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OPTIMAL CONTROL OF A LUMPY SKIN DISEASE TRANSMISSION MODEL WITH DISINFECTION AND CULLING STRATEGIES

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Abstract. Lumpy skin disease (LSD) is a viral infection that affects buffaloes and cattle across various regions, including both tropical and temperate climates, and has been reported in some wild ruminants. It is caused by the Lumpy Skin Disease Virus (LSDV), a member of the Capripoxvirus genus. This study presents a mathematical model of lumpy skin disease (LSD) that captures the transmission dynamics among multiple cattle populations, mosquito vectors, and environmental viral loads. To effectively curb the spread of the infection, an optimal control framework is applied for evaluating various parameters to identify the best transmission-reduction strategy. Specifically, the interventions include environmental disinfection to lower viral presence and the culling of infected cattle. The proposed strategy aims to simultaneously minimize the number of infected cows, the environmental virus concentration, and the economic expenses associated with these interventions. Utilizing Pontryagin's minimum principle, the study successfully resolves this optimization problem. The derived strategies are subsequently analyzed through numerical simulations—employing the modified forward-backward sweep method—under four conditions: no action, solely disinfection, solely culling, and a combined approach. The findings indicate that the simultaneous application of disinfection and culling is the most effective strategy for eliminating the disease.

Keywords: lumpy skin disease; mathematical modelling; optimal control; disinfection; culling.

2020 AMS Subject Classification: 34A34, 92B05, 49K15.

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1. INTRODUCTION

Lumpy Skin Disease (LSD) in cattle is triggered by the LSD virus, characterized as an enveloped, ovoid, double-stranded DNA virus classified under the *Capripoxvirus* genus within the *Poxviridae* family [1]. The dissemination of this virus relies heavily on arthropod vectors; for instance, *Aedes aegypti* mosquitoes are documented transmitters of the LSD virus, comparable to the established role of *Stomoxys calcitrans* in the mechanical transmission of sheeppox virus (SPPV). Beyond vector-driven infection, the disease can also be spread via iatrogenic routes, highlighting the considerable role humans play in transmitting the virus through medical procedures and injections [2].

Since its first detection in Zambia in 1929, LSD is known to spread indirectly through contaminated environments, water, and farming tools [3]. The virus survives for up to 11 days in milk, blood, and saliva, and 22 days in semen, which puts nursing calves at risk [4]. While all cattle are susceptible regardless of age or sex [5], morbidity (around 10%) and mortality (1-3%) rates in endemic areas depend heavily on herd immunity, breed, vectors, and viral strains [6]. Importantly, LSD does not infect humans; however, it is a highly contagious disease that spreads rapidly among cattle once an outbreak begins [7].

Lumpy Skin Disease (LSD) leads to substantial agricultural deficits [8], with severe economic repercussions stemming largely from irreversible damage to cattle hides [9]. The clinical profile of the disease includes fever, skin edema, enlarged lymph nodes, and the emergence of nodules across the skin and mucous membranes. These distinctive lesions present as firm, flat-topped papules measuring 0.5 to 5 cm, which can often merge together [10]. Infected cattle frequently suffer from prolonged lethargy, driving further financial losses through decreased milk yields, weight reduction, infertility, spontaneous abortion, and mortality [11]. Because vectors drive transmission, outbreaks peak alongside insect populations during wet summer and autumn seasons, following an incubation period of up to 28 days in natural infections [12]. Due to overlapping symptoms with other illnesses, initial visual diagnoses based on skin nodules require laboratory validation. Given its cross-border spread and heavy economic toll on the cattle

industry, the World Organization for Animal Health (OIE) classifies LSD as a notifiable disease [13]. Current management strategies involve veterinary care and insecticide applications to reduce vector-cattle interactions.

Mathematical modeling has emerged as a fundamental approach in infectious disease studies, and informing the development of effective control strategies. Various strategies for controlling LSD have been proposed. For instance, cow vaccination is recommended in areas with significant mosquito populations and high disease prevalence [14]. However, vaccination is less common in areas with rare outbreaks and is not universally recommended, partly because it can be expensive for farmers [15]. Other control approaches focus on addressing sources and transmission routes, such as eliminating breeding areas and using insecticides for mosquito control [2]. Nevertheless, insecticides sprayed into the environment can also kill beneficial insects, such as pollinators, thereby negatively impacting biodiversity and requiring careful consideration.

Regarding LSD modeling, the SEIR framework has been used to study disease dynamics and the effect of vaccination on reducing disease in cow populations [16]. However, that model did not include a separate vaccination compartment nor verify its reliability by checking fundamental properties. To address these limitations, several researchers have extended the LSD model by incorporating vaccination as an explicit control measure. For example, Butt et al. [12] formulated an SVEIR model with optimal control to evaluate the impact of vaccination on LSD transmission. Furthermore, an LSD model developed based on the SIR framework, incorporating three transmission routes: direct contact between cows, mosquito bites, and environmental persistence of the virus from infected cows [17]. However, their model did not include culling of infected cows, nor did it account for disease-induced mortality. To fill this gap, Handayani et al. [18] proposed an LSD model that introduced a culling parameter and performed a comprehensive dynamic analysis. That model established two equilibrium points—the disease-free equilibrium (DFE) and the endemic equilibrium (EE). When the basic reproduction number (R_0) is less than one, the DFE is locally and globally asymptotically stable [18]. Sensitivity analysis was also conducted to determine the influence of model parameters on (R_0). In their model, all parameters are constant, indicating that when (R_0) exceeds one, an endemic equilibrium will occur. In their formulation, all parameters are considered constant, and the model

demonstrates that an endemic equilibrium emerges when the basic reproduction number ($\mathcal{R}_0$) exceeds one.

To prevent the risk of disease persistence and endemic state, the implementation of effective control measures is essential. We introduces time-dependent parameters specifically, the culling rate of infected cows (η_c) is reformulated as a time-dependent function ($u_2(t)$), serving as a control measure for managing infected cow populations. Similarly, the disinfection rate of the virus (θ) is modified from a constant to a time-dependent function ($u_1(t)$), representing disinfection control for the environmental virus population. To determine the optimal control strategies, several mathematical methods can be employed; one of these is Pontryagin's Minimum Principle, a well-established approach in optimal control theory. Based on the preceding discussion, this study develops an optimal control model for LSD by incorporating both disinfection and culling interventions. Culling infected cows is regarded as a necessary measure, as these animals are no longer viable for breeding due to the formation of nodules on the skin and internal organs, rendering their hides worthless and their meat unsuitable for consumption [19]. Accurate and early diagnosis is crucial to controlling outbreaks [20]. By integrating disinfection and culling controls, this research aims to reduce the number of infected cows, the concentration of virus in the environment, and the resulting economic losses.

2. THE MATHEMATICAL MODEL

We developed the Handayani et al. [18] LSD model with the addition of disinfection and culling as a control. The LSD transmission mechanism involves two populations: cows and mosquitoes. The cow population is divided into three subpopulations: susceptible cows (S_c), infected cows (I_c), and recovered cows (R_c). Meanwhile, the mosquito population consists of two subpopulations: susceptible mosquitoes (S_v) and infected mosquitoes (I_v). L represents the presence of the virus in the environment. The model considers three main infection pathways: direct transmission from infected to susceptible cow at rate β , transmission through mosquitoes that acquire the virus from infected cow at rate β_c , and environmental transmission where susceptible cow ingests the virus shed by infected cow during grazing or drinking at rate β_l . The parameters $\pi_c, \mu_c, \gamma_c, \pi_v, \mu_v, \beta_v, \nu_c, \theta$ represent cow recruitment, natural death of cows, recovery rate, mosquito recruitment, natural death of mosquitoes, infection occurring through bites on an

infected cow, the environmental viral load, and disinfection rate. The compartmental structure of our model is illustrated in Fig. 1.

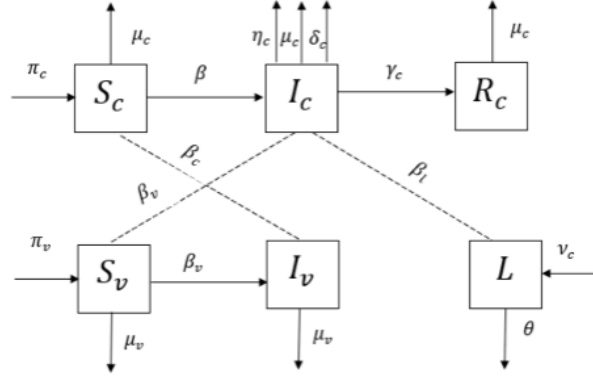


FIGURE 1. Schematic diagram of the LSD model.

The solid arrows in the diagram illustrate the outflows from each compartment, whereas the dashed arrows represent the interactions among the compartments. Considering these processes, the dynamics of their LSD transmission can be expressed through the following system of equations:

$$\begin{aligned}
 \frac{dS_c}{dt} &= \pi_c - \beta S_c I_c - \beta_c S_c I_v - \beta_l S_c L - \mu_c S_c, \\
 \frac{dI_c}{dt} &= \beta S_c I_c + \beta_c S_c I_v + \beta_l S_c L - \gamma_c I_c - \mu_c I_c - \delta_c I_c - \eta_c I_c, \\
 \frac{dR_c}{dt} &= \gamma_c I_c - \mu_c R_c, \\
 \frac{dS_v}{dt} &= \pi_v - \beta_v S_v I_c - \mu_v S_v, \\
 \frac{dI_v}{dt} &= \beta_v S_v I_c - \mu_v I_v, \\
 \frac{dL}{dt} &= v_c I_c - \theta L.
 \end{aligned}
 \tag{1}$$

From system (1), Handayani et al. [18] get two equilibrium points, i.e. disease-free (E^0) and the-endemic equilibrium point (E^*) are established. The disease-free equilibrium point is given by

$$E^0 = \left(\frac{\pi_c}{\mu_c}, 0, 0, \frac{\pi_v}{\mu_v}, 0, 0 \right).$$

The endemic equilibrium point also can be determined from system 1 as follows

$$E^* = \left(\frac{(\pi_c - (\gamma_c + \mu_c + \delta_c + \eta_c)I_c^*)}{\mu_c}, I_c^*, \frac{\gamma_c I_c^*}{\mu_c}, \frac{\pi_v}{\mu_v}, \frac{\pi_v}{\beta_v I_c^* + \mu_v}, \frac{v_c I_c^*}{\theta} \right),$$

where I_c^* is the solution to the equation

$$P_1(I_c^*)^2 + P_2 I_c^* + P_3 = 0,$$

with

$$\begin{aligned} P_1 &= -\beta(\gamma_c + \mu_c + \delta_c + \eta_c)\theta\mu_v\beta_v - \beta_l v_c(\gamma_c + \mu_c + \delta_c + \eta_c)\mu_v\beta_v, \\ P_2 &= \beta\pi_c\theta\mu_v\beta_v - \beta(\gamma_c + \mu_c + \delta_c + \eta_c)\theta\mu_v^2 - \beta_c\beta_v\pi_v(\gamma_c + \mu_c + \delta_c + \eta_c)\theta \\ &\quad + \beta_l\pi_c v_c\mu_v\beta_v - \beta_l\mu_v^2 v_c(\gamma_c + \mu_c + \delta_c + \eta_c) - (\gamma_c + \mu_c + \delta_c + \eta_c)\mu_c\mu_v\theta\beta_v, \\ P_3 &= \beta\pi_c\theta\mu_v^2 + \beta_v\pi_c\pi_v\theta + \beta_l\pi_c\mu_v^2 v_c + (\gamma_c + \mu_c + \delta_c + \eta_c)\mu_c\mu_v^2\theta. \end{aligned}$$

The stability of the disease free equilibrium point is locally and globally asymptotically when $\mathcal{R}_0 < 1$. Meanwhile, the endemic equilibrium points was shown locally asymptotically stable depend on Routh-Hurwitz condition. According to the results of $\mathcal{R}_0$'s sensitivity analysis Handayani et al. [18] and in order to control the spread of the LSD transmission, it is imperative to focus more on reducing β have positive high sensitivity impact on $\mathcal{R}_0$. Therefore, in this article the model construction stage involves changing the disinfection rate θ and the culling rate of infected cattle η_c in model (1), which are constant, to time-dependent values.

$$\begin{aligned} \frac{dS_c}{dt} &= \pi_c - \beta S_c I_c - \beta_c S_c I_v - \beta_l S_c L - \mu_c S_c, \\ \frac{dI_c}{dt} &= \beta S_c I_c + \beta_c S_c I_v + \beta_l S_c L - \gamma_c I_c - \mu_c I_c - \delta_c I_c - u_2(t)I_c, \\ \frac{dR_c}{dt} &= \gamma_c I_c - \mu_c R_c, \\ \frac{dS_v}{dt} &= \pi_v - \beta_v S_v I_c - \mu_v S_v, \\ \frac{dI_v}{dt} &= \beta_v S_v I_c - \mu_v I_v, \\ \frac{dL}{dt} &= v_c I_c - u_1(t)L, \end{aligned} \tag{2}$$

with the following non-negative initial conditions:

$$\begin{aligned}
(3) \quad & S_c(0) = S_{c0} \geq 0, \quad I_c(0) = I_{c0} \geq 0, \quad R_c(0) = R_{c0} \geq 0, \\
& S_v(0) = S_{v0} \geq 0, \quad I_v(0) = I_{v0} \geq 0, \quad L(0) = L_0 \geq 0,
\end{aligned}$$

Where all initial values lie in the interval $[0, 1]$, we also assume that, at the end of the control period T , the proportion of individuals in each subpopulation is not determined, namely $S_c(T)$, $I_c(T)$, $R_c(T)$, $S_v(T)$, $I_v(T)$, and $L(T)$ are all free.

3. OPTIMAL CONTROL PROBLEM

The objective of this research is to determine the optimal implementation of two control measures, namely disinfection to reduce environmental viral contamination and culling to mitigate the infected population, while incorporating the economic costs of implementing these interventions. The next step is to construct a Hamilton function, and the equations of state and costs are determined. Finally, Pontryagin's Minimum Principle is applied, a condition necessary to obtain optimal control that minimizes the objective function.

The objective of implementing these control variables—specifically, environmental disinfection (u_1) and the culling of infected cattle (u_2)—is to optimally suppress the transmission of LSD while simultaneously minimizing the associated operational costs. To achieve this, a non-linear cost function is formulated, utilizing a quadratic form for the control efforts to reflect the non-linear nature of intervention expenses. Mathematically, the performance index (or objective functional) for model (2) is defined as follows:

$$(4) \quad \min J = \int_0^T \left(a_1 I_c(t) + a_2 L(t) + \frac{1}{2} w_1 u_1^2(t) + \frac{1}{2} w_2 u_2^2(t) \right) dt,$$

where T represents the total duration of the control strategy. The coefficients a_1 and a_2 serve as the weighting factors for the infected cattle and environmental virus, respectively, while w_1 and w_2 are the relative weight costs for the control variables $u_1(t)$ and $u_2(t)$.

To apply Pontryagin's minimum principle, the Hamiltonian function (H) must be defined to link the system dynamics, control inputs, and costate variables. The Hamiltonian is crucial for determining the optimal control strategy, as it provides the necessary optimality conditions by integrating the state equations with the objective functional. This framework ensures that the control variables minimize the system's overall cost within the given constraints. Based on

model (2) and objective functional (4), H is explicitly formulated as:

$$(5) \quad H(t, x, u, \lambda_j) = a_1 I_c + a_2 L + \frac{1}{2} w_1 u_1^2(t) + \frac{1}{2} w_2 u_2^2(t) + \sum_{i=1}^6 \lambda_j g_i(t, x, u),$$

where $\lambda_j = \lambda_j(t)$ for $j = 1, 2, \dots, 6$ are adjoint variables (costate variables). To solve the optimal control problem, this study applies Pontryagin's minimum principle to derive the necessary optimality conditions. The principle states that minimizing an objective functional requires the existence of adjoint variables λ_j , ensuring that the Hamiltonian, H , achieves its absolute minimum with respect to the control input, $H(t, x^*, u^*, \lambda_j^*) \leq H(t, x, u, \lambda_j)$, at all times. This relationship allows the vectors for the control and adjoint variables to be explicitly defined as:

$$(6) \quad \frac{\partial H}{\partial u_i} = 0$$

for $i = 1, 2$. The following conditions establish the dynamic relationship between the state and adjoint variables using the Hamiltonian. This formulation ensures mathematical consistency within the optimization framework: the state dynamics dictate the system's behavior over time, while the adjoint dynamics measure how sensitive the cost functional is to changes in the state variables, which can be expressed as:

$$(7) \quad \frac{d\lambda_j}{dt} = -\frac{\partial H}{\partial x_j},$$

$$(8) \quad \frac{dx_j}{dt} = \frac{\partial H}{\partial \lambda_j},$$

for $j = 1, 2, \dots, 6$ and $x_i \in \{S_c, I_c, R_c, S_v, I_v, L\}$. While condition (8) gives back the dynamical system (2), condition (6) specifies the optimal controls, and condition (7) characterizes the adjoint system as given in Theorem 1.

Theorem 1. *The optimal controls u_1^*, u_2^* , which minimize the objective functional, are given by*

$$(9) \quad u_1^*(t) = \min \left(u_{1,\max}, \max \left(0, \frac{\lambda_6 L}{w_1} \right) \right)$$

$$(10) \quad u_2^*(t) = \min \left(u_{2,\max}, \max \left(0, \frac{\lambda_2 I_c}{w_2} \right) \right)$$

Proof. We get the Hamiltonian H , which is defined as follows

$$\begin{aligned}
 H(t, x, u, \lambda_j) = & a_1 I_c + a_2 L + \frac{1}{2} w_1 u_1^2(t) + \frac{1}{2} w_2 u_2^2(t) \\
 & + \lambda_1 (\pi_c - \beta S_c I_c - \beta_c S_c I_v - \beta_l S_c L - \mu_c S_c) \\
 & + \lambda_2 (\beta S_c I_c + \beta_c S_c I_v + \beta_l S_c L - \gamma_c I_c - \mu_c I_c - \delta_c I_c - u_2 I_c) \\
 & + \lambda_3 (\gamma_c I_c - \mu_c R_c) \\
 & + \lambda_4 (\pi_v - \beta_v S_v I_c - \mu_v S_v) \\
 & + \lambda_5 (\beta_v S_v I_c - \mu_v I_v) \\
 & + \lambda_6 (v_c I_c - u_1 L)
 \end{aligned}
 \tag{11}$$

with terminal time

$$\lambda_j(T) = 0, \quad j = 1, 2, \dots, 6 \tag{12}$$

The following necessary conditions define the dynamic relationship between the state and adjoint variables through the Hamiltonian function.

$$\begin{aligned}
 \lambda_1' &= \lambda_1 \beta I_c + \lambda_1 \beta_c I_v + \lambda_1 \beta_l L + \lambda_1 \mu_c - \lambda_2 \beta I_c - \lambda_2 \beta_c I_v - \lambda_2 \beta_l L \\
 \lambda_2' &= -a_1 + \lambda_1 \beta S_c - \lambda_2 \beta S_c + \lambda_2 \gamma_c + \lambda_2 \mu_c + \lambda_2 \delta_c + \lambda_2 u_2 - \lambda_3 \gamma_c + \lambda_4 \beta_v S_v - \lambda_5 \beta_v S_v - \lambda_6 v_c \\
 \lambda_3' &= -\lambda_3 \mu_c \\
 \lambda_4' &= -\lambda_4 \beta_v I_c - \lambda_4 \mu_v + \lambda_5 \beta_v I_c \\
 \lambda_5' &= \lambda_1 \beta_c S_c - \lambda_2 \beta_c S_c + \lambda_5 \mu_v \\
 \lambda_6' &= -a_2 + \lambda_1 \beta_l S_c - \lambda_2 \beta_l S_c + \lambda_6 u_1
 \end{aligned}
 \tag{13}$$

Optimal controls (9) and (10) are obtained by application of (6)

$$u_1^* = \frac{\lambda_6 L}{w_1} \tag{14}$$

$$u_2^* = \frac{\lambda_2 I_c}{w_2} \tag{15}$$

After projection to the bounds, the results are confirmed. $\square$

4. NUMERICAL SIMULATIONS AND INTERPRETATIONS

Numerical simulation with control applies the forward-backward sweep method, a numerical method for optimal control problems. This method boasts high accuracy because it utilizes the fourth-order forward Runge-Kutta method and the fourth-order backward Runge-Kutta method using MATLAB software. Next, a control strategy is implemented on the model using a combination of $u_1(t)$ and $u_2(t)$. Finally, the numerical simulation results are interpreted in consideration of biological phenomena.

The optimal controls and resulting state trajectories are computed by numerically integrating the state equations (2), the adjoint equations (13), and the control characterizations (14) and (15). This forms a classic two-point boundary value problem, as the state system relies on specified initial conditions (3), while the adjoint system depends on given terminal conditions (12). To handle this complexity, the Runge-Kutta method is combined with the forward-backward sweep technique [21], providing a practical framework for obtaining optimal control solutions.

To evaluate the effects of the two control measures on the transmission dynamics of LSD, numerical simulations were performed using MATLAB software. The simulations include three control strategies, designed as combinations of prevention and disease eradication efforts to mitigate the spread of LSD. This analysis aims to assess the effectiveness of both control types in reducing disease transmission. The simulation begins by assigning initial values to each population, namely $S_c(0) = 480$, $I_c(0) = 5$, $R_c(0) = 0$, $S_v(0) = 1000$, $I_v(0) = 10$, $L(0) = 0$. The parameters employed in the numerical simulation are based on previous studies on LSD transmission. The parameter values used in the numerical simulations are summarized in Table 1.

TABLE 1. The parameter value

Parameter	Value	Source
π_c	1	Alfwzan et al. [17]
π_v	2	Alfwzan et al. [17]
β	0.0008	Handayani et al. [18]
β_c	0.0004	Handayani et al. [18]
β_v	0.0006	Handayani et al. [18]
β_l	0.0005	Handayani et al. [18]
γ_c	0.1	Alfwzan et al. [17]
μ_c	0.01	Alfwzan et al. [17]
μ_v	0.05	Alfwzan et al. [17]
δ_c	0.01	Alfwzan et al. [17]
ν_c	0.05	Alfwzan et al. [17]
u_2	1	Assumed
u_1	1	Assumed

This study considers three specific strategies, including u_1 only, u_2 only, and combination of both u_1 and u_2 , to manage the dynamics of LSD. The implementation of culling can improve the effectiveness of control measures by reducing the number of infected cattle. Additionally, disinfection programs can help minimize the presence of the virus in the environment, including in cages, vehicles, equipment, personnel, and other potentially contaminated areas [2].

The first strategy, numerical simulations are performed using only the disinfection control (u_1) for 200 days. It was assumed that the infected cow and virus in the environment weights are given by $a_1 = 5000$ and $a_2 = 50$, while the control cost weights are $w_1 = 2$ and $w_2 = 2$.

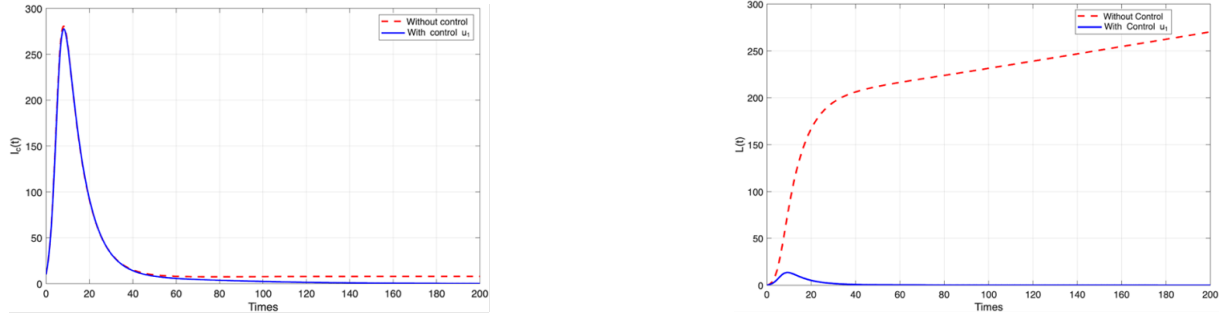


FIGURE 2. State variables without and with control.

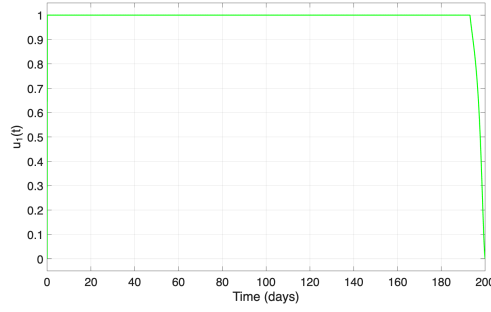


FIGURE 3. Optimal control profile for the disinfection rate.

In Figure 2, the implementation of disinfection control u_1 slightly reduces the peak of infected cows compared to the uncontrolled case. However, its most significant effect is observed on the environmental virus concentration $L(t)$, which is drastically suppressed and tends toward zero over time. In contrast, without control, the environmental viral load increases and stabilizes at a relatively high level. This indicates that disinfection is highly effective in reducing environmental contamination, although its direct impact on infected cows is limited.

The optimal control profile for disinfection, shown in Figure 3, reveals that the control variable u_1 remains at its upper bound for almost the entire time horizon. This suggests that the optimal strategy is to apply disinfection at maximum intensity throughout the intervention period. Biologically, this reflects the significant role of environmental transmission, where continuous and strong disinfection is required to effectively suppress the presence of pathogens in the environment. After optimization, the value of the objective functional for the first strategy is 22612192.79.

Next, for the second strategy, numerical simulations are performed over a 200-day period using only the culling control (u_2). With the infected cow and virus in the environment, the objective function uses control weights $a_1 = 5000$ and $a_2 = 50$, while the control cost weights are set to $w_1 = 0$ and $w_2 = 2$, respectively.

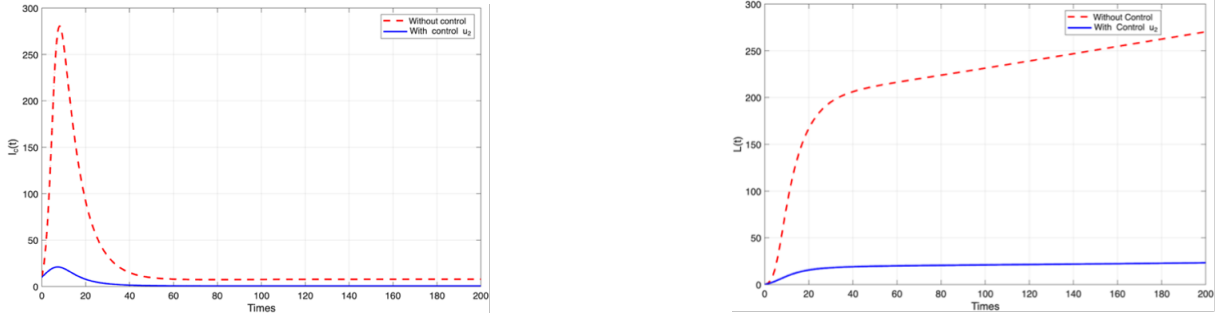


FIGURE 4. State variables without and with control.

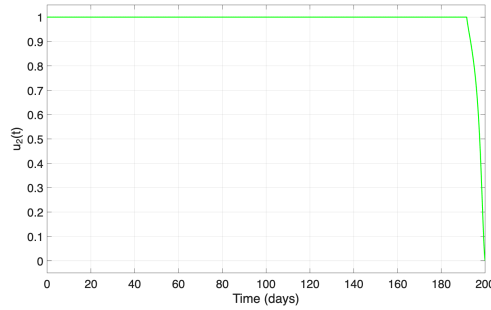


FIGURE 5. Optimal control profile for the culling rate.

Figure 4 presents the effect of the culling control u_2 on the system. The results demonstrate a substantial reduction in the number of infected cows when culling is applied, as the infected population rapidly declines to near zero. Additionally, the environmental virus concentration is also reduced, although not completely eliminated. This suggests that culling is highly effective in removing the primary source of infection—namely, infected cows—thereby significantly interrupting the transmission process.

The corresponding optimal control profile in Figure 5 shows that u_2 remains at its maximum level throughout the simulation period. This indicates that continuous and aggressive culling is the most effective strategy for controlling the outbreak. From a biological perspective, this highlights the importance of promptly removing infected individuals to prevent further disease

spread. The value of functional objectives after optimisation for second strategies is 2527107.68. The last strategies, numerical simulations are performed using the disinfection control (u_1) and culling control (u_2) for 200 days. With weights $w_1 = 2$ and $w_2 = 2$ in the objective function.

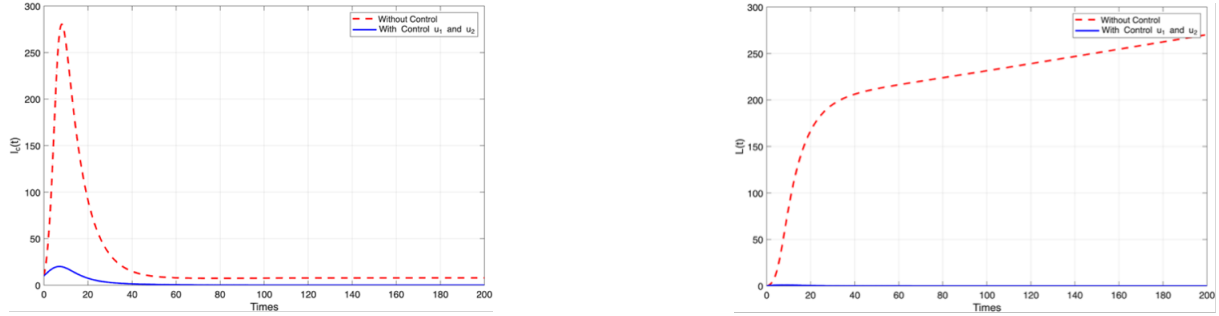


FIGURE 6. State variables without and with control.

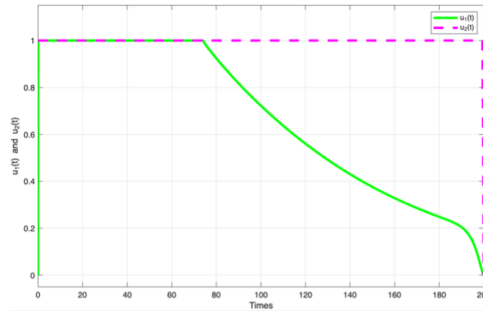


FIGURE 7. Optimal control profile for the disinfection rate and culling rate)

The combined control strategy, depicted in Figure 6, provides the most effective outcome among all scenarios. Both the infected cow population and the environmental virus concentration are reduced to negligible levels in a shorter time compared to single-control strategies. This demonstrates a synergistic effect between disinfection and culling, where the simultaneous removal of infection sources and environmental pathogens leads to optimal disease control.

Finally, Figure 7 illustrates the time-dependent behavior of both control variables under the combined strategy. The culling control $u_2(t)$ remains at its maximum value throughout the time period, while the disinfection control $u_1(t)$ starts at its peak and gradually decreases over time. This indicates that intensive disinfection is most crucial during the early stages of the outbreak, when environmental contamination is high, and can be reduced as the system approaches a disease-free state. These results suggest that optimal intervention

strategies should be dynamic, with strong early actions followed by gradual relaxation as the epidemic is brought under control. The value of functional objectives after optimization for last strategies is 19688420.77. From this study examines three specific strategies, including the value of functional objectives, for managing the dynamics of LSD. The best strategies is use both control with the value of functional objectives 19688420.77.

5. CONCLUSION

In this study, a mathematical model describing the transmission dynamics of LSD was analyzed, incorporating both environmental and host-related transmission pathways. The results show that different control strategies exhibit varying levels of effectiveness in reducing disease spread. Disinfection is highly effective in controlling environmental contamination, while culling plays a crucial role in reducing the number of infected cows. Among all strategies considered, the combined implementation of disinfection and culling provides the most efficient control outcome, leading to a rapid decline in both infection levels and environmental virus concentration. The optimal control analysis further suggests that intervention strategies should be dynamic. Intensive control measures are required during the early stages of an outbreak, particularly for disinfection, while sustained culling remains essential throughout the control period. Overall, these findings highlight the importance of integrated and time-dependent control strategies in managing infectious diseases. The results of this study can serve as a useful reference for policymakers in designing effective disease control programs, particularly in systems where both environmental and host transmission pathways are present.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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