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COMMON FIXED POINT THEOREMS FOR GENERALIZED CONTRACTION MAPPINGS IN MODULAR METRIC SPACES

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Abstract: The notion of modular metric spaces being a natural generalization of classical modulars over linear spaces Lebesgue, Orlicz, Musielak-Orlicz, Lorentz, Orlicz-Lorentz, and Calderon-Lozanovskii spaces was recently introduced. Chistyakov [4, 6] introduced and studied the concept of modular metric spaces and proved fixed point theorems for contractive map in Modular spaces. It is related to contracting rather "generalized average velocities" than metric distances, and the successive approximations of fixed points converge to the fixed points in a weaker sense as compared to metric convergence. In this paper, we prove some unique common fixed point theorems for generalized contraction type mappings for six self occasionally weakly compatible mappings in modular metric spaces.

Keywords: Modular, modular metric space, occasionally weakly compatible, fixed point, contraction mapping.

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1. Introduction

The study of fixed and common fixed points of mappings satisfying a certain metrical contractive conditions attracted many researchers. Let (X, d) be a metric space. A mapping $T: X \to X$ is a contraction if $d(Tx, Ty) \leq kd(x, y)$, for all $x, y \in X$, where $0 \leq k < 1$. The Banach's contraction mapping principle appeared in explicit form in Banach's thesis in 1922 [3]. Since its simplicity and usefulness, it has become a very popular tool in solving existence problems in many branches of mathematical analysis. Banach contraction principle has been extended in many different directions; see [7-18]. Sessa [20] initiated the notion of weakly commuting. Then Jungck

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generalized this idea, first to compatible mappings [8] and then to weakly compatible mappings [9]. Here it may be pointed out that a pair of self mappings without coincidence point is also weakly compatible as the requirement of weak compatibility is met out vacuously. But such pairs are uninteresting in common fixed point considerations as opposed to a pair of weakly compatible mappings with at least one coincidence point which one may term as nontrivial weakly compatible pair. In an attempt to coin a proper generalization of nontrivial weakly compatible pair, Al-Thagafi and Shahzad [2] introduced the notion of occasionally weakly compatible pair (abbreviated as OWC in the sequel). Abbas and Rhoades [1] proved some common fixed point theorems for occasionally weakly compatible mappings satisfying a generalized contractive condition.

The notion of modular spaces, as a generalization of metric spaces, was introduced by Nakano [17] and was intensively developed by Koshi and Shimogaki [11], Yamamuro [22] and others. The main idea behind this new concept is the physical interpretation of the modular. Informally speaking whereas a metric on a set represent finite nonnegative distances between two points of the set, a modular on a set attributes a non negative (possibly, infinite valued) 'field of (generalized) velocities': to each 'time' $\lambda > 0$ (the absolute value of), an average velocity $\omega_\lambda(x, y)$ is associated in such way that in order to cover the 'distance' between points $x, y \epsilon X$, it takes time λ to move from x to y with velocity$\omega_\lambda(x, y)$. A lot of mathematicians are interested fixed points of modular spaces. Further the most complete development of these theories are due to Luxemburg [12], Musielk and Orlicz [13], Mazur [16], Turpin [21] and there collaborators.

In 2008, Chistyakov [4] introduced the notion of modular metric spaces generated by F-modular and developed the theory of this space. In 2010 Chistyakov [5] defined the notion of modular on an arbitrary set and develop the theory of metric spaces generated by modular such that called the modular metric spaces. Recently, Mongkolkeha et al. [14, 15] has introduced some notions and established some fixed point results in modular metric spaces. In this paper, we study and prove the existence of fixed point theorems for contraction mappings in modular metric spaces and generalized the result of Mongkolkeha et al. [14, 15] and Rahimpoor et al. [19].

2. Preliminaries

We will start with a brief recollection of basic concepts and facts in modular spaces and modular metric spaces (see [4, 5, 6]).

Definition 2.1. Let X be a vector space over R (or C). A functional $\rho : X \to [0,\infty]$ is called a modular if for arbitrary x and y, elements of X satisfying the following three conditions:

(A.1) $\rho(x) = 0$ $if\ and\ only\ if\ x = 0.$

(A.2) $\rho(\alpha x) = \rho(x)$ $for\ all\ scalar\ \alpha\ with\ |\alpha| = 1;$

(A.3) $\rho(\alpha x + \beta y) \leq \rho(x) + \rho(y), whenever\ \alpha, \beta \geq 0, \alpha + \beta = 1.$

If we replace (A.3) by

(A.4) $\rho(\alpha x + \beta y) \leq \alpha^s \rho(x) + \beta^s \rho(y), for\ \alpha, \beta \geq 0, \alpha^s + \beta^s = 1\ with\ an\ s\epsilon(0,1],$

then the modular ρ is called s-convex modular, and if s = 1, ρ is called a convex modular.

If ρ is modular in X, then the set defined by

$$X_\rho = \{x \in X : \rho(\lambda x) \to 0\ as\ \lambda \to 0^+\} \quad (2.1)$$

is called a modular space. X_ρ is a vector subspace of X it can be equipped with an F - norm defined by setting $\|x\|_\rho = \inf\{\lambda > 0 : \rho(\frac{x}{\lambda}) \leq \lambda\}$, $\quad x \epsilon X_\rho$. (2.2)

In addition, if ρ is convex, then the modular space X_ρ coincides with

$$X_\rho^* = \{x \in X : \exists \lambda = \lambda(x) > 0\ such\ that\ \rho(\lambda x) < \infty\} \quad (2.3)$$

and the functional $\|x\|_\rho^* = \inf\{\lambda > 0 : \rho(\frac{x}{\lambda}) \leq 1\}$ is an ordinary norm on X_ρ^* which is equivalence to $\|x\|_\rho$(see [13]).

Let X be a non empty set, $\lambda \in (0,\infty)$and due to the disparity of the arguments, function $\omega : (0,\infty)\times X\times X \to [0,\infty]$ will be written as $\omega_\lambda(x,y) = \omega(\lambda,x,y)$ for all $\lambda > 0$ and $x, y \in X$.

Definition 2.2. Let X be a non empty set. A function $\omega : (0,\infty) \times X\times X \to [0,\infty]$ is said to be a metric modular on X if it satisfies the following three axioms:

(i) given $x, y \in X, \omega_\lambda(x,y) = 0\ for\ all\ \lambda > 0\ if\ and\ only\ if\ x = y;$

(ii) $\omega_\lambda(x,y) = \omega_\lambda(y,x)\ for\ all\ \lambda > 0$ and $x, y \in X;$

(iii) $\omega_{\lambda+\mu}(x,y) \leq \omega_\lambda(x,z) + \omega_\mu(z,y)\ for\ all\ \lambda, \mu > 0$ and $x, y, z \in X.$

If instead of (i), we have only the condition

(i’) $\omega_\lambda(x,x) = 0\ for\ all\ \lambda > 0$ and $x \in X$, then ω is said to be a (metric) pseudo modular on X and if ω satisfies (i’)and

(i_s) given x, y ϵ X, if there exists a number $\lambda > 0$, possibly depending on x and y, such that $\omega_\lambda(x,y) = 0$, then x = y, with this condition ω is called a strict modular on X.

A modular (pseudo modular, strict modular) w on X is said to be convex if, instead of (iii), we replace the following condition:

(iv) for all $\lambda > 0$, $\mu > 0$ and x, y, z ∈ X it satisfies the inequality

$$\omega_{\lambda+\mu}(x,y) = \frac{\lambda}{\lambda+\mu}\omega_\lambda(x,z) + \frac{\mu}{\lambda+\mu}\omega_\mu(z,y) \text{ for all } \lambda,\mu > 0 \text{and } x,y,z \in X.$$

Clearly, if ω is a strict modular, then ω is a modular, which in turn implies ω is a pseudo modular on X, and similar implications hold for convex ω.The essential property of a (pseudo) modular ω on a set X is a following given $x, y \epsilon X$, the function $0 < \lambda \to \omega_\lambda(x,y) \epsilon [0,\infty]$ is non increasing on $(0,\infty)$. In fact, if $0 < \mu < \lambda$, then (iii), (i') and (ii) imply

$$\omega_\lambda(x,y) \le \omega_{\lambda-\mu}(x,x) + \omega_\mu(x,y) = \omega_\mu(x,y) \tag{2.4}$$

It follows that at each point $\lambda > 0$ the right limit $\omega_{\lambda+0}(x,y) := \lim_{\varepsilon\to+0} \omega_{\lambda+\varepsilon}(x,y)$ and the left limit $\omega_{\lambda-0}(x,y) := \lim_{\varepsilon\to+0} \omega_{\lambda-\varepsilon}(x,y)$ exist in [0,∞] and the following two inequalities hold:

$$\omega_{\lambda+0}(x,y) \le \omega_\lambda(x,y) \le \omega_{\lambda-0}(x,y) \tag{2.5}$$

From [4, 5], we know that, if $x_0 \in X$, the set

$$X_\omega = \{x \in X : \lim_{\lambda\to\infty} \omega_\lambda(x,x_0) = 0\}$$

is a metric space, called a modular space, whose metric is given by

$$d_\omega^0(x,y) = \inf\{\lambda > 0 : \omega_\lambda(x,y) \le \lambda\}\, for\ all\ x,y \in X_\omega.$$

Moreover, if ω is convex, the modular set X_ω is equal to

$$X_\omega^* = \{x \in X : \exists\, \lambda = \lambda(x) > 0 \text{ such that } \omega_\lambda(x,x_0) < \infty\}$$

and metrizable by $d_\omega^*(x,y) = \inf\{\lambda > 0 : \omega_\lambda(x,y) \le 1\}\, for\ all\ x,y \in X_\omega^*$.

We know that if X is a real linear space, $\rho : X \to [0,\infty]$ and

$$\omega_\lambda(x,y) = \rho\left(\frac{x-y}{\lambda}\right) for\ all\ \lambda > 0\ and\ x,y \in X, \tag{2.6}$$

then ρ is modular (convex modular) on X in the sense of (A.1) - (A.4) if and only if ω is metric modular (convex metric modular, respectively) on X. On the other hand, if ω satisfy the following two conditions:

(i) $\omega_\lambda(\mu x, 0) = \omega_{\lambda/\mu}(x,0)\ for\ all\ \lambda,\mu > 0\ and\ x \in X,$

(ii) $\omega_\lambda(x+z, y+z) = \omega_\lambda(x,y) for\ all\ \lambda > 0\ and\ x,y,z \in X$, if we set $\rho(x) = \omega_1(x,0)$ with (2.6) holds, where $x \in X$, then

(a) $X_\rho = X_\omega$ is a linear subspace of X and the functional $\|x\|_\rho = d_\omega^0(x,0), x\epsilon\ _\rho$, is an F-norm on X_ρ;

(b) If ω is convex, $X_\rho^* \equiv X_\omega^*(0) = X_\rho$ is a linear subspace of X and the functional $\|x\|_\rho = d_\omega^*(x, 0)$, $x \epsilon X_\rho^*$, is an norm on X_ρ^*.

Similar assertions hold if replace the word modular by pseudo modular. If ω is metric modular in X, we called the set X_ω is modular metric space.

By the idea of property in metric spaces and modular spaces, we defined the following:

Definition 2.3.[14] Let X_ω be a modular metric space.

(1) The sequence $(x_n)_{n\epsilon N}$ in X_ω is said to be convergent to $x \epsilon X_\omega$ if

$$\omega_\lambda(x_n, x) \to 0 \; as \; n \to \infty \; for \; all \; \lambda > o \, .$$

(2) The sequence $(x_n)_{n\epsilon N}$ in X_ω is said to be Cauchy if

$$\omega_\lambda(x_m, x_n) \to 0 \; as \; m, n \to \infty \; for \; all \; \lambda > o \, .$$

(3) A subset C of X_ω is said to be closed if the limit of the convergent sequence of C always belong to C.

(4) A subset C of X_ω is said to be complete if any Cauchy sequence in C is a convergent sequence and its limit in C.

(5) A subset C of X_ω is said to be bounded if for all $\lambda > o$

$$\delta_\omega(C) = \sup\{\omega_\lambda(x, y); x, y \,\epsilon\, C\} < \infty.$$

We recall the following definitions in metric spaces.

Definition 2.4. Let X be a set, f, g self maps of X. A point x in X is called a coincidence point of f and g iff fx = gx. We shall call w = fx = gx, a point of coincidence of f and g.

Definition 2.5. Two maps S and T are said to be weakly compatible if they commute at coincidence points.

Al -Thagafi and Shahzad [2] gave a proper generalization of nontrivial weakly compatible maps which have a coincidence point.

Definition 2.6. [2] Two self maps f and g of a set X are occasionally weakly compatible (owc) iff there is a point x in X which is a coincidence point of f and g at which f and g commute.

We also use the following lemma from Jungck and Rhoades[10].

Lemma 2.1.[10] Let X be a set. Let f, g be owc self maps of X. If f and g have a unique point of coincidence, w := fx = gx, then w is a unique common fixed point of f and g.

Thus we define the above definitions in modular metric spaces as follows:

Definition 2.7. Let X_ω be a modular metric space. Let f, g self maps of X_ω. A point x in X_ω is called a coincidence point of f and g iff fx = gx. We shall call w = fx = gx a point of coincidence of f and g.

Definition 2.8. Let X_ω be a modular metric space. Two maps S and T of X_ω are said to be weakly compatible if they commute at coincidence points.

Definition 2.9. Let X_ω be a modular metric space. Two self maps f and g of X_ω are occasionally weakly compatible (owc) iff there is a point x in X_ω which is a coincidence point of f and g at which f and g commute.

Lemma 2.2. Let X_ω be a modular metric space and f, g owc self maps of X_ω. If f and g have a unique point of coincidence, w:= fx = gx, then w is a unique common fixed point of f and g.

The purpose of this paper is to prove some common fixed point theorems for owc mappings satisfying generalized contraction in modular metric spaces which generalizes several results from the literature. In process, our results generalize several fixed point theorems in following respects.

(i) The class of spaces is widened from the class of metric spaces to the class of modular metric spaces.

(iii) The condition on completeness/compactness of the space is completely relaxed.

(iv) The condition of weak compatibility is weakened to OWC.

(v) The condition of the required containment of the ranges of the involved mappings is not essential.

(vi) The condition of continuity of the involved mappings is also relaxed.

3. Main results

Theorem 3.1. Let X_ω be a modular metric space and I, J, R, S, T, U : $X_\omega \to X_\omega$ be self mapping of X_ω such that the pairs (SR, I) and (TU, J) are occasionally weakly compatible. Suppose there exists numbers $a, b, c\ , d\epsilon\ [0,1)$ with at least one of a, b, c, d > 0 such that the following assertion for all $x, y\ \epsilon\ X_\omega$ and λ >0 hold:

(3.1.1) $(a + b + c + 2d) < 1;$ for all $0 \le a, b, c, d < 1;$

(3.1.2) $\omega_\lambda\,(SRx, TUy) \le a\omega_\lambda(Ix\ ,Jy) + b\omega_\lambda(SRx, Ix) + c\omega_\lambda(TUy, Jy) + d[\omega_\lambda(SRx, Jy) + \omega_{2\lambda}(TUy, Ix)];$

(3.1.3) $\omega_\lambda\,(SRx, TUy) < \infty.$

Then SR, TU, I and J have a common fixed point in X_ω. Furthermore if the pairs (S,R), (S,I), (R,I),(T,J),(T,U), (U,J) are commuting pairs of mappings then I, J, R, S, T and U have a unique common fixed point in X_ω.

Proof. Since the pair (SR, I) and (TU, J) are occasionally weakly compatible then there exists ϵX_ω: $SRu = Iu$ and $Jv = TUv$. Moreover; SR(Iu) =I(SRu) and TU(Jv) =J(TUv).

Now we can assert that $SRu = TUv$, if not then by (3.1.2)

$$\omega_\lambda(SRu, TUv) \leq a\omega_\lambda(Iu, Jv) + b\omega_\lambda(SRu, Iu) + c\omega_\lambda(TUv, Jv) + d[\omega_\lambda(SRu, Jv) + \omega_{2\lambda}(TUv, Iu)]$$

$$\omega_\lambda(SRu, TUv) \leq a\omega_\lambda(Iu, Jv) + b\omega_\lambda(Iu, Iu) + c\omega_\lambda(Jv, Jv) + d[\omega_\lambda(Iu, Jv) + \omega_{2\lambda}(Jv, Iu)]$$

$$\omega_\lambda(SRu, TUv) \leq a\omega_\lambda(Iu, Jv) + d[\omega_\lambda(Iu, Jv) + \omega_{2\lambda}(Iu, Jv)]$$

By definition of metric modular and the inequality (2.4)

$$\omega_\lambda(SRu, TUv) \leq (a+d)\omega_\lambda(Iu, Jv) + d\omega_{2\lambda}(Iu, Jv)$$

$$\omega_\lambda(SRu, TUv) \leq (a+d)\omega_\lambda(Iu, Jv) + d[\omega_\lambda(Iu, Iu) + \omega_\lambda(Iu, Jv)]$$

$$\omega_\lambda(SRu, TUv) = (a+d)\omega_\lambda(Iu, Jv) + d\omega_\lambda(Iu, Jv)$$

$$\omega_\lambda(SR\quad, TU\quad) = (a+2d)\omega_\lambda(Iu, Jv) = (a+2d)\omega_\lambda(SRu, TUv)$$

or $(1-a-2d)\omega_\lambda(SRu, TUv) \leq 0$, which is a contradiction.

Hence $SRu = TUv$ and thus $SRu = Iu = TU \quad = Jv$. (3.2)

Moreover, if there is another point z such that $SRz = Iz$, and using condition (3.1.2)

$$\omega_\lambda(SRz, TUv) \leq a\omega_\lambda(Iz, Jv) + b\omega_\lambda(SR\quad, Iz) + c\omega_\lambda(TUv, Jv) + d[\omega_\lambda(SRz, Jv) + \omega_{2\lambda}(TU\quad, Iz)]$$

$$\omega_\lambda(SRz, TUv) \leq a\omega_\lambda(SRz, TUv) + b\omega_\lambda(SR\quad, SRz) + c\omega_\lambda(TUv, TUv) + d[\omega_\lambda(SRz, TUv) + \omega_{2\lambda}(SRz, TU\quad)]$$

$\omega_\lambda(SRz, TUv) \leq (a+2d)\omega_\lambda(SRz, TUv)$, which is contradiction.

Hence we get $SRz = Iz = TUv = Jv$. (3.3)

Thus from equation (3.2) and (3.3) it follows that $SRz = SRu$.This implies $z = u$.

Hence, $w = SR \quad = Iu$ for some $w \epsilon X_\omega$ is the unique point of coincidence of SR and I.

Then by lemma 2.2, w is a unique common fixed point of SR and I.

Hence $SRw = Iw = w$.

Similarly, there is another common fixed point $w' \epsilon X_\omega : w' = TUw' = Jw'$.

For the uniqueness, suppose $w' \neq w$, then by (3.1.2) we have

$$\omega_\lambda(SRw, TUw') \leq a\omega_\lambda(Iw, Jw') + b\omega_\lambda(SRw, Iz) + c\omega_\lambda(TUw', Jw') + d[\omega_\lambda(SRw, Jw') + \omega_{2\lambda}(TUw', Iw)]$$

$$\omega_\lambda(w, w') \leq a\omega_\lambda(w, w') + d[\omega_\lambda(w, w') + \omega_{2\lambda}(w', w)],$$

we get $\omega_\lambda(w, w') \leq (a+2d)\omega_\lambda(w, w')$, which is contradiction. Hence w$= w'$.

Hence w is a unique common fixed point of SR, TU, I and J.

Furthermore, if we take pairs (S, R), (S, I), (R, I), (T, J), (T, U), (U, J) are commuting pairs then

$\mathrm{Sw} = \mathrm{S(SRw)} = \mathrm{S(RS)w} = \mathrm{SR(Sw)}$

$\mathrm{Sw} = \mathrm{S(Iw)} = \mathrm{S(RS)w} = \mathrm{I(Sw)}$

$\mathrm{Rw} = \mathrm{R(SRw)} = \mathrm{RS(Rw)} = \mathrm{SR(Rw)}$

$\mathrm{Rw} = \mathrm{R(Iw)} = \mathrm{(Rw)},$

this shows that Sw and Rw is common fixed point of (SR, I) and this gives

$$\mathrm{SRw} = \mathrm{Sw} = \mathrm{Rw} = \mathrm{Iw} = \mathrm{w}\,.$$

Similarly, we have $\mathrm{TUw} = \mathrm{Tw} = \mathrm{Uw} = \mathrm{Jw} = \mathrm{w}$.

Hence, w is a unique common fixed point of S, R, I, J, T, U.

Corollary 3.1. Let X_ω be a modular metric space and I, J, S and T: $X_\omega \to X_\omega$ be self mapping of X_ω such that the pairs (S, I) and (T, J) are occasionally weakly compatible. Suppose there exists numbers $a, b, c\ , d \epsilon\ [0,1)$ with at least one of a, b, c, d > 0 such that the following assertion for all $x, y\ \epsilon\ X_\omega$ and λ >0 hold:

(3.2.1) $(a + b + c + 2d) < 1;$ for all $0 \leq a, b, c, d < 1;$

(3.2.2) $\omega_\lambda\,(Sx, Ty) \leq a\omega_\lambda(Ix\ , Jy) + b\omega_\lambda(Sx, Ix) + c\omega_\lambda(Ty, Jy) + d[\omega_\lambda(Sx, Jy) + \omega_{2\lambda}(Ty, Ix)];$

(3.2.3) $\omega_\lambda\,(Sx, Ty) < \infty$.

Then S, T, I and J have a unique common fixed point in X_ω.

Proof. If we put R= U = Ix_ω where Ix_ω is an identity mapping on X_ω, the result follows from theorem 3.1.

Corollary 3.2. Let X_ω be a modular metric space and S and T : $X_\omega \to X_\omega$ be self mapping of X_ω such that the S and T are occasionally weakly compatible. Suppose there exists numbers $a, b, c\ , d \epsilon\ [0,1)$ with at least one of a, b, c, d > 0 such that the following assertion for all $x, y\ \epsilon\ X_\omega$ and λ >0 hold:

(3.3.1) $(a + b + c + 2d) < 1;$ for all $0 \leq a, b, c, d < 1;$

(3.3.2) $\omega_\lambda\,(Tx, Ty) \leq a\omega_\lambda(Sx\ , Sy) + b\omega_\lambda(Sx, Tx) + c\omega_\lambda(Sy, Ty) + d[\omega_\lambda(Sx, Ty) + \omega_{2\lambda}(Sy, Tx)];$

(3.3.3) $\omega_\lambda\,(Sx, Ty) < \infty$.

Then S and T have a unique common fixed point in X_ω.

Proof. If we put I=J =S, and S=T in (3.2.2) the result follows from theorem 3.1.

Remark 1. The theorem 3.1 remains true if the inequality (3.1.1), (3.1.2) are replaced by the following inequality-

(i) $\omega_\lambda(SRx, TUy) \leq a\omega_\lambda(Ix, Jy) + b\omega_\lambda(SRx, Ix) + c\omega_\lambda(TUy, Jy) + d\omega_\lambda(SRx, Jy) + e\omega_{2\lambda}(TUy, Ix)$;
with $(a + b + c + d + e) < 1$; for all $0 \leq a, b, c, d, e < 1$;

(ii) $\omega_\lambda(SRx, TUy) \leq a\omega_\lambda(Ix, Jy) + b[\omega_\lambda(SRx, Ix) + \omega_\lambda(TUy, Jy)] + c[\omega_\lambda(SRx, Jy) + \omega_{2\lambda}(TUy, Ix)]$;
with $(a + 2b + 2c) < 1$; for all $0 \leq a, b, c < 1$;

(iii) We consider a function $\varphi: R^+ \to R^+$ satisfying $0 < \varphi(t) < t$ *and* $\lim_{t\to o} \varphi^n(t) = 0$, for each t > o

$$\omega_\lambda(SRx, TUy) \leq \varphi[max\{\omega_\lambda(Ix, Jy), \omega_\lambda(SRx, Jy), \omega_\lambda(TUy, Ix), \frac{1}{2}(\omega_\lambda(SRx, Ix) + \omega_\lambda(TUy, Jy))\}]$$

Remark 2. If in sequel, in conditions (i), (ii), and (iii) of remark.1, we put R=U = identity mapping, we obtain the results for four mappings S, T, I and J.

Conclusion. Some common fixed point theorem for six self mappings in modular metric space for occasionally weakly compatible mappings has been establish, which improves and extends similar known results in the existing literature of fixed point theory. The main result in theorem 3.1and corollaries 3.1 and 3.2 are new in modular metric spaces.

Conflict of Interests.

The authors declare that there is no conflict of interests.

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