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VIRUS PROPAGATION OF TWO-DIMENSIONAL ADVECTION DISPERSION EQUATION IN HETEROGENEOUS MEDIUM: A COMPARATIVE STUDY

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Abstract: In this study we model the virus propagation with mathematical framework that predicts its behavior. Specifically, the advection-dispersion equation (ADE) is employed to understand the dynamics of virus spread and potential remedies. Numerical and Analytical solutions of two-dimensional advection dispersion equation are obtained to describe how the viruses spread from a point source along flow domain in heterogeneous medium. Waterborne disease, associated with contaminated water poses serious health issues. The dispersion co-efficient or flow velocity of the medium reacts inversely with the retardation term. To conceptualize temporal dependence, various combinations of linear, exponential, and asymptotic forms can be used. By introducing new space and time coordinates, the spatial-temporal coefficients of the ADE are simplified to constant values. The numerical solution of the virus transport problem is obtained by FTCS explicit method (Forward Time Central space method) and Laplace Integral transform technique is used to obtain the analytical solution and their graphical representation is made by MATLAB.

Keywords: Advection-dispersion equation; Groundwater; Heterogeneity; Virus Propagation.

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1. INTRODUCTION

Groundwater is one of the most abundant resources on Earth. It is also the largest source of freshwater. As a vital economic resource, groundwater provides more than 85% of public water supplies, primarily sourced from wells. It serves as a crucial drinking water source for billions of people worldwide. Ecologically, groundwater plays a key role in sustaining aquatic habitats, regulating river flows, and supporting plant growth, especially in arid regions. Additionally, groundwater helps maintain soil moisture, preventing desertification. It also acts as a buffer against droughts and climate change, ensuring water availability during times of crisis. Groundwater contamination is an escalating concern as societies reliance on groundwater continues to grow. As we are all aware, contamination of aquifers has become a significant issue in nearly every region of the world.

Water borne disease, associated with contaminated water poses serious health issues. To effectively understand and analyze biological and environmental systems, it is crucial to develop and study mathematical models grounded in both hypothetical and real-world data, as well as experimental observations. The emergence of pandemics such as AIDS, COVID-19, MERS-CoV-2 and others has raised global concerns, highlighting the importance of these models in predicting, managing, and ultimately mitigating the impacts of infectious diseases. By applying mathematical models, we can gain valuable insights into the spread of diseases, the effectiveness of interventions, and the potential long-term effects on populations, thus aiding public health responses and policymaking [1]. In recent decades, numerous mathematical models have been developed and analyzed to study the depletion of environment and biological resources, including pollution, industrial discharge and pollution dynamics to understand the impact of various factors. An analytical study of flow against dispersion is an early, widely cited contribution to understand how clean water flow can limit the spread of contaminants in porous media [2-6]. Exact solution for one dimensional and two-dimensional advective diffusive transport from a varying pulse type homogeneousas well as heterogeneous mediumoffering benchmark solution for groundwater and river pollution modelling [7-15]. One such nonlinear mathematical model has been created and investigated to assess how technological efforts are influenced by time delay [16-19].The mathematical model for the propagation of viruses or contaminants is based on two different theories: the Bloch flow equation, and the second is Fick's law of diffusion, which incorporates the principle of mass conservation [20-23]. The effect of carrier dependent infectious disease increases with the growth of carrier populations caused by conducive household discharge [24-27]. When a species contaminates its own surroundings internally generated toxicant can both depress total population and expand a deformed, non-reproductive subclass [17]. A time fractional model was developed for anomalous transport for multispecies reactive system and Multispecies convection-dispersion transport equation with variable parameters [28-29]. This study [30] presents a mathematical model using a variable-order fractional diffusion equation to accurately predict the spread of the SARS-CoV-2 virus and other environmental pollutants, providing valuable insights for urban and health planners to manage future outbreaks. A mathematical model that describes the localization and spreads of influenza A virus within the human respiratory tract, considering both diffusion and advection of the virus and includes cellular regeneration and immune response components to study spatial infection dynamics [31]. The impact of fluid viscosity and flow transition on nonlinear filtration through porous media [32]. A Simple analytical solution for oxygen transfer was discussed into anaerobic groundwater [33]. Time dependent injection characteristics are typical design variables for remediation and barrier systems, two-dimensional analytical solutions offer efficient tools to explore these effects in practice[34]. In the context of Middle East, recent literature along with guidance and various recommendations for respiratory syndrome-associated corona virus (MERS-CoV) and Ebola virus, drawing medical evidence [35-41].Two-dimensional model for a parent contaminant and its sequential degradation products in a homogeneous aquifer, explicitly incorporating rate limited sorption and arbitrary time dependent inlet boundary condition with prediction closely matched a Laplace Transform finite difference numerical model, confirming accuracy and stability [42]. Influenza transmission with transmission and treatment were discussed numerically and analytically [43-44]. A three-dimensional mathematical model to study how both anthropogenic and natural contaminant sources coexist and affect groundwater[45]. Various theories and applications are discussed in the literature[46-49].

2. MATHEMATICAL FORMULATION AND ANALYTICAL SOLUTION

Longitudinal and lateral direction at the origin considered as the x and y axes respectively. Let V be the concentration of the virus at any time $t$and $u(x,y), v(y,t)$ be the velocity components at position (x,y) also $D(x,t)$ and $D(y,t)$ be the dispersion coefficient along the x and y axes respectively. The partial differential equation in two-dimensional virus transport problem in heterogeneous medium is defined as:

$$\frac{\partial}{\partial x}\left(D(x,t)\frac{\partial V}{\partial x}-u(x,t)V\right)+\frac{\partial}{\partial y}\left(D(y,t)\frac{\partial V}{\partial x}-v(y,t)V\right)+\gamma(x,y,t)=R(x,y,t)\frac{\partial V}{\partial t} \tag{1}$$

Since the system is heterogeneous in nature, therefore the dispersion coefficient and advection parameter is assumed to be temporally and spatially dependent. The dispersion co-efficient or flow velocity of the medium reacts inversely with the retardation term (adsorption coefficient),

$$\begin{aligned} &D(x,t)=D_{x0}f(mt)(1+ax)^2;u(x,t)=u_{x0}f(mt)(1+ax)\\ &D(y,t)=D_{y0}f(mt)(1+ay)^2;v(y,t)=u_{y0}f(mt)(1+ay)\\ &\gamma=\gamma_0 f(mt);R=\frac{R_0}{f(mt)}\end{aligned} \tag{2}$$

Initially if the domain is porous then velocity varies with time and position. The input source is taken in two different intervals. Thus, initial and boundary conditions are expressed in the following ways:

$$V(x,y,t)=V_i+\log\left(\frac{\gamma}{u(x,t)}\right)+\log\left(\frac{\gamma}{v(y,t)}\right)\quad t=0;x,y\geq 0 \tag{3}$$

$$V\left(x,y,t\right)=V_c \qquad 0<t<t_0 \qquad \left.\begin{matrix} \\ \end{matrix}\right\} x=0,\, y=0 \tag{4}$$
$$=0 \qquad t>t_0$$

$$\frac{\partial V}{\partial x}=\frac{\partial V}{\partial y}=0 \qquad x\to\infty, y\to\infty, t\geq 0 \tag{5}$$

Using Equation(2), equation (1) can be reduced to

$$f\left(mt\right)\left\{\frac{\partial}{\partial x}\left(D_{x0}\left(1+ax\right)^2\frac{\partial V}{\partial x}-u_{x0}\left(1+ax\right)V\right)+\frac{\partial}{\partial y}\left(D_{y0}\left(1+ay\right)^2\frac{\partial V}{\partial y}-u_{y0}\left(1+ay\right)V\right)\right\}+\gamma_0 f\left(mt\right)=\frac{R_0}{f\left(mt\right)}\frac{\partial V}{\partial t}$$

$$\Rightarrow D_{x0}\left(1+ax\right)^2\frac{\partial^2 V}{\partial x^2}+\left(2D_{x0}-u_{x0}\right)\left(1+ax\right)\frac{\partial V}{\partial x}-au_{x0}V+D_{y0}\left(1+ay\right)^2\frac{\partial^2 V}{\partial y^2}+\left(2D_{y0}-u_{y0}\right)\left(1+ay\right)\frac{\partial V}{\partial y}-au_{y0}V+\gamma_0=R_0\frac{\partial V}{\partial T} \tag{6}$$

Where T is given by [47]as

$$T=\int_0^t f^2\left(mt\right)dt \tag{7}$$

Introducing the new variable [50]

$$X=\frac{1}{a(1+ax)};Y=\frac{1}{a(1+ay)} \tag{8}$$

With the help of Eqns. (7) and (8), equation (6) is transformed into

$$\Rightarrow D_{x0}a^2X^2\frac{\partial^2 V}{\partial X^2}+D_{y0}a^2Y^2\frac{\partial^2 V}{\partial Y^2}+au_{x0}X\frac{\partial V}{\partial X}+au_{y0}Y\frac{\partial V}{\partial Y}-a(u_{x0}+u_{y0})V+\gamma_0=R_0\frac{\partial V}{\partial T} \tag{9}$$

Use the transformation

$$z_1=-\log aX\,;z_2=-\log aY \tag{10}$$

to convert the equation with variable coefficient to constant coefficient.

On applying equation (10) into equation (9) along with the initial and boundary conditions eqns. (3)-(5), these are converted into following equations:

$$\Rightarrow D_{x0}a^2\frac{\partial^2 V}{\partial {z_1}^2}+D_{y0}a^2\frac{\partial^2 V}{\partial {z_2}^2}+a\left(aD_{x0}-u_{x0}\right)\frac{\partial V}{\partial z_1}+a\left(aD_{y0}-u_{y0}\right)\frac{\partial V}{\partial z_1}-a(u_{x0}+u_{y0})V+\gamma_0=R_0\frac{\partial V}{\partial T} \tag{11}$$

$$V\left(z_1, z_2, T\right) = V_{GM} - z_1 - z_2; \qquad \mathrm{T} = 0; z_1, z_2 \geq 0, \qquad V_{GM} = V_i + \log\left(\frac{\gamma_0}{u_{x0}}\right) + \log\left(\frac{\gamma_0}{u_{y0}}\right) \tag{12}$$

$$\left.\begin{array}{ll} V\left(z_1, z_2, T\right) = V_c & 0 < \mathrm{T} < \mathrm{T}_0 \\ \quad = 0 & \mathrm{T} > T_0 \end{array}\right\} z_1 = 0, z_2 = 0 \tag{13}$$

$$\frac{\partial V}{\partial z_1} = \frac{\partial V}{\partial z_2} = 0 \qquad z_1 \to \infty, z_2 \to \infty, T \geq 0 \tag{14}$$

Using $z = z_1 + z_2$ to transform the two-dimensional problem into one-dimensional equation. Thus eqns. (11)-(14), become

$$D\frac{\partial^2 V}{\partial z^2} - W\frac{\partial V}{\partial z} - UV + \gamma_0 = R_0\frac{\partial V}{\partial T} \tag{15}$$

Subject to

$$V\left(z, T\right) = V_{GM} - z; \qquad \mathrm{T} = 0; \quad z \geq 0, \tag{16}$$

$$\left.\begin{array}{ll} V\left(z, T\right) = V_c & 0 < \mathrm{T} < \mathrm{T}_0 \\ \quad = 0 & \mathrm{T} > T_0 \end{array}\right\} z = 0 \tag{17}$$

$$\frac{\partial V}{\partial z} = 0 \qquad z \to \infty, \quad T \geq 0 \tag{18}$$

Where $D = a^2\left(D_{x0} + D_{x0}\right), U = a\left(u_{x0} + u_{y0}\right), W = D - U$.

Consider the transformation

$$V(z,T) = H(z,T)\exp\left(\alpha z + \beta T\right) + \frac{\gamma_0}{U}; \;\; \alpha = \frac{W}{2D}; \beta = -\frac{1}{R_0}\left(\frac{W^2}{4D} + U\right) \tag{19}$$

The given set of equations are converted into following form:

$$\frac{\partial^2 H}{\partial z^2} = \eta\frac{\partial H}{\partial T} \qquad ;\eta = \frac{R_0}{D} \tag{20}$$

$$H\left(z, T\right) = \left(V_{GM} - \frac{\gamma_0}{U} - z\right)\exp(-\alpha z); \qquad \mathrm{T} = 0; \quad z \geq 0, \tag{21}$$

$$\left.\begin{aligned} H(z,T) &= \left(V_c - \frac{\gamma_0}{U}\right)\exp(-\beta T); && 0 < \mathrm{T} < \mathrm{T}_0 \\ &= -\frac{\gamma_0}{U}\exp(-\beta T); && \mathrm{T} > T_0 \end{aligned}\right\}, z = 0 \tag{22}$$

$$\frac{\partial H}{\partial z} + \alpha H = 0 \qquad z \to \infty,\ \ T \ge 0 \tag{23}$$

Apply Laplace transform on equation(20), it is converted as follows:

$$\frac{\partial^2 \overline{H}}{\partial z^2} - \eta p \overline{H} = -\eta\left(V_{GM} - \frac{\gamma_0}{U} - z\right)\exp(-\alpha z)$$

On solving the above equations, we get

$$\overline{H}(z,p) = Ae^{\sqrt{\eta p}z} + Be^{-\sqrt{\eta p}z} + \left[\frac{\eta\left(V_{GM} - \frac{\gamma_0}{U}\right)}{\eta p - \alpha^2} + \frac{2\alpha\eta}{\left(\alpha^2 - \eta p\right)^2}\right]\exp(-\alpha z) + \frac{\eta z \exp(-\alpha z)}{\alpha^2 - \eta p} \tag{24}$$

On using initial and boundary conditions given in eqns.(21)-(23) the values of arbitrary constants A and B can be obtained as follows

$$A = 0 \text{ and } \qquad B = \begin{cases} \left(V_c - \frac{\gamma_0}{U}\right)\left(\frac{1 - e^{-(p+\beta)\mathrm{T}_0}}{p+\beta}\right) - \frac{\eta\left(V_{GM} - \frac{\gamma_0}{U}\right)}{\eta p - \alpha^2} - \frac{2\alpha\eta}{\left(\eta p - \alpha^2\right)^2} & ; 0 < T < T_0 \\ -\frac{\gamma_0}{U}\frac{e^{-(p+\beta)\mathrm{T}_0}}{p+\beta} - \frac{\eta\left(V_{GM} - \frac{\gamma_0}{U}\right)}{\eta p - \alpha^2} - \frac{2\alpha\eta}{\left(\eta p - \alpha^2\right)^2} & ; T > T_0 \end{cases} \tag{25}$$

Substituting the values of A and B in equation (24) and then applying inverse transformation given in equation (19) and then inverse Laplace transform consequently, the analytical solution can be described as

$$V(z,T) = \begin{cases} \left(V_c - \frac{\gamma_0}{U}\right)V_1(z,T) - \left(V_c - \frac{\gamma_0}{U}\right)V_2(z,T) - \left(V_{GM} - \frac{\gamma_0}{U}\right)V_3(z,T) - \frac{2\alpha}{\eta}V_4(z,T) \\ + \left(V_{GM} - \frac{\gamma_0}{U}\right)\exp(-\alpha z)V_5(z,T) + \frac{2\alpha}{\eta}\exp(-\alpha z)V_6(z,T) - z\exp(-\alpha z)V_5(z,T) + \frac{\gamma_0}{U} & ; 0 < T < T_0 \\ -\frac{\gamma_0}{U}V_2(z,T) - \left(V_{GM} - \frac{\gamma_0}{U}\right)V_3(z,T) - \frac{2\alpha}{\eta}V_4(z,T) \\ + \left(V_{GM} - \frac{\gamma_0}{U}\right)\exp(-\alpha z)V_5(z,T) + \frac{2\alpha}{\eta}\exp(-\alpha z)V_6(z,T) - z\exp(-\alpha z)V_5(z,T) + \frac{\gamma_0}{U} & ; \quad T > T_0 \end{cases} \tag{26}$$

On Combining the two input sources and expressing $V(z,T)$ in the form of unit step function as

$$\begin{aligned} V(z,T) &= \left(V_c - \frac{\gamma_0}{U}\right)V_1(z,T) - \left(V_c - \frac{\gamma_0}{U}\right)V_2(z,T) + \left[\left(V_c - \frac{2\gamma_0}{U}\right)V_2(z,T) - \left(V_c - \frac{\gamma_0}{U}\right)V_1(z,T)\right]U(T-T_0,0) \\ &\quad -\left(V_{GM} - \frac{\gamma_0}{U}\right)V_3(z,T) - \frac{2\alpha}{\eta}V_4(z,T) + \left(V_{GM} - \frac{\gamma_0}{U}\right)\exp(-\alpha z)V_5(z,T) \\ &\quad + \frac{2\alpha}{\eta}\exp(-\alpha z)V_6(z,T) - z\exp(-\alpha z)V_5(z,T) + \frac{\gamma_0}{U} \end{aligned} \tag{27}$$

Where

$$V_1(z,T) = \frac{1}{2}\left[\begin{aligned} &\exp\left(\alpha z - \sqrt{-\beta\eta}\,z\right) erfc\left(\frac{\sqrt{\eta}\,z}{2\sqrt{T}} - \sqrt{-\beta T}\right) \\ &+ \exp\left(\alpha z + \sqrt{-\beta\eta}\,z\right) erfc\left(\frac{\sqrt{\eta}\,z}{2\sqrt{T}} + \sqrt{-\beta T}\right) \end{aligned}\right] \tag{28}$$

$$V_2(z,T) = \frac{1}{2}\left[\begin{aligned} &\exp\left(\alpha z - \sqrt{-\beta\eta}z\right) erfc\left(\frac{\sqrt{\eta}z}{2\sqrt{T-T_0}} - \sqrt{-\beta(T-T_0)}\right) \\ &+ \exp\left(\alpha z + \sqrt{-\beta\eta}z\right) erfc\left(\frac{\sqrt{\eta}z}{2\sqrt{(T-T_0)}} + \sqrt{-\beta(T-T_0)}\right) \end{aligned}\right] \tag{29}$$

$$V_3(z,T) = \frac{1}{2}\left[\begin{aligned} &\exp\left(\frac{\alpha^2}{\eta}T + \beta T\right) erfc\left(\frac{\sqrt{\eta}z}{2\sqrt{T}} - \alpha\sqrt{\frac{T}{\eta}}\right) \\ &+ \exp\left(\frac{\alpha^2}{\eta}T + 2\alpha z + \beta T\right) erfc\left(\frac{\sqrt{\eta}z}{2\sqrt{T}} + \alpha\sqrt{\frac{T}{\eta}}\right) \end{aligned}\right] \tag{30}$$

$$V_4(z,T) = \frac{1}{4\alpha}\left[\begin{aligned} &(2\alpha T - \eta z)\exp\left(\frac{\alpha^2}{\eta}T + \beta T\right) erfc\left(\frac{\sqrt{\eta}z}{2\sqrt{T}} - \alpha\sqrt{\frac{T}{\eta}}\right) \\ &+ (2\alpha T + \eta z)\exp\left(\frac{\alpha^2}{\eta}T + 2\alpha z + \beta T\right) erfc\left(\frac{\sqrt{\eta}z}{2\sqrt{T}} + \alpha\sqrt{\frac{T}{\eta}}\right) \end{aligned}\right] \tag{31}$$

$$V_5(z,T) = \exp\left(\frac{\alpha^2}{\eta}T + \alpha z + \beta T\right) \tag{32}$$

$$V_6(z,T) = T\exp\left(\frac{\alpha^2}{\eta}T + \alpha z + \beta T\right) \tag{33}$$

3. NUMERICAL SOLUTION

To solve the physical problems numerical methods such as: Finite difference method and Finite element method are widely used by researchers. The numerical solution of the virus transport problem is obtained by FTCS explicit method (Forward Time Central space method). It is first order accurate in time and second order accurate in space. In the present problem the region is semi-infinite in both the dimensions. To find the numerical solution, convert the given domain into finite domain using the transformation

$$\begin{aligned} X' &= 1-\exp(-x) \Rightarrow x = -\log(1-X') \\ Y' &= 1-\exp(-y) \Rightarrow y = -\log(1-Y') \end{aligned} \tag{34}$$

It is assumed that

$$\begin{aligned} X_i' &= X_{i-1}' + \Delta X'; \quad X'_0 = 0, \Delta X' = 0.1 \\ Y_j' &= Y_{j-1}' + \Delta Y'; \quad \mathrm{Y}'_0 = 0, \Delta Y' = 0.1, i = j = 1,2,3,\ldots, M \end{aligned} \tag{35}$$

The time domain is represented as

$$T_k = T_{k\text{-}1} + \Delta T; \quad \mathrm{T}_0 = 0,\ \Delta T = 0.0001,\ \ \mathrm{k} = 1,2,3,\ldots,\ \mathrm{N} \tag{36}$$

Using the above substitution, equation(6) along with the initial and boundary conditions eqns. (3)-(5) is represented as

$$\begin{aligned} \frac{R_0\left(V_{i,j,k+1}-V_{i,j,k}\right)}{\Delta T} &= D_{x0}\left(1-a\log\left(1-X_i'\right)\right)^2\left[\left(1-X_i'\right)^2\frac{\left(V_{i+1,j,k}-2V_{i,j,k}+V_{i-1,j,k}\right)}{\left(\Delta X'\right)^2}-\left(1-X_i'\right)\frac{\left(V_{i+1,j,k}-V_{i-1,j,k}\right)}{2\Delta X'}\right] \\ &+D_{y0}\left(1-a\log\left(1-Y_j'\right)\right)^2\left[\left(1-Y_j'\right)^2\frac{\left(V_{i,j+1,k}-2V_{i,j,k}+V_{i,j-1,k}\right)}{\left(\Delta Y'\right)^2}-\left(1-Y_j'\right)\frac{\left(V_{i,j+1,k}-V_{i,j-1,k}\right)}{2\Delta Y'}\right] \\ &+\left(2D_{x0}-u_{x0}\right)\left(1-a\log\left(1-X_i'\right)\right)\left(1-X_i'\right)\frac{\left(V_{i+1,j,k}-V_{i-1,j,k}\right)}{2\Delta X'} \\ &+\left(2D_{y0}-u_{y0}\right)\left(1-a\log\left(1-Y_j'\right)\right)\left(1-Y_j'\right)\frac{\left(V_{i,j+1,k}-V_{i,j-1,k}\right)}{2\Delta Y'}-a\left(u_{x0}+u_{y0}\right)V_{i,j,k}+\gamma_0 \end{aligned} \tag{37}$$

$$V_{i,j,0} = V_{GM} - \log\left(1-a\log\left(1-X_i'\right)\right) - \log\left(1-a\log\left(1-Y_j'\right)\right)\ ; \quad i,j \geq 0 \tag{38}$$

$$\begin{aligned} V_{0,0,k} &= V_c \qquad ; \quad 0 < k < k_0 \\ &= 0 \qquad ; \quad k_0 < k < \mathrm{N} \end{aligned} \tag{39}$$

$$V_{M,j,k} = V_{M-1,j,k} \quad and \quad V_{i,N,k} = V_{i,N-1,k}\ ; \ \ i,j > 0 \tag{40}$$

The stability criteria for the method to be conditionally stable is

$$\frac{\Delta T}{\left(\Delta X'\right)^2} \le \frac{1}{2} \quad and \quad \frac{\Delta T}{\left(\Delta Y'\right)^2} \le \frac{1}{2} \tag{41}$$

4. RESULT AND DISCUSSION

Natural disasters such as earthquakes, forest fires, floods, and tsunamis, infectious pandemic diseases are emerging as a major threat to humans and other living species. The rapid worldwide transmission of nCoV has highlighted concerns regarding global preparedness for such health emergencies. Therefore, proactive measures and effective preparedness strategies are essential to protect humanity from contagious diseases. COVID-19, caused by SARS-CoV-2, spread to humans through zoonotic transmission. Furthermore, inadequate knowledge of proper sanitation practices and low levels of literacy among people significantly contributed to the spread of nCoVinfections[1].Analytical and numerical solutions of eqs. (27) and (37) are computed for the set of different parameters and their comparison has been done in the below figures. The input values for the purpose of numerical computation are considered from the Literature [11, 12, 45] $V_i = 0.01, V_c = 1, \gamma_0 = 0.02, a = 1.0,$ $D_{x0} = D_{y0} = 0.07$, $u_{x0} = 0.03, u_{y0} = 0.003$, $m = 0.1, R_0 = 0.25$, $t = 1.0$ (year). The different curves are drawn for $f(mt) = \exp(-mt)$ and solutions are obtained for uniform input concentration with pulse type at t_0=1.2. The uniform input concentration is considered between $0 < t < t_0$ and after t_0 the source is eliminated. Initially the region has some concentration varies with time and distance. Figure 1 shows the pattern of concentration for different values of $t < t_0$ as 0.4, 0.7 and 1.0 when the uniform input concentration exists. Figure 2 represents the concentration values for $t > t_0$ as 1.4, 1.7 and 2.2 after the source is removed. Figure 3 and Figure 4 represent the profile for the analytical and numerical solution obtained for different values of t respectively. From figure 5 that depicts the concentration behavior for various D, it is observed that the concentration decreases rapidly on increasing the dispersion co-efficient. Figure 6 explains the concentration pattern for virus, it predicts that if the seepage velocity is high, concentration decreases rapidly and vice versa.

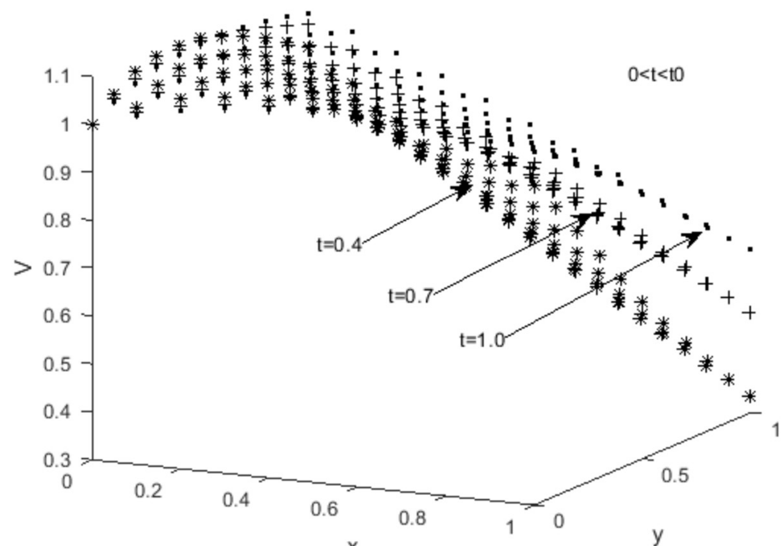

Figure 1 represents the concentration profile for analytical solution, it shows that if the source of input exists at 0<t<t_0 then the virus propagation decreases rapidly for small time interval as compare to long duration.

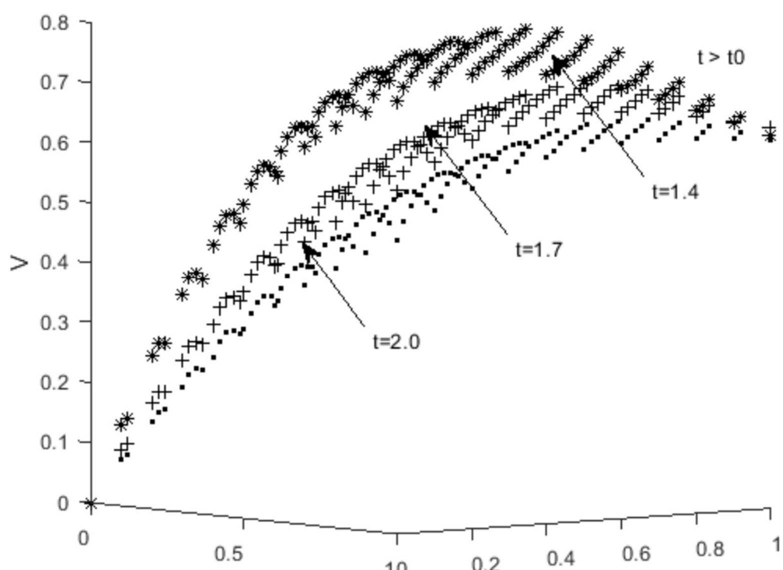

Figure 2 depicts the virus propagation for t>t_0, this is the case at which the input source has been removed initially. It is observed that the virus has a high impact for less time. On increasing the time interval the effect of virus is subsidized.

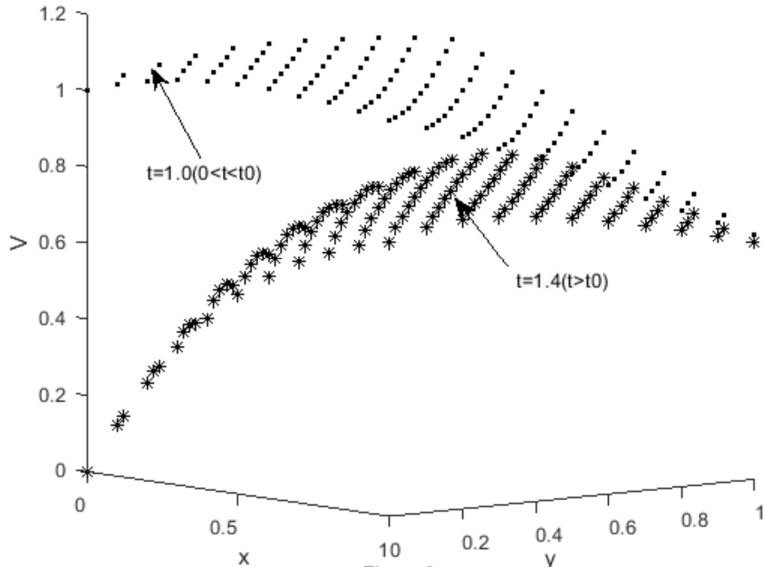

Figure 3

Figure 3 represents the analytical profile for virus in presence as well as in the absence of input source. It is observed that in presence of input source the graph reaches upto maximum level of virus propagation as compared to in the absence of input source.

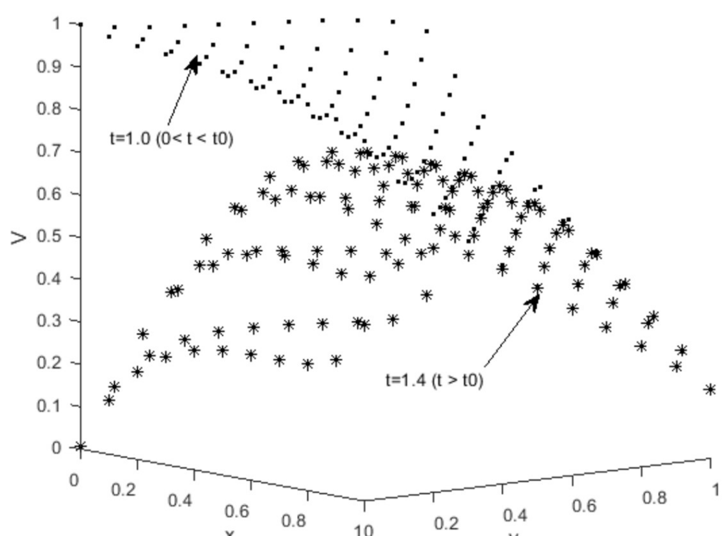

Figure 4 (Numerical Solution)

Figure 4 shows the numerical profile for virus propagation in presence as well as in the absence of input source. It is clearly seen that both analytical and numerical results follow the same profile that validates the mathematical solution numerically.

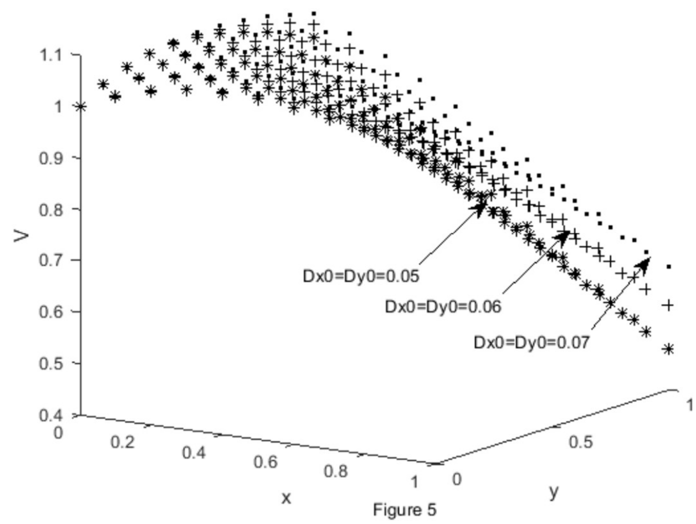

Figure 5 observed the virus propagation for different values of D. It is observed that on decreasing dispersivity parameter the concentration decreases more rapidly.

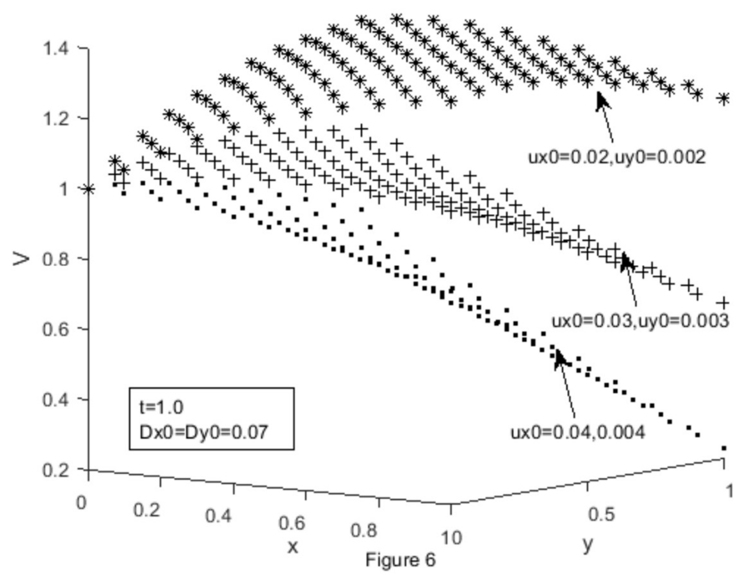

Figure 6 represents the virus propagation for different values of seepage velocity. It shows that if seepage velocity is high, concentration decreases rapidly and vice versa.

5. CONCLUSION

This study investigates the spread of infectious viruses throughout a two-dimensional flow domain, such as the human body, air, or water systems. The viral transmission from a point source is analytically characterized by solving the two-dimensional advection–dispersion equation (ADE) in both longitudinal and transverse directions of the flow field. The coefficients of the ADE are considered spatio-temporally varying to represent the heterogeneity of the medium and the unsteady nature of environmental flow conditions. The adsorption (retardation) coefficient exhibits an inverse relationship with the time-dependent dispersion coefficient and flow velocity. Specifically, lower retardation occurs when the dispersion coefficient or flow velocity increases, resulting in faster virus propagation, whereas higher retardation reduces the spread of the virus. Various temporal dependencies, including linear, exponential, and asymptotic forms, are considered to analyze the transmission behavior under different environmental conditions. The analytical solution is obtained using transformed space–time coordinates through the Laplace transform technique. The proposed two-dimensional mathematical model can serve as an effective predictive tool for environmentalists, biologists, chemists, and public health researchers in understanding and controlling infectious disease transmission in complex flow environments.

AUTHOR CONTRIBUTIONS

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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