type (P).

Proposition 2.14. If S and T are continuous mappings in an intuitionistic fuzzy metric space X into itself. Then

i. If S and T are compatible mappings if and only if they are compatible mappings of type (P).
ii. S and T are compatible of type (A) if and only if they are compatible mappings of type (P).

Proposition 2.15. Let S and T be mappings from an intuitionistic fuzzy metric spaces (X, M, N, *, ◊) into itself. If a pair {S, T} is compatible of type (A) on X and if $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z \text{ for some } z \in X$, then we have

I. $M(TSx_n, Sz, \mathrm{t}) \to 1$ and $N(TSx_n, Sz, \mathrm{t}) \to 0 as\ n \to \infty\ if\ S\ is\ continuous.$

II. $M(STx_n, Tz, \mathrm{t}) \to 1$ and $N(STx_n, Tz, \mathrm{t}) \to 0 as\ n \to \infty\ if\ S\ is\ continuous.$

III. $STz = TSz\ and\ Sz = Tz\ if\ S\ and\ T\ are\ continuous\ at\ z.$

For the proof of the above propositions one can use same techniques as in Pathak et al. [10] and Alaca et al.[2]., Turkoglu et al.[15].

Lemma 2.1.[2] Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space, and for all x, y ∈ X, and t > 0 if for a number k ∈ (0, 1), M(x, y, kt) ≥ M(x, y, t) and , N(x, y, kt) ≤ N(x, y, t) then x = y.

3. Main Result.

Theorem 3.1. Suppose S, T and A are three self mappings of a complete intuitionistic fuzzy metric space (X, M, N, *, ◊) into itself satisfying the following conditions:

(I). S(X) ∪ T(X) ⊂ A(X);

(II). $M(Sx, Tx, t) \geq \alpha_1 M(Ax, Ay, t) + \alpha_2 M(Sx, Ax, t) + \alpha_3 M(Ty, Ay, t) + \alpha_4 M(Ax, Ty, t) + \alpha_5 M(Ay, Sx, t)$ and

$N(Sx, Tx, t) \leq \alpha_1 N(Ax, Ay, t) + \alpha_2 N(Sx, Ax, t) + \alpha_3 N(Ty, Ay, t) + \alpha_4 N(Ax, Ty, t) + \alpha_5 N(Ay, Sx, t)$

for all $x, y \in X\ and\ \alpha_1, \alpha_2, \alpha_3, \alpha_4\ and\ \alpha_5$ are non negative number such that $(\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4 + \alpha_5) < 1$.

(III). One of S, T and A is continuous;

(IV). (S, A) and (T, A) are compatible mappings of Type (A).

Then S, T and A have a unique common fixed point in X.

Proof. Let $x_0 \in X$ arbitrary. Construct a sequence$\{x_n\}$, as follows

$$\mathrm{A}x_{2n+1} = Sx_{2n}\ ,\ \mathrm{A}x_{2n+2} = Tx_{2n+1},\ \ for\ all\ n = 0,1,2, \ldots \quad (1)$$

$M(\mathrm{A}x_{2n+1}, \mathrm{A}x_{2n+2}, t) = M(Sx_{2n}, Tx_{2n+1}, t\ ) \geq \alpha_1 M(\mathrm{A}x_{2n}, \mathrm{A}x_{2n+1}, t) + \alpha_2 M(\mathrm{S}x_{2n}, \mathrm{A}x_{2n}, t) + \alpha_3 M(\mathrm{T}x_{2n+1}, \mathrm{A}x_{2n+1}, t) + \alpha_4 M(\mathrm{A}x_{2n}, \mathrm{T}x_{2n+1}, t) + \alpha_5 M(\mathrm{A}x_{2n+1}, \mathrm{A}x_{2n+2}, t)$

$$\geq (\alpha_1 + \alpha_2 + \alpha_4)M(\mathrm{A}x_{2n}, \mathrm{A}x_{2n+1}, t) + (\alpha_3 + \alpha_4)M(\mathrm{A}x_{2n+1}, \mathrm{A}x_{2n+2}, t)$$

and $N(\mathrm{A}x_{2n+1}, \mathrm{A}x_{2n+2}, t) = N(Sx_{2n}, Tx_{2n+1}, t) \leq \alpha_1 N(\mathrm{A}x_{2n}, \mathrm{A}x_{2n+1}, t) +$
$\alpha_2 N(\mathrm{S}x_{2n}, \mathrm{A}x_{2n}, t) + \alpha_3 N(\mathrm{T}x_{2n+1}, \mathrm{A}x_{2n+1}, t) + \alpha_4 N(\mathrm{A}x_{2n}, \mathrm{T}x_{2n+1}, t) +$
$\alpha_5 N(\mathrm{A}x_{2n+1}, \mathrm{A}x_{2n+2}, t)$
$\leq (\alpha_1 + \alpha_2 + \alpha_4)N(\mathrm{A}x_{2n}, \mathrm{A}x_{2n+1}, t) + (\alpha_3 + \alpha_4)N(\mathrm{A}x_{2n+1}, \mathrm{A}x_{2n+2}, t),$
it follows

$$M(\mathrm{A}x_{2n+1}, \mathrm{A}x_{2n+2}, t) \geq \frac{(\alpha_1 + \alpha_2 + \alpha_4)}{(1 - \alpha_3 - \alpha_4)} M(\mathrm{A}x_{2n}, \mathrm{A}x_{2n+1}, t)$$

and

$$N(\mathrm{A}x_{2n+1}, \mathrm{A}x_{2n+2}, t) \leq \frac{(\alpha_1 + \alpha_2 + \alpha_4)}{(1 - \alpha_3 - \alpha_4)} N(\mathrm{A}x_{2n}, \mathrm{A}x_{2n+1}, t)$$

or

$$M(\mathrm{A}x_{2n+1}, \mathrm{A}x_{2n+2}, t) \geq h\, M(\mathrm{A}x_{2n}, \mathrm{A}x_{2n+1}, t)$$

and

$$N(\mathrm{A}x_{2n+1}, \mathrm{A}x_{2n+2}, t) \leq h\, N(\mathrm{A}x_{2n}, \mathrm{A}x_{2n+1}, t)$$

where $h = \frac{(\alpha_1 + \alpha_2 + \alpha_4)}{(1 - \alpha_3 - \alpha_4)} < 1$

Similarly, we can show that

$$M(\mathrm{A}x_{2n+1}, \mathrm{A}x_{2n+2}, t) \geq h^{2n+1}\, M(\mathrm{A}x_0, \mathrm{A}x_1, t)$$

and

$$N(\mathrm{A}x_{2n+1}, \mathrm{A}x_{2n+2}, t) \leq h^{2n+1}\, N(\mathrm{A}x_0, \mathrm{A}x_1, t)$$

For k > n

$$M(\mathrm{A}x_n, \mathrm{A}x_{n+k}, t) \geq h\, M(\mathrm{A}x_{n+i-1}, \mathrm{A}x_{n+i}, t)$$
$$M(\mathrm{A}x_n, \mathrm{A}x_{n+k}, t) \geq h^{n+i-1}\, M(\mathrm{A}x_0, \mathrm{A}x_1, t)$$
$$\geq M(\mathrm{A}x_0, \mathrm{A}x_1, t) \to 1 \text{ as } n \to \infty$$

and

$$N(\mathrm{A}x_n, \mathrm{A}x_{n+k}, t) \leq h\, N(\mathrm{A}x_{n+i-1}, \mathrm{A}x_{n+i}, t)$$
$$N(\mathrm{A}x_n, \mathrm{A}x_{n+k}, t) \leq h^{n+i-1}\, N(\mathrm{A}x_0, \mathrm{A}x_1, t)$$
$$\leq N(\mathrm{A}x_0, \mathrm{A}x_1, t) \to 0 \text{ as } n \to \infty$$

Hence $\{\mathrm{A}x_n\}$ is a Cauchy sequence. Since X is complete intuitionistic fuzzy metric space, there exists a point $z \in X$ such that $\mathrm{A}x_n \to z$. Then the subsequence of $\{\mathrm{A}x_n\}$, $\{\mathrm{S}x_{2n}\}$ and $\{\mathrm{T}x_{2n+1}\}$ also converges to z. i.e. $\{\mathrm{S}x_{2n}\} \to z$ and $\{\mathrm{T}x_{2n+1}\} \to z$.

Suppose that A is continuous, and the pair (S, A) is compatible mappings of type (A).

Then from condition (II), we have,

$$M(\mathrm{SA}x_{2n}, T\mathrm{A}x_{2n+2}, t) \geq \alpha_1 M(\mathrm{AA}x_{2n}, \mathrm{A}x_{2n+1}, t) + \alpha_2 M(\mathrm{SA}x_{2n}, \mathrm{A}Ax_{2n}, t) +$$
$$\alpha_3 M(\mathrm{T}x_{2n+1}, \mathrm{A}x_{2n+1}, t) + \alpha_4 M(\mathrm{AA}x_{2n}, \mathrm{T}x_{2n+1}, t) + \alpha_5 M(\mathrm{A}x_{2n+1}, S\mathrm{A}x_{2n}, t)$$

and

$$N(\mathrm{SA}x_{2n}, T\mathrm{A}x_{2n+2}, t) \leq \alpha_1 N(\mathrm{AA}x_{2n}, \mathrm{A}x_{2n+1}, t) + \alpha_2 N(\mathrm{SA}x_{2n}, \mathrm{A}Ax_{2n}, t) +$$
$$\alpha_3 N(\mathrm{T}x_{2n+1}, \mathrm{A}x_{2n+1}, t) + \alpha_4 N(\mathrm{AA}x_{2n}, \mathrm{T}x_{2n+1}, t) + \alpha_5 N(\mathrm{A}x_{2n+1}, S\mathrm{A}x_{2n}, t)$$

Since A is continuous, $\mathrm{AA}x_{2n} \to Az \ as \ n \to \infty$. Since the pair (S, A) is compatible of type (A), then $S\mathrm{A}x_{2n} \to Az \ as \ n \to \infty$.

Letting $n \to \infty$, we have

$$M(\mathrm{Az}, z, t) \geq \alpha_1 M(\mathrm{Az}, \mathrm{z}, t) + \alpha_2 M(\mathrm{Az}, Az, t) + \alpha_3 M(\mathrm{z}, \mathrm{z}, t) + \alpha_4 M(\mathrm{Az}, \mathrm{z}, t) + \alpha_5 M(\mathrm{z}, \mathrm{Az}, t)$$

and

$$N(\mathrm{Az}, z, t) \leq \alpha_1 N(\mathrm{Az}, \mathrm{z}, t) + \alpha_2 N(\mathrm{Az}, Az, t) + \alpha_3 N(\mathrm{z}, \mathrm{z}, t) + \alpha_4 N(\mathrm{Az}, \mathrm{z}, t) + \alpha_5 N(\mathrm{z}, \mathrm{Az}, t)$$

Hence $M(\mathrm{Az}, \mathrm{z}, t) = 1, N(\mathrm{Az}, \mathrm{z}, t) = 0$ and Az = z, Since $0 \leq (\alpha_1 + \alpha_4 + \alpha_5) < 1$.

Again we have

$$M(Sz, \mathrm{T}x_{2n+1}, t)$$
$$\geq \alpha_1 M(\mathrm{Az}, \mathrm{A}x_{2n+1}, t) + \alpha_2 M(\mathrm{Sz}, \mathrm{Az}, t) + \alpha_3 M(\mathrm{T}x_{2n+1}, \mathrm{A}x_{2n+1}, t)$$
$$+ \alpha_4 M(\mathrm{Az}, \mathrm{T}x_{2n+1}, t) + \alpha_5 M(\mathrm{A}x_{2n+1}, \mathrm{Sz}, t)$$

$$N(Sz, \mathrm{T}x_{2n+1}, t)$$
$$\leq \alpha_1 N(\mathrm{Az}, \mathrm{A}x_{2n+1}, t) + \alpha_2 N(\mathrm{Sz}, \mathrm{Az}, t) + \alpha_3 N(\mathrm{T}x_{2n+1}, \mathrm{A}x_{2n+1}, t)$$
$$+ \alpha_4 N(\mathrm{Az}, \mathrm{T}x_{2n+1}, t) + \alpha_5 N(\mathrm{A}x_{2n+1}, \mathrm{Sz}, t)$$

Letting n$\to \infty$ and using Az = z, we have

$$M(Sz, z, t) \geq \alpha_1 M(z, \mathrm{z}, t) + \alpha_2 M(\mathrm{Sz}, \mathrm{z}, t) + \alpha_3 M(\mathrm{z}, \mathrm{z}, t) + \alpha_4 M(z, Sz, t) + \alpha_5 M(\mathrm{z}, \mathrm{Sz}, t)$$
$$= (\alpha_2 + \alpha_5) M(Sz, z, t)$$

and

$$N(Sz, z, t) \leq \alpha_1 N(z, z, t) + \alpha_2 N(\mathrm{Sz}, \mathrm{z}, t) + \alpha_3 N(\mathrm{z}, \mathrm{z}, t) + \alpha_4 N(z, Sz, t) + \alpha_5 N(\mathrm{z}, \mathrm{Sz}, t)$$
$$= (\alpha_2 + \alpha_5) N(Sz, z, t)$$

Hence $M(\mathrm{Sz}, \mathrm{z}, t) = 1, N(\mathrm{Sz}, \mathrm{z}, t) = 0 \ and \ Sz = z$, since $(\alpha_2 + \alpha_5) < 1$.

So we have $$Az = Sz = z.$$

By condition (II), we have

$$M(z,Tz,t) = M(Sz,Tz,t)$$
$$\geq \alpha_1 M(Az,A\,z,t) + \alpha_2 M(\mathrm{Sz},Az,t) + \alpha_3 M(\mathrm{Tz},Az,t) + \alpha_4 M(Az,Tz,t)$$
$$+ \alpha_5 M(\mathrm{Az},\mathrm{Sz},t) = (\alpha_3 + \alpha_4) M(Tz,z,t)$$

and

$$N(z,Tz,t) = N(Sz,Tz,t)$$
$$\leq \alpha_1 N(Az,A\,z,t) + \alpha_2 N(\mathrm{Sz},Az,t) + \alpha_3 N(\mathrm{Tz},Az,t) + \alpha_4 N(Az,Tz,t)$$
$$+ \alpha_5 N(\mathrm{Az},\mathrm{Sz},t) = (\alpha_3 + \alpha_4) N(Tz,z,t)$$

Hence $M(z,Tz,t) = 1, N(z,Tz,t) = 0$ $i.e. z = Tz$, since $(\alpha_3 + \alpha_4) < 1$.

Showing that z is common fixed point of S, T and A. Similarly, we can show that z is a common fixed point of S, T and A when the pair (T, A) is compatible mapping of type (A).

In order to prove the uniqueness of fixed point, let z and z' be two common fixed point of S, T and A. So we have $z = Sz = Tz = Az$ and $z' = Sz' = Tz' = Az'$

$$M(z,z',t) = M(Sz,Tz',t)$$
$$\geq \alpha_1 M(Az,Tz',t) + \alpha_2 M(\mathrm{Sz},Az,t) + \alpha_3 M(\mathrm{T}z',Az',t) + \alpha_4 M(Az,Tz',t)$$
$$+ \alpha_5 M(\mathrm{A}z',\mathrm{Sz},t)$$
$$= \alpha_1 M(z,z',t) + \alpha_2 M(z,\mathrm{z},t) + \alpha_3 M(z',z',t) + \alpha_4 M(z,z',t) + \alpha_5 M(z',\mathrm{z},t)$$
$$= (\alpha_1 + \alpha_4 + \alpha_5) M(z,z',t)$$

and

$$N(z,z',t) = N(Sz,Tz',t)$$
$$\leq \alpha_1 N(Az,Tz',t) + \alpha_2 N(\mathrm{Sz},Az,t) + \alpha_3 N(\mathrm{T}z',Az',t) + \alpha_4 N(Az,Tz',t)$$
$$+ \alpha_5 N(\mathrm{A}z',\mathrm{Sz},t)$$
$$= \alpha_1 N(z,z',t) + \alpha_2 N(z,\mathrm{z},t) + \alpha_3 N(z',z',t) + \alpha_4 N(z,z',t) + \alpha_5 N(z',\mathrm{z},t)$$
$$= (\alpha_1 + \alpha_4 + \alpha_5) N(z,z',t).$$

Hence $M(z,z',t) = 1, N(z,z',t) = 0$ $and\ z = z', since\ (\alpha_1 + \alpha_4 + \alpha_5) < 1$.

Thus, the common fixed point is unique.

Corollary 3.1. Suppose S, T and A are three self mappings of a complete intuitionistic fuzzy metric space (X, M, N, *, ◊) into itself satisfying the following conditions:

(I). S(X) ∪ T(X) ⊂ A(X);

(II). $M(Sx,Tx,t) \geq \alpha_1 M(Ax,Ay,t) + \alpha_2 M(Sx,Ax,t) + \alpha_3 M(Ty,Ay,t) + \alpha_4 M(Ax,Ty,t) + \alpha_5 M(Ay,Sx,t)$ and

$N(Sx,Tx,t) \leq \alpha_1 N(Ax,Ay,t) + \alpha_2 N(Sx,Ax,t) + \alpha_3 N(Ty,Ay,t) + \alpha_4 N(Ax,Ty,t) + \alpha_5 N(Ay,Sx,t)$

for all $x, y \in X$ and $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and α_5 are non negative number such that $(\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4 + \alpha_5) < 1$.

(III). One of S, T and A is continuous;

(IV). (S, A) and (T, A) are compatible mappings of Type (P).

Then S, T and A have a unique common fixed point.

Proof. The proof of the corollary is a direct consequence follows from the proposition 2.13, proposition 2.14 and theorem 3.1.

Theorem 3.2. Suppose S, A, T and B are four self mappings of a complete intuitionistic fuzzy metric space (X, M, N, *, ◊) into itself satisfying the following conditions:

(I). S(X) ⊂ B(X), T(X) ⊂ A(X);

(II). $M(Sx,Ty,t) \geq \alpha_1 M(Ax,Ay,t) + \alpha_2 M(Ax,Sx,t) + \alpha_3 M(By,Ty,t) + \alpha_4 M(Ax,Ty,t) + \alpha_5 M(By,Sx,t)$

and

$N(Sx,Ty,t) \leq \alpha_1 N(Ax,Ay,t) + \alpha_2 N(Ax,Sx,t) + \alpha_3 N(By,Ty,t) + \alpha_4 N(Ax,Ty,t) + \alpha_5 N(By,Sx,t)$

for all $x, y \in X$ and $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and α_5 are non negative number such that

$$(\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4 + \alpha_5) < 1.$$

(III). One of S, A, T and B is continuous;

(IV). (S, A) and (T, B) are compatible mappings of Type (A).

Then S, A, T and B have a unique common fixed point.

Proof. Let $x_0 \in X$ arbitrary. Choose a point x_1 in X such that $Sx_0 = Bx_1$.This can be done since S(X) ⊂ B(X). Let a point x_2 in X such that $Tx_1 = Ax_2$. This can be done since T(X) ⊂ A(X). In general, we can choose $x_{2n}, x_{2n+1}, x_{2n+2}, \ldots$, such that $Sx_{2n} = Bx_{2n+1}$ and $Tx_{2n+1} = Ax_{2n+2}$ $for\ all\ n = 0,1,2, \ldots$ So that we obtain a sequence $Sx_0, Tx_1, Sx_2, Tx_3, \ldots$ $for\ all\ n = 0,1,2, \ldots$ (2)

Using condition (II), we have

$M(Sx_{2n+1}, Tx_{2n+1}, t) \geq \alpha_1 M(Ax_{2n}, Bx_{2n+1}, t) + \alpha_2 M(Ax_{2n}, Sx_{2n}, t) + \alpha_3 M(Bx_{2n+1}, Tx_{2n+1}, t) + \alpha_4 M(Ax_{2n}, Tx_{2n+1}, t) + \alpha_5 M(Bx_{2n+1}, Sx_{2n}, t)$

$$= \alpha_1 M(\mathrm{T}x_{2n-1}, \mathrm{S}x_{2n}, t) + \alpha_2 M(\mathrm{T}x_{2n-1}, \mathrm{S}x_{2n}, t) + \alpha_3 M(\mathrm{S}x_{2n}, \mathrm{T}x_{2n+1}, t) + \alpha_4 M(\mathrm{T}x_{2n-1}, \mathrm{T}x_{2n+1}, t) + \alpha_5 M(\mathrm{S}x_{2n}, \mathrm{S}x_{2n}, t)$$

and

$$N(\mathrm{S}x_{2n+1}, \mathrm{T}x_{2n+1}, t) \le \alpha_1 N(\mathrm{A}x_{2n}, \mathrm{B}x_{2n+1}, t) + \alpha_2 N(\mathrm{A}x_{2n}, \mathrm{S}x_{2n}, t) + \alpha_3 N(\mathrm{B}x_{2n+1}, \mathrm{T}x_{2n+1}, t) + \alpha_4 N(\mathrm{A}x_{2n}, \mathrm{T}x_{2n+1}, t) + \alpha_5 N(\mathrm{B}x_{2n+1}, \mathrm{S}x_{2n}, t)$$
$$= \alpha_1 N(\mathrm{T}x_{2n-1}, \mathrm{S}x_{2n}, t) + \alpha_2 N(\mathrm{T}x_{2n-1}, \mathrm{S}x_{2n}, t) + \alpha_3 N(\mathrm{S}x_{2n}, \mathrm{T}x_{2n+1}, t) + \alpha_4 N(\mathrm{T}x_{2n-1}, \mathrm{T}x_{2n+1}, t) + \alpha_5 N(\mathrm{S}x_{2n}, \mathrm{S}x_{2n}, t)$$

it follows

$$M(\mathrm{S}x_{2n+1}, \mathrm{T}x_{2n+1}, t) \ge \alpha_1 M(\mathrm{T}x_{2n-1}, \mathrm{S}x_{2n}, t) + \alpha_2 M(\mathrm{T}x_{2n-1}, \mathrm{S}x_{2n}, t) + \alpha_3 M(\mathrm{S}x_{2n}, \mathrm{T}x_{2n+1}, t) + \alpha_4 M(\mathrm{T}x_{2n-1}, \mathrm{S}x_{2n}, t) + \alpha_4 M(\mathrm{S}x_{2n}, \mathrm{T}x_{2n+1}, t)$$

and

$$N(\mathrm{S}x_{2n+1}, \mathrm{T}x_{2n+1}, t) \le \alpha_1 N(\mathrm{T}x_{2n-1}, \mathrm{S}x_{2n}, t) + \alpha_2 N(\mathrm{T}x_{2n-1}, \mathrm{S}x_{2n}, t) + \alpha_3 N(\mathrm{S}x_{2n}, \mathrm{T}x_{2n+1}, t) + \alpha_4 N(\mathrm{T}x_{2n-1}, \mathrm{S}x_{2n}, t) + \alpha_4 N(\mathrm{S}x_{2n}, \mathrm{T}x_{2n+1}, t)$$

it follows

$$M(\mathrm{S}x_{2n+1}, \mathrm{T}x_{2n+1}, t) \ge (\alpha_1 + \alpha_2 + \alpha_4) M(\mathrm{T}x_{2n-1}, \mathrm{S}x_{2n}, t) + (\alpha_3 + \alpha_4) M(\mathrm{S}x_{2n}, \mathrm{T}x_{2n+1}, t)$$

and

$$N(\mathrm{S}x_{2n+1}, \mathrm{T}x_{2n+1}, t) \le (\alpha_1 + \alpha_2 + \alpha_4) N(\mathrm{T}x_{2n-1}, \mathrm{S}x_{2n}, t) + (\alpha_3 + \alpha_4) N(\mathrm{S}x_{2n}, \mathrm{T}x_{2n+1}, t)$$

Hence

$$M(\mathrm{S}x_{2n}, \mathrm{T}x_{2n+1}, t) \ge \frac{(\alpha_1 + \alpha_2 + \alpha_4)}{(1 - \alpha_3 - \alpha_4)} M(\mathrm{S}x_{2n}, \mathrm{T}x_{2n-1}, t)$$

and

$$N(\mathrm{S}x_{2n}, \mathrm{A}x_{2n+1}, t) \le \frac{(\alpha_1 + \alpha_2 + \alpha_4)}{(1 - \alpha_3 - \alpha_4)} N(\mathrm{S}x_{2n}, \mathrm{T}x_{2n-1}, t)$$

or

$$M(\mathrm{S}x_{2n}, \mathrm{T}x_{2n+1}, t) \ge k M(\mathrm{S}x_{2n}, \mathrm{T}x_{2n-1}, t)$$

and

$$N(\mathrm{S}x_{2n}, \mathrm{T}x_{2n+1}, t) \le k N(\mathrm{S}x_{2n}, \mathrm{T}x_{2n-1}, t)$$

where $k = \frac{(\alpha_1 + \alpha_2 + \alpha_4)}{1 - \alpha_3 - \alpha_4} < 1$

Similarly we can show that

$$M(\mathrm{S}x_{2n}, \mathrm{T}x_{2n-1}, t) \geq kM(\mathrm{S}x_{2n-2}, \mathrm{T}x_{2n-1}, t)$$

and

$$N(\mathrm{S}x_{2n}, \mathrm{T}x_{2n-1}, t) \leq kN(\mathrm{S}x_{2n-2}, \mathrm{T}x_{2n-1}, t)$$

therefore

$$M(\mathrm{S}x_{2n}, \mathrm{T}x_{2n+1}, t) \geq k^2\, M(\mathrm{S}x_{2n-2}, \mathrm{T}x_{2n-1}, t) \geq k^{2n}M(\mathrm{S}x_0, \mathrm{T}x_1, t)$$

and

$$N(\mathrm{S}x_{2n}, \mathrm{T}x_{2n+1}, t) \leq k^2\, N(\mathrm{S}x_{2n-2}, \mathrm{T}x_{2n-1}, t) \leq k^{2n}N(\mathrm{S}x_0, \mathrm{T}x_1, t)$$

which implies that the sequence (2) is a Cauchy sequence and since (X, M,N, *,◊) is complete, so the sequence (2) has a limit point z in X. Hence, sub sequences $\{\mathrm{S}x_{2n}\}, \{Bx_{2n-1}\}$ $\{\mathrm{T}x_{2n-1}\}$ and $\{\mathrm{A}x_{2n}\}$also converges to the point z in X. Suppose that A is continuous, then $\mathrm{AA}x_{2n} \to Az$ and $\mathrm{AS}x_{2n} \to Az \ as\ n \to \infty$. Since the pair (S, A) is compatible mappings of type (A), we get $\mathrm{SA}x_{2n} \to Az \ as\ n \to \infty$.

Now by (II)

$$M(SAx_{2n}, Tx_{2n+1}, t)$$
$$\geq \alpha_1 M(AAx_{2n}, Bx_{2n+1}, t) + \alpha_2 M(AAx_{2n}, SAx_{2n+1}, t)$$
$$+ \alpha_3 M(Bx_{2n+1}, Tx_{2n+1}, t) + \alpha_4 M(AAx_{2n}, Tx_{2n+1}, t) + \alpha_5 M(Bx_{2n+1}, SAx_{2n}, t)$$

and $N(SAx_{2n}, Tx_{2n+1}, t) \leq \alpha_1 N(AAx_{2n}, Bx_{2n+1}, t) + \alpha_2 N(AAx_{2n}, SAx_{2n+1}, t) + \alpha_3 N(Bx_{2n+1}, Tx_{2n+1}, t) + \alpha_4 N(AAx_{2n}, Tx_{2n+1}, t) + \alpha_5 N(Bx_{2n+1}, SAx_{2n}, t)$

Letting $n \to \infty$, we have

$M(\mathrm{A}z, z, t) \geq \alpha_1 M(Az, z, t) + \alpha_2 M(Az, Az, t) + \alpha_3 M(z, z, t) + \alpha_4 M(Az, z, t) + \alpha_5 M(z, Az, t)$
$= (\alpha_1 + \alpha_4 + \alpha_5) M(Az, z, t)$

and $N(\mathrm{A}z, z, t) \leq \alpha_1 N(Az, z, t) + \alpha_2 N(Az, Az, t) + \alpha_3 N(z, z, t) + \alpha_4 N(Az, z, t) + \alpha_5 N(z, Az, t) = (\alpha_1 + \alpha_4 + \alpha_5) N(Az, z, t)$

This gives $M(Az, z, t) = 1, N(Az, z, t) = 0$, Since $0 \leq \alpha_1 + \alpha_4 + \alpha_5 < 1$. Hence Az = z. Further

$M(Sz, Tx_{2n+1}, t) \geq \alpha_1 M(Az, Bx_{2n+1}, t) + \alpha_2 M(Az, Sz, t) + \alpha_3 M(Bx_{2n+1}, Tx_{2n+1}, t) + \alpha_4 M(Az, Tx_{2n+1}, t) + \alpha_5 M(Bx_{2n+1}, Sz, t)$

and $N(Sz, Tx_{2n+1}, t) \leq \alpha_1 N(Az, Bx_{2n+1}, t) + \alpha_2 N(Az, Sz, t) + \alpha_3 N(Bx_{2n+1}, Tx_{2n+1}, t) + \alpha_4 N(Az, Tx_{2n+1}, t) + \alpha_5 N(Bx_{2n+1}, Sz, t)$

letting $Bx_{2n+1}, Tx_{2n+1} \to z \ as\ n \to \infty$ Az = z we get

$M(Sz,z,t) \geq \alpha_1 M(z,z,t) + \alpha_2 M(z,Sz,t) + \alpha_3 M(z,z,t) + \alpha_4 M(z,z,t) + \alpha_5 M(z,Sz,t) = (\alpha_2 + \alpha_5) M(Sz,z,t)$

and $N(Sz,z,t) \leq \alpha_1 N(z,z,t) + \alpha_2 N(z,Sz,t) + \alpha_3 N(z,z,t) + \alpha_4 N(z,z,t) + \alpha_5 N(z,Sz,t)$

$= (\alpha_2 + \alpha_5) N(Sz,z,t)$

Hence, $M(Sz,z,t) = 1$ *and* $N(Sz,z,t) = 0$ i.e. $Sz = z$, since $0 \leq (\alpha_2 + \alpha_5) < 1$.

Thus, $Az = Sz = z$.

$S(X) \subset B(X)$, there is a point $u \in X$ such that $z = Sz = Bu$. Now by (II)

$M(z,Tu,t) = M(Sz,Tu,t) \geq \alpha_1 M(z,Bu,t) + \alpha_2 M(Az,Sz,t) + \alpha_3 M(Bu,Tu,t) + \alpha_4 M(Az,Tu,t) + \alpha_5 M(Bu,Sz,t)$

$= \alpha_1 M(z,z,t) + \alpha_2 M(z,z,t) + \alpha_3 M(z,Tu,t) + \alpha_4 M(z,Tu,t) + \alpha_5 M(z,z,t)$

$= (\alpha_3 + \alpha_4) M(z,Tu,t)$

and $N(z,Tu,t) = N(Sz,Tu,t) \leq \alpha_1 N(z,Bu,t) + \alpha_2 N(Az,Sz,t) + \alpha_3 N(Bu,Tu,t) + \alpha_4 N(Az,Tu,t) + \alpha_5 N(Bu,Sz,t)$

$= \alpha_1 N(z,z,t) + \alpha_2 N(z,z,t) + \alpha_3 N(z,Tu,t) + \alpha_4 N(z,Tu,t) + \alpha_5 N(z,z,t)$

$= (\alpha_3 + \alpha_4) N(z,Tu,t)$

hence $M(z,Tu,t) = 1$ *and* $N(z,Tu,t) = 0$ i.e. $Tu = z = Bu$,

since $0 \leq (\alpha_3 + \alpha_4) < 1$.Take $y_n = u \; for \; n \geq 1$.

Then, $Ty_n \to Tu = z \; and \; By_n \to Bu = z \; as \; n \to \infty$.

Since the pair (T, B) is compatible mapping of type (A), we get

$$\lim_{n\to\infty} M(BTy_n, TTy_n, t) = 1 \; and \; \lim_{n\to\infty} N(BTy_n, TTy_n, t) = 0 \; implies$$

$$M(Tz,Bz,t) = 1 \; and \; N(Tz,Bz,t) = 0$$

since $Ty_n \to z \; and \; By_n = z$ for all $n \geq 1$. Hence Tz = Bz. Now

$$\begin{aligned} M(z,Tz,t) &= M(Sz,Tz,t) \\ &\geq \alpha_1 M(Az,Bz,t) + \alpha_2 M(Az,Sz,t) + \alpha_3 M(Bz,Tz,t) + \alpha_4 M(Az,Tz,t) \\ &\quad + \alpha_5 M(Bz,Sz,t) \\ &= \alpha_1 M(z,Tz,t) + \alpha_2 M(z,z,t) + \alpha_3 M(Tz,Tz,t) + \alpha_4 M(z,Tz,t) + \alpha_5 M(Tz,z,t) \\ &= (\alpha_1 + \alpha_4 + \alpha_5) M(z,Tz,t) \end{aligned}$$

and

$$N(z,Tz,t) = N(Sz,Tz,t)$$
$$\leq \alpha_1 N(Az,Bz,t) + \alpha_2 N(Az,Sz,t) + \alpha_3 N(Bz,Tz,t) + \alpha_4 N(Az,Tz,t) + \alpha_5 N(Bz,Sz,t)$$
$$= \alpha_1 N(z,Tz,t) + \alpha_2 N(z,z,t) + \alpha_3 N(Tz,Tz,t) + \alpha_4 N(z,Tz,t) + \alpha_5 N(Tz,z,t)$$
$$= (\alpha_1 + \alpha_4 + \alpha_5) N(z,Tz,t)$$

since $0 \leq (\alpha_1 + \alpha_4 + \alpha_5) < 1$,we get $Tz = z$. Hence $z = Tz = Bz$,

therefore z is a common fixed point of S, A, T and B when the continuity of A is assumed.

Now suppose that S is continuous. Then $SSx_{2n} \to Sz, SAx_{2n} \to Sz$ as $n \to \infty$.

By condition (II), we have

$$M(SSx_{2n}, Tx_{2n+1}, t)$$
$$\geq \alpha_1 M(ASx_{2n}, Bx_{2n+1}, t) + \alpha_2 M(ASx_{2n}, SSx_{2n+1}, t) + \alpha_3 M(Bx_{2n+1}, Tx_{2n+1}, t) + \alpha_4 M(ASx_{2n}, Tx_{2n+1}, t) + \alpha_5 M(Bx_{2n+1}, SSx_{2n}, t)$$

and

$$N(SSx_{2n}, Tx_{2n+1}, t)$$
$$\leq \alpha_1 N(ASx_{2n}, Bx_{2n+1}, t) + \alpha_2 N(ASx_{2n}, SSx_{2n+1}, t) + \alpha_3 N(Bx_{2n+1}, Tx_{2n+1}, t) + \alpha_4 N(ASx_{2n}, Tx_{2n+1}, t) + \alpha_5 N(Bx_{2n+1}, SSx_{2n}, t)$$

Letting $n \to \infty$ and using the compatible mappings of type (A) of the pair (S, A), we get

$$ASx_n \to Sz$$
$$M(Sz,z,t) \geq \alpha_1 M(Sz,z,t) + \alpha_2 M(Sz,Sz,t) + \alpha_3 M(z,z,t) + \alpha_4 M(Sz,z,t) + \alpha_5 M(z,Sz,t)$$
$$= (\alpha_1 + \alpha_4 + \alpha_5) M(Sz,z,t)$$

and

$$N(Sz,z,t) \leq \alpha_1 N(Sz,z,t) + \alpha_2 N(Sz,Sz,t) + \alpha_3 N(z,z,t) + \alpha_4 N(Sz,z,t) + \alpha_5 N(z,Sz,t)$$
$$= (\alpha_1 + \alpha_4 + \alpha_5) N(Sz,z,t)$$

Since $0 \leq (\alpha_1 + \alpha_4 + \alpha_5) < 1$,we get Sz = z.

Since S(X) ⊂ B(X), there is a point $v \in X$ *such that*

$$z = Sz = Bv.$$

By condition (II), we have

$$M(SSx_{2n}, Tu, t)$$
$$\geq \alpha_1 M(ASx_{2n}, Bu, t) + \alpha_2 M(ASx_{2n}, SSx_{2n}, t) + \alpha_3 M(Bu, Tu, t) + \alpha_4 M(ASx_{2n}, Tu, t) + \alpha_5 M(Bu, SSx_{2n}, t)$$

and

$N(SSx_{2n}, Tu, t) \leq \alpha_1 N(ASx_{2n}, Bu, t) + \alpha_2 N(ASx_{2n}, SSx_{2n}, t) + \alpha_3 N(Bu, Tu, t) +$
$\alpha_4 N(ASx_{2n}, Tu, t) + \alpha_5 N(Bu, SSx_{2n}, t)$.

Taking $n \to \infty$

$$\begin{aligned} M(z, Tv, t) = {} & M(Sz, Tv, t) \\ & \geq \alpha_1 M(z, z, t) + \alpha_2 M(z, z, t) + \alpha_3 M(z, Tv, t) + \alpha_4 M(z, Tv, t) + \alpha_5 M(z, z, t) \\ & = (\alpha_3 + \alpha_4) M(z, Tv, t) \end{aligned}$$

and

$$\begin{aligned} N(z, Tv, t) = {} & N(Sz, Tv, t) \\ & \leq \alpha_1 N(z, z, t) + \alpha_2 N(z, z, t) + \alpha_3 N(z, Tv, t) + \alpha_4 N(z, Tv, t) + \alpha_5 N(z, z, t) \\ & = (\alpha_3 + \alpha_4) N(z, Tv, t) \end{aligned}$$

since $0 \leq (\alpha_3 + \alpha_4) < 1$.we get $Tv = z$.

Thus $z = Bv = Tv$ let $y_n = v$ then $Ty_n \to Tv = z$ and $By_n \to Tv = z \; as \; n \to \infty$.

Since the pair (T, B) is compatible mapping of type (A), we get

$\lim_{n\to\infty} M(BTy_n, TTy_n, t) = 1$ and $\lim_{n\to\infty} N(BTy_n, TTy_n, t) = 0$

this gives $BTv = TTv$ or $Bz = Tz$, further

$$\begin{aligned} M(Sx_{2n}, Tz, t) \geq {} & \alpha_1 M(Ax_{2n}, Bz, t) + \alpha_2 M(Ax_{2n}, Sx_{2n}, t) + \alpha_3 M(Bz, Tz, t) \\ & + \alpha_4 M(Ax_{2n}, Tz, t) + \alpha_5 M(Bz, Sx_{2n}, t) \end{aligned}$$

and

$$\begin{aligned} N(Sx_{2n}, Tz, t) \leq {} & \alpha_1 N(Ax_{2n}, Bz, t) + \alpha_2 N(Ax_{2n}, Sx_{2n}, t) + \alpha_3 N(Bz, Tz, t) + \alpha_4 N(Ax_{2n}, Tz, t) \\ & + \alpha_5 N(Bz, Sx_{2n}, t) \end{aligned}$$

Letting $n \to \infty$, we get

$$\begin{aligned} M(z, Tz, t) \geq {} & \alpha_1 M(z, Tz, t) + \alpha_2 M(z, z, t) + \alpha_3 M(Tz, Tz, t) + \alpha_4 M(z, Tz, t) \\ & + \alpha_5 M(Tz, z, t) = (\alpha_1 + \alpha_4 + \alpha_5) M(z, Tz, t) \end{aligned}$$

and

$$\begin{aligned} N(z, Tz, t) \leq {} & \alpha_1 N(z, Tz, t) + \alpha_2 N(z, z, t) + \alpha_3 N(Tz, Tz, t) + \alpha_4 N(z, Tz, t) \\ & + \alpha_5 N(Tz, z, t) = (\alpha_1 + \alpha_4 + \alpha_5) N(z, Tz, t) \end{aligned}$$

since $0 \leq (\alpha_1 + \alpha_4 + \alpha_5) < 1$, we get $Tz = z$.

Again we have $T(X) \subset A(X)$, there is a point $w \in X$ such that $z = Tz = Aw$.Now

$$M(Sw,z,t) = M(Sw,Tz,t)$$
$$\geq \alpha_1 M(Aw,Bz,t) + \alpha_2 M(Aw,Sw,t) + \alpha_3 M(Bz,Tz,t) + \alpha_4 M(Aw,Tz,t) + \alpha_5 M(Bz,Sw,t)$$
$$= \alpha_1 M(z,z,t) + \alpha_2 M(z,Sw,t) + \alpha_3 M(z,z,t) + \alpha_4 M(z,Tz,t) + \alpha_5 M(z,Sw,t)$$
$$= (\alpha_2 + \alpha_5) M(z,Sw,t)$$

and

$$N(Sw,z,t) = N(Sw,Tz,t)$$
$$\leq \alpha_1 N(Aw,Bz,t) + \alpha_2 N(Aw,Sw,t) + \alpha_3 N(Bz,Tz,t) + \alpha_4 N(Aw,Tz,t) + \alpha_5 N(Bz,Sw,t)$$
$$= \alpha_1 N(z,z,t) + \alpha_2 N(z,Sw,t) + \alpha_3 N(z,z,t) + \alpha_4 N(z,Tz,t) + \alpha_5 N(z,Sw,t)$$
$$= (\alpha_2 + \alpha_5) N(z,Sw,t)$$

since $0 \leq (\alpha_2 + \alpha_5) < 1$. we get Sw = z. Take $y_n = w$ then $Sy_n \rightarrow Sw = z, Ay_n \rightarrow Aw = z$

Since the pair (S, A) is compatible mappings of type (A),

we get $\lim_{n\rightarrow\infty} M(ASy_n, SSy_n, t) = 1$ *and* $\lim_{n\rightarrow\infty} N(ASy_n, SSy_n, t) = 0$

this implies that $AAw = SSw$ *or* $Az = Sz$.

Thus we have $z = Sz = Az = Bz = Tz$. Hence z is a common fixed point of S, A, T and B.

Similarly we can prove the same result when B or T is continuous. In order to prove the uniqueness of fixed point, let z *and* z' be two common fixed point of S, A, T and B. So we have

$z = Sz = Az = Bz = Tz$ *and* $z' = Sz' = Az' = Bz' = Tz'$

By condition (II)

$$M(z,z',t) = M(Sz,Tz',t)$$
$$\geq \alpha_1 M(Az,Bz',t) + \alpha_2 M(Az,Sz,t) + \alpha_3 M(Bz',Tz',t) + \alpha_4 M(Az,Tz',t) + \alpha_5 M(Bz',Sz,t)$$
$$= \alpha_1 M(z,z',t) + \alpha_2 M(z,z,t) + \alpha_3 M(z',z',t) + \alpha_4 M(z,z',t) + \alpha_5 M(z',z,t)$$
$$= (\alpha_1 + \alpha_4 + \alpha_5) M(z',z,t)$$

and

$$N(z,z',t) = N(Sz,Tz',t)$$
$$\leq \alpha_1 N(Az,Bz',t) + \alpha_2 N(Az,Sz,t) + \alpha_3 N(Bz',Tz',t) + \alpha_4 N(Az,Tz',t) + \alpha_5 N(Bz',Sz,t)$$
$$= \alpha_1 N(z,z',t) + \alpha_2 N(z,z,t) + \alpha_3 N(z',z',t) + \alpha_4 N(z,z',t) + \alpha_5 N(z',z,t)$$

$$= (\alpha_1 + \alpha_4 + \alpha_5)N(z', z, t)$$

$0 \leq (\alpha_1 + \alpha_4 + \alpha_5) < 1$, we have $z = z'$.

This completes the proof.

Corollary 3.2. Suppose S, A, T and B are four self mappings of a complete intuitionistic fuzzy metric space (X, M, N, *, ◊) into itself satisfying the following conditions:

(I). S(X) ⊂ B(X), T(X) ⊂ A(X);

(II). $M(Sx, Ty, t) \geq \alpha_1 M(Ax, Ay, t) + \alpha_2 M(Ax, Sx, t) + \alpha_3 M(By, Ty, t) + \alpha_4 M(Ax, Ty, t) + \alpha_5 M(By, Sx, t)$

and

$N(Sx, Ty, t) \leq \alpha_1 N(Ax, Ay, t) + \alpha_2 N(Ax, Sx, t) + \alpha_3 N(By, Ty, t) + \alpha_4 N(Ax, Ty, t) + \alpha_5 N(By, Sx, t)$

for all $x, y \in X$ and $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and α_5 are non negative number such that

$$\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4 + \alpha_5 < 1.$$

(III). One of S, A, T and B is continuous;

(IV). (S, A) and (T, B) are compatible mappings of Type (P).

Then S, A, T and B have unique common fixed point.

Proof. The proof of the corollary is a direct consequence follows from the proposition 2.13, proposition 2.14 and theorem 3.2.

Conflicts of Interests

The author declares that there is no conflict of interests

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