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EFFECTS OF MIGRATION AND SELECTIVE HARVESTING ON THE PREY-PREDATOR POPULATION DYNAMICS IN THREE DIFFERENT AREAS

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Abstract: This paper develops a nonlinear mathematical model to describe the population dynamics of butini fish (*Glossogobius matanensis*) as prey and cork fish (*Channa striata*) as predator across three connected lake areas. The model incorporates migration and selective harvesting processes, and examines the existence and local stability of the interior equilibrium point using analytical and numerical approaches. Numerical simulations are presented to visualize the trajectories of prey and predator populations, confirming the analytical results. The findings reveal that both species can coexist sustainably and that selective harvesting under specific effort levels enhances prey recovery without destabilizing predator dynamics.

Keywords: predator–prey model; migration; selective harvesting; population dynamics; stability analysis; sustainable resource management; mathematical modelling.

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1. INTRODUCTION

Nonlinear system dynamics, including system behaviour and stability, has been applied in various fields and for multiple purposes focusing on local and global stability; for examples, see [1-7]. Population dynamics is one of the challenging objects of study in ecological mathematics. The interactions between populations, such as predation, competition, and mutualism, have ecological consequences [24-25]. Population as a proper stock also has social and economic impacts. The study of the dynamic behaviour of populations becomes more complex and comprehensive because people must manage stock to protect the population from extinction. Apart from that, harvesting the population also provides economic benefits over a certain period.

Previous researchers have extensively explored the modelling of predator and prey population dynamics, considering a range of factors such as stage structure, migration, diffusion, harvesting, and taxes. Some studies have focused on population dynamics involving one predator with two prey or two predators with one prey. Herlambang *et al.* proposed a model that incorporates reserve areas and taxes to curb the overexploitation of the population [8]. In [9], the author studied population dynamics in nature reserve areas with harvesting and identified a condition that could maximize the present value of the harvesting effort. The structural behaviour of the stages of predator and prey population dynamics in two environmental areas with harvesting in fishing-free areas has been discussed in [10], leading to the identification of certain conditions that provide optimal harvest values. Our research builds on these findings and offers new insights into the effects of migration and selective harvesting on prey-predator population dynamics, contributing to the understanding of this complex issue and providing valuable information for population management and protection from extinction.

The effect of selective harvesting on the dynamics of predator and prey populations has been observed for some purposes. Some researchers have examined only the prey being harvested [10-13]. The studies of predator and prey models when only the predator is harvested can be seen in [14-18]. Some other researchers have studied predator-prey models by considering both populations being harvested; the examples can be seen in [15, 19-20]. Predator and prey models with exploitation were often associated with the economic point of view, including maximum profit and present value problems; one can see examples in [8-9,19].

In Malili Lake Complex, South Sulawesi, Indonesia, butini fish (*Glossogobius matanansis*), which lives at the bottom of the lakes, and its predator cork fish (*Channa striata*) are sources of food for the surrounding community [21]. The dynamics of butini fish as an endemic and its

predator must be appropriately managed to prevent the fish from extinction. Based on the findings of the researchers above and as a strategy to manage the endemic butini fish and its predator, we consider the dynamical behaviors of both predator and prey populations, where the prey lives in the three connected areas, three lakes, namely Matano, Mahalona, and Towuti. The prey and its predator can migrate in one way from the first area to the second area and then to the third area through a river. The economically valuable cork fish (predator) and butini fish (prey) in the third area (Towuti Lake) are harvested with constant harvesting efforts. We study the presence of an interior equilibrium point and its stability, including the effect of migration and selective harvesting on the dynamics of prey and predator. The plots of trajectories of prey and population are visualized to confirm the analytic result.

2. MATERIAL AND METHODS

We consider predator and prey populations in an environment involving three areas: Matano, Mahalona, and Towuti. The three areas are considered to have the same conditions but different in carrying capacity. The third area is the largest of the three lakes. The butini fish (B) is the prey, and the cork fish (G) is its predator, which can migrate from the first area to the third area. The growth of the prey population is assumed to follow the logistic equation. The predator population is considered only to eat the prey in that area. The phenomena of the linkage between prey and its predator and the dynamics are depicted in Figure 1.

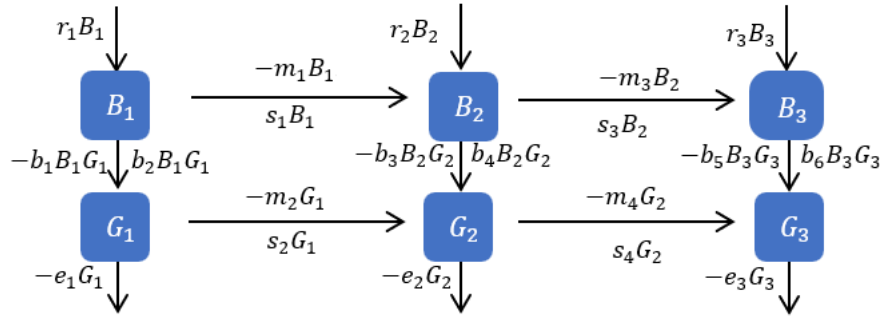


Figure 1. Prey and predator populations interaction diagram in three areas

Based on the assumption and Figure 1, the growth rates of three predators and three prey in the three areas are stated in the form of the equations system as follows

$$\frac{dB_1}{dt} = r_1 B_1 - a_1 B_1^2 - m_1 B_1 - b_1 B_1 G_1 \quad (1)$$

$$\frac{dG_1}{dt} = -e_1 G_1 - m_2 G_1 + b_2 B_1 G_1 \quad (2)$$

$$\frac{dB_2}{dt} = r_2 B_2 - a_2 B_2^2 + s_1 B_1 - m_3 B_2 - b_3 B_2 G_2 \quad (3)$$

$$\frac{dG_2}{dt} = -e_2 G_2 + s_2 G_1 - m_4 G_2 + b_4 B_2 G_2 \quad (4)$$

$$\frac{dB_3}{dt} = r_3 B_3 - a_3 B_3^2 + s_3 B_2 - b_5 B_3 G_3 \quad (5)$$

$$\frac{dG_3}{dt} = -e_3 G_3 + s_4 G_2 + b_6 B_3 G_3. \quad (6)$$

The symbol B_i states the size of the prey population in area i , for $i=1,2,3$. The symbol G_i states the predator population size in area i , for $i=1,2,3$. The symbol $\frac{dB_i}{dt}$ denotes the growth rate of the prey population in area i , for $i=1,2,3$. The symbol $\frac{dG_i}{dt}$ denotes the growth rate of the predator population in area i , for $i=1,2,3$. The parameters r_1 , r_2 , and r_3 are the intrinsic growth of the prey population in its area, respectively. The parameters a_1 , a_2 , and a_3 measure the intraspecific interaction of the prey in its area, respectively. The parameters b_1 , b_2 , and b_3 measure the negative effect of predation on the prey population in its area, respectively. The parameters b_1 , b_3 , and b_5 measure the negative effect of predation on the prey population in its area, respectively. The parameters b_2 , b_4 , and b_6 measure the positive effect of predation on the predator population in its area, respectively. The parameters e_1 , e_2 , and e_3 state the natural mortality of the predator population in its area, respectively. The parameters m_1 and m_2 measure the effect of migration on the growth of prey and its predator in the first area. The parameters s_1 and s_2 measure the effect of migration on the growth of prey and its predator in the second area. The parameter m_3 , m_4 , s_3 , and s_4 are similar meaning to the previous explanation.

These two fish are the local people's favourite food, and therefore, both fish are harvested by the people. Under certain considerations, only the fish in the third area are harvested. In this model,

we consider the harvesting rate to be proportional to the population size with the same effort (E_1) .

Under these assumptions, a model with selective harvesting is obtained. Equation (1) – (4) do not change, while equation (5) – (6) are developed and become

$$\frac{dB_3}{dt} = r_3 B_3 - a_3 B_3^2 + s_3 B_2 - b_5 B_3 G_3 - q_1 E_1 B_3 \quad (5.a)$$

$$\frac{dG_3}{dt} = -e_3 G_3 + s_4 G_2 + b_6 B_3 G_3 - q_2 E_1 G_3. \quad (6.a)$$

The symbols q_1 and q_2 denote the catchability coefficient for the prey and predator populations.

3. RESULTS EXISTENCE AND STABILITY OF INTERIOR EQUILIBRIUM POINT

In order to get the interior equilibrium points of the model (1) – (6), we set the system of equations

equal to zero, $\frac{dB_i}{dt} = \frac{dG_i}{dt} = 0$, for $i=1,2,3$, and solve them in terms of B_i and G_i .

$$B_1(r_1 - a_1 B_1 - m_1 - b_1 G_1) = 0 \quad (7)$$

$$G_1(-e_1 - m_2 + b_2 B_1) = 0 \quad (8)$$

$$B_2(r_2 - a_2 B_2 - m_3 - b_3 G_2) + s_1 B_1 = 0 \quad (9)$$

$$G_2(-e_2 + s_2 G_1 - m_4 + b_4 B_2) + s_2 G_1 = 0 \quad (10)$$

$$B_3(r_3 - a_3 B_3 - b_5 G_3) + s_3 B_2 = 0 \quad (11)$$

$$G_3(-e_3 + b_6 B_3) + s_4 G_2 = 0. \quad (12)$$

From equation (8) and equation (7) we get $B_1 = B_1^* = \frac{e_1 + m_2}{b_2}$ and $G_1 = G_1^* = \frac{r_1 - m_1 - a_1 B_1^*}{b_1}$

under the condition $r_1 - m_1 - a_1 B_1^* > 0$. After substituting $B_1 = B_1^*$ and $G_1 = G_1^*$ into equation (9)

and equation (10) and solving the two equations in term of B_2 and G_2 , we get $B_2 = B_2^*$ and

$G_2 = G_2^*$, where B_2^* is the positive root of the polynomial $A_3 Z^3 + A_2 Z^2 + A_1 Z + A_0 = 0$, with

$$A_3 = a_2 b_4, \quad A_2 = -a_2(e_2 + m_4) - b_4(r_2 - m_3), \quad A_1 = -b_4 s_1 B_1^* - b_3 s_2 G_1^* + (e_2 + m_4)(r_2 - m_3),$$

$$A_0 = s_1 B_1^* + (e_2 + m_4), \text{ and } G_2^* = \frac{a_2(-B_2^*)^2 + s_1 B_1^* + (r_2 - m_3) B_2^*}{b_3 B_2^*}.$$

After substituting $B_2 = B_2^*$ and $G_2 = G_2^*$ into equation (11) and equation (12) and solving the two equations in terms of B_3 and G_3 , we get $B_3 = B_3^*$ and $G_3 = G_3^*$. The value of B_3^* is the positive root of the polynomial $P_3Z^3 + P_2Z^2 + P_1Z + P_0 = 0$, with $P_3 = a_3b_6$, $P_2 = -a_3e_3 - b_6r_3$,

$$P_1 = -B_2^*b_6s_3 - G_2^*b_5s_4 + e_3r_3, \quad P_0 = -B_2^*e_3s_3, \quad G_3^* = \frac{a_3(-B_3^*) + s_3B_2^* + r_3B_3^*}{b_5B_3^*}. \quad \text{Therefore, under a}$$

specific condition of the parameter values of the model we get an interior equilibrium point $E^* = (B_1^*, G_1^*, B_2^*, G_2^*, B_3^*, G_3^*)$.

The model is a nonlinear system of differential equations, and it is not easy to analyze the global stability of the interior equilibrium point E^* . Thus, we must consider and analyze the local stability of the interior equilibrium point. The Jacobian matrix of the model is given by

$$J = \begin{pmatrix} M_{11} & -b_1B_1 & 0 & 0 & 0 & 0 \\ b_2G_1 & b_2B_1 - e_1 - m_2 & 0 & 0 & 0 & 0 \\ s_1 & 0 & M_{33} & -b_3B_2 & 0 & 0 \\ 0 & s_2 & b_4G_2 & b_4B_2 - e_2 - m_4 & 0 & 0 \\ 0 & 0 & s_3 & 0 & M_{55} & -b_5B_3 \\ 0 & 0 & 0 & s_4 & b_6G_3 & b_6B_3 - e_3 \end{pmatrix},$$

where $M_{11} = r_1 - m_1 - b_1G_1 - 2a_1B_1$, $M_{33} = r_2 - m_3 - b_3G_2 - 2a_2B_2$, and $M_{55} = r_3 - b_5G_3 - 2a_3B_3$.

The Jacobian matrix of the model evaluated at the equilibrium point E^* is given by

$$J = \begin{pmatrix} M_{11} & -b_1B_1^* & 0 & 0 & 0 & 0 \\ b_2G_1^* & M_{22} & 0 & 0 & 0 & 0 \\ s_1 & 0 & M_{33} & -b_3B_2^* & 0 & 0 \\ 0 & s_2 & b_4G_2^* & M_{44} & 0 & 0 \\ 0 & 0 & s_3 & 0 & M_{55} & -b_5B_3^* \\ 0 & 0 & 0 & s_4 & b_6G_3^* & M_{66} \end{pmatrix},$$

where $M_{11} = r_1 - m_1 - b_1G_1^* - 2a_1B_1^*$, $M_{22} = b_2B_1^* - e_1 - m_2$, $M_{33} = r_2 - m_3 - b_3G_2^* - 2a_2B_2^*$,

$M_{44} = b_4B_2^* - e_2 - m_4$, $M_{55} = r_3 - b_5G_3^* - 2a_3B_3^*$, and $M_{66} = b_6B_3^* - e_3$. Polynomial characteristic

of Jacobian matrix J_{E^*} is given by

$$f(\lambda) = \left[\lambda^2 + (-M_{11} - M_{22})\lambda + b_1 G_1^* b_2 B_1^* + M_{11} M_{22} \right] \times \\ \left[\lambda^2 + (-M_{33} - M_{44})\lambda + b_2 G_2^* b_2 B_2^* + M_{33} M_{44} \right] \times \\ \left[\lambda^2 + (-M_{55} - M_{66})\lambda + b_5 G_3^* b_6 B_3^* + M_{55} M_{66} \right].$$

Following the Routh-Hurwitz stability criteria [22], all of the eigenvalues of Jacobian matrix J_{E^*} have negative real parts when $M_{11} + M_{22} < 0$, $b_1 G_1^* b_2 B_1^* + M_{11} M_{22} > 0$, $M_{33} + M_{44} < 0$, $b_2 G_2^* b_2 B_2^* + M_{33} M_{44} > 0$, $M_{55} + M_{66} < 0$, and $b_5 G_3^* b_6 B_3^* + M_{55} M_{66} > 0$.

The interior equilibrium point for the model with harvesting is given by $E^\oplus = (B_1^\oplus, G_1^\oplus, B_2^\oplus, G_2^\oplus, B_3^\oplus, G_3^\oplus)$, where $B_1^\oplus = B_1^*$, $G_1^\oplus = G_1^*$, $B_2^\oplus = B_2^*$, $G_2^\oplus = G_2^*$. Substituting $B_2 = B_2^*$ and $G_2 = G_2^*$ into Equation (5a) and (6a) equals zero, and solving the two equations in term of B_3 and G_3 , we get $B_3 = B_3^\oplus$ and $G_3 = G_3^\oplus$. The value of $B_3^\oplus$ is the positive root of the polynomial $S_3 Z^3 + S_2 Z^2 + S_1 Z + S_0 = 0$, with $S_3 = a_3 b_6$, $S_2 = -a_3(e_3 + q_2 E_1) - b_6(r_3 - q_1 E_1)$, $S_1 = -B_2^\oplus b_6 s_3 - G_2^\oplus b_5 s_4 + (e_3 + q_2 E_1)(r_3 - q_1 E_1)$, and $S_0 = B_2^\oplus s_3(e_3 + q_2 E_1)$. While the value of G_3^* is given by $G_3^* = \frac{a_3(-B_3^\oplus)^2 + s_3 B_2^\oplus + (r_3 - q_1 E_1) B_3^\oplus}{b_5 B_3^\oplus}$.

Therefore, under a certain condition of parameter values of the model, we get an interior equilibrium point $E^\oplus = (B_1^\oplus, G_1^\oplus, B_2^\oplus, G_2^\oplus, B_3^\oplus, G_3^\oplus)$. The Jacobian matrix of the model evaluated at the equilibrium point $E^\oplus$ is given by

$$J_{E^\oplus} = \begin{pmatrix} N_{11} & -b_1 B_1^\oplus & 0 & 0 & 0 & 0 \\ b_2 G_1^\oplus & N_{22} & 0 & 0 & 0 & 0 \\ s_1 & 0 & N_{33} & -b_3 B_2^\oplus & 0 & 0 \\ 0 & s_2 & b_4 G_2^\oplus & N_{44} & 0 & 0 \\ 0 & 0 & s_3 & 0 & N_{55} & -b_5 B_3^\oplus \\ 0 & 0 & 0 & s_4 & b_6 G_3^\oplus & N_{66} \end{pmatrix},$$

where $N_{11} = r_1 - m_1 - b_1 G_1^\oplus - 2a_1 B_1^\oplus$, $N_{22} = b_2 B_1^\oplus - e_1 - m_2$, $N_{33} = r_2 - m_3 - b_3 G_2^\oplus - 2a_2 B_2^\oplus$,

$$N_{44} = b_4 B_2^\oplus - e_2 - m_4, \quad N_{55} = (r_3 - q_1 E_1) - b_5 G_3^\oplus - 2a_3 B_3^\oplus, \quad \text{and} \quad N_{66} = b_6 B_3^\oplus - (e_3 + q_2 E_1).$$

Polynomial characteristic of the Jacobian matrix $J_{E^\oplus}$ is given by

$$\begin{aligned} f(\lambda) = & \left[\lambda^2 + (-N_{11} - N_{22})\lambda + b_1 G_1^\oplus b_2 B_1^\oplus + N_{11} N_{22} \right] \times \\ & \left[\lambda^2 + (-N_{33} - N_{44})\lambda + b_2 G_2^\oplus b_2 B_2^\oplus + N_{33} N_{44} \right] \times \\ & \left[\lambda^2 + (-N_{55} - N_{66})\lambda + b_5 G_3^\oplus b_6 B_3^\oplus + N_{55} N_{66} \right]. \end{aligned}$$

Following the Routh-Hurwitz stability criteria [23], all of the eigenvalues of the Jacobian matrik $J_{E^\oplus}$ have negative real parts when $N_{11} + N_{22} < 0$, $b_1 G_1^\oplus b_2 B_1^\oplus + N_{11} N_{22} > 0$, $N_{33} + N_{44} < 0$, $b_2 G_2^\oplus b_2 B_2^\oplus + N_{33} N_{44} > 0$, $N_{55} + N_{66} < 0$, and $b_5 G_3^\oplus b_6 B_3^\oplus + N_{55} N_{66} > 0$.

4. SIMULATIONS

The values of the parameters model used for simulation with appropriate units are $r_1 = 15$, $r_2 = 5$,

$$r_3 = 15, \quad a_1 = \frac{1}{10000000}, \quad a_2 = \frac{1}{100000}, \quad a_3 = \frac{1}{23000000}, \quad b_1 = 0.0001, \quad b_2 = 0.00000001,$$

$$b_3 = 0.0001, \quad b_4 = 0.00000001, \quad b_5 = 0.0001, \quad b_6 = 0.00000001, \quad m_1 = 0.0000005,$$

$$m_2 = 0.0000005, \quad s_1 = 0.00000015, \quad s_2 = 0.00000015, \quad m_3 = 0.000005, \quad m_4 = 0.000005,$$

$$s_3 = 0.00000015, \quad s_4 = 0.00000015, \quad e_1 = 0.005, \quad e_2 = 0.0005, \quad e_3 = 0.005, \quad q_1 = 1, \quad \text{and} \quad q_2 = 1.$$

Some parameter values are assumed, and others are adopted from (Dubey *et al.* 2018; Didiharyono *et al.* 2023). This gives an interior equilibrium point E^* of the model as $E^* = (500050, 149499, 50450, 44954, 499955, 149783)$. Substituting the equilibrium point

into the Jacobian matrix, we get

$$J_{E^*} = \begin{pmatrix} -0.05005 & -50.0050 & 0 & 0 & 0 & 0 \\ 0.00149 & 0 & 0 & 0 & 0 & 0 \\ 0.00000015 & 0 & -0.50450 & -5.04501 & 0 & 0 \\ 0 & 0.00000015 & 0.0004495 & 0.00000049 & 0 & 0 \\ 0 & 0 & 0.0000015 & 0 & -0.021737 & -49.99549 \\ 0 & 0 & 0 & 0.0000015 & 0.0014978 & -0.00000045 \end{pmatrix},$$

with $\det(J_{E^*}) = 0.00001269798427$.

MIGRATION AND SELECTIVE HARVESTING

From the Jacobian matrix J_{E^*} , we have polynomial characteristic $f(\lambda) = 1.000000042\lambda^6 + 0.576245200\lambda^5 + 0.189196436\lambda^4 + 0.08156169\lambda^3 + 0.0086676\lambda^2 + 0.00282788\lambda + 0.00001269798$, which can be factorized as $f(\lambda) = 1.000000042\{\lambda^2 + 0.5042814458\lambda + 0.002274305\} + \{\lambda^2 + 0.0490107719\lambda + 0.0725479277\}\{\lambda^2 + 0.02295295804\lambda + 0.076959279\}$.

From $f(\lambda) = 0$ we get eigenvalues, namely $-0.0250025 \pm 0.272272515i$, -0.49996637917 , -0.00453677365 , $-0.0108688901 \pm 0.2734345418i$. Since the real parts of eigenvalues are all negative, then we conclude that the equilibrium point $E^* = (B_1^*, G_1^*, B_2^*, G_2^*, B_3^*, G_3^*)$ is locally asymptotically stable. The trajectories of the prey and its predator around the equilibrium point are illustrated in Figures 2, 3, and 4.

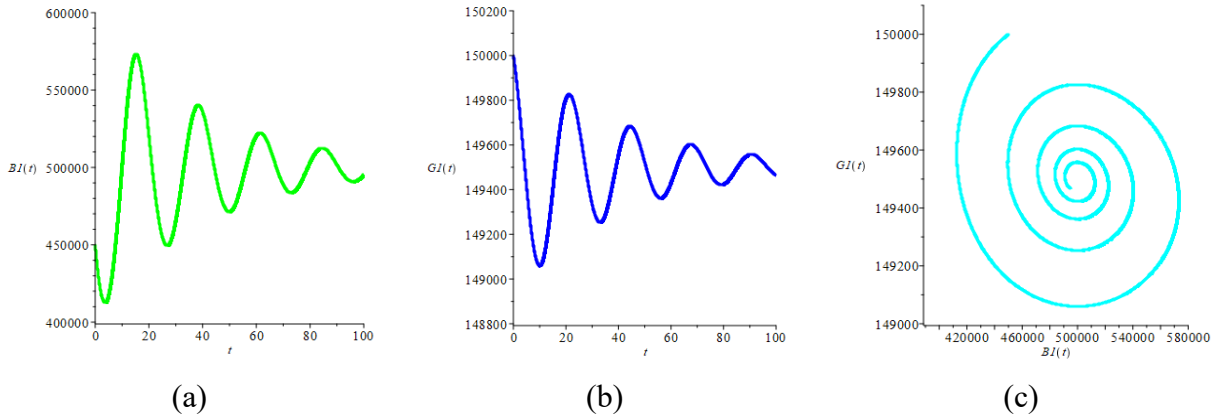


Figure 2. The dynamics of prey and predator in the first area

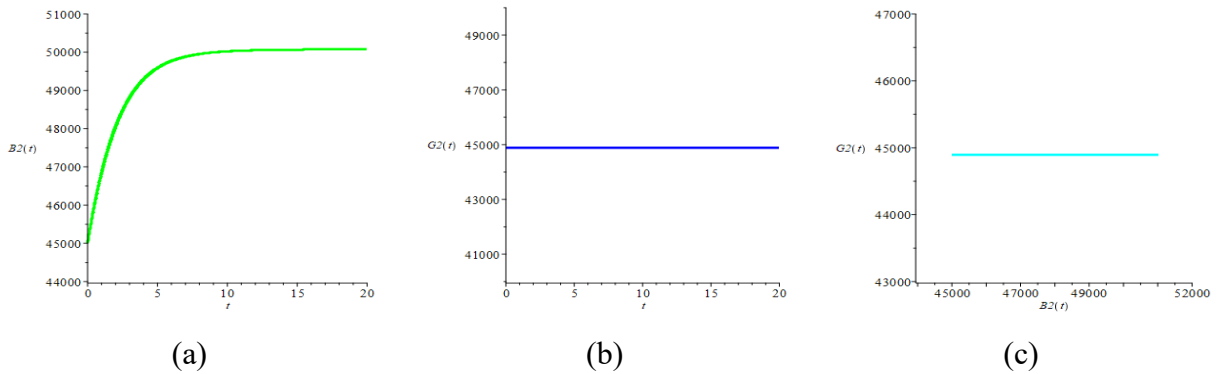


Figure 3. The dynamics of prey and predator in the second area

Figures 2 and 4 show that the prey and predator populations in the first and third areas oscillate around the equilibrium point and gradually toward the equilibrium point. Likewise, the prey and predator populations in the second area both of populations move monotonically toward the equilibrium point, Figure 3. The plots show that the prey and predator populations in the three areas will remain sustainable even if predation occurs in three areas.

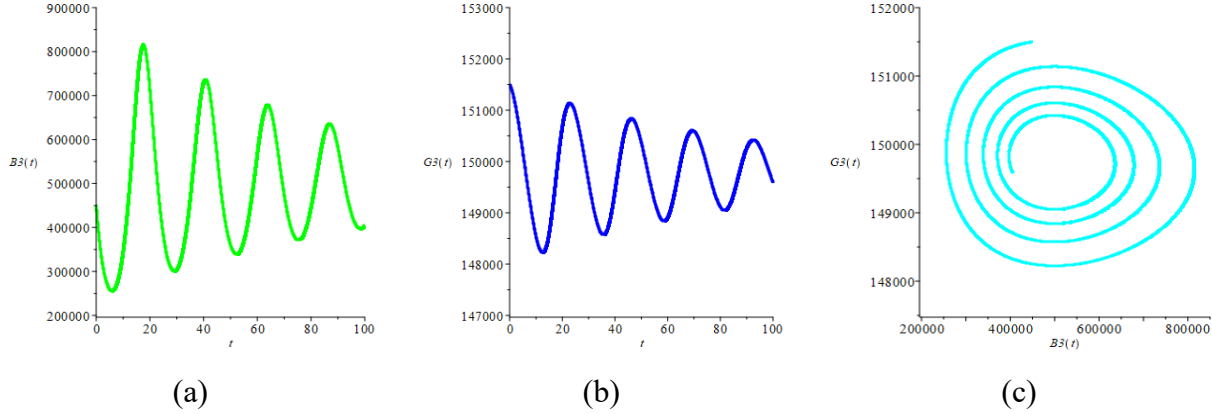


Figure 4. The dynamics of prey and predator in the third area

From the equilibrium point, we know that $B_1 = B_1^* = \frac{e_1 + m_2}{b_2}$ and $G_1 = G_1^* = \frac{r_1 - m_1 - a_1 B_1^*}{b_1}$. It

is easy to see that when m_2 increases the value of B_1^* also increases. On the contrary, if m_1 and m_2 increase, the value of G_1^* decreases. The effects of migration rates from the second to third areas have yet to be explicitly known. From the simulation, we investigate the effect of the migration rate from the first area to the second area. The effect of migration can be seen in changes in the value of the equilibrium point and its stability by increasing the migration rate gradually, as given in Table 1.

From Table 1, it is known that when the values of m_1 , m_2 , s_1 and s_2 are increased gradually, then the values of the equilibrium point B_1^* increases, G_1^* decreases, B_2^* decreases, G_2^* increases, B_3^* decreases, and G_3^* moves down and then up. The value of B_1^* may increase even though there is migration because, at the same time, the predator also migrates at the same rate, which causes the level of predation to decrease. Gradually increasing the migration rate from the first area to the second area and ensuring no change in the migration rate from the second area to

the third area allows the value of G_2^* to rise. Increasing the value of G_2^* causes the value of B_2^* to move down. There is an interesting phenomenon in the third area because the value of B_3^* decreases while G_3^* falls, then rises again, even though the changes are insignificant.

Table 1. The effect of migration to the equilibrium point and its stability

Migration rates m_1, m_2	Equilibrium point	Eigenvalues
$m_1 = 0.0001;$ $m_2 = 0.0001;$ $s_1 = 0.00008;$ $s_2 = 0.00008$	$B_1^* = 510000; G_1^* = 149489,$ $B_2^* = 25314; G_2^* = 47484.593,$ $B_3^* = 499952.446; G_1^* = 149783.301$	$-0.025500 \pm 0.274934i$ $-0.249945, -0.005066i,$ $-0.010868 \pm 0.273434i$
$m_1 = 0.00015;$ $m_2 = 0.00015;$ $s_1 = 0.00012;$ $s_2 = 0.00012$	$B_1^* = 515000; G_1^* = 149483,$ $B_2^* = 13651.224; G_2^* = 48680.098,$ $B_3^* = 499951.249; G_1^* = 149783.301$	$-0.025750 \pm 0.276262i$ $-0.136145, -0.005262i,$ $-0.010868 \pm 0.273433i$
$m_1 = 0.0002;$ $m_2 = 0.0002;$ $s_1 = 0.00016;$ $s_2 = 0.00016$	$B_1^* = 520000; G_1^* = 149478,$ $B_2^* = 2702.872; G_2^* = 50037.483,$ $B_3^* = 499949.889; G_1^* = 149782.630$	$-0.026000 \pm 0.277583i$ $-0.055345, -0.002942i,$ $-0.010868 \pm 0.273434i$

The effect of harvesting can be seen in changes in the value of the equilibrium point and its stability by increasing the migration rate gradually, as given in Table 2. From the existence of the interior equilibrium point, we know that the values of the equilibrium point for $B_1^\oplus, G_1^\oplus, B_2^\oplus$, and $G_2^\oplus$ do not depend on the values of the equilibrium point $B_3^\oplus$ and $G_3^\oplus$. Thus, the values of harvesting effort (E_1) in the third area, do not change the values of equilibrium points in the first and the second areas. From the simulation, we know that by gradually increasing the values of harvesting effort, the value $B_3^\oplus$ increases significantly. On the contrary, the values of $G_3^\oplus$ decreases insignificantly. The equilibrium point does not change in type and stability. The harvesting efforts increase the values of real and imaginary parts of the last eigenvalues. The changes in eigenvalue increase the frequency and amplitude of the trajectories for prey and its

predator in the third area. These results imply that controlled migration enhances prey resilience by reducing over predation in source areas. Similarly, moderate harvesting in the terminal area supports long-term population sustainability without ecological collapse. The trajectories of prey and its predator with various harvesting values are illustrated in Figures 5, 6, and 7.

Table 2. The effect of harvesting on the equilibrium point and its stability

Harvesting effort E_1	Equilibrium point	Eigenvalues
$E_1 = 0$	$B_1^\oplus = 500050; G_1^\oplus = 149499,$ $B_2^\oplus = 50450; G_2^\oplus = 44954,$ $B_3^\oplus = 499955; G_1^\oplus = 149783$	$-0.025002 \pm 0.272272i, -0.499966,$ $-0.0045367, -0.010868 \pm 0.273434i$
$E_1 = 0.001$	$B_1^\oplus = 500050; G_1^\oplus = 149499,$ $B_2^\oplus = 50450; G_2^\oplus = 44954,$ $B_3^\oplus = 599955; G_1^\oplus = 149729$	$-0.025002 \pm 0.272272i, -0.499966,$ $-0.0045377, -0.012042 \pm 0.299433i$
$E_1 = 0.01$	$B_1^\oplus = 500050; G_1^\oplus = 149499,$ $B_2^\oplus = 50450; G_2^\oplus = 44954,$ $B_3^\oplus = 1499955; G_1^\oplus = 149248$	$-0.025002 \pm 0.272272i, -0.499966,$ $-0.004536, -0.022607 \pm 0.472018i$

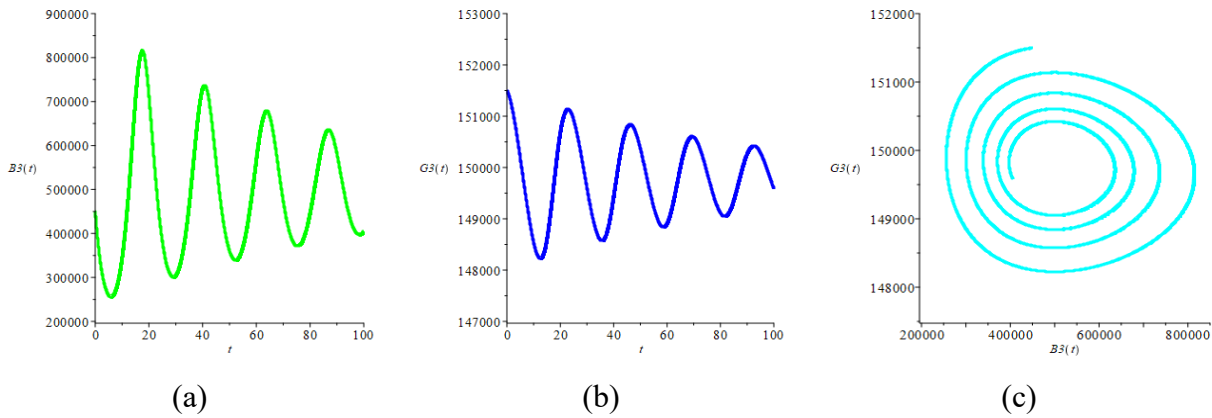


Figure 5. The dynamics of prey and predator in the third area without harvesting ($E_1 = 0$)

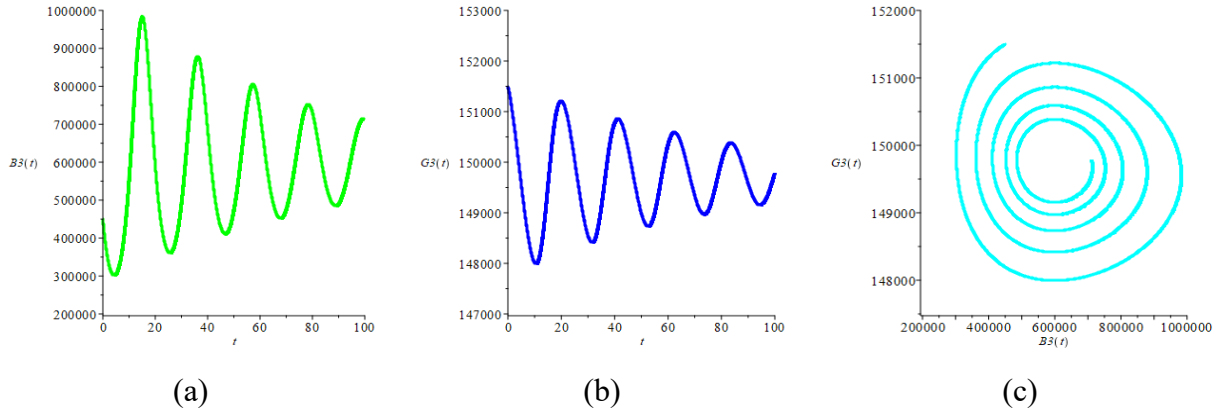


Figure 6. The dynamics of prey and predator in the third area with harvesting ($E_1 = 0.001$)

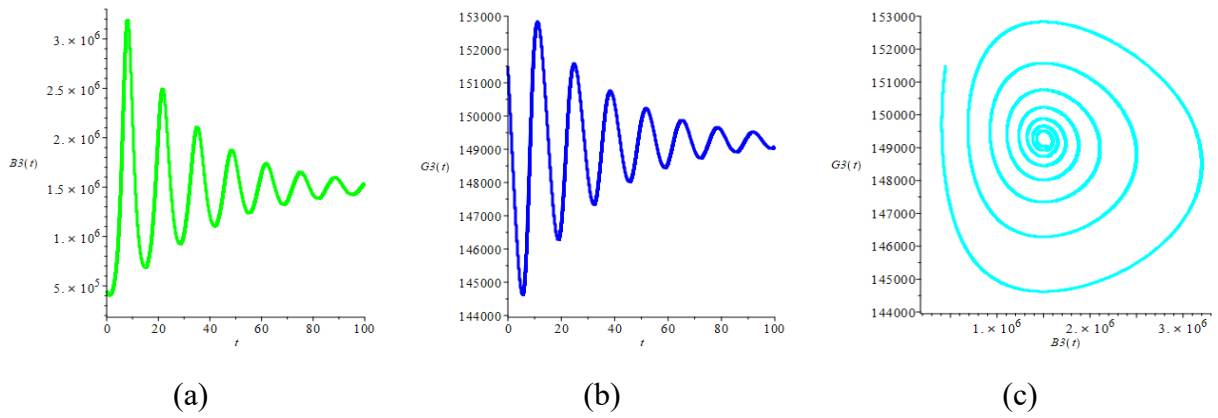


Figure 7. The dynamics of prey and predator in the third area with harvesting ($E_1 = 0.01$)

Increasing harvesting effort also causes the equilibrium point value for the prey population in the third area to increase. On the other hand, harvesting causes the equilibrium point value for the predator population to decrease. With the results of this analysis and simulation, harvesting both populations in the third area with the same effort can be used as a strategy to prevent the prey population from becoming extinct instead of increasing the prey population. Harvesting also reduces the number of predator populations, but not significantly.

5. CONCLUSION

A nonlinear mathematical model describing the population dynamics of *butini* fish (*Glossogobius matanensis*) as prey and cork fish (*Channa striata*) as predator across three interconnected lake areas was developed and analyzed. The model integrates both migration and selective harvesting

processes and demonstrates that an interior equilibrium point can exist under biologically feasible conditions. Analytical and numerical results reveal that the equilibrium point is locally asymptotically stable for suitable parameter values. In the first and third areas, both populations exhibit damped oscillations converging to equilibrium, whereas in the second area they approach equilibrium monotonically. These dynamics confirm the long-term coexistence of the two species despite migration and harvesting pressures.

Increasing migration rates from the first to the second area enhances the prey population in the source region while slightly reducing predator density, suggesting that dispersal can buffer prey abundance against overpredation. Conversely, applying equal harvesting effort to both species in the third area increases prey abundance and marginally decreases predator numbers without destabilizing the system. Hence, selective harvesting in the third area can serve as a sustainable management approach to preserve endemic prey populations while maintaining predator balance. Future work could extend this framework by incorporating time delays, stochastic effects, fractional-order dynamics, or optimal control formulations to provide deeper insights into ecological and economic sustainability.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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