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EXISTENCE OF COMMON COUPLED FIXED POINT RESULTS IN GENERALIZED E-FUZZY b-METRIC SPACES

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Abstract. In the present manuscript, we first initiate and establish some topological properties along with some contractions for generalized E-fuzzy b-metric spaces (E-Fb-MS). Also, we demonstrated the results of coupled coincidence fixed-points under the ϕ-contraction map in complete generalized E-Fb-MS. An application related to an integral equation in the generalized space is also substantiated. These outcomes of fixed point and fuzzy set theory extend the existing results in the literature.

Keywords: fixed point; E-fuzzy b-metric space; coupled fixed point; coupled coincidence point; contraction map.
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1. Introduction and Preliminaries

The Banach contraction principle (*BCP*) for the existence and uniqueness of solutions to a variety of problems in various breaches of pure and applied mathematics was introduced by Banach [1] in 1922. Many eminent authors gave the generalizations of *BCP* by altering the contraction conditions. The fuzzy set was initially given by Zadeh [24] in 1965. George and Veeramani (*GV*) [5] in 1994 modified the concept of fuzzy metric spaces (*FMS*), which was invented by Kramosil and Michalek (*KM*) [9] in 1975.

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Bhaskar and Lakshmikantham [2] in 2006 defined the concept of a coupled fixed point (*FP*) for a single map and proved various coupled fixed point theorems (*FPT*) in the metric space (*MS*). In addition, they gave some related applications in the solution of periodic boundary value problems. Sessa [17] discussed the weak commutativity conditions in *MS*. Weakly compatible and compatible maps in menger spaces were elaborated by Fang [4] in 2009. Xin-Qi-Hu [23] in 2011 established a common *FP* result for mapping under ϕ - contraction in *FMS*. Sedghi *et al.* [16] in 2012 discussed coupled *FPT* for contractions in *FMS*. Various authors ([3], [6], [7] and [15]) gave different types of contractions, consequences and common *FP* outcomes in generalized metric and fuzzy metric spaces. Khan *et al.* [8] proved common *FP* outcomes in *FMS* for compatible maps of types (*A-1*) and (*A-2*). Common *FPT* in cone *S-MS* was extended by Singh *et al.* [18]. Rathee and Singh [13] in 2025 demonstrated *FPT* in V-fuzzy b-metric spaces using the generalized *CLR*-property. Using the concept of *G-MS* introduced by Mustafa [12], Sukanya and Jose [21] in 2018 proposed *E-FMS* and proved some related *FPT*. Sukanya and Jose [22] in 2023 established coupled *FP* results in complete *E-FMS*.

Getting motivation from these authors, Kumar and Manjeet [10] in 2025 introduced a new notion of generalized E-fuzzy b-metric space (E-Fb-MS) using *E-FMS* and *b-FMS* and proved *FPT* for three pairs of self-maps. The purpose of this paper is to establish some new topology and contractions for generalized E-Fb-MS including some coupled coincidence *FPT* under ϕ-contraction in complete generalized E-Fb-MS. Also, give an application in integral equations.

Definition 1.1 ([24]). Let $\check{\mathfrak{A}}$ be the universal set. A subset $\tilde{\mathrm{A}}$ of $\check{\mathfrak{A}}$ with the membership function $\beth_{\tilde{\mathrm{A}}}(\varpi)$ that can take any value in the interval $[0,1]$ is called a fuzzy set.

Definition 1.2 ([14]). A continuous t-norm (t-conorm) is a binary operation $\widehat{\mathfrak{S}} : [0,1] \times [0,1] \to [0,1]$ that satisfies the following conditions for all $\eth_1, \eth_2, \eth_3,\ \eth_4 \in \check{\mathfrak{A}}$:

(T_1) $\hat{\mathfrak{S}}$ is continuous,

(T_2) $\hat{\mathfrak{S}}$ is commutative i.e., $\hat{\mathfrak{S}}(\eth_1, \eth_2) = \hat{\mathfrak{S}}(\eth_2, \eth_1)$,

(T_3) $\hat{\mathfrak{S}}$ is associative i.e., $\hat{\mathfrak{S}}\left(\eth_1, \hat{\mathfrak{S}}(\eth_2, \eth_3)\right) = \hat{\mathfrak{S}}\left(\hat{\mathfrak{S}}(\eth_1, \eth_2), \eth_3\right)$,

(T_4) $\hat{\mathfrak{S}}(\eth_1, 1) = \eth_1$,

(T_5) $\hat{\mathfrak{S}}$ is monotonic i.e., $\hat{\mathfrak{S}}(\eth_1,\ \eth_2) \leq \hat{\mathfrak{S}}(\eth_3,\ \eth_4)$ whenever $\eth_1 \leq \eth_2$ $and \eth_3 \leq \eth_4$.

Definition 1.3 ([5]). The 3-tuple $\left(\check{\mathfrak{A}},\acute{\mathbb{M}},\widehat{\mathfrak{S}}\right)$ is known as *FMS* if $\widehat{\mathfrak{S}}$ is a t-conorm, $\check{\mathfrak{A}}$ is an arbitrary set and $\acute{\mathbb{M}}$ is a fuzzy set in $\check{\mathfrak{A}}\times\check{\mathfrak{A}}\times[0,\infty)$ satisfying the following axioms for every $\varpi,w,\xi\in\check{\mathfrak{A}}$ and $s,t>0$:

(FM$_1$) $\acute{\mathbb{M}}(\varpi,w,t)>0$,

(FM$_2$) $\acute{\mathbb{M}}(\varpi,w,t)=1$ iff $\varpi=w$,

(FM$_3$) $\acute{\mathbb{M}}(\varpi,w,t)=\acute{\mathbb{M}}(w,\varpi,t)$,

(FM$_4$) $\widehat{\mathfrak{S}}\left(\acute{\mathbb{M}}(\varpi,w,t),\acute{\mathbb{M}}(w,\xi,s)\right)\leq\acute{\mathbb{M}}(\varpi,\xi,t+s)$,

(FM$_5$) $\acute{\mathbb{M}}(\varpi,w,\bullet):[0,\infty)\to[0,1]$ is continuous.

$\acute{\mathbb{M}}(\varpi,w,t)$ denotes the degree of nearness between ϖ and w with respect to t.

Definition 1.4 ([21]). The 3-tuple $\left(\check{\mathfrak{A}},\acute{\mathscr{E}},\widehat{\mathfrak{S}}\right)$ is known as *E-FMS* if $\widehat{\mathfrak{S}}$ is a t-conorm, $\check{\mathfrak{A}}$ is an arbitrary set and $\acute{\mathscr{E}}$ is a fuzzy set in $\check{\mathfrak{A}}\times\check{\mathfrak{A}}\times\check{\mathfrak{A}}\times(0,\infty)$ satisfying the following conditions for all $\varpi,w,\xi,\iota\in\check{\mathfrak{A}}$ and $s,t>0$:

(E$_{FM}$1) $\acute{\mathscr{E}}(\varpi,w,\xi,t)>0$,

(E$_{FM}$2) $\acute{\mathscr{E}}(\varpi,\varpi,w,t)\geq\acute{\mathscr{E}}(\varpi,w,\xi,t)$ with $w\neq\xi$,

(E$_{FM}$3) $\acute{\mathscr{E}}(\varpi,w,\xi,t)=1$ if and only if $\varpi=w=\xi$,

(E$_{FM}$4) $\acute{\mathscr{E}}(\varpi,w,\xi,t)=\acute{\mathscr{E}}(p(\varpi,w,\xi),t)$, where p is a permutation function,

(E$_{FM}$ 5) $\widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}(\varpi,\iota,\xi,t),\acute{\mathscr{E}}(\iota,w,\xi,s)\right)\leq\acute{\mathscr{E}}(\varpi,w,\xi,t+s)$,

(E$_{FM}$ 6) $\acute{\mathscr{E}}(\varpi,w,\xi,\bullet):(0,\infty)\to[0,1]$ is continuous.

Definition 1.5 ([19]). Let $\left(\check{\mathfrak{A}}_1,\acute{\mathscr{E}}_1,\widehat{\mathfrak{S}}\right)$ and $\left(\check{\mathfrak{A}}_2,\acute{\mathscr{E}}_2,\widehat{\mathfrak{S}}\right)$ be two *E-FMS*'s. A function $\widetilde{\wp}:\check{\mathfrak{A}}_1\to\check{\mathfrak{A}}_2$ is said to be continuous at a point $\mathfrak{a}\in\check{\mathfrak{A}}_1$ if for all $r>0$ there exist $\mathscr{S}>0$ such that $\acute{\mathscr{E}}_2\left(\widetilde{\wp}(\varpi),\widetilde{\wp}(\mathfrak{a}),\widetilde{\wp}(\mathfrak{a}),s\right)>1-r$, whenever $\acute{\mathscr{E}}_1(\varpi,\mathfrak{a},\mathfrak{a},t)>1-\mathscr{S}$.

Definition 1.6 ([19]). Let $\left(\check{\mathfrak{A}}_1,\acute{\mathscr{E}}_1,\widehat{\mathfrak{S}}\right)$ and $\left(\check{\mathfrak{A}}_2,\acute{\mathscr{E}}_2,\widehat{\mathfrak{S}}\right)$ be two *E-FMS*'s. A function $\widetilde{\wp}:\check{\mathfrak{A}}_1\to\check{\mathfrak{A}}_2$ is said to be uniformly continuous if for all $r>0$ there exist $\mathscr{S}>0$ such that $\acute{\mathscr{E}}_2\left(\widetilde{\wp}(\varpi),\widetilde{\wp}(w),\widetilde{\wp}(\xi),s\right)>1-r$, whenever $\acute{\mathscr{E}}_1(\varpi,w,\xi,t)>1-\mathscr{S}$.

Definition 1.7 ([19])**.** Let $\left(\check{\mathfrak{A}},\acute{\mathscr{E}},\widehat{\mathfrak{S}}\right)$ be a *E-FMS*. A mapping $\widetilde{\wp}\colon \check{\mathfrak{A}} \to \check{\mathfrak{A}}$ is said to be contraction on $\check{\mathfrak{A}}$ if there exists some κ with $0<\kappa<1$ such that:

$$\left(\frac{1}{\acute{\mathscr{E}}\left(\widetilde{\wp}\varpi,\widetilde{\wp}w,\widetilde{\wp}\xi,t\right)}-1\right)<\kappa\left(\frac{1}{\acute{\mathscr{E}}\left(\varpi,w,\xi,t\right)}-1\right).$$

Definition 1.8 ([20])**.** Let $\left(\check{\mathfrak{A}},\acute{\mathscr{E}},\widehat{\mathfrak{S}}\right)$ be a *E-FMS*. An open ball with center $\mathfrak{p}_0$ and radius $\mathfrak{z}$ is given by $\widehat{\mathscr{B}}_{\acute{\mathscr{E}}}(\mathfrak{p}_0,\mathfrak{z},t)=\left\{\varpi\in\check{\mathfrak{A}}\ :\acute{\mathscr{E}}(\mathfrak{p}_0,\varpi,\varpi,t)>1-\mathfrak{z}\right\}$.

In 2025, Kumar and Manjeet [10], introduced the notion of generalized E-Fb-MS in the following way:

Definition 1.9 ([10])**.** The 3-tuple $\left(\check{\mathfrak{A}},\acute{\mathscr{E}},\widehat{\mathfrak{S}}\right)$ is defined as generalized E-Fb-MS or E-Fb-MS if $\widehat{\mathfrak{S}}$ is a t-conorm, $\check{\mathfrak{A}}$ is an arbitrary set and $\acute{\mathscr{E}}$ is a fuzzy set in $\check{\mathfrak{A}}^3\times(0,\infty)$ satisfies the following axioms for every $\varpi,w,\xi,\imath\in\check{\mathfrak{A}}$, $s,t>0$ and real number $b\geq 1$:

($E_{FbM}1$) $\acute{\mathscr{E}}\left(\varpi,w,\xi,\frac{t}{b}\right)>0$,

($E_{FbM}2$) $\acute{\mathscr{E}}\left(\varpi,\varpi,w,\frac{t}{b}\right)\geq\acute{\mathscr{E}}\left(\varpi,w,\xi,\frac{t}{b}\right)$ with $w\neq\xi$,

($E_{FbM}3$) $\acute{\mathscr{E}}\left(\varpi,w,\xi,\frac{t}{b}\right)=1\Leftrightarrow\varpi=w=\xi$,

($E_{FbM}4$) $\acute{\mathscr{E}}\left(\varpi,w,\xi,\frac{t}{b}\right)=\acute{\mathscr{E}}\left(p\left(\varpi,w,\xi\right),\frac{t}{b}\right)$, where p is a permutation function,

($E_{FbM}5$) $\widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\varpi,\imath,\xi,\frac{t}{b}\right),\acute{\mathscr{E}}\left(\imath,w,\xi,\frac{s}{b}\right)\right)\leq\acute{\mathscr{E}}\left(\varpi,w,\xi,\frac{t+s}{b}\right)$,

($E_{FbM}6$) $\acute{\mathscr{E}}(\varpi,w,\xi,\bullet):(0,\infty)\to[0,1]$ is continuous.

Example 1.10 ([10])**.** Let $\check{\mathfrak{A}}=\mathbb{R}$, $\overline{\mathbb{G}}$ is a G-metric on $\check{\mathfrak{A}}$ and $\acute{\mathscr{E}}$ is a fuzzy set in $\check{\mathfrak{A}}^3\times(0,\infty)$ satisfying:

$$\acute{\mathscr{E}}\left(\varpi,w,\xi,\frac{t}{b}\right)=\left[\exp\left(\frac{b\overline{\mathbb{G}}(\varpi,w,\xi)}{t}\right)\right]^{-1}$$

for every $\varpi,w,\xi\in\check{\mathfrak{A}}$, $t\in(0,\infty)$, $b\geq 1$ and t-conorm $\widehat{\mathfrak{S}}(\mathfrak{d}_1,\mathfrak{d}_2)=\mathfrak{d}_1\mathfrak{d}_2$ for all $\mathfrak{d}_1,\mathfrak{d}_2\in[0,1]$. Then, $\acute{\mathscr{E}}$ is a generalized E-Fb-MS on $\check{\mathfrak{A}}$.

Definition 1.11 ([10])**.** Let $\left(\check{\mathfrak{A}},\acute{\mathscr{E}},\widehat{\mathfrak{S}}\right)$ be a generalized E-Fb-MS. A sequence $\{\mathfrak{p}_n\}$ from $\check{\mathfrak{A}}$ is said to be convergent to $\varpi\in\check{\mathfrak{A}}$ iff $\acute{\mathscr{E}}\left(\mathfrak{p}_n,\ \varpi,\varpi,\frac{t}{b}\right)\to 1$ as $n\to\infty$ for every $t>0$ and $b\geq 1$.

Definition 1.12 ([10])**.** Let $\left(\check{\mathfrak{A}},\acute{\mathscr{E}},\widehat{\mathfrak{S}}\right)$ be a generalized E-Fb-MS. A sequence $\{\mathfrak{p}_n\}$ from $\check{\mathfrak{A}}$ is said to be a Cauchy sequence if and only if for any $r\in(0,\ 1)$, there exists a natural number

n_0 s.t. $\acute{\mathscr{E}}\left(\mathfrak{p}_n, \mathfrak{p}_m, \ \mathfrak{p}_l, \frac{t}{b}\right) > \ 1-r$ for all $n, m, l \ \geq \ n_0$, $t > 0$ and $b \geq 1$.

Definition 1.13 ([10])**.** A generalized E-Fb-MS $\left(\check{\mathfrak{A}}, \acute{\mathscr{E}}, \widehat{\mathfrak{S}}\right)$ in which every Cauchy sequence is convergent is said to be complete E-Fb-MS.

Lemma 1.14 ([10])**.** If $\left(\check{\mathfrak{A}}, \acute{\mathscr{E}}, \widehat{\mathfrak{S}}\right)$ be a generalized E-Fb-MS then $\acute{\mathscr{E}}\left(\boldsymbol{\varpi}, w, \xi, \frac{t}{b}\right)$ is non-decreasing w.r.t. to t for all $\boldsymbol{\varpi}, w, \xi, \imath \in \check{\mathfrak{A}}$, $s, t > 0$ and $b \geq 1$.

Lemma 1.15 ([10])**.** Let $\left(\check{\mathfrak{A}}, \acute{\mathscr{E}}, \widehat{\mathfrak{S}}\right)$ be a generalized E-Fb-MS and if there exists a constant k such that $0 < k < 1$, $t \in (0, \infty)$ and $b \geq 1$ satisfies $\acute{\mathscr{E}}\left(\boldsymbol{\varpi}, w, \xi, \frac{kt}{b}\right) \geq \acute{\mathscr{E}}\left(\boldsymbol{\varpi}, w, \xi, \frac{t}{b}\right)$ for all $\boldsymbol{\varpi}, w, \xi \in \check{\mathfrak{A}}$ then $\boldsymbol{\varpi} = w = \xi$.

Definition 1.16 ([2])**.** Let $\check{\mathfrak{A}}$ be a nonempty set, the element $(\boldsymbol{\varpi}, w) \in \check{\mathfrak{A}} \times \check{\mathfrak{A}}$ is called a coupled *FP* of a map $\widetilde{\wp} : \check{\mathfrak{A}} \times \check{\mathfrak{A}} \to \check{\mathfrak{A}}$ if $\widetilde{\wp}(\boldsymbol{\varpi}, w) = \boldsymbol{\varpi}$ and $\widetilde{\wp}(w, \boldsymbol{\varpi}) = w$.

Definition 1.17 ([11])**.** Let $\check{\mathfrak{A}}$ be a nonempty set, the element $(\boldsymbol{\varpi}, w) \in \check{\mathfrak{A}} \times \check{\mathfrak{A}}$ is called a coupled coincidence point of the mappings $\widetilde{\wp} : \check{\mathfrak{A}} \times \check{\mathfrak{A}} \to \check{\mathfrak{A}}$ and $\check{\mathfrak{h}} : \ \check{\mathfrak{A}} \to \check{\mathfrak{A}}$ if $\widetilde{\wp}(\boldsymbol{\varpi}, w) = \check{\mathfrak{h}}\boldsymbol{\varpi}$ and $\widetilde{\wp}(w, \boldsymbol{\varpi}) = \check{\mathfrak{h}}w$.

Definition 1.18 ([4])**.** An element $\mathfrak{p}_1 \in \check{\mathfrak{A}}$ is called common *FP* of the mappings $\widetilde{\wp} : \check{\mathfrak{A}} \times \check{\mathfrak{A}} \to \check{\mathfrak{A}}$ and $\check{\mathfrak{h}} : \ \check{\mathfrak{A}} \to \check{\mathfrak{A}}$ if $\widetilde{\wp}(\boldsymbol{\varpi}, \boldsymbol{\varpi}) = \check{\mathfrak{h}}\boldsymbol{\varpi} = \boldsymbol{\varpi}$.

Definition 1.19 ([11])**.** The map $\widetilde{\wp} : \check{\mathfrak{A}} \times \check{\mathfrak{A}} \to \check{\mathfrak{A}}$ and self-map $\check{\mathfrak{h}} : \ \check{\mathfrak{A}} \to \check{\mathfrak{A}}$ are said to be commutative if $\check{\mathfrak{h}}\widetilde{\wp}(\boldsymbol{\varpi}, w) = \widetilde{\wp}\left(\check{\mathfrak{h}}\boldsymbol{\varpi}, \check{\mathfrak{h}}w\right)$.

2. Topological Definitions and Contraction theorems

In this section, we firstly introduce some topologies related to generalized E-Fb-MS: continuity, uniform continuity, open ball, closed ball, sequentially convergent, sub-sequentially convergent and some new contractions. Then, we validate a new contraction result.

Definition 2.1: Let $\left(\breve{\mathfrak{A}}_1, \acute{\mathscr{E}}_1, \widehat{\mathfrak{S}}\right)$ and $\left(\breve{\mathfrak{A}}_2, \acute{\mathscr{E}}_2, \widehat{\mathfrak{S}}\right)$ be two generalized E-Fb-MS's. A function $\widetilde{\wp}\colon \breve{\mathfrak{A}}_1 \to \breve{\mathfrak{A}}_2$ is said to be continuous at a point $\mathfrak{a} \in \breve{\mathfrak{A}}_1$ if for all $r > 0,\ t \in (0,\infty)$ and $b \geq 1,\ \exists\, \mathscr{S} > 0$ s.t. $\acute{\mathscr{E}}_2\left(\widetilde{\wp}(\boldsymbol{\varpi}), \widetilde{\wp}(\mathfrak{a}), \widetilde{\wp}(\mathfrak{a}), \frac{s}{b}\right) > 1 - r$, if $\acute{\mathscr{E}}_1\left(\boldsymbol{\varpi}, \mathfrak{a}, \mathfrak{a}, \frac{t}{b}\right) > 1 - \mathscr{S}$.

Definition 2.2: Let $\left(\breve{\mathfrak{A}}_1, \acute{\mathscr{E}}_1, \widehat{\mathfrak{S}}\right)$ and $\left(\breve{\mathfrak{A}}_2, \acute{\mathscr{E}}_2, \widehat{\mathfrak{S}}\right)$ be two generalized E-Fb-MS's. A function $\widetilde{\wp}\colon \breve{\mathfrak{A}}_1 \to \breve{\mathfrak{A}}_2$ is said to be uniformly continuous if for any $r > 0,\ t \in (0,\infty)$ and $b \geq 1$ there exist $\mathscr{S} > 0$ s.t. $\acute{\mathscr{E}}_2\left(\widetilde{\wp}(\boldsymbol{\varpi}), \widetilde{\wp}(w), \widetilde{\wp}(\xi), \frac{s}{b}\right) > 1 - r$, whenever $\acute{\mathscr{E}}_2\left(\boldsymbol{\varpi}, w, \xi, \frac{t}{b}\right) > 1 - \mathscr{S}$.

Definition 2.3: Let $\left(\breve{\mathfrak{A}}, \acute{\mathscr{E}}, \widehat{\mathfrak{S}}\right)$ be generalized E-Fb-MS. An open ball with centre $\mathfrak{p}_0,\ b \geq 1$ and radius $\mathfrak{z}$ is given by $\widehat{\mathscr{B}}_{\acute{\mathscr{E}}}\left(\mathfrak{p}_0, \mathfrak{z}, \frac{t}{b}\right) = \left\{\boldsymbol{\varpi} \in \breve{\mathfrak{A}}\ : \acute{\mathscr{E}}\left(\mathfrak{p}_0, \boldsymbol{\varpi}, \boldsymbol{\varpi}, \frac{t}{b}\right) > 1 - \mathfrak{z}\right\}$.

Definition 2.4: Let $\left(\breve{\mathfrak{A}}, \acute{\mathscr{E}}, \widehat{\mathfrak{S}}\right)$ be generalized E-Fb-MS. A closed ball with centre $\mathfrak{p}_0,\ b \geq 1$ and radius $\mathfrak{z}$ is given by $\widehat{\mathscr{B}}_{\acute{\mathscr{E}}}\left[\mathfrak{p}_0, \mathfrak{z}, \frac{t}{b}\right] = \left\{\boldsymbol{\varpi} \in \breve{\mathfrak{A}}\ : \acute{\mathscr{E}}\left(\mathfrak{p}_0, \boldsymbol{\varpi}, \boldsymbol{\varpi}, \frac{t}{b}\right) \geq 1 - \mathfrak{z}\right\}$.

Definition 2.5: Let $\left(\breve{\mathfrak{A}}, \acute{\mathscr{E}}, \widehat{\mathfrak{S}}\right)$ be a generalized E-Fb-MS. A mapping $\widetilde{\wp}\colon \breve{\mathfrak{A}} \to \breve{\mathfrak{A}}$ is said to be contraction on $\breve{\mathfrak{A}}$ if there exists some κ with $0 < \kappa < 1,\ t \in (0,\infty)$ and $b \geq 1$ s.t.:

$$\left(\frac{1}{\acute{\mathscr{E}}\left(\widetilde{\wp}\boldsymbol{\varpi}, \widetilde{\wp}w, \widetilde{\wp}\xi, \frac{t}{b}\right)} - 1\right) \leq \kappa\left(\frac{1}{\acute{\mathscr{E}}\left(\boldsymbol{\varpi}, w, \xi, \frac{t}{b}\right)} - 1\right) \tag{1}$$

Definition 2.6: Let $\left(\breve{\mathfrak{A}}, \acute{\mathscr{E}}, \widehat{\mathfrak{S}}\right)$ be generalized E-Fb-MS. A mapping $\widetilde{\wp}\colon \breve{\mathfrak{A}} \to \breve{\mathfrak{A}}$ is said to be sequentially convergent for every sequence $\{\mathfrak{p}_m\}$ in $\breve{\mathfrak{A}}$, if $\left\{\widetilde{\wp}\mathfrak{p}_m\right\}$ is convergent then $\{\mathfrak{p}_m\}$ is also convergent.

Definition 2.7: Let $\left(\breve{\mathfrak{A}}, \acute{\mathscr{E}}, \widehat{\mathfrak{S}}\right)$ be generalized E-Fb-MS. A mapping $\widetilde{\wp}\colon \breve{\mathfrak{A}} \to \breve{\mathfrak{A}}$ is said to be sub-sequentially convergent for all sequence $\{\mathfrak{p}_m\}$ in $\breve{\mathfrak{A}}$, if $\left\{\widetilde{\wp}\mathfrak{p}_m\right\}$ is convergent then $\{\mathfrak{p}_m\}$ is a convergent sub-sequence.

Definition 2.8: Let $\left(\breve{\mathfrak{A}}, \acute{\mathscr{E}}, \widehat{\mathfrak{S}}\right)$ be a compact generalized E-Fb-MS then every self-map $\widetilde{\wp}\colon \breve{\mathfrak{A}} \to \breve{\mathfrak{A}}$ is said to be sub-sequentially convergent and every continuous function $\widetilde{\wp}\colon \breve{\mathfrak{A}} \to \breve{\mathfrak{A}}$ is sequentially convergent.

Definition 2.9: Let $\left(\check{\mathfrak{A}},\acute{\mathscr{E}},\widehat{\mathfrak{S}}\right)$ be a generalized E-Fb-MS and $\acute{T},\widetilde{\wp}:\check{\mathfrak{A}}\to\check{\mathfrak{A}}$ be two self-maps. A self-map $\widetilde{\wp}:\check{\mathfrak{A}}\to\check{\mathfrak{A}}$ is said to be E-fuzzy T-contraction if there exists κ s.t. $0<\kappa<1$ and for all $\varpi,w,\xi\in\check{\mathfrak{A}}$, $t>0$ and $b\geq 1$ satisfies:

$$\acute{\mathscr{E}}\left(\acute{T}\widetilde{\wp}\varpi,\acute{T}\widetilde{\wp}w,\acute{T}\widetilde{\wp}\xi,\frac{\kappa t}{b}\right)\geq\acute{\mathscr{E}}\left(\acute{T}\varpi,\acute{T}w,\acute{T}\xi,\frac{t}{b}\right) \tag{2}$$

Lemma 2.10: A contraction mapping $\widetilde{\wp}$ on $\left(\check{\mathfrak{A}},\acute{\mathscr{E}},\widehat{\mathfrak{S}}\right)$ is both continuous and uniformly continuous.

Proof: Let $\widetilde{\wp}$ be a contraction mapping on $\left(\check{\mathfrak{A}},\acute{\mathscr{E}},\widehat{\mathfrak{S}}\right)$. Then, by equation (1) for some κ with $0<\kappa<1$, $t\in(0,\infty)$ and $b\geq 1$, one can have

$$\left(\frac{1}{\acute{\mathscr{E}}\left(\widetilde{\wp}\varpi,\widetilde{\wp}w,\widetilde{\wp}\xi,\frac{t}{b}\right)}-1\right)\leq\kappa\left(\frac{1}{\acute{\mathscr{E}}\left(\varpi,w,\xi,\frac{t}{b}\right)}-1\right),$$

$$\left(\frac{1}{\acute{\mathscr{E}}\left(\widetilde{\wp}\varpi,\widetilde{\wp}w,\widetilde{\wp}\xi,\frac{t}{b}\right)}-1\right)\leq\kappa\left(\frac{1-\acute{\mathscr{E}}\left(\varpi,w,\xi,\frac{t}{b}\right)}{\acute{\mathscr{E}}\left(\varpi,w,\xi,\frac{t}{b}\right)}\right),$$

$$\left(\frac{1}{\acute{\mathscr{E}}\left(\widetilde{\wp}\varpi,\widetilde{\wp}w,\widetilde{\wp}\xi,\frac{t}{b}\right)}-1\right)\leq\kappa\left(\frac{1-\mathscr{S}}{\mathscr{S}}\right),$$

$$\left(\frac{1}{\acute{\mathscr{E}}\left(\widetilde{\wp}\varpi,\widetilde{\wp}w,\widetilde{\wp}\xi,\frac{t}{b}\right)}-1\right)\leq\kappa r_0,$$

$$\frac{1}{\acute{\mathscr{E}}\left(\widetilde{\wp}\varpi,\widetilde{\wp}w,\widetilde{\wp}\xi,\frac{t}{b}\right)}\leq 1+\kappa r_0,$$

$$\frac{1}{1+\kappa r_0}\leq\acute{\mathscr{E}}\left(\widetilde{\wp}\varpi,\widetilde{\wp}w,\widetilde{\wp}\xi,\frac{t}{b}\right),$$

$$1-\frac{\kappa r_0}{1+\kappa r_0}\leq\acute{\mathscr{E}}\left(\widetilde{\wp}\varpi,\widetilde{\wp}w,\widetilde{\wp}\xi,\frac{t}{b}\right),$$

$$1-r\leq\acute{\mathscr{E}}\left(\widetilde{\wp}\varpi,\widetilde{\wp}w,\widetilde{\wp}\xi,\frac{t}{b}\right).$$

Hence, $\widetilde{\wp}$ is uniformly continuous.

Theorem 2.11: Let $\left(\check{\mathfrak{A}}, \acute{\mathscr{E}}, \widehat{\mathfrak{S}}\right)$ be a complete generalized E-Fb-MS and $\widetilde{\wp}$ be a contraction mapping on $\check{\mathfrak{A}}$, i.e. for some κ with $0<\kappa<1$, $t \in (0,\infty)$ and $b \geq 1$,

$$\left(\frac{1}{\acute{\mathscr{E}}\left(\widetilde{\wp}\varpi, \widetilde{\wp}w, \widetilde{\wp}\xi, \frac{t}{b}\right)} - 1\right) \leq \kappa\left(\frac{1}{\acute{\mathscr{E}}\left(\varpi, w, \xi, \frac{t}{b}\right)} - 1\right). \tag{3}$$

Then, mapping $\widetilde{\wp}$ has a *UFP*.

Proof: Let $\mathfrak{p}_0 \in \check{\mathfrak{A}}$, consider a sequence in $\check{\mathfrak{A}}$ such that

$$\mathfrak{p}_1 = \widetilde{\wp}\mathfrak{p}_0,$$

$$\mathfrak{p}_2 = \widetilde{\wp}\mathfrak{p}_1 = \widetilde{\wp}^2\mathfrak{p}_0,$$

$$\mathfrak{p}_3 = \widetilde{\wp}\mathfrak{p}_2 = \widetilde{\wp}^2\mathfrak{p}_0,$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\mathfrak{p}_m = \widetilde{\wp}\mathfrak{p}_{m-1} = \widetilde{\wp}^m\mathfrak{p}_0.$$

Now, by definition of contraction, one cane have

$$\left(\frac{1}{\acute{\mathscr{E}}\left(\mathfrak{p}_m, \mathfrak{p}_n, \mathfrak{p}_l, \frac{t}{b}\right)} - 1\right) = \left(\frac{1}{\acute{\mathscr{E}}\left(\widetilde{\wp}\mathfrak{p}_{m-1}, \widetilde{\wp}\mathfrak{p}_{n-1}, \widetilde{\wp}\mathfrak{p}_{l-1}, \frac{t}{b}\right)} - 1\right)$$

$$\leq \kappa\left(\frac{1}{\acute{\mathscr{E}}\left(\mathfrak{p}_{m-1}, \mathfrak{p}_{n-1}, \mathfrak{p}_{l-1}, \frac{t}{b}\right)} - 1\right) \leq \kappa\left(\frac{1}{\acute{\mathscr{E}}\left(\widetilde{\wp}\mathfrak{p}_{m-2}, \widetilde{\wp}\mathfrak{p}_{n-2}, \widetilde{\wp}\mathfrak{p}_{l-2}, \frac{t}{b}\right)} - 1\right),$$

$$\leq \kappa^2\left(\frac{1}{\acute{\mathscr{E}}\left(\mathfrak{p}_{m-2}, \mathfrak{p}_{n-2}, \mathfrak{p}_{l-2}, \frac{t}{b}\right)} - 1\right),$$

$$\ldots\ldots \quad \ldots \quad \ldots$$

$$\leq \kappa^m\left(\frac{1}{\acute{\mathscr{E}}\left(\mathfrak{p}_0, \mathfrak{p}_{n-m}, \mathfrak{p}_{l-m}, \frac{t}{b}\right)} - 1\right).$$

Considering limit as $m \to \infty$, one can get

$$\acute{\mathscr{E}}\left(\mathfrak{p}_m, \mathfrak{p}_n, \mathfrak{p}_l, \frac{t}{b}\right) \to 1.$$

Hence, $\{\mathfrak{p}_m\}$ is a Cauchy sequence, since $\check{\mathfrak{A}}$ is complete, there exists a $\varpi \in \check{\mathfrak{A}}$ s.t. $\{\mathfrak{p}_m\} \to \varpi$. Now, we show that $\widetilde{\wp}(\mathfrak{p}_m) \to \widetilde{\wp}(\varpi)$ as $m \to \infty$, we obtain

$$\left(\frac{1}{\acute{\mathscr{E}}\left(\widetilde{\wp}\varpi, \widetilde{\wp}\mathfrak{p}_m, \widetilde{\wp}\mathfrak{p}_n, \frac{t}{b}\right)} - 1\right) \leq \kappa\left(\frac{1}{\acute{\mathscr{E}}\left(\varpi, \mathfrak{p}_m, \mathfrak{p}_n, \frac{t}{b}\right)} - 1\right) \to 0.$$

Thus,

$$\frac{1}{\acute{\mathscr{E}}\left(\widetilde{\wp}\varpi, \widetilde{\wp}\mathfrak{p}_m, \widetilde{\wp}\mathfrak{p}_n, , \frac{t}{b}\right)} - 1 \to 0,$$

$$\acute{\mathscr{E}}\left(\widetilde{\wp}\varpi, \widetilde{\wp}\mathfrak{p}_m, \widetilde{\wp}\mathfrak{p}_n, , \frac{t}{b}\right) \to 1,$$

$$i.e., \widetilde{\wp}(\mathfrak{p}_m) \to \widetilde{\wp}(\varpi).$$

Now, we establish $\widetilde{\wp}(\varpi) = \varpi$,

$$\left(\frac{1}{\acute{\mathscr{E}}\left(\widetilde{\wp}\mathfrak{p}_m, \widetilde{\wp}\mathfrak{p}_m, \widetilde{\wp}\mathfrak{p}_n, \frac{t}{b}\right)} - 1\right) = \left(\frac{1}{\acute{\mathscr{E}}\left(\widetilde{\wp}\mathfrak{p}_{m-1}, \widetilde{\wp}\mathfrak{p}_m, \widetilde{\wp}\mathfrak{p}_n, \frac{t}{b}\right)} - 1\right)$$

$$\leq \kappa\left(\frac{1}{\acute{\mathscr{E}}\left(\mathfrak{p}_{m-1}, \mathfrak{p}_m, \mathfrak{p}_n, \frac{t}{b}\right)} - 1\right)$$

$$\leq \kappa^2\left(\frac{1}{\acute{\mathscr{E}}\left(\mathfrak{p}_{m-2}, \mathfrak{p}_{m-1}, \mathfrak{p}_{n-1}, \frac{t}{b}\right)} - 1\right)$$

$$\ldots\ldots \quad \ldots \quad \ldots$$

$$\leq \kappa^{m-1}\left(\frac{1}{\acute{\mathscr{E}}\left(\mathfrak{p}_0, \varpi, \mathfrak{p}_{n-m}, \frac{t}{b}\right)} - 1\right),$$

assuming limit as $m \to \infty$, one can have

$$\left(\frac{1}{\acute{\mathscr{E}}\left(\varpi, \widetilde{\wp}\varpi, \widetilde{\wp}\varpi, , \frac{t}{b}\right)} - 1\right) \to 0,$$

$$\acute{\mathscr{E}}\left(\varpi, \widetilde{\wp}\varpi, \widetilde{\wp}\varpi, , \frac{t}{b}\right) = 1,$$

$$\widetilde{\wp}(\varpi) = \varpi.$$

Hence, ϖ is a *FP* of mapping $\widetilde{\wp}$.

Uniqueness: Let $\mathfrak{u}_o$ be another *FP*, then $\widetilde{\wp}(\mathfrak{u}_o) = \mathfrak{u}_o$. Now, from given hypothesis, we get

$$\left(\frac{1}{\acute{\mathscr{E}}\left(\varpi, \mathfrak{u}_o, \varpi, , \frac{t}{b}\right)} - 1\right) = \left(\frac{1}{\acute{\mathscr{E}}\left(\widetilde{\wp}\varpi, \widetilde{\wp}\mathfrak{u}_o, \widetilde{\wp}\varpi, , \frac{t}{b}\right)} - 1\right)$$

$$\leq \kappa\left(\frac{1}{\acute{\mathscr{E}}\left(\varpi, \mathfrak{u}_o, \varpi, , \frac{t}{b}\right)} - 1\right),$$

Therefore, we obtain

$$\acute{\mathscr{E}}\left(\varpi, \mathfrak{u}_o, \varpi, , \frac{t}{b}\right) = 1.$$

Hence, $\mathfrak{u}_o = \varpi$.

Corollary 2.12: Let $\widetilde{\wp}$ be a self-mapping of complete generalized E-Fb-MS $\check{\mathfrak{A}}$. If $\widetilde{\wp}$ be a contraction on a closed ball $\widehat{\mathscr{B}}_{\acute{\mathscr{E}}}\left[\mathfrak{p}_0, \mathfrak{z}, \frac{t}{b}\right]$, then there exists a *UFP* of $\widetilde{\wp}$ in $\widehat{\mathscr{B}}_{\acute{\mathscr{E}}}\left[\mathfrak{p}_0, \mathfrak{z}, \frac{t}{b}\right]$.

Proof: Consider a sequence $\{\mathfrak{p}_m\}$ in such a manner that

$$\mathfrak{p}_1 = \widetilde{\wp}\mathfrak{p}_0,$$

$$\mathfrak{p}_2 = \widetilde{\wp}\mathfrak{p}_1 = \widetilde{\wp}^2\mathfrak{p}_0,$$

$$\mathfrak{p}_3 = \widetilde{\wp}\mathfrak{p}_2 = \widetilde{\wp}^2\mathfrak{p}_0,$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\mathfrak{p}_m = \widetilde{\wp}\mathfrak{p}_{m-1} = \widetilde{\wp}^m\mathfrak{p}_0.$$

Putting $m = 0$ and $l = n$ in the inequality of Theorem 2.11, we obtain

$$\left(\frac{1}{\acute{\mathscr{E}}\left(\widetilde{\wp}\mathfrak{p}_0, \widetilde{\wp}\mathfrak{p}_n, \widetilde{\wp}\mathfrak{p}_n, \frac{t}{b}\right)} - 1\right) \leq \kappa\left(\frac{1}{\acute{\mathscr{E}}\left(\mathfrak{p}_0, \mathfrak{p}_n, \mathfrak{p}_n, \frac{t}{b}\right)} - 1\right),$$

$$\left(\frac{1}{\acute{\mathscr{E}}\left(\mathfrak{p}_0, \mathfrak{p}_n, \mathfrak{p}_n, \frac{t}{b}\right)}(1-\kappa)\right) \leq (1-\kappa),$$

$$\acute{\mathscr{E}}\left(\mathfrak{p}_0, \mathfrak{p}_n, \mathfrak{p}_n, \frac{t}{b}\right) \geq 1 > 1 - \mathfrak{z}, \text{ for } 0 < \mathfrak{z} < 1.$$

Hence, $\mathfrak{p}_n \in \widehat{\mathscr{B}}_{\acute{\mathscr{E}}}\left(\mathfrak{p}_0, \mathfrak{z}, \frac{t}{b}\right)$, i.e., $\{\mathfrak{p}_n\}$ is a sequence in $\widehat{\mathscr{B}}_{\acute{\mathscr{E}}}\left(\mathfrak{p}_0, \mathfrak{z}, \frac{t}{b}\right)$.

Now, by using Theorem 2.11 we say $\mathfrak{p}_n \to \varpi$.

Since $\widehat{\mathscr{B}}_{\acute{\mathscr{E}}}\left[\mathfrak{p}_0,\mathfrak{z},\frac{t}{b}\right]$ is closed, so $\varpi\in\widehat{\mathscr{B}}_{\acute{\mathscr{E}}}\left(\mathfrak{p}_0,\mathfrak{z},\frac{t}{b}\right)$.
Hence, mapping $\widetilde{\wp}$ has a *UFP* in $\widehat{\mathscr{B}}_{\acute{\mathscr{E}}}\left[\mathfrak{p}_0,\mathfrak{z},\frac{t}{b}\right]$.

Theorem 2.13: Let $\left(\check{\mathfrak{A}},\acute{\mathscr{E}},\widehat{\mathfrak{S}}\right)$ be a complete generalized E-Fb-MS, $\acute{T}:\check{\mathfrak{A}}\to\check{\mathfrak{A}}$ be one-one, continuous and subsequentially convergent mapping. Also, for all $\varpi,w,\xi\in\check{\mathfrak{A}}$, $b\geq 1$, $\acute{\mathscr{E}}\left(\varpi,w,\xi,\frac{t}{b}\right)\to 1$ as $\frac{t}{b}\to\infty$. Then for each E-fuzzy T-contraction mapping a continuous $\widetilde{\wp}$: $\check{\mathfrak{A}}\to\check{\mathfrak{A}}$, $\widetilde{\wp}$ has a *UFP*.
Further, if $\acute{T}$ is sequentially convergent then for every $\mathfrak{p}_0\in\check{\mathfrak{A}}$ the sequence of iterates $\left\{\widetilde{\wp}^m\mathfrak{p}_0\right\}$ converges to $\mathfrak{p}_0$ *FP* in $\check{\mathfrak{A}}$.

Proof: Initially, we defined an iterative sequence $\{\mathfrak{p}_m\}$ by $\mathfrak{p}_{m+1}=\widetilde{\wp}\mathfrak{p}_m$ or $\widetilde{\wp}^m\mathfrak{p}_0=\mathfrak{p}_m$ for non-negative integer m starting from $\mathfrak{p}_0\in\check{\mathfrak{A}}$. Then for any integer $m\in\mathbb{N}$, $b\geq 1$ and $t>0$ by induction, one can get

$$\acute{\mathscr{E}}\left(\acute{T}\mathfrak{p}_m,\acute{T}\mathfrak{p}_{m+1},\acute{T}\mathfrak{p}_{m+1},\frac{\kappa t}{b}\right)\geq\acute{\mathscr{E}}\left(\acute{T}\widetilde{\wp}\mathfrak{p}_{m-1},\acute{T}\widetilde{\wp}\mathfrak{p}_m,\acute{T}\widetilde{\wp}\mathfrak{p}_m,\frac{\kappa t}{b}\right)$$

$$\geq\acute{\mathscr{E}}\left(\acute{T}\mathfrak{p}_0,\acute{T}\mathfrak{p}_1,\acute{T}\mathfrak{p}_1,\frac{t}{\kappa^{m-1}b}\right).\tag{4}$$

Thus for any $n\in\mathbb{N}$ from equation (4), we get

$$\acute{\mathscr{E}}\left(\acute{T}\mathfrak{p}_m,\acute{T}\mathfrak{p}_{m+n},\acute{T}\mathfrak{p}_{m+n},\frac{t}{b}\right)$$

$$\geq\widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\acute{T}\mathfrak{p}_m,\acute{T}\mathfrak{p}_{m+1},\acute{T}\mathfrak{p}_{m+1},\frac{t}{b}\right),\ldots,\acute{\mathscr{E}}\left(\acute{T}\mathfrak{p}_{m+n-1},\acute{T}\mathfrak{p}_{m+n},\acute{T}\mathfrak{p}_{m+n},\frac{t}{b}\right)\right),$$

$$\geq\widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\acute{T}\mathfrak{p}_0,\acute{T}\mathfrak{p}_1,\acute{T}\mathfrak{p}_1,\frac{t}{n\kappa^m b}\right),\ldots,\acute{\mathscr{E}}\left(\acute{T}\mathfrak{p}_0,\acute{T}\mathfrak{p}_1,\acute{T}\mathfrak{p}_1,\frac{t}{n\kappa^m b}\right)\right).\tag{5}$$

Since, $\acute{\mathscr{E}}\left(\varpi,w,\xi,\frac{t}{b}\right)\to 1$ as $\frac{t}{b}\to\infty$ then form equation (5), one can get

$$\lim_{m\to\infty}\acute{\mathscr{E}}\left(\acute{T}\mathfrak{p}_m,\acute{T}\mathfrak{p}_{m+n},\acute{T}\mathfrak{p}_{m+n},\frac{t}{b}\right)\geq\widehat{\mathfrak{S}}(1,1,\ldots 1)=1$$

Thus $\left\{\acute{T}\mathfrak{p}_m\right\}$ or $\left\{\acute{T}\widetilde{\wp}^m\mathfrak{p}_0\right\}$ is a Cauchy sequence and hence is convergent sequence. So there exists $w\in\check{\mathfrak{A}}$ such as

$$\lim_{m\to\infty}\acute{T}\widetilde{\wp}^m\mathfrak{p}_0=w\tag{6}$$

Since $\acute{T}$ is a subsequentially convergent $\left\{\widetilde{\wp}^m\mathfrak{p}_0\right\}$ has a convergent subsequence.

Therefore, there exist $\varpi \in \check{\mathfrak{A}}$ and $\{m_k\}_{k=1}^{\infty}$ s.t. $\lim_{k\to\infty} \widetilde{\wp}^{m_k}\mathfrak{p}_0 = \varpi$.

Thus, $\lim_{k\to\infty} \acute{T}\widetilde{\wp}^{m_k}\mathfrak{p}_0 = \acute{T}\varpi$.

Hence, $\acute{T}\varpi = w$.

Since $\widetilde{\wp}$ is continuous and $\lim_{k\to\infty} \widetilde{\wp}^{m_k}\mathfrak{p}_0 = \varpi$ then $\lim_{k\to\infty} \widetilde{\wp}^{m_k+1}\mathfrak{p}_0 = \widetilde{\wp}\varpi$ implies that $\lim_{k\to\infty} \acute{T}\widetilde{\wp}^{m_k+1}\mathfrak{p}_0 = \acute{T}\widetilde{\wp}\varpi$.

In similar manner to equation (6), $\lim_{k\to\infty} \acute{T}\widetilde{\wp}^{m_k+1}\mathfrak{p}_0 = w$ i.e., $\acute{T}\widetilde{\wp}\varpi = w$.

Since $\acute{T}$ is one-one, therefore $\widetilde{\wp}\varpi = \varpi$.

Hence, mapping $\widetilde{\wp}$ has a *FP*.

Uniqueness: Let $\mathfrak{u}_o$ be another *FP* of $\widetilde{\wp}$. Then, considering as limit $m \to \infty$ we have

$$1 \geq \acute{\mathscr{E}}\left(\acute{T}\varpi, \acute{T}\mathfrak{u}_o, \acute{T}\varpi, \frac{\kappa t}{b}\right)$$

$$= \acute{\mathscr{E}}\left(\acute{T}\widetilde{\wp}\varpi, \acute{T}\widetilde{\wp}\mathfrak{u}_o, \acute{T}\widetilde{\wp}\varpi, \frac{\kappa t}{b}\right) \geq \acute{\mathscr{E}}\left(\acute{T}\varpi, \acute{T}\mathfrak{u}_o, \acute{T}\varpi, \frac{t}{\kappa b}\right)$$

$$= \acute{\mathscr{E}}\left(\acute{T}\widetilde{\wp}\varpi, \acute{T}\widetilde{\wp}\mathfrak{u}_o, \acute{T}\widetilde{\wp}\varpi, \frac{t}{\kappa b}\right)$$

$$\geq \acute{\mathscr{E}}\left(\acute{T}\varpi, \acute{T}\mathfrak{u}_o, \acute{T}\varpi, \frac{t}{\kappa^2 b}\right) \geq \cdots \geq \acute{\mathscr{E}}\left(\acute{T}\varpi, \acute{T}\mathfrak{u}_o, \acute{T}\varpi, \frac{t}{\kappa^m b}\right) \to 1.$$

Thus, by Lemma 1.15 we get $\acute{T}\varpi = \acute{T}\mathfrak{u}_o$.

Also, $\acute{T}$ is one-one, implies that $\varpi = \mathfrak{u}_o$.

Hence, mapping $\widetilde{\wp}$ has a *UFP*.

3. Coupled Fixed Point Results

In this section, we prove coupled *FPT* for ϕ-contraction in complete generalized *E*-F*b*-MS. Let ϕ be the function $\phi : (0,\infty) \to (0,\infty)$ such that $0 < \phi(t) < t$ and $\lim_{m\to\infty} \phi^m(t) = 0$ for all $t > 0$.

In this section, we denote for every $m \in \mathbb{N}$:

$$\left[\acute{\mathscr{E}}\left(\varpi, w, \xi, \frac{t}{b}\right)\right]^m = \underbrace{\widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\varpi, w, \xi, \frac{t}{b}\right), \acute{\mathscr{E}}\left(\varpi, w, \xi, \frac{t}{b}\right), \ldots, \acute{\mathscr{E}}\left(\varpi, w, \xi, \frac{t}{b}\right)\right)}_{m-\text{times}}.$$

Theorem 3.1: Let $\left(\check{\mathfrak{A}},\acute{\mathscr{E}},\widehat{\mathfrak{S}}\right)$ be a complete generalized E-Fb-MS. Defined the maps $\widetilde{\wp}:\check{\mathfrak{A}}\times\check{\mathfrak{A}}\to\check{\mathfrak{A}}$ and $\check{\mathfrak{h}}:\ \check{\mathfrak{A}}\to\check{\mathfrak{A}}$ s.t. they satisfies:

$$\acute{\mathscr{E}}\left(\widetilde{\wp}(\varpi,w),\widetilde{\wp}(\xi,y),\widetilde{\wp}(u,\mathrm{g}),\phi\left(\frac{t}{b}\right)\right)$$

$$\geq\widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\check{\mathfrak{h}}(\varpi),\check{\mathfrak{h}}(\xi),\check{\mathfrak{h}}(u),\frac{t}{b}\right),\acute{\mathscr{E}}\left(\check{\mathfrak{h}}(w),\check{\mathfrak{h}}(y),\check{\mathfrak{h}}(\mathrm{g}),\frac{t}{b}\right)\right),$$

for every $\varpi,w,\xi,y,u,\mathrm{g}\in\check{\mathfrak{A}},\ t\in(0,\infty)$ and $b\in[1,\infty)$.

Suppose that mappings $\widetilde{\wp}$ and $\check{\mathfrak{h}}$ have the following conditions:

$(A_{3.1.1})$ $\widetilde{\wp}\left(\check{\mathfrak{A}}\times\check{\mathfrak{A}}\right)\subseteq\check{\mathfrak{h}}(\check{\mathfrak{A}})$,

$(A_{3.1.2})$ $\check{\mathfrak{h}}$ is a continuous mapping,

$(A_{3.1.3})$ $\check{\mathfrak{h}}$ commutes with $\widetilde{\wp}$.

Then, there exists a *UFP*, $\varpi\in\check{\mathfrak{A}}$ s.t. $\varpi=\check{\mathfrak{h}}\varpi=\widetilde{\wp}(\varpi,\varpi)$.

Proof: Since, $\widetilde{\wp}\left(\check{\mathfrak{A}}\times\check{\mathfrak{A}}\right)\subseteq\check{\mathfrak{h}}(\check{\mathfrak{A}})$ we can construct two sequences $\{\check{\mathfrak{h}}\mathfrak{p}_m\}$ and $\{\check{\mathfrak{h}}\mathfrak{q}_m\}$ from $\check{\mathfrak{A}}$ such that $\check{\mathfrak{h}}(\mathfrak{p}_{m+1})=\widetilde{\wp}(\mathfrak{p}_m,\mathfrak{q}_m)$, $\check{\mathfrak{h}}(\mathfrak{q}_{m+1})=\widetilde{\wp}(\mathfrak{q}_m,\mathfrak{p}_m)$, for every $m\geq 0$.

Now, for every $r\in(0,1)$ there exists $\mathfrak{s}\in(0,1)$ s.t.

$$\underbrace{\widehat{\mathfrak{S}}\left((1-\mathfrak{s}),(1-\mathfrak{s}),\ldots,(1-\mathfrak{s})\right)}_{k}>(1-r)$$

for all $k\in\mathbb{N}$.

Therefore,

$$\lim_{m\to\infty}\acute{\mathscr{E}}\left(\varpi,w,\xi,\frac{t}{b}\right)=1\text{ for all }\varpi,w,\xi\in\check{\mathfrak{A}},\ b\geq 1\text{ and }t>0. \tag{7}$$

Thus, for any $t>0,b\geq 1$ and $\mathfrak{s}>0$, there exists $t_0>0$ s.t.:

$$\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_0,\check{\mathfrak{h}}\mathfrak{p}_1,\check{\mathfrak{h}}\mathfrak{p}_2,\frac{t_0}{b}\right)>(1-\mathfrak{s}),\ \acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_0,\check{\mathfrak{h}}\mathfrak{q}_1,\check{\mathfrak{h}}\mathfrak{q}_2,\frac{t_0}{b}\right)>(1-\mathfrak{s})$$

and

$$\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_2,\check{\mathfrak{h}}\mathfrak{p}_1,\check{\mathfrak{h}}\mathfrak{p}_0,\frac{t_0}{b}\right)>(1-\mathfrak{s}),\ \acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_2,\check{\mathfrak{h}}\mathfrak{q}_1,\check{\mathfrak{h}}\mathfrak{q}_0,\frac{t_0}{b}\right)>(1-\mathfrak{s}).$$

For a ϕ-function there exists $k\in\mathbb{N}$, s.t.

$$\sum_{m=k}^{\infty}\phi^m\left(\frac{t}{b}\right)<\frac{t}{2b},$$

$$\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_1,\check{\mathfrak{h}}\mathfrak{p}_2,\check{\mathfrak{h}}\mathfrak{p}_3,\phi\left(\frac{t_0}{b}\right)\right)=\acute{\mathscr{E}}\left(\widetilde{\wp}(\mathfrak{p}_0,\mathfrak{q}_0),\widetilde{\wp}(\mathfrak{p}_1,\mathfrak{q}_1),\widetilde{\wp}(\mathfrak{p}_2,\mathfrak{q}_2),\phi\left(\frac{t_0}{b}\right)\right)$$

$$\geq \widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_0, \check{\mathfrak{h}}\mathfrak{p}_1, \check{\mathfrak{h}}\mathfrak{p}_2, \frac{t_0}{b}\right), \acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_0, \check{\mathfrak{h}}\mathfrak{q}_1, \check{\mathfrak{h}}\mathfrak{q}_2, \frac{t_0}{b}\right)\right).$$

In the similar way, one can obtain

$$\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_1, \check{\mathfrak{h}}\mathfrak{q}_2, \check{\mathfrak{h}}\mathfrak{q}_3, \phi\left(\frac{t_0}{b}\right)\right) \geq \widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_0, \check{\mathfrak{h}}\mathfrak{q}_1, \check{\mathfrak{h}}\mathfrak{q}_2, \frac{t_0}{b}\right), \acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_0, \check{\mathfrak{h}}\mathfrak{p}_1, \check{\mathfrak{h}}\mathfrak{p}_2, \frac{t_0}{b}\right)\right).$$

Now,

$$\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_2, \check{\mathfrak{h}}\mathfrak{p}_3, \check{\mathfrak{h}}\mathfrak{p}_4, \phi^2\left(\frac{t_0}{b}\right)\right) = \acute{\mathscr{E}}\left(\widetilde{\wp}(\mathfrak{p}_1, \mathfrak{q}_1), \widetilde{\wp}(\mathfrak{p}_2, \mathfrak{q}_2), \widetilde{\wp}(\mathfrak{p}_3, \mathfrak{q}_3), \phi^2\left(\frac{t_0}{b}\right)\right)$$

$$\geq \widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_1, \check{\mathfrak{h}}\mathfrak{p}_2, \check{\mathfrak{h}}\mathfrak{p}_3, \phi\left(\frac{t_0}{b}\right)\right), \acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_1, \check{\mathfrak{h}}\mathfrak{q}_2, \check{\mathfrak{h}}\mathfrak{q}_3, , \frac{t_0}{b}\right)\right),$$

$$\geq \widehat{\mathfrak{S}}\left(\left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_0, \check{\mathfrak{h}}\mathfrak{p}_1, \check{\mathfrak{h}}\mathfrak{p}_2, \frac{t_0}{b}\right)\right]^2, \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_0, \check{\mathfrak{h}}\mathfrak{q}_1, \check{\mathfrak{h}}\mathfrak{q}_2, \frac{t_0}{b}\right)\right]^2\right).$$

Similarly,

$$\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_1, \check{\mathfrak{h}}\mathfrak{q}_2, \check{\mathfrak{h}}\mathfrak{q}_3, \phi^2\left(\frac{t_0}{b}\right)\right) \geq \widehat{\mathfrak{S}}\left(\left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_0, \check{\mathfrak{h}}\mathfrak{q}_1, \check{\mathfrak{h}}\mathfrak{q}_2, \frac{t_0}{b}\right)\right]^2, \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_0, \check{\mathfrak{h}}\mathfrak{p}_1, \check{\mathfrak{h}}\mathfrak{p}_2, \frac{t_0}{b}\right)\right]^2\right),$$

continuing like this, we get

$$\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_m, \check{\mathfrak{h}}\mathfrak{p}_{m+1}, \check{\mathfrak{h}}\mathfrak{p}_{m+2}, \phi^m\left(\frac{t_0}{b}\right)\right)$$

$$\geq \widehat{\mathfrak{S}}\left(\left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_0, \check{\mathfrak{h}}\mathfrak{p}_1, \check{\mathfrak{h}}\mathfrak{p}_2, \frac{t_0}{b}\right)\right]^{2^{m-1}}, \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_0, \check{\mathfrak{h}}\mathfrak{q}_1, \check{\mathfrak{h}}\mathfrak{q}_2, \frac{t_0}{b}\right)\right]^{2^{m-1}}\right)$$

and

$$\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_m, \check{\mathfrak{h}}\mathfrak{q}_{m+1}, \check{\mathfrak{h}}\mathfrak{q}_{m+2}, \phi^m\left(\frac{t_0}{b}\right)\right)$$

$$\geq \widehat{\mathfrak{S}}\left(\left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_0, \check{\mathfrak{h}}\mathfrak{q}_1, \check{\mathfrak{h}}\mathfrak{q}_2, \frac{t_0}{b}\right)\right]^{2^{m-1}}, \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_0, \check{\mathfrak{h}}\mathfrak{p}_1, \check{\mathfrak{h}}\mathfrak{p}_2, \frac{t_0}{b}\right)\right]^{2^{m-1}}\right).$$

In the similar manner, we obtain

$$\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{m+2}, \check{\mathfrak{h}}\mathfrak{p}_{m+1}, \check{\mathfrak{h}}\mathfrak{p}_m, \phi^m\left(\frac{t_0}{b}\right)\right)$$

$$\geq \widehat{\mathfrak{S}}\left(\left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_2, \check{\mathfrak{h}}\mathfrak{p}_1, \check{\mathfrak{h}}\mathfrak{p}_1, \frac{t_0}{b}\right)\right]^{2^{m-1}}, \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_2, \check{\mathfrak{h}}\mathfrak{q}_1, \check{\mathfrak{h}}\mathfrak{q}_0, \frac{t_0}{b}\right)\right]^{2^{m-1}}\right),$$

also

$$\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_{m+2}, \check{\mathfrak{h}}\mathfrak{q}_{m+1}, \check{\mathfrak{h}}\mathfrak{q}_{m}, \phi^{m}\left(\frac{t_0}{b}\right)\right)$$

$$\geq \widehat{\mathfrak{S}}\left(\left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_{2}, \check{\mathfrak{h}}\mathfrak{q}_{1}, \check{\mathfrak{h}}\mathfrak{q}_{0}, \frac{t_0}{b}\right)\right]^{2^{m-1}}, \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{2}, \check{\mathfrak{h}}\mathfrak{p}_{1}, \check{\mathfrak{h}}\mathfrak{p}_{1}, \frac{t_0}{b}\right)\right]^{2^{m-1}}\right).$$

Now, for $n, m \in \mathbb{N}$ with $n > m > k_0$,

$$\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{n}, \check{\mathfrak{h}}\mathfrak{p}_{n}, \check{\mathfrak{h}}\mathfrak{p}_{m}, \frac{t}{b}\right) \geq \widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{m+1}, \check{\mathfrak{h}}\mathfrak{p}_{n}, \check{\mathfrak{h}}\mathfrak{p}_{m}, \frac{t}{2b}\right), \acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{n}, \check{\mathfrak{h}}\mathfrak{p}_{m+1}, \check{\mathfrak{h}}\mathfrak{p}_{m}, \frac{t}{2b}\right)\right),$$

$$\geq \widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{m+1}, \check{\mathfrak{h}}\mathfrak{p}_{n}, \check{\mathfrak{h}}\mathfrak{p}_{m}, \sum_{k=m}^{n-2}\phi^{k}\left(\frac{t_0}{b}\right)\right), \acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{n}, \check{\mathfrak{h}}\mathfrak{p}_{m+1}, \check{\mathfrak{h}}\mathfrak{p}_{m}, \sum_{k=m}^{n-2}\phi^{k}\left(\frac{t_0}{b}\right)\right)\right),$$

$$\geq \widehat{\mathfrak{S}}\begin{pmatrix} \acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{m}, \check{\mathfrak{h}}\mathfrak{p}_{m+1}, \check{\mathfrak{h}}\mathfrak{p}_{m+2}, \phi^{m}\left(\frac{t_0}{b}\right)\right), \acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{m+1}, \check{\mathfrak{h}}\mathfrak{p}_{m+2}, \check{\mathfrak{h}}\mathfrak{p}_{m+3}, \phi^{m}\left(\frac{t_0}{b}\right)\right) \\ ,\dots, \\ \acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{n-2}, \check{\mathfrak{h}}\mathfrak{p}_{n-1}, \check{\mathfrak{h}}\mathfrak{p}_{n}, \phi^{n-2}\left(\frac{t_0}{b}\right)\right) \\ \acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{m+2}, \check{\mathfrak{h}}\mathfrak{p}_{m+1}, \check{\mathfrak{h}}\mathfrak{p}_{m}, \phi^{m}\left(\frac{t_0}{b}\right)\right), \acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{m+3}, \check{\mathfrak{h}}\mathfrak{p}_{m+2}, \check{\mathfrak{h}}\mathfrak{p}_{m+1}, \phi^{m}\left(\frac{t_0}{b}\right)\right) \\ ,\dots, \\ \acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{n}, \check{\mathfrak{h}}\mathfrak{p}_{n-1}, \check{\mathfrak{h}}\mathfrak{p}_{n-2}, \phi^{n-2}\left(\frac{t_0}{b}\right)\right) \end{pmatrix},$$

$$\geq \widehat{\mathfrak{S}}\begin{pmatrix} \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{0}, \check{\mathfrak{h}}\mathfrak{p}_{1}, \check{\mathfrak{h}}\mathfrak{p}_{2}, \frac{t_0}{b}\right)\right]^{2^{m-1}}, \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_{0}, \check{\mathfrak{h}}\mathfrak{q}_{1}, \check{\mathfrak{h}}\mathfrak{q}_{2}, \frac{t_0}{b}\right)\right]^{2^{m-1}}, \\ \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{0}, \check{\mathfrak{h}}\mathfrak{p}_{1}, \check{\mathfrak{h}}\mathfrak{p}_{2}, \frac{t_0}{b}\right)\right]^{2^{m}}, \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_{0}, \check{\mathfrak{h}}\mathfrak{q}_{1}, \check{\mathfrak{h}}\mathfrak{q}_{2}, \frac{t_0}{b}\right)\right]^{2^{m}}, \dots, \\ \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{0}, \check{\mathfrak{h}}\mathfrak{p}_{1}, \check{\mathfrak{h}}\mathfrak{p}_{2}, \frac{t_0}{b}\right)\right]^{2^{n-3}}, \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_{0}, \check{\mathfrak{h}}\mathfrak{q}_{1}, \check{\mathfrak{h}}\mathfrak{q}_{2}, \frac{t_0}{b}\right)\right]^{2^{n-3}}, \\ \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{2}, \check{\mathfrak{h}}\mathfrak{p}_{1}, \check{\mathfrak{h}}\mathfrak{p}_{0}, \frac{t_0}{b}\right)\right]^{2^{m-1}}, \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_{2}, \check{\mathfrak{h}}\mathfrak{q}_{1}, \check{\mathfrak{h}}\mathfrak{q}_{0}, \frac{t_0}{b}\right)\right]^{2^{m-1}} \\ \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{2}, \check{\mathfrak{h}}\mathfrak{p}_{1}, \check{\mathfrak{h}}\mathfrak{p}_{0}, \frac{t_0}{b}\right)\right]^{2^{m}}, \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_{2}, \check{\mathfrak{h}}\mathfrak{q}_{1}, \check{\mathfrak{h}}\mathfrak{q}_{0}, \frac{t_0}{b}\right)\right]^{2^{m}}, \dots, \\ \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{2}, \check{\mathfrak{h}}\mathfrak{p}_{1}, \check{\mathfrak{h}}\mathfrak{p}_{0}, \frac{t_0}{b}\right)\right]^{2^{n-3}}, \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_{2}, \check{\mathfrak{h}}\mathfrak{q}_{1}, \check{\mathfrak{h}}\mathfrak{q}_{0}, \frac{t_0}{b}\right)\right]^{2^{n-3}} \end{pmatrix},$$

$$\geq \widehat{\mathfrak{S}}\begin{pmatrix} \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{0}, \check{\mathfrak{h}}\mathfrak{p}_{1}, \check{\mathfrak{h}}\mathfrak{p}_{2}, \frac{t_0}{b}\right)\right]^{2^{\frac{(n-m-1)}{2}(n+m-4)}}, \\ \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_{0}, \check{\mathfrak{h}}\mathfrak{q}_{1}, \check{\mathfrak{h}}\mathfrak{q}_{2}, \frac{t_0}{b}\right)\right]^{2^{\frac{(n-m-1)}{2}(n+m-4)}}, \\ \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{2}, \check{\mathfrak{h}}\mathfrak{p}_{1}, \check{\mathfrak{h}}\mathfrak{p}_{0}, \frac{t_0}{b}\right)\right]^{2^{\frac{(n-m-1)}{2}(n+m-4)}}, \\ \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_{2}, \check{\mathfrak{h}}\mathfrak{q}_{1}, \check{\mathfrak{h}}\mathfrak{q}_{0}, \frac{t_0}{b}\right)\right]^{2^{\frac{(n-m-1)}{2}(n+m-4)}} \end{pmatrix},$$

$$> \underbrace{\widehat{\mathfrak{S}}\left((1-\mathfrak{s}),(1-\mathfrak{s}),\ldots,(1-\mathfrak{s})\right)}_{2^{\frac{(n-m-1)}{2}(n+m-4)}} > (1-r).$$

Hence, $\{\check{\mathfrak{h}}\mathfrak{p}_m\}$ is a Cauchy sequence, similarly we can show that $\{\check{\mathfrak{h}}\mathfrak{q}_m\}$ is a Cauchy sequence. As $\acute{\mathscr{E}}$ is a complete in E-Fb-MS $\check{\mathfrak{A}}$, one can get $\{\check{\mathfrak{h}}\mathfrak{p}_m\}$ converges to some $\varpi \in \check{\mathfrak{A}}$ and $\{\check{\mathfrak{h}}\mathfrak{q}_m\}$ converges to some $w \in \check{\mathfrak{A}}$.

Thus,

$$\lim_{m\to\infty} \widetilde{\wp}(\mathfrak{p}_m,\mathfrak{q}_m) = \lim_{m\to\infty} \check{\mathfrak{h}}(\mathfrak{p}_m) = \varpi$$

and

$$\lim_{m\to\infty} \widetilde{\wp}(\mathfrak{q}_m,\mathfrak{p}_m) = \lim_{m\to\infty} \check{\mathfrak{h}}(\mathfrak{q}_m) = w.$$

Since $\check{\mathfrak{h}}$ is continuous, we have $\{\check{\mathfrak{h}}\check{\mathfrak{h}}\mathfrak{p}_m\}$ converges to $\check{\mathfrak{h}}\varpi$ and $\{\check{\mathfrak{h}}\check{\mathfrak{h}}\mathfrak{q}_m\}$ converges to $\check{\mathfrak{h}}w$. Also, since both $\widetilde{\wp}$ and $\check{\mathfrak{h}}$ commute we have

$$\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\check{\mathfrak{h}}\mathfrak{p}_m,\widetilde{\wp}(\varpi,w),\widetilde{\wp}(\varpi,w),\frac{t}{b}\right) = \acute{\mathscr{E}}\left(\widetilde{\wp}\left(\check{\mathfrak{h}}\mathfrak{p}_m,\check{\mathfrak{h}}\mathfrak{q}_m\right),\widetilde{\wp}(\varpi,w),\widetilde{\wp}(\varpi,w),\frac{t}{b}\right),$$

$$\geq \acute{\mathscr{E}}\left(\widetilde{\wp}\left(\check{\mathfrak{h}}\mathfrak{p}_m,\check{\mathfrak{h}}\mathfrak{q}_m\right),\widetilde{\wp}(\varpi,w),\widetilde{\wp}(\varpi,w),\phi\left(\frac{t}{b}\right)\right),$$

$$\geq \widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\check{\mathfrak{h}}\mathfrak{p}_m,\check{\mathfrak{h}}\varpi,\check{\mathfrak{h}}\varpi,\frac{t}{b}\right),\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\check{\mathfrak{h}}\mathfrak{q}_m,\check{\mathfrak{h}}w,\check{\mathfrak{h}}w,\frac{t}{b}\right)\right).$$

Since, $\acute{\mathscr{E}}$ is continuous, let us assuming as $m \to \infty$,

$$\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\varpi,\widetilde{\wp}(\varpi,w),\widetilde{\wp}(\varpi,w),\frac{t}{b}\right) \geq \widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\varpi,\check{\mathfrak{h}}\varpi,\check{\mathfrak{h}}\varpi,\frac{t}{b}\right),\acute{\mathscr{E}}\left(\check{\mathfrak{h}}w,\check{\mathfrak{h}}w,\check{\mathfrak{h}}w,\frac{t}{b}\right)\right).$$

Hence, $\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\varpi,\widetilde{\wp}(\varpi,w),\widetilde{\wp}(\varpi,w),\frac{t}{b}\right) = 1$ *i.e.*, $\widetilde{\wp}(\varpi,w) = \check{\mathfrak{h}}\varpi$ and similarly one can obtain $\widetilde{\wp}(w,\varpi) = \check{\mathfrak{h}}w$.

Thus, (ϖ,w) is a coupled coincidence point of $\widetilde{\wp}$ and $\check{\mathfrak{h}}$.

For every $r \in (0,\infty)$ there exists $\mathfrak{s} \in (0,\infty)$ s.t. for all $k \in \mathbb{N}$

$$\underbrace{\widehat{\mathfrak{S}}\left((1-\mathfrak{s}),(1-\mathfrak{s}),\ldots,(1-\mathfrak{s})\right)}_{k} > (1-r).$$

Using equation (6) for any $t > 0, b \geq 1$ and $\mathfrak{s} > 0$ there exists $t_0 > 0$ s.t.

$$\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\varpi,w,w,\frac{t_0}{b}\right) > (1-\mathfrak{s}) \text{ and } \acute{\mathscr{E}}\left(\check{\mathfrak{h}}w,\varpi,\varpi,\frac{t_0}{b}\right) > (1-\mathfrak{s}).$$

Since, $\phi^m\left(\frac{t_0}{b}\right) \to 0$ there exists $k \in \mathbb{N}$ s.t. $\phi^m\left(\frac{t_0}{b}\right) < \frac{t}{b}$ for all $m \geq k$.

$$\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\boldsymbol{\varpi}, \check{\mathfrak{h}}\mathfrak{q}_{m+1}, \check{\mathfrak{h}}\mathfrak{q}_{m+1}, \phi\left(\frac{t_0}{b}\right)\right) = \acute{\mathscr{E}}\left(\widetilde{\wp}(\boldsymbol{\varpi}, w), \widetilde{\wp}(\mathfrak{q}_m, \mathfrak{p}_m), \widetilde{\wp}(\mathfrak{q}_m, \mathfrak{p}_m), \phi\left(\frac{t_0}{b}\right)\right),$$

$$\geq \widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\boldsymbol{\varpi}, \check{\mathfrak{h}}\mathfrak{q}_m, \check{\mathfrak{h}}\mathfrak{q}_m, \frac{t_0}{b}\right), \acute{\mathscr{E}}\left(\check{\mathfrak{h}}w, \check{\mathfrak{h}}\mathfrak{p}_m, \check{\mathfrak{h}}\mathfrak{p}_m, \frac{t_0}{b}\right)\right),$$

assuming limit as $m \to \infty$,

$$\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\boldsymbol{\varpi}, w, w, \phi\left(\frac{t_0}{b}\right)\right) \geq \widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\boldsymbol{\varpi}, w, w, \frac{t_0}{b}\right), \acute{\mathscr{E}}\left(\check{\mathfrak{h}}w, \boldsymbol{\varpi}, \boldsymbol{\varpi}, \frac{t_0}{b}\right)\right).$$

Similarly,

$$\acute{\mathscr{E}}\left(\check{\mathfrak{h}}w, \boldsymbol{\varpi}, \boldsymbol{\varpi}, \phi\left(\frac{t_0}{b}\right)\right) \geq \widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\check{\mathfrak{h}}w, \boldsymbol{\varpi}, \boldsymbol{\varpi}, \frac{t_0}{b}\right), \acute{\mathscr{E}}\left(\check{\mathfrak{h}}\boldsymbol{\varpi}, w, w, \frac{t_0}{b}\right)\right).$$

$$\widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\boldsymbol{\varpi}, w, w, \phi\left(\frac{t_0}{b}\right)\right), \acute{\mathscr{E}}\left(\check{\mathfrak{h}}w, \boldsymbol{\varpi}, \boldsymbol{\varpi}, \phi\left(\frac{t_0}{b}\right)\right)\right)$$

$$\geq \widehat{\mathfrak{S}}\left(\left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\boldsymbol{\varpi}, w, w, \frac{t_0}{b}\right)\right]^2, \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}w, \boldsymbol{\varpi}, \boldsymbol{\varpi}, \frac{t_0}{b}\right)\right]^2\right).$$

Continuing this process, we get

$$\widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\boldsymbol{\varpi}, w, w, \phi^m\left(\frac{t_0}{b}\right)\right), \acute{\mathscr{E}}\left(\check{\mathfrak{h}}w, \boldsymbol{\varpi}, \boldsymbol{\varpi}, \phi^m\left(\frac{t_0}{b}\right)\right)\right)$$

$$\geq \widehat{\mathfrak{S}}\left(\left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\boldsymbol{\varpi}, w, w, \frac{t_0}{b}\right)\right]^{2^m}, \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}w, \boldsymbol{\varpi}, \boldsymbol{\varpi}, \frac{t_0}{b}\right)\right]^{2^m}\right)$$

for all $m \in \mathbb{N}$.

Hence,

$$\widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\boldsymbol{\varpi}, w, w, \frac{t}{b}\right), \acute{\mathscr{E}}\left(\check{\mathfrak{h}}w, \boldsymbol{\varpi}, \boldsymbol{\varpi}, \frac{t}{b}\right)\right)$$

$$\geq \widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\boldsymbol{\varpi}, w, w, \phi^k\left(\frac{t_0}{b}\right)\right), \acute{\mathscr{E}}\left(\check{\mathfrak{h}}w, \boldsymbol{\varpi}, \boldsymbol{\varpi}, \phi^k\left(\frac{t_0}{b}\right)\right)\right),$$

$$\geq \widehat{\mathfrak{S}}\left(\left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\boldsymbol{\varpi}, w, w, \frac{t_0}{b}\right)\right]^{2^k}, \left[\acute{\mathscr{E}}\left(\check{\mathfrak{h}}w, \boldsymbol{\varpi}, \boldsymbol{\varpi}, \frac{t_0}{b}\right)\right]^{2^k}\right),$$

$$\geq \underbrace{\widehat{\mathfrak{S}}\left((1-\mathfrak{s}), (1-\mathfrak{s}), \ldots, (1-\mathfrak{s})\right)}_{2^{2k}} > (1-r).$$

Thus, for any $r > 0$, $\widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\boldsymbol{\varpi}, w, w, \frac{t}{b}\right), \acute{\mathscr{E}}\left(\check{\mathfrak{h}}w, \boldsymbol{\varpi}, \boldsymbol{\varpi}, \frac{t}{b}\right)\right) > (1-r)$ for every $t > 0$ and $b \geq 1$.

Therefore, $\check{\mathfrak{h}}w = \boldsymbol{\varpi}$ and $\check{\mathfrak{h}}\boldsymbol{\varpi} = w$.

Now, for any $r > 0$ there exists $\mathfrak{s} \in (0, \infty)$ s.t.

$$\underbrace{\widehat{\mathfrak{S}}\left((1-\mathfrak{s}),(1-\mathfrak{s}),\ldots,(1-\mathfrak{s})\right)}_{k} > (1-r), \text{ for all } k \in \mathbb{N}.$$

Using equation (8) for any $t > 0, b \geq 1$ and $\mathfrak{s} > 0$ there exists $t_0 > 0$ s.t.

$$\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\varpi, w, w, \frac{t_0}{b}\right) > (1-\mathfrak{s}) \text{ and } \acute{\mathscr{E}}\left(\check{\mathfrak{h}}w, \varpi, \varpi, \frac{t_0}{b}\right) > (1-\mathfrak{s}).$$

Since $\phi^m\left(\frac{t_0}{b}\right) \to 0$ there exists $k \in \mathbb{N}$ such that $\phi^m\left(\frac{t_0}{b}\right) < \frac{t}{b}$ for all $m \geq k$.

$$\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_{m+1}, \check{\mathfrak{h}}\mathfrak{q}_{m+1}, \check{\mathfrak{h}}\mathfrak{q}_{m+1}, \phi\left(\frac{t_0}{b}\right)\right)$$

$$= \acute{\mathscr{E}}\left(\widetilde{\wp}(\mathfrak{p}_m, \mathfrak{q}_m), \widetilde{\wp}(\mathfrak{q}_m, \mathfrak{p}_m), \widetilde{\wp}(\mathfrak{q}_m, \mathfrak{p}_m), \phi\left(\frac{t_0}{b}\right)\right)$$

$$\geq \widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{p}_m, \check{\mathfrak{h}}\mathfrak{q}_m, \check{\mathfrak{h}}\mathfrak{q}_m, \frac{t_0}{b}\right), \acute{\mathscr{E}}\left(\check{\mathfrak{h}}\mathfrak{q}_m, \check{\mathfrak{h}}\mathfrak{p}_m, \check{\mathfrak{h}}\mathfrak{p}_m, \frac{t_0}{b}\right)\right),$$

taking limit as $m \to \infty$

$$\acute{\mathscr{E}}\left(\varpi, w, w, \phi\left(\frac{t_0}{b}\right)\right) \geq \widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\varpi, w, w, \frac{t_0}{b}\right), \acute{\mathscr{E}}\left(w, \varpi, \varpi, \frac{t_0}{b}\right)\right),$$

continuing this process, we get

$$\acute{\mathscr{E}}\left(\varpi, w, w, \phi^m\left(\frac{t_0}{b}\right)\right) \geq \widehat{\mathfrak{S}}\left(\left[\acute{\mathscr{E}}\left(\varpi, w, w, \frac{t_0}{b}\right)\right]^m, \left[\acute{\mathscr{E}}\left(w, \varpi, \varpi, \frac{t_0}{b}\right)\right]^m\right).$$

Thus,

$$\acute{\mathscr{E}}\left(\varpi, w, w, \frac{t}{b}\right) \geq \acute{\mathscr{E}}\left(\varpi, w, w, \phi^m\left(\frac{t_0}{b}\right)\right)$$

$$\geq \widehat{\mathfrak{S}}\left(\left[\acute{\mathscr{E}}\left(\varpi, w, w, \frac{t_0}{b}\right)\right]^m, \left[\acute{\mathscr{E}}\left(w, \varpi, \varpi, \frac{t_0}{b}\right)\right]^m\right),$$

$$> \underbrace{\widehat{\mathfrak{S}}\left((1-\mathfrak{s}),(1-\mathfrak{s}),\ldots,(1-\mathfrak{s})\right)}_{k} > (1-r).$$

Hence, for all $r > 0, t > 0$, $\acute{\mathscr{E}}\left(\varpi, w, w, \frac{t}{b}\right) > (1-r)$ implies $\varpi = w$.

Thus, $\check{\mathfrak{h}}(\varpi) = \widetilde{\wp}(\varpi, \varpi) = \varpi$.

Therefore, (ϖ, ϖ) is the unique *i.e.*, mappings $\widetilde{\wp}$ and $\check{\mathfrak{h}}$ have a unique common *FP* in $\check{\mathfrak{A}}$.

Corollary 3.2: Let $\left(\check{\mathfrak{A}}, \acute{\mathscr{E}}, \widehat{\mathfrak{S}}\right)$ be a complete generalized E-Fb-MS. The Defined $\widetilde{\wp}: \check{\mathfrak{A}} \times \check{\mathfrak{A}} \to \check{\mathfrak{A}}$ and $\check{\mathfrak{h}}: \check{\mathfrak{A}} \to \check{\mathfrak{A}}$ are maps s.t. that satisfy:

$$\acute{\mathscr{E}}\left(\widetilde{\wp}(\varpi, w), \widetilde{\wp}(\xi, y), \widetilde{\wp}(u, \mathrm{g}), \phi\left(\frac{t}{b}\right)\right) \geq \widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\varpi, \xi, u, \frac{t}{b}\right), \acute{\mathscr{E}}\left(w, y, \mathrm{g}, \frac{t}{b}\right)\right)$$

for every $\varpi, w, \xi, y, u, \mathrm{g} \in \check{\mathfrak{A}}$, $t \in (0, \infty)$ and $b \in [1, \infty)$.

Then there exists a unique $\varpi \in \check{\mathfrak{A}}$ s.t. $\widetilde{\wp}(\varpi, \varpi) = \varpi$.

Proof: Consider $\check{\mathfrak{h}}: \check{\mathfrak{A}} \to \check{\mathfrak{A}}$ as identity map in Theorem 3.1, we arrive at the conclusion.

Corollary 3.3: Let $\left(\check{\mathfrak{A}}, \acute{\mathscr{E}}, \widehat{\mathfrak{S}}\right)$ be a complete generalized E-Fb-MS and $\widetilde{\wp}: \check{\mathfrak{A}} \times \check{\mathfrak{A}} \to \check{\mathfrak{A}}$ and $\check{\mathfrak{h}}:$ $\check{\mathfrak{A}} \to \check{\mathfrak{A}}$ be mappings such that satisfies:

$$\acute{\mathscr{E}}\left(\widetilde{\wp}(\varpi, w), \widetilde{\wp}(\xi, y), \widetilde{\wp}(u, \mathrm{g}), \frac{kt}{b}\right) \geq \widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\varpi, \xi, u, \frac{t}{b}\right), \acute{\mathscr{E}}\left(w, y, \mathrm{g}, \frac{t}{b}\right)\right)$$

for every $\varpi, w, \xi, y, u, \mathrm{g} \in \check{\mathfrak{A}}$, $t \in (0, \infty)$ and $b \in [1, \infty)$.

Then there exists a unique $\varpi \in \check{\mathfrak{A}}$ s.t. $\widetilde{\wp}(\varpi, \varpi) = \varpi$.

Proof: Consider $\phi\left(\frac{t}{b}\right) = \frac{kt}{b}$ as in Corollary 3.2, one can arrive at the conclusion.

Example: 3.4: Let $\check{\mathfrak{A}} = [0, 1]$. Define t-norm $\widehat{\mathfrak{S}}(\mathfrak{d}_1, \mathfrak{d}_2) = \mathfrak{d}_1 \mathfrak{d}_2$ for every $\mathfrak{d}_1, \mathfrak{d}_2 \in [0, 1]$, $t \in (0, \infty)$ and $b \geq 1$, $\acute{\mathscr{E}}: \check{\mathfrak{A}}^3 \times (0, \infty) \to \check{\mathfrak{A}}$ as:

$$\acute{\mathscr{E}}\left(\varpi, w, \xi, \frac{t}{b}\right) = \left[\exp\left(\frac{\overline{\mathbb{G}}(\varpi, w, \xi)}{t/b}\right)\right]^{-1}$$

where,

$$\overline{\mathbb{G}}(\varpi, w, \xi) = |\varpi - w| + |w - \xi| + |\xi - \varpi|.$$

Consider $\phi\left(\frac{t}{b}\right) = \frac{t}{6b}$ and maps $\widetilde{\wp}: \check{\mathfrak{A}} \times \check{\mathfrak{A}} \to \check{\mathfrak{A}}$, $\check{\mathfrak{h}}: \check{\mathfrak{A}} \to \check{\mathfrak{A}}$ be defined as $\check{\mathfrak{h}}(\varpi) = \varpi$, $\widetilde{\wp}(\varpi, w) = \frac{\varpi w}{6}$, for all $\varpi, w \in \check{\mathfrak{A}}$.

Then there exists a common *UFP* of $\widetilde{\wp}$ and $\check{\mathfrak{h}}$.

Solution: By the given conditions, we have the following

$$\overline{\mathbb{G}}\left(\widetilde{\wp}(\varpi, w), \widetilde{\wp}(\xi, y), \widetilde{\wp}(u, \mathrm{g})\right) = \left|\frac{\varpi w}{6} - \frac{\xi y}{6}\right| + \left|\frac{\xi y}{6} - \frac{u\mathrm{g}}{6}\right| + \left|\frac{u\mathrm{g}}{6} - \frac{\varpi w}{6}\right|,$$

$$\leq \frac{1}{6}\left\{(|\varpi - \xi| + |\xi - u| + |u - \varpi|) + (|w - y| + |y - \mathrm{g}| + |\mathrm{g} - w|)\right\},$$

$$\leq \frac{1}{6}\left\{\overline{\mathbb{G}}\left(\check{\mathfrak{h}}\varpi,\check{\mathfrak{h}}\xi,\check{\mathfrak{h}}u\right)\ +\overline{\mathbb{G}}\left(\check{\mathfrak{h}}w,\check{\mathfrak{h}}y,\check{\mathfrak{h}}\mathrm{g}\right)\right\}$$

$$\begin{aligned}\acute{\mathscr{E}}\left(\widetilde{\wp}(\varpi,w),\widetilde{\wp}(\xi,y),\widetilde{\wp}(u,\mathrm{g}),\phi\left(\frac{t}{b}\right)\right) &= \left[\exp\left(\frac{\overline{\mathbb{G}}\left(\widetilde{\wp}(\varpi,w),\widetilde{\wp}(\xi,y),\widetilde{\wp}(u,\mathrm{g})\right)}{\phi\left(\frac{t}{b}\right)}\right)\right]^{-1},\\ &\geq \left[\exp\left(\frac{\frac{1}{6}\left\{\overline{\mathbb{G}}\left(\check{\mathfrak{h}}\varpi,\check{\mathfrak{h}}\xi,\check{\mathfrak{h}}u\right)\ +\overline{\mathbb{G}}\left(\check{\mathfrak{h}}w,\check{\mathfrak{h}}y,\check{\mathfrak{h}}\mathrm{g}\right)\right\}}{\frac{t}{6b}}\right)\right]^{-1},\\ &\geq \widehat{\mathfrak{S}}\left(\left[\exp\left(\frac{\left\{\overline{\mathbb{G}}\left(\check{\mathfrak{h}}\varpi,\check{\mathfrak{h}}\xi,\check{\mathfrak{h}}u\right)\right\}}{\frac{t}{b}}\right)\right]^{-1},\left[\exp\left(\frac{\left\{\overline{\mathbb{G}}\left(\check{\mathfrak{h}}w,\check{\mathfrak{h}}y,\check{\mathfrak{h}}\mathrm{g}\right)\right\}}{\frac{t}{b}}\right)\right]^{-1}\right),\\ &\geq \widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\check{\mathfrak{h}}\varpi,\check{\mathfrak{h}}\xi,\check{\mathfrak{h}}u,\frac{t}{b}\right),\acute{\mathscr{E}}\left(\check{\mathfrak{h}}w,\check{\mathfrak{h}}y,\check{\mathfrak{h}}\mathrm{g},\frac{t}{b}\right)\right).\end{aligned}$$

Therefore, it satisfies all conditions of the Theorem 3.1.

Hence, maps have a common *UFP*.

Theorem 3.5: Let $\left(\check{\mathfrak{A}},\acute{\mathscr{E}},\widehat{\mathfrak{S}}\right)$ be a complete generalized E-Fb-MS. Define the mappings $\widetilde{\wp}$: $\check{\mathfrak{A}}\times\check{\mathfrak{A}}\to\check{\mathfrak{A}}$ and $\check{\mathfrak{h}}$: $\check{\mathfrak{A}}\to\check{\mathfrak{A}}$ s.t. they satisfies:

$$\acute{\mathscr{E}}\left(\widetilde{\wp}(\varpi,w),\widetilde{\wp}(\xi,y),\widetilde{\wp}(u,\mathrm{g}),\phi\left(\frac{t}{b}\right)\right)$$

$$\geq \min\left\{\acute{\mathscr{E}}\left(\check{\mathfrak{h}}(\varpi),\check{\mathfrak{h}}(\xi),\check{\mathfrak{h}}(u),\frac{t}{b}\right),\acute{\mathscr{E}}\left(\check{\mathfrak{h}}(w),\check{\mathfrak{h}}(y),\check{\mathfrak{h}}(\mathrm{g}),\frac{t}{b}\right)\right\}, \tag{8}$$

for every $\varpi,w,\xi,y,u,\mathrm{g}\in\check{\mathfrak{A}},\ t\in(0,\infty)$ and $b\in[1,\infty)$. Suppose that $\widetilde{\wp}$ and $\check{\mathfrak{h}}$ also satisfy the following conditions:

$(A_{3.5.1})$ $\widetilde{\wp}\left(\check{\mathfrak{A}}\times\check{\mathfrak{A}}\right)\subseteq\check{\mathfrak{h}}(\check{\mathfrak{A}})$,

$(A_{3.5.2})$ $\check{\mathfrak{h}}$ is a continuous map,

$(A_{3.5.3})$ $\check{\mathfrak{h}}$ commutes with $\widetilde{\wp}$.

Then, there exists a *UFP* $\varpi\in\check{\mathfrak{A}}$ s.t. $\varpi=\check{\mathfrak{h}}\varpi=\widetilde{\wp}(\varpi,\varpi)$.

Proof: This can be proved in a similar way as Theorem 3.1.

4. APPLICATION

In this section, using our results proved in Section 3, we establish the existence and uniqueness of the solution of an integral equation.

Theorem 4.1: Consider $\check{\mathfrak{A}} = C([0,1])$. Defined $\acute{\mathscr{E}} : \check{\mathfrak{A}} \times \check{\mathfrak{A}} \times \check{\mathfrak{A}} \times (0,\infty) \to (0,1]$ for all $\varpi, w, \xi, y, u, \mathrm{g} \in \check{\mathfrak{A}}$, $t \in (0,\infty)$, $b \in [1,\infty)$ and t-conorm $\widehat{\mathfrak{S}}(\mathfrak{d}_1, \mathfrak{d}_2) = \mathfrak{d}_1\mathfrak{d}_2$ for all $\mathfrak{d}_1, \mathfrak{d}_2 \in [0,1]$ as:

$$\acute{\mathscr{E}}\left(\varpi, w, \xi, \frac{\tilde{t}}{b}\right) = \left[\exp\left(\frac{\overline{\mathbb{G}}(\varpi, w, \xi)}{\tilde{t}/b}\right)\right]^{-1}$$

where $\overline{\mathbb{G}} : \check{\mathfrak{A}} \times \check{\mathfrak{A}} \times \check{\mathfrak{A}} \to \mathbb{R}$ is:

$$\overline{\mathbb{G}}(\varpi, w, \xi) = \sup_{\sigma\in[0,1]} |\varpi(\sigma) - w(\sigma)| + \sup_{\sigma\in[0,1]} |w(\sigma) - \xi(\sigma)| + \sup_{\sigma\in[0,1]} |\xi(\sigma) - \varpi(\sigma)|.$$

Then, clearly $\left(\check{\mathfrak{A}}, \acute{\mathscr{E}}, \widehat{\mathfrak{S}}\right)$ be a complete generalized E-Fb-MS.
Assume that the self-mappings $\phi : (0,\infty) \to (0,\infty)$, $\check{\mathfrak{h}} : \check{\mathfrak{A}} \to \check{\mathfrak{A}}$ can be defined as $\phi\left(\frac{\tilde{t}}{b}\right) = \frac{r\tilde{t}}{b}$, for any $r \in (0,1)$ and $\check{\mathfrak{h}}(\varpi) = \varpi$.

Suppose the integral equation,

$$\widetilde{\wp}(\varpi, w)(\sigma) = \zeta_1(\sigma) + \int_0^1 \Upsilon(\sigma, \mathfrak{X}) \mathscr{Y}(\mathfrak{X}, \varpi(\mathfrak{X}), w(\mathfrak{X}))\, d\mathfrak{X}, \sigma \in [0,1], \tag{9}$$

with the following conditions:

($A_{4.1.1}$) $\mathscr{Y} : [0,1] \times (0,\infty) \times (0,\infty) \to (0,\infty)$ and $\zeta_1 : [0,1] \to (0,\infty)$ are continuous,

($A_{4.1.2}$) there exists $0 < r < 1$ such that for all $\varpi, w, \xi, y \in (0,\infty)$ and $\mathfrak{X} \in [0,1]$

$$|\mathscr{Y}(\mathfrak{X}, \varpi, w) - \mathscr{Y}(\mathfrak{X}, \xi, y)| \le r(|\varpi - \xi| + |w - y|),$$

($A_{4.1.3}$) for all $\sigma, \mathfrak{X} \in [0,1]$, $\sup_{\sigma\in[0,1]} \int_0^1 k(\sigma, \mathfrak{X})\, d\mathfrak{X} < 1$.

Then, there exists a unique solution of the integral equation.

Proof: For,

$$\overline{\mathbb{G}}\left(\widetilde{\wp}(\varpi, w)(\sigma), \widetilde{\wp}(\xi, y)(\sigma), \widetilde{\wp}(u, \mathrm{g})(\sigma)\right)$$

$$= \sup_{\sigma\in[0,1]} \left|\widetilde{\wp}(\varpi, w)(\sigma) - \widetilde{\wp}(\xi, y)(\sigma)\right|$$

$$+ \sup_{\sigma\in[0,1]} \left|\widetilde{\wp}(\xi, y)(\sigma) - \widetilde{\wp}(u, \mathrm{g})(\sigma)\right|$$

$$+ \sup_{\sigma\in[0,1]} \left|\widetilde{\wp}(u, \mathrm{g})(\sigma) - \widetilde{\wp}(\varpi, w)(\sigma)\right|,$$

$$\le \sup_{\sigma\in[0,1]} \left|\int_0^1 k(\sigma, \mathfrak{X}) \mathscr{Y}(\mathfrak{X}, \varpi(\mathfrak{X}), w(\mathfrak{X}))\, d\mathfrak{X} - \int_0^1 k(\sigma, \mathfrak{X}) \mathscr{Y}(\mathfrak{X}, \xi(\mathfrak{X}), y(\mathfrak{X}))\, d\mathfrak{X}\right|$$

$$+\sup_{\sigma\in[0,1]}\left|\int_0^1 k(\sigma,\mathfrak{X})\mathscr{Y}(\mathfrak{X},\xi(\mathfrak{X}),y(\mathfrak{X}))\,d\mathfrak{X}-\int_0^1 k(\sigma,\mathfrak{X})\mathscr{Y}(\mathfrak{X},u(\mathfrak{X}),\mathrm{g}(\mathfrak{X}))\,d\mathfrak{X}\right|$$

$$+\sup_{\sigma\in[0,1]}\left|\int_0^1 k(\sigma,\mathfrak{X})\mathscr{Y}(\mathfrak{X},u(\mathfrak{X}),\mathrm{g}(\mathfrak{X}))\,d\mathfrak{X}-\int_0^1 k(\sigma,\mathfrak{X})\mathscr{Y}(\mathfrak{X},\varpi(\mathfrak{X}),w(\mathfrak{X}))\,d\mathfrak{X}\right|,$$

$$\leq \sup_{\sigma\in[0,1]}\left|\int_0^1 k(\sigma,\mathfrak{X})\{\mathscr{Y}(\mathfrak{X},\varpi(\mathfrak{X}),w(\mathfrak{X}))-\mathscr{Y}(\mathfrak{X},\xi(\mathfrak{X}),y(\mathfrak{X}))\}\,d\mathfrak{X}\right|$$

$$+\sup_{\sigma\in[0,1]}\left|\int_0^1 k(\sigma,\mathfrak{X})\{\mathscr{Y}(\mathfrak{X},\xi(\mathfrak{X}),y(\mathfrak{X}))-\mathscr{Y}(\mathfrak{X},u(\mathfrak{X}),\mathrm{g}(\mathfrak{X}))\}\,d\mathfrak{X}\right|$$

$$+\sup_{\sigma\in[0,1]}\left|\int_0^1 k(\sigma,\mathfrak{X})\{\mathscr{Y}(\mathfrak{X},u(\mathfrak{X}),\mathrm{g}(\mathfrak{X}))-\mathscr{Y}(\mathfrak{X},\varpi(\mathfrak{X}),w(\mathfrak{X}))\}\,d\mathfrak{X}\right|,$$

$$\leq \sup_{\sigma\in[0,1]}\int_0^1 k(\sigma,\mathfrak{X})\,d\mathfrak{X}\left\{\begin{array}{c}\sup_{\sigma\in[0,1]}|\varpi(\sigma)-\xi(\sigma)|\ +\sup_{\sigma\in[0,1]}|w(\sigma)-y(\sigma)|\\ +\sup_{\sigma\in[0,1]}|\xi(\sigma)-u(\sigma)|\\ +\sup_{\sigma\in[0,1]}|y(\sigma)-\mathrm{g}(\sigma)|\\ +\sup_{\sigma\in[0,1]}|\mathrm{g}(\sigma)-w(\sigma)|\end{array}\right\},$$

$$\leq r\left\{\overline{\mathbb{G}}(\varpi,\xi,u)\ +\overline{\mathbb{G}}(w,y,\mathrm{g})\right\}.$$

Then,

$$\acute{\mathscr{E}}\left(\widetilde{\wp}(\varpi,w)(t/b),\widetilde{\wp}(\xi,y)(t/b),\widetilde{\wp}(u,\mathrm{g})(t/b),\phi(t/b)\right)$$

$$=\left[\exp\left(\frac{\overline{\mathbb{G}}\left(\widetilde{\wp}(\varpi,w)(t/b),\widetilde{\wp}(\xi,y)(t/b),\widetilde{\wp}(u,\mathrm{g})(t/b)\right)}{\phi(t/b)}\right)\right]^{-1},$$

$$\geq\left[\exp\left(\frac{r\left\{\overline{\mathbb{G}}(\varpi,\xi,u)\ +\overline{\mathbb{G}}(w,y,\mathrm{g})\right\}}{(rt/b}\right)\right]^{-1},$$

$$\geq\widehat{\mathfrak{S}}\left(\left[\exp\left(\frac{\overline{\mathbb{G}}(\varpi,\xi,u)}{t/b}\right)\right]^{-1},\left[\exp\left(\frac{\overline{\mathbb{G}}(w,y,\mathrm{g})}{t/b}\right)\right]^{-1}\right),$$

$$\geq\widehat{\mathfrak{S}}\left(\acute{\mathscr{E}}\left(\varpi,\xi,u,\frac{\tilde{t}}{b}\right),\acute{\mathscr{E}}\left(w,y,\mathrm{g},\frac{\tilde{t}}{b}\right)\right).$$

Thus, it satisfies all conditions of Theorem 3.1.

Hence, there exists a unique $\varpi\in C([0,1])$ as the solution of the given integral equation.

5. Conclusion

In this paper, we define some topological properties and contractions for generalized E-Fb-MS. Also, establish certain new contraction theorems and some coupled coincidence *FPT* under ϕ-contraction in complete generalized E-Fb-MS. For validation of our results, we include examples and an application in the integral equation. Our result presented in this paper generalizes, improves, and extends some known *FPT* exists for the various generalized metric and fuzzy metric spaces.

Abbreviations

E-Fb-MS: E-fuzzy b-metric spaces; *FPT*: fixed point theorems; *UFP*: unique fixed point; *FP*: fixed point; *s.t.*: such that.

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Conflict of Interests

The authors declare that there is no conflict of interests.

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