Available online at http://scik.org
Commun. Math. Biol. Neurosci. 2013, 2013:2
ISSN 2052-2541

EXISTENCE AND STABILITY OF SOLUTIONS FOR A DIFFUSIVE PREDATOR-PREY MODEL WITH PREDATOR CANNIBALISM

YUJUAN JIAO

College of Mathematics and Computer Science, Northwest University for Nationalities,
Lanzhou 730124, China

Abstract. In this paper, we consider a diffusive predator-prey model with predator cannibalism. Using the energy estimates and Gagliardo-Nirenberg-type inequalities, the existence and uniform boundedness of global solutions for the model are proved. Meanwhile, the sufficient conditions for global asymptotic stability of the positive equilibrium for this model are given by constructing a Lyapunov function.

Keywords: cannibalism; cross-diffusion; global solution; uniform boundedness; stability.

2000 AMS Subject Classification: 35K57, 35B35, 92D25

1. Introduction

Cannibalism is a ubiquitous biological phenomenon, particularly important among many arthropod and fish species [9]. In a survey article on evolution and intraspecific predation, Polis demonstrated that cannibalism is an interesting and important mechanism in population dynamics [18]. Cushing listed three dynamic phenomena duo to cannibalism: (1) Stabilizing self-regulation of the cannibalistic population; (2) survival of the population in circumstances in which absence of cannibalism would result in extinction; (3) cannibalism can be the source of multiple steady states and consequently of hysteresis

Received April 30, 2013

effects [5]. In 2009, Sun, Zhang and Jin in [24] considered the following reaction diffusion equation with predator cannibalism

$$\begin{cases} u_{1t} - D_{u_1} \triangle u_1 = r u_1 (1 - \frac{u_1}{k}) - \frac{a u_1 u_2}{1 + a h u_1}, \\ u_{2t} - D_{u_2} \triangle u_2 = \frac{b a u_1 u_2}{1 + a h u_1} - e u_2 - \frac{a u_2}{1 + a h u_1}, \end{cases} \tag{1.1}$$

where a, b, e, h, k, r and $D_{u_i} (i = 1, 2)$ are positive constants, $u_1(x,t), u_2(x,t)$ denote the density of prey and predator species; r, k are the intrinsic growth rates and environment capacity, respectively. b is transform factor from prey to predator, h denotes handling time, e is death rate of the predator. D_{u_1} and D_{u_2} are called diffusion coefficients, $a u_2^2/(1 + a h u_1)$ is the rate of intra-specific competition of the predator. For more details on the backgrounds about this system, one can see [24].

Rescaling the system (1.1) such that

$u \mapsto a h u_1,\ v \mapsto (a/r) u_2,\ t \mapsto tr,\ l \mapsto b/(hr),\ m \mapsto ahk,\ s \mapsto d/r,\ d_1 \mapsto D_{u_1}/(ahr),\ d_2 \mapsto D_{u_2}/a$ yield

$$\begin{cases} u_t = d_1 \Delta u + u(1 - \frac{u}{m}) - \frac{uv}{1+u}, & x \in \Omega,\ t > 0, \\ v_t = d_2 \Delta v + \frac{luv}{1+u} - sv - \frac{v^2}{1+u}, & x \in \Omega,\ t > 0, \\ \partial_n u = \partial_n v = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x),\ v(x,0) = v_0(x), & x \in \Omega, \end{cases} \tag{1.2}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, n is the outward unit normal vector of the boundary $\partial\Omega$, $\partial_n = \partial/\partial n$. The functions $u_0(x)$ and $v_0(x)$ are nonnegative which are not identically zero.

The system (1.2) has a positive equilibrium $E^* = (u^*, v^*)$ if and only if

$$m(l - s) > s, \tag{1.3}$$

where

$$u^* = \frac{m(s - l + 1) - 1 + \sqrt{m^2(l - s - 1)^2 + 2m(l + s + 1) + 1}}{2},$$

$$v^* = (1 + u^*)(1 - \frac{u^*}{m}).$$

In [24], Turning pattern was proved under different assumptions by numerical simulation.

In recent years, the SKT type cross-diffusion systems have attracted the attention of a great number of investigators and have been successfully developed on the theoretical backgrounds. The above work mainly concentrates on: (1) The instability and stability induced by cross-diffusion, and the existence of nonconstant positive steady-state solutions [13, 19, 23]; (2) the global existence of strong solutions [7, 8, 10, 14, 15, 16, 25, 27]; (3) the global existence of weak solutions based on semi-discretization or finite element approximation [4, 6, 11, 12]; and (4) the dynamical behaviors [14, 15], etc.

We are concerned with the following predator-prey model with predator cannibalism with full cross-diffusion

$$\begin{cases} u_t = \Delta(d_1u + \alpha_{11}u^2 + \alpha_{12}uv) + u(1-\frac{u}{m}) - \frac{uv}{1+u}, & x \in \Omega,\ t>0, \\ v_t = \Delta(d_2v + \alpha_{21}uv + \alpha_{22}v^2) + \frac{luv}{1+u} - sv - \frac{v^2}{1+u}, & x \in \Omega,\ t>0, \\ \partial_n u = \partial_n v = 0, & x \in \partial\Omega,\ t>0, \\ u(x,0) = u_0(x),\ v(x,0) = v_0(x), & x \in \Omega, \end{cases} \tag{1.4}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, n is the outward unit normal vector of the boundary $\partial\Omega$. The initial data u_0 and v_0 are continuous nonnegative functions which are not identically zero. The homogeneous Neumann boundary condition indicates that the system is self-contained with zero population flux across the boundary. The parameters d_1, d_2 are the diffusion rates, α_{ii} $(i=1,2)$ are referred as self-diffusion pressures, and α_{ij} $(i,j=1,2,\ i\neq j)$ are cross-diffusion pressures. For more details on the backgrounds about self-diffusion and cross-diffusion, one can see [10].

The local existence of solutions for the system (1.4) is an immediate consequence of a series of important papers [1, 2, 3] by Amann. Roughly speaking, if $u_0(x)$ and $v_0(x)$ in $W_p^1(\Omega)$ with $p>n$, then (1.4) has a unique nonnegative solution $u, v \in C\big([0,T), W_p^1(\Omega)\big) \cap C^\infty\big((0,T), C^\infty(\Omega)\big)$, where $T \in (0,\infty]$ is the maximal existence time for the solution. If the solution (u,v) satisfies the estimate

$$\sup\left\{\|u(\cdot,t)\|_{W_p^1(\Omega)},\ \|v(\cdot,t)\|_{W_p^1(\Omega)} : 0<t<T\right\} < \infty,$$

then $T=+\infty$. Moreover, if $u_0(x), v_0(x)\in W_p^2(\Omega)$, then $u,v\in C\big([0,\infty), W_p^2(\Omega)\big)$.

For the following SKT system

$$\begin{cases} u_t = d_1\Delta[(1+\alpha v+\gamma u)u] + au(1-u-cv), & x\in\Omega,\ t>0,\\ v_t = d_2\Delta[(1+\delta v)v] + bv(1-du-v), & x\in\Omega,\ t>0,\\ \partial_n u = \partial_n v = 0, & x\in\partial\Omega,\ t>0,\\ u(x,0)=u_0(x),\ v(x,0)=v_0(x), & x\in\Omega. \end{cases} \tag{P}$$

Yamada in [27] proposed four open problems:

(1) The global existence of solutions of (P) in the case $\delta>0$ and the space dimension $N\geq 6$;

(2) the global existence in the case $\gamma=0$;

(3) in order to study the asymptotic behavior of u,v as $t\to\infty$, need to establish the uniform boundedness of global solutions; and

(4) the global existence of solutions for the following full SKT system

$$\begin{cases} u_t = d_1\Delta[(1+\alpha v+\gamma u)u] + au(1-u-cv), & x\in\Omega,\ t>0,\\ v_t = d_2\Delta[(1+\beta u+\delta v)v] + bv(1-du-v), & x\in\Omega,\ t>0,\\ \partial_n u = \partial_n v = 0, & x\in\partial\Omega,\ t>0,\\ u(x,0)=u_0(x),\ v(x,0)=v_0(x), & x\in\Omega \end{cases}$$

with $\alpha,\ \gamma,\ \beta,\ \delta>0$.

Very few global existence results for (1.4) are known. The main purpose of this paper is to establish the uniform boundedness of global solutions for the system (1.4) in one space dimension. For convenience, we consider the following system

$$\begin{cases} u_t = (d_1u+\alpha_{11}u^2+\alpha_{12}uv)_{xx} + u(1-\frac{u}{m}) - \frac{uv}{1+u}, & 0<x<1,\ t>0,\\ v_t = (d_2v+\alpha_{21}uv+\alpha_{22}v^2)_{xx} + \frac{luv}{1+u} - sv - \frac{v^2}{1+u}, & 0<x<1,\ t>0,\\ u_x(x,t)=v_x(x,t)=0, & x=0,1,\ t>0,\\ u(x,0)=u_0(x),\ v(x,0)=v_0(x), & 0<x<1. \end{cases} \tag{1.5}$$

We firstly investigate the global existence and the uniform boundedness of the solutions for (1.5), then prove the global asymptotic stability of the positive equilibrium (u^*, v^*) of (1.5) by an important lemma from [26]. The proof is complete and complement to the uniform convergence theorems in papers [20, 21, 22] .

It is obvious that (u^*, v^*) is the unique positive equilibrium of the system (1.5) if (1.3) holds.

For simplicity, we denote $\|\cdot\|_{W_p^k(0,1)}$ by $|\cdot|_{k,p}$ and $\|\cdot\|_{L^p(0,1)}$ by $|\cdot|_p$. Our main results are as follows.

Theorem 1.1. Let $u_0, v_0 \in W_2^2(0,1)$, (u,v) is the unique nonnegative solution of system (1.5) in the maximal existence interval $[0,T)$. Assume that

$$8\alpha_{11}\alpha_{21} > \alpha_{12}^2, \quad 8\alpha_{22}\alpha_{12} > \alpha_{21}^2. \tag{1.6}$$

Then there exist $t_0 > 0$ and positive constants M, M' which depend on $d_i, \alpha_{ij}, l, m, s$, such that

$$\sup\left\{|u(\cdot,t)|_{1,2},\ |v(\cdot,t)|_{1,2} : t \in (t_0,T)\right\} \le M', \tag{1.7}$$

$$\max\left\{u(x,t),\ v(x,t) : (x,t) \in [0,1]\times(t_0,T)\right\} \le M, \tag{1.8}$$

and $T = +\infty$. Moreover, in the case that $d_1, d_2 \ge 1,\ d_2/d_1 \in [\underline{d}, \overline{d}]$, where $\underline{d}$ and $\overline{d}$ are positive constants, M', M depend on $\underline{d}, \overline{d}$, but do not depend on d_1, d_2.

Remark 1.1. Since the continuous embedding $H^1(\Omega) \hookrightarrow L^\infty(\Omega)$ holds only in one dimensional space , we can only establish the uniform maximum-norn estimates about time for the solution in one dimensional space.

Theorem 1.2. Assume that all conditions in Theorem 1.1 are satisfied. Assume further that

$$4d_1d_2u^*v^* > M^2\left(\alpha_{12}u^* + \alpha_{21}v^*\right)^2, \tag{1.9}$$

$$4(1+u^*)^2 > m\left[4v^*(1+u^*) + (1+u^*+v^*-l)^2\right] \tag{1.10}$$

and (1.3) hold, M is given by (1.8). Then the unique positive equilibrium (u^*, v^*) of (1.5) is globally asymptotically stable.

Remark 1.2. The system (1.5) has no nonconstant positive steady-state if all conditions of Theorem 1.2 hold.

2. Global solutions

In order to establish the uniform W_2^1-estimates of the solutions for system (1.5), the following Gagliardo-Nirenberg-type inequalities and the corresponding corollary play important roles (see [17]).

Theorem 2.1. Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with $\partial\Omega \in C^m$. For every function $u \in W_r^m(\Omega), 1 \le q, r \le \infty$, the derivative $D^j u$ $(0 \le j < m)$ satisfies the inequality

$$|D^j u|_p \le C\big(|D^m u|_r^a |u|_q^{1-a} + |u|_q\big),$$

provided one of the following three conditions is satisfied: (1) $r \le q$, (2) $0 < n(r-q)/(mrq) < 1$, or (3) $n(r-q)/(mrq) = 1$ and $m - n/q$ is not a nonnegative integer, where $1/p = j/n + a(1/r - m/n) + (1-a)/q$ for all $a \in [j/m, 1)$, and the positive constant C depends on n, m, j, q, r, a.

Corollary 1. There exists a positive constant C such that

$$|u|_2 \le C\big(|u_x|_2^{1/3}|u|_1^{2/3} + |u|_1\big), \qquad \forall u \in W_2^1(0,1), \tag{2.1}$$

$$|u|_4 \le C\big(|u_x|_2^{1/2}|u|_1^{1/2} + |u|_1\big), \qquad \forall u \in W_2^1(0,1), \tag{2.2}$$

$$|u|_{\frac{5}{2}} \le C\big(|u_x|_2^{2/5}|u|_1^{3/5} + |u|_1\big), \qquad \forall u \in W_2^1(0,1), \tag{2.3}$$

$$|u_x|_2 \le C\big(|u_{xx}|_2^{3/5}|u|_1^{2/5} + |u|_1\big), \qquad \forall u \in W_2^2(0,1). \tag{2.4}$$

Throughout this paper, we always denote that C is Sobolev embedding constant or other kind of universal constant, A_j, B_j, C_j are some positive constants which depend only on α_{ij} $(i,j = 1,2), l, m$ and s, K_j are positive constants depending on d_i, α_{ij} $(i,j = 1,2), l, m$

and s . When $d_1, d_2 \geq 1, d_2/d_1 \in [\underline{d}, \overline{d}], K_j$ depend on $\underline{d}, \overline{d}$, but do not depend on d_1 and d_2.

Proof of Theorem 1.1. Step 1, estimate $|u|_1, |v|_1$.

Taking integration of the two equations in (1.5) over $(0,1)$, respectively, and letting $z = lu + v$, we have

$$\frac{d}{dt}\int_0^1 z dx = \int_0^1 \left[lu(1-\frac{u}{m}) - sv - \frac{v^2}{1+u} \right] dx \leq \int_0^1 \left(lu - \frac{l}{m}u^2 \right) dx \leq \frac{lm}{4}.$$

So, there exists a positive constant M_0 which depends on l, m and s, such that

$$\int_0^1 u\ dx, \int_0^1 v\ dx \leq M_0, \quad t \geq \tau_0. \tag{2.5}$$

Moreover, there exists a positive constant M_0' which depends on l, m, s and L^1-norm of u_0, v_0, such that

$$\int_0^1 u\ dx, \int_0^1 v\ dx \leq M_0', \quad t \geq 0. \tag{2.5$'$}$$

Step 2, estimate $|u|_2, |v|_2$.

Multiplying the first two equations in system (1.5) by u, v, respectively, and integrating over $(0,1)$, we have

$$\frac{1}{2}\frac{d}{dt}\int_0^1 u^2 dx \leq -d_1 \int_0^1 u_x^2 dx - \int_0^1 \left[(2\alpha_{11}u + \alpha_{12}v)u_x^2 + \alpha_{12}v_x u u_x \right] dx + \int_0^1 u^2 dx,$$
$$\frac{1}{2}\frac{d}{dt}\int_0^1 v^2 dx \leq -d_2 \int_0^1 v_x^2 dx - \int_0^1 \left[(\alpha_{21}u + 2\alpha_{22}v)v_x^2 + \alpha_{21}u_x v v_x \right] dx + l\int_0^1 v^2 dx,$$

from which it follows that

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\int_0^1 (u^2+v^2)dx &\leq -d\int_0^1 (u_x^2+v_x^2)dx + \int_0^1 u^2 dx + l\int_0^1 v^2 dx - \int_0^1 q(u_x,v_x)dx \\ &\leq -d\int_0^1 (u_x^2+v_x^2)dx + (1+l)\int_0^1 (u^2+v^2)dx - \int_0^1 q(u_x,v_x)dx,\end{aligned}$$

where $d = \min\{d_1, d_2\}$. Some tedious calculations yield that

$$q(u_x, v_x) = (2\alpha_{11}u + \alpha_{12}v)u_x^2 + (\alpha_{12}u + \alpha_{21}v)u_x v_x + (\alpha_{21}u + 2\alpha_{22}v)v_x^2$$

is positive definite quadratic form of u_x, v_x if (1.6) holds. So (1.6) implies that

$$\frac{1}{2}\frac{d}{dt}\int_0^1 \left(u^2+v^2\right) dx \leq -d\int_0^1 \left(u_x^2+v_x^2\right) dx + (1+l)\int_0^1 \left(u^2+v^2\right) dx. \tag{2.6}$$

Now, we proceed in the following two cases.

(i) $t \geq \tau_0$. The inequality (2.1) implies that $|u|_2^6 \leq C\left(|u_x|_2^2|u|_1^4 + |u|_1^6\right) \leq CM_0^4\left(|u_x|_2^2 + M_0^2\right)$. So we have $\int_0^1 u_x^2 dx \geq \frac{1}{CM_0^4}\left(\int_0^1 u^2 dx\right)^3 - M_0^2$, and

$$-\int_0^1 \left(u_x^2 + v_x^2\right) dx \leq -\frac{1}{9CM_0^4}\left[\int_0^1 (u^2+v^2)dx\right]^3 + 2M_0^2. \tag{2.7}$$

It follows from (2.6) and (2.7) that

$$\frac{1}{2}\frac{d}{dt}\int_0^1 \left(u^2+v^2\right) dx \leq d\left\{-C_2\left[\int_0^1 \left(u^2+v^2\right) dx\right]^3 + 2M_0^2 + \frac{1}{d}(1+l)\int_0^1 \left(u^2+v^2\right) dx\right\}. \tag{2.8}$$

This means that there exist positive constants τ_1 and M_1 depending on d_i $(i,j=1,2), l, m$ and s, such that

$$\int_0^1 u^2 dx, \int_0^1 v^2 dx \leq M_1,\ t \geq \tau_1. \tag{2.9}$$

When $d \geq 1$, M_1 is independent of d because the zero point of the right-hand side in (2.8) can be estimated by positive constants independent of d.

(ii) $t \geq 0$. Repeating estimates in (i) by $(2.5)'$, we can obtain that there exists a positive constant M_1' depending on d_i $(i,j=1,2), l, m, s$ and the L^1, L^2-norm of u_0, v_0, such that

$$\int_0^1 u^2 dx, \int_0^1 v^2 dx \leq M_1', \quad t \geq 0. \tag{2.9$'$}$$

When $d \geq 1, M_1'$ is independent of d.

Step 3, estimate $|u_x|_2, |v_x|_2$.

Introducing the following scaling

$$\widetilde{u} = \frac{u}{d_1},\ \widetilde{v} = \frac{v}{d_1},\ \widetilde{t} = d_1 t. \tag{2.10}$$

Denoting $\xi = d_2/d_1$, and using u, v, t instead of $\widetilde{u}, \widetilde{v}, \widetilde{t}$, respectively, then system (1.5) is reduced to

$$\begin{cases} u_t = P_{xx} + f(u,v), & 0<x<1,\ t>0, \\ v_t = Q_{xx} + g(u,v), & 0<x<1,\ t>0, \\ u_x(x,t) = v_x(x,t) = 0, & x=0,1,\ t>0, \\ u(x,0) = \widetilde{u}_0(x),\ v(x,0) = \widetilde{v}_0(x), & 0<x<1, \end{cases} \tag{2.11}$$

where

$P = u + \alpha_{11}u^2 + \alpha_{12}uv,\ Q = \xi v + \alpha_{21}uv + \alpha_{22}v^2,\ f(u,v) = d_1^{-1}u - \frac{u^2}{m} - \frac{uv}{1+d_1u},\ g(u,v) = \frac{luv}{1+d_1u} - sd_1^{-1}v - \frac{v^2}{1+d_1u}$.

We still proceed in the following two cases.

(i) $t \geq \tau_1^* = d_1\tau_1$. It is clear that

$$\begin{aligned}
&\int_0^1 u\ dx, \int_0^1 v\ dx \leq M_0 d_1^{-1}, \\
&\int_0^1 u^2 dx, \int_0^1 v^2 dx \leq M_1 d_1^{-2}, \\
&|P|_1, |Q|_1 \leq A_1 K_1 d_1^{-1},
\end{aligned} \tag{2.12}$$

where $K_1 = (1+\xi) + M_1 d_1^{-1}$ and $A_1 = \max\{M_0, \alpha_{11}+\alpha_{12}, \alpha_{21}+\alpha_{22}\}$.

Multiplying the first two equations in (2.11) by P_t, Q_t, integrating them over the domain $(0,1)$, respectively, and then adding up the two integration equalities, we have

$$\begin{aligned}
\frac{1}{2}\overline{y}'(t) = &-\int_0^1 u_t^2 dx - \xi\int_0^1 v_t^2 dx - \int_0^1 q(u_t, v_t)dx \\
&+ \int_0^1 \left[(1+2\alpha_{11}u+\alpha_{12}v)u_t f + \alpha_{12}uv_t f\right] dx + \int_0^1 \left[(\xi+\alpha_{21}u+2\alpha_{22}v)v_t g + \alpha_{21}vu_t g\right] dx
\end{aligned}$$

where $\overline{y}(t) = \int_0^1 (P_x^2 + Q_x^2)dx$. It is not hard verify by (1.6) that there exists a positive constant C_3 depending only on α_{ij} $(i,j=1,2)$, such that

$$q(u_t, v_t) \geq C_3(u+v)\left(u_t^2 + v_t^2\right).$$

Thus,

$$\begin{aligned}
\frac{1}{2}\overline{y}'(t) \leq &-\int_0^1 u_t^2 dx - \xi\int_0^1 v_t^2 dx - C_3\int_0^1 (u+v)(u_t^2+v_t^2)dx \\
&+ \int_0^1 \left[(1+2\alpha_{11}u+\alpha_{12}v)u_t f + \alpha_{12}uv_t f\right] dx + \int_0^1 \left[(\xi+\alpha_{21}u+2\alpha_{22}v)v_t g + \alpha_{21}vu_t g\right] dx.
\end{aligned} \tag{2.13}$$

Using Young inequality, Hölder inequality and (2.12), we can obtain the following estimates

$$
\begin{aligned}
&\int_0^1 u^3 \ \mathrm{d}x \le \left(\int_0^1 u^2 \ \mathrm{d}x\right)^{2/3} \left(\int_0^1 u^5 \ \mathrm{d}x\right)^{1/3} \le M_1^{2/3} d_1^{-4/3} \left(\int_0^1 u^5 \ \mathrm{d}x\right)^{1/3}, \\
&\int_0^1 u^4 \ \mathrm{d}x \le \left(\int_0^1 u^2 \ \mathrm{d}x\right)^{1/3} \left(\int_0^1 u^5 \ \mathrm{d}x\right)^{2/3} \le M_1^{1/3} d_1^{-2/3} \left(\int_0^1 u^5 \ \mathrm{d}x\right)^{2/3}, \\
&\int_0^1 uv^2 \ \mathrm{d}x \le \left(\int_0^1 u^2 \ \mathrm{d}x\right)^{1/2} \left(\int_0^1 v^2 \ \mathrm{d}x\right)^{1/6} \left(\int_0^1 v^5 \ \mathrm{d}x\right)^{1/3} \le M_1^{2/3} d_1^{-4/3} \left(\int_0^1 v^5 \ \mathrm{d}x\right)^{1/3}, \\
&\int_0^1 uv^3 \ \mathrm{d}x \le \frac{3}{4}\int_0^1 \left(u^4+v^4\right) \ \mathrm{d}x \le \frac{3}{4} M_1^{1/3} d_1^{-2/3} \left[\left(\int_0^1 u^5 \ \mathrm{d}x\right)^{2/3} + \left(\int_0^1 v^5 \ \mathrm{d}x\right)^{2/3}\right], \\
&\int_0^1 uv^4 \ \mathrm{d}x \le \frac{1}{5}\int_0^1 u^5 \ \mathrm{d}x + \frac{4}{5}\int_0^1 v^5 \ \mathrm{d}x \le \frac{4}{5}\int_0^1 \left(u^5+v^5\right) \ \mathrm{d}x, \\
&\int_0^1 u^2v^2 \ \mathrm{d}x \le \frac{1}{2}\int_0^1 u^4 \ \mathrm{d}x + \frac{1}{2}\int_0^1 v^4 \ \mathrm{d}x \le \frac{1}{2} M_1^{1/3} d_1^{-2/3} \left[\left(\int_0^1 u^5 \ \mathrm{d}x\right)^{2/3} + \left(\int_0^1 v^5 \ \mathrm{d}x\right)^{2/3}\right], \\
&\int_0^1 u^2v^3 \ \mathrm{d}x \le \frac{2}{5}\int_0^1 u^5 \ \mathrm{d}x + \frac{3}{5}\int_0^1 v^5 \ \mathrm{d}x \le \frac{3}{5}\int_0^1 \left(u^5+v^5\right) \ \mathrm{d}x.
\end{aligned}
\tag{2.14}
$$

Applying the above estimates and Gagliardo-Nirenberg-type inequalities to the terms on the right-hand side of (2.13), we have

$$
\begin{aligned}
&-\int_0^1 u_t^2 dx \le -\frac{1}{2}\int_0^1 P_{xx}^2 dx + \int_0^1 f^2 dx, \\
&-\xi \int_0^1 v_t^2 dx \le -\frac{\xi}{2}\int_0^1 Q_{xx}^2 dx + \xi \int_0^1 g^2 dx,
\end{aligned}
$$

$$
\begin{aligned}
\int_0^1 f^2 dx &\le \int_0^1 \left(d_1^{-2}u^2 + m^{-2}u^4 + u^2v^2 + 2m^{-1}u^3 v\right) dx \\
&\le M_1 d_1^{-4} + \left(m^{-2} + \frac{1}{2} + 2m^{-1}\right) M_1^{1/3} d_1^{-2/3} \left[\left(\int_0^1 u^5 \mathrm{d}x\right)^{2/3} + \left(\int_0^1 v^5 \mathrm{d}x\right)^{2/3}\right],
\end{aligned}
$$

$$
\begin{aligned}
\xi \int_0^1 g^2 dx \leq & \, \xi \int_0^1 \left(l^2 u^2 v^2 + s^2 d_1^{-2} v^2 + v^4 + 2 s d_1^{-1} v^3\right) dx \\
\leq & \, \xi s^2 M_1 d_1^{-4} + 2 \xi s M_1^{2/3} d_1^{-7/3} \left(\int_0^1 v^5 \mathrm{d}x\right)^{1/3} \\
& + (\frac{1}{2}\xi l^2 + \xi) M_1^{1/3} d_1^{-2/3} \left[\left(\int_0^1 u^5 \mathrm{d}x\right)^{2/3} + \left(\int_0^1 v^5 \mathrm{d}x\right)^{2/3}\right]
\end{aligned}
$$

and

$$
\begin{aligned}
& - \int_0^1 u_t^2 dx - \xi \int_0^1 v_t^2 dx \\
\leq & - \frac{1}{2} \int_0^1 P_{xx}^2 dx - \frac{\xi}{2} \int_0^1 Q_{xx}^2 dx + \left(1 + \xi s^2\right) M_1 d_1^{-4} + 2 \xi s M_1^{2/3} d_1^{-7/3} \left(\int_0^1 v^5 \mathrm{d}x\right)^{1/3} \\
& + \left(m^{-2} + \frac{1}{2} + 2m^{-1} + \frac{1}{2}\xi l^2 + \xi\right) M_1^{1/3} d_1^{-2/3} \left[\left(\int_0^1 u^5 \mathrm{d}x\right)^{2/3} + \left(\int_0^1 v^5 \mathrm{d}x\right)^{2/3}\right].
\end{aligned}
\tag{2.15}
$$

Similarly, we can obtain

$$
\begin{aligned}
& \int_0^1 u_t f dx \\
\leq & \int_0^1 u_t \left(d_1^{-1} u + m^{-1} u^2 + uv\right) dx \\
\leq & \frac{d_1^{-2}}{2\varepsilon} \int_0^1 u dx + \frac{\varepsilon}{2} \int_0^1 u u_t^2 dx + \frac{m^{-2}}{2\varepsilon} \int_0^1 u^3 dx + \frac{\varepsilon}{2} \int_0^1 u u_t^2 dx + \frac{1}{2\varepsilon} \int_0^1 u v^2 dx + \frac{\varepsilon}{2} \int_0^1 u u_t^2 dx \\
\leq & \frac{1}{2\varepsilon} M_0 d_1^{-3} + \frac{1 + m^{-2}}{2\varepsilon} M_1^{2/3} d_1^{-4/3} \left[\left(\int_0^1 u^5 \mathrm{d}x\right)^{1/3} + \left(\int_0^1 v^5 \mathrm{d}x\right)^{1/3}\right] + \frac{3\varepsilon}{2} \int_0^1 u u_t^2 dx,
\end{aligned}
$$

$$
\begin{aligned}
& 2\alpha_{11} \int_0^1 u u_t f dx \\
\leq & 2\alpha_{11} \int_0^1 u u_t \left(d_1^{-1} u + m^{-1} u^2 + uv\right) dx \\
\leq & \frac{\alpha_{11}^2 d_1^{-2}}{\varepsilon} \int_0^1 u^3 dx + \varepsilon \int_0^1 u u_t^2 dx + \frac{\alpha_{11}^2 m^{-2}}{\varepsilon} \int_0^1 u^5 dx + \varepsilon \int_0^1 u u_t^2 dx \quad + \frac{\alpha_{11}^2}{\varepsilon} \int_0^1 u^3 v^2 dx + \varepsilon \int_0^1 u u_t^2 dx \\
\leq & \frac{\alpha_{11}^2}{\varepsilon} M_1^{2/3} d_1^{-10/3} \left(\int_0^1 u^5 dx\right)^{1/3} + \frac{\alpha_{11}^2 (1 + m^{-2})}{\varepsilon} \left(\int_0^1 u^5 \mathrm{d}x + \int_0^1 v^5 \mathrm{d}x\right) + 3\varepsilon \int_0^1 u u_t^2 dx,
\end{aligned}
$$

$$
\begin{aligned}
&\alpha_{12}\int_0^1 vu_t f dx\\
\leq&\alpha_{12}\int_0^1 vu_t\left(d_1^{-1}u+m^{-1}u^2+uv\right)dx\\
\leq&\frac{\alpha_{12}^2 d_1^{-2}}{2\varepsilon}\int_0^1 uv^2dx+\frac{\varepsilon}{2}\int_0^1 uu_t^2dx+\frac{\alpha_{12}^2 m^{-2}}{2\varepsilon}\int_0^1 u^3v^2dx+\frac{\varepsilon}{2}\int_0^1 uu_t^2dx+\frac{\alpha_{12}^2}{2\varepsilon}\int_0^1 uv^4dx+\frac{\varepsilon}{2}\int_0^1 uu_t^2dx\\
\leq&\frac{\alpha_{12}^2}{2\varepsilon}M_1^{2/3}d_1^{-10/3}\left(\int_0^1 u^5dx\right)^{1/3}+\frac{\alpha_{12}^2(1+m^{-2})}{2\varepsilon}\left(\int_0^1 u^5\mathrm{d}x+\int_0^1 v^5\mathrm{d}x\right)+\frac{3\varepsilon}{2}\int_0^1 uu_t^2dx,
\end{aligned}
$$

$$
\begin{aligned}
&\alpha_{12}\int_0^1 uv_t f dx\\
\leq&\alpha_{12}\int_0^1 uv_t\left(d_1^{-1}u+m^{-1}u^2+uv\right)dx\\
\leq&\frac{\alpha_{12}^2 d_1^{-2}}{2\varepsilon}\int_0^1 u^3dx+\frac{\varepsilon}{2}\int_0^1 uv_t^2dx+\frac{\alpha_{12}^2 m^{-2}}{2\varepsilon}\int_0^1 u^5dx+\frac{\varepsilon}{2}\int_0^1 uv_t^2dx+\frac{\alpha_{12}^2}{2\varepsilon}\int_0^1 u^3v^2dx+\frac{\varepsilon}{2}\int_0^1 uv_t^2dx\\
\leq&\frac{\alpha_{12}^2}{2\varepsilon}M_1^{2/3}d_1^{-10/3}\left(\int_0^1 u^5dx\right)^{1/3}+\frac{\alpha_{12}^2(1+m^{-2})}{2\varepsilon}\left(\int_0^1 u^5\mathrm{d}x+\int_0^1 v^5\mathrm{d}x\right)+\frac{3\varepsilon}{2}\int_0^1 uv_t^2dx,
\end{aligned}
$$

$$
\begin{aligned}
&\xi\int_0^1 v_t g dx\\
\leq&\ \xi\int_0^1 v_t\left(luv+sd_1^{-1}v+v^2\right)dx\\
\leq&\frac{\xi^2 l^2}{2\varepsilon}\int_0^1 u^2v\ dx+\frac{\varepsilon}{2}\int_0^1 vv_t^2dx+\frac{\xi^2 s^2 d_1^{-2}}{2\varepsilon}\int_0^1 v\ dx+\frac{\varepsilon}{2}\int_0^1 vv_t^2dx+\frac{\xi^2}{2\varepsilon}\int_0^1 v^3dx+\frac{\varepsilon}{2}\int_0^1 vv_t^2dx\\
\leq&\frac{\xi^2 s^2}{2\varepsilon}M_0 d_1^{-3}+\frac{\xi^2\left(1+l^2\right)}{2\varepsilon}M_1^{2/3}d_1^{-4/3}\left[\left(\int_0^1 u^5\mathrm{d}x\right)^{1/3}+\left(\int_0^1 v^5\mathrm{d}x\right)^{1/3}\right]+\frac{3\varepsilon}{2}\int_0^1 vv_t^2dx,
\end{aligned}
$$

$$
\begin{aligned}
&\alpha_{21}\int_0^1 uv_t g dx\\
\leq&\alpha_{21}\int_0^1 uv_t\left(luv+sd_1^{-1}v+v^2\right)dx\\
\leq&\frac{\alpha_{21}^2 l^2}{2\varepsilon}\int_0^1 u^4v\ dx+\frac{\varepsilon}{2}\int_0^1 vv_t^2dx+\frac{\alpha_{21}^2 s^2 d_1^{-2}}{2\varepsilon}\int_0^1 u^2v\ dx+\frac{\varepsilon}{2}\int_0^1 vv_t^2dx+\frac{\alpha_{21}^2}{2\varepsilon}\int_0^1 u^2v^3dx+\frac{\varepsilon}{2}\int_0^1 vv_t^2dx\\
\leq&\frac{\alpha_{21}^2 s^2}{2\varepsilon}M_1^{2/3}d_1^{-10/3}\left(\int_0^1 u^5dx\right)^{1/3}+\frac{\alpha_{21}^2(1+l^2)}{2\varepsilon}\left(\int_0^1 u^5\mathrm{d}x+\int_0^1 v^5\mathrm{d}x\right)+\frac{3\varepsilon}{2}\int_0^1 vv_t^2dx,
\end{aligned}
$$

$$
\begin{aligned}
&2\alpha_{22}\int_0^1 vv_t g dx\\
\leq &2\alpha_{22}\int_0^1 vv_t\left(luv+sd_1^{-1}v+v^2\right)dx\\
&\leq \frac{\alpha_{22}^2 l^2}{\varepsilon}\int_0^1 u^2v^3dx+\varepsilon\int_0^1 vv_t^2dx+\frac{\alpha_{22}^2s^2d_1^{-2}}{\varepsilon}\int_0^1 v^3dx+\varepsilon\int_0^1 vv_t^2dx+\frac{\alpha_{22}^2}{\varepsilon}\int_0^1 v^5dx+\varepsilon\int_0^1 vv_t^2dx\\
&\leq \frac{\alpha_{22}^2s^2}{\varepsilon}M_1^{2/3}d_1^{-10/3}\left(\int_0^1 u^5dx\right)^{1/3}+\frac{\alpha_{22}^2(1+l^2)}{\varepsilon}\left(\int_0^1 u^5\mathrm{d}x+\int_0^1 v^5\mathrm{d}x\right)+3\varepsilon\int_0^1 vv_t^2dx,
\end{aligned}
$$

$$
\begin{aligned}
&\alpha_{21}\int_0^1 vu_t g dx\\
\leq &\alpha_{21}\int_0^1 vu_t\left(luv+sd_1^{-1}v+v^2\right)dx\\
\leq &\frac{\alpha_{21}^2 l^2}{2\varepsilon}\int_0^1 u^2v^3dx+\frac{\varepsilon}{2}\int_0^1 vu_t^2dx+\frac{\alpha_{21}^2s^2d_1^{-2}}{2\varepsilon}\int_0^1 v^3dx+\frac{\varepsilon}{2}\int_0^1 vu_t^2dx+\frac{\alpha_{21}^2}{2\varepsilon}\int_0^1 v^5dx+\frac{\varepsilon}{2}\int_0^1 vu_t^2dx\\
\leq &\frac{\alpha_{21}^2s^2}{2\varepsilon}M_1^{2/3}d_1^{-10/3}\left(\int_0^1 v^5dx\right)^{1/3}+\frac{\alpha_{21}^2(1+l^2)}{2\varepsilon}\left(\int_0^1 u^5\mathrm{d}x+\int_0^1 v^5\mathrm{d}x\right)+\frac{3\varepsilon}{2}\int_0^1 vu_t^2dx.
\end{aligned}
$$

By the above inequalities and the condition (1.6), we have

$$
\begin{aligned}
&\int_0^1\left[\left(1+2\alpha_{11}u+\alpha_{12}v\right)u_tf+\alpha_{12}uv_tf\right]dx+\int_0^1\left[\left(\xi+\alpha_{21}u+2\alpha_{22}v\right)v_tg+\alpha_{21}vu_tg\right]dx\\
&\quad\leq \overline{\lambda}\varepsilon\int_0^1(u+v)\left(u_t^2+v_t^2\right)dx+\frac{C_4}{\varepsilon}\left(1+\xi^2\right)M_0d_1^{-3}\\
&\qquad+\frac{C_5}{\varepsilon}\left(1+\xi^2+d_1^{-2}\right)M_1^{2/3}d_1^{-4/3}\left[\int_0^1\left(u^5+v^5\right)dx\right]^{1/3}+\frac{C_6}{\varepsilon}\int_0^1\left(u^5+v^5\right)dx,
\end{aligned}
\tag{2.16}
$$

where $\overline{\lambda}$ is a constant. Choose a small enough positive number $\varepsilon(\alpha_{ij},l,m,s)$ $(i,j=1,2)$, such that $\overline{\lambda}\varepsilon<C_3$. Substituting inequalities (2.15) and (2.16) into (2.13), one can obtain

$$
\frac{1}{2}\overline{y}'(t)\leq-\frac{1}{2}\int_0^1 P_{xx}^2dx-\frac{\xi}{2}\int_0^1 Q_{xx}^2dx+B_1K_2d_1^{-3}+B_2K_3d_1^{-4/3}Y^{1/3}+B_3K_4d_1^{-2/3}Y^{2/3}+B_4Y,
\tag{2.17}
$$

where

$Y=\int_0^1\left(u^5+v^5\right)dx$, $K_2=(1+\xi^2)M_0+(1+\xi)M_1d_1^{-1}$, $K_3=\left(1+\xi^2+d_1^{-2}+\xi d_1^{-1}\right)M_1^{2/3}$, $K_4=(1+\xi)M_1^{1/3}$.

Clearly,

$$
P\geq\alpha_{11}u^2,\quad Q\geq\alpha_{22}v^2.
$$

It follows from (2.12) and (2.3) to functions $P,\ Q$ that

$$\begin{aligned}
&Y \le B_5 \int_0^1 \left(P^{5/2} + Q^{5/2}\right) dx \le B_6 K_1^{3/2} d_1^{-3/2}\ \overline{y}^{1/2} + B_6 K_1^{5/2} d_1^{-5/2},\\
&Y^{1/3} \le B_7 K_1^{1/2} d_1^{-1/2}\ \overline{y}^{1/6} + B_7 K_1^{5/6} d_1^{-5/6},\\
&Y^{2/3} \le B_8 K_1 d_1^{-1}\ \overline{y}^{1/3} + B_8 K_1^{5/3} d_1^{-5/3}.
\end{aligned} \tag{2.18}$$

Moreover, one can obtain by (2.4) and (2.12) that

$$-\frac{1}{2}\int_0^1 P_{xx}^2 dx - \frac{\xi}{2}\int_0^1 Q_{xx}^2 dx \le -B_9 \min\left\{1,\ \xi\right\} K_1^{-4/3} d_1^{4/3} \overline{y}^{5/3} + (1+\xi)\, K_1^2 d_1^{-2}. \tag{2.19}$$

Combining (2.16), (2.18) and (2.19), we have

$$\begin{aligned}
\frac{1}{2}\overline{y}'(t) \le &- A_1 \min\{1,\ \xi\} K_1^{-4/3} d_1^{4/3} \overline{y}^{5/3}\\
&+ A_2\left[(1+\xi) K_1^2 d_1^{-2} + K_2 d_1^{-3} + K_1^{5/2} d_1^{-5/2} + K_3 d_1^{-13/6} + K_1^{5/2} d_1^{-5/2}\right]\\
&+ A_3 K_1^{3/2} d_1^{-3/2}\ \overline{y}^{1/2} + A_4 K_1 K_4 d_1^{-3/2} \overline{y}^{1/3} + A_5 K_1^{1/2} K_3 d_1^{-11/6}\ \overline{y}^{1/6}.
\end{aligned} \tag{2.20}$$

Multiplying inequality (2.20) by d_1^2, we have

$$\begin{aligned}
\frac{1}{2} y'(t) \le &- A_1 \min\{1,\ \xi\} K_1^{-4/3} y^{5/3}\\
&+ A_2\left[(1+\xi)\, K_1^2 + K_2 d_1^{-1} + K_1^{5/2} d_1^{-1/2} + K_3 d_1^{-1/6} + K_1^{5/2} d_1^{-1/2}\right]\\
&+ A_3 K_1^{3/2} d_1^{-1/2} y^{1/2} + A_4 K_1 K_4 d_1^{-1/6} y^{1/3} + A_5 K_1^{1/2} K_3 d_1^{-1/6} y^{1/6},
\end{aligned} \tag{2.21}$$

where $y = \int_0^1 \left[(d_1 P_x)^2 + (d_1 Q_x)^2\right] dx$. The inequality (2.21) implies that there exist $\widetilde{\tau}_2 > 0$ and positive constant $\widetilde{M}_2$ depending on $d_i, \alpha_{ij}\ (i,j = 1,2), l, m$ and s, such that

$$\int_0^1 (d_1 P_x)^2\, dx, \int_0^1 (d_1 Q_x)^2\, dx \le \widetilde{M}_2,\ t \ge \widetilde{\tau}_2. \tag{2.22}$$

In the case that $d_1, d_2 \ge 1$, $\xi \in [\underline{d}, \overline{d}]$, the coefficients of inequality (2.20) can be estimated by some constants which depend on $\underline{d}, \overline{d}$, but do not depend on d_1, d_2. So $\widetilde{M}_2$ depends on $\alpha_{ij}\ (i,j = 1,2), l, m, s, \underline{d}$ and $\overline{d}$, but it is irrelevant to d_1, d_2, when $d_1, d_2 \ge 1$ and $\xi \in [\underline{d}, \overline{d}]$. Since

$$\begin{pmatrix} P_x \\ Q_x \end{pmatrix} = \begin{pmatrix} P_u & P_v \\ Q_u & Q_v \end{pmatrix} \begin{pmatrix} u_x \\ v_x \end{pmatrix},$$

we can transform the formulations of u_x, v_x into fraction representations, then distribute the denominators of the absolute value of the fractions to the numerators item and enlarge the term concerning with u_x, v_x to obtain

$$|d_1 u_x| + |d_1 v_x| \le L\big(|d_1 P_x| + |d_1 Q_x|\big), \quad 0 < x < 1, t > 0, \tag{2.23}$$

where L is a constant depending only on ξ, α_{ij} $(i,j = 1,2)$. After scaling back and contacting estimates (2.22) and (2.23), there exist positive constant M_2 depending on d_i, α_{ij} $(i,j = 1,2), l, m, s$ and $\tau_2 > 0$, such that

$$\int_0^1 u_x^2 dx, \int_0^1 v_x^2 dx \le M_2, \quad t \ge \tau_2. \tag{2.24}$$

When $d_1, d_2 \ge 1$ and $\xi \in \big[\underline{d}, \overline{d}\big]$, M_2 is independent of d_1, d_2.

(ii) $t \ge 0$. Modifying the dependency of the coefficients in inequalities (2.12)-(2.14), namely replacing M_0, M_1 with M_0', M_1', there exists a positive constant M_2' depending on d_i, α_{ij} $(i,j = 1,2), l, m, s$ and the W_2^1-norm of u_0, v_0, such that

$$\int_0^1 u_x^2 dx, \int_0^1 v_x^2 dx \le M_2', \quad t \ge 0. \tag{2.24$'$}$$

Furthermore, in the case that $d_1, d_2 \ge 1$, $\xi \in \big[\underline{d}, \overline{d}\big]$, M_2' depends on $\underline{d}, \overline{d}$, but does not depend on d_1, d_2.

Summarizing estimates $(2.5), (2.9), (2.24)$ and Sobolev embedding theorem, there exist positive constants M, M' depending only on d_i, α_{ij} $(i,j = 1,2), l, m$ and s, such that (1.10) and (1.11) hold. In particular, $M, M^{'}$ depend only on α_{ij} $(i,j = 1,2), l, m, s, \underline{d}$ and $\overline{d}$, but do not depend on d_1, d_2, when $d_1, d_2 \ge 1$ and $\xi \in \big[\underline{d}, \overline{d}\big]$. Similarly, according to $(2.5)', (2.9)', (2.24)'$, there exists a positive constant M'' depending on d_i, α_{ij} $(i,j = 1,2), l, m, s$ and the initial functions u_0, v_0 , such that

$$|u(\cdot,t)|_{1,2}, |v(\cdot,t)|_{1,2} \le M'', \quad t \ge 0.$$

Further, in the case that $d_1, d_2 \ge 1$, $\xi \in \big[\underline{d}, \overline{d}\big]$, M'' depends only on $\underline{d}, \overline{d}$, but does not depend on d_1, d_2. Thus, $T = +\infty$. This completes the proof of Theorem 1.1.

3. Global stability

In order to obtain the uniform convergence of the solution to system (1.2), we recall the following result which can be found in [26].

Lemma 3.1. Let a and b be positive constants. Assume that $\varphi, \psi \in C^1\left([a,+\infty)\right), \psi(t) \geq 0$ and φ is bounded from below. If $\varphi'(t) \leq -b\psi(t)$ and $\psi'(t)$ is bounded from above in $[a,+\infty)$, then $\lim\limits_{t\to\infty} \psi(t) = 0$.

Proof of Theorem 1.2. Let (u,v) be a solution for the system (1.5) with initial functions $u_0(x), v_0(x) \geq 0$. From the strong maximum principle for parabolic equations, it is not hard to verify that $u, v > 0$ for $t > 0$. Define the function

$$H(u,v) = \int_0^1 \left(u - u^* - u^* \ln \frac{u}{u^*}\right) dx + \int_0^1 \left(v - v^* - v^* \ln \frac{v}{v^*}\right) dx.$$

Then the time derivative of $H(u,v)$ for the system (1.5) satisfies

$$\begin{aligned}
&\frac{dH}{dt} = \int_0^1 \frac{u-u^*}{u} u_t dx + \int_0^1 \frac{v-v^*}{v} v_t dx \\
= &-\int_0^1 \left[\frac{u^*}{u^2}\left(d_1 + 2\alpha_{11}u + \alpha_{12}v\right)u_x^2 + (\frac{\alpha_{12}u^*}{u} + \frac{\alpha_{21}v^*}{v})u_x v_x + \frac{\alpha v^*}{v^2}\left(d_2 + \alpha_{21}u + 2\alpha_{22}v\right)v_x^2\right]dx \\
&- \int_0^1 \left[\left(\frac{1}{m} - \frac{v^*}{(1+u^*)(1+u)}\right)(u-u^*)^2 + \frac{1+u^*+v^*-l}{(1+u^*)(1+u)}(u-u^*)(v-v^*) + \frac{1}{1+u}(v-v^*)^2\right] dx.
\end{aligned} \tag{3.1}$$

The first integrand in the right hand of (3.1) is positive definite if

$$4u^*v^*(d_1 + 2\alpha_{11}u + \alpha_{12}v)(d_2 + \alpha_{21}u + 2\alpha_{22}v) > \ (\alpha_{12}u^*v + \alpha_{21}v^*u)^2, \tag{3.2}$$

and the condition (1.9) implies (3.2). The second integrand in the right hand of (3.1) is positive definite if

$$4(1+u^*)\left[(1+u^*)(1+u) - mv^*\right] > m\left(1+u^*+v^*-l\right)^2, \tag{3.3}$$

and the condition (1.10) implies (3.3). Consequently, there exists $\delta > 0$, such that

$$\frac{dH}{dt} \leq -\delta \int_0^1 [(u-u^*)^2 + (v-v^*)^2]dx, \quad \frac{dH}{dt} \leq\ 0, \quad (u,v) \neq (u^*,v^*). \tag{3.4}$$

By the maximum-norm estimate in Theorem 1.1 and some tedious calculations, we can verify $(d/dt)\int_0^1 [(u-u^*)^2 + (v-v^*)^2]dx$ is bounded from above. Then from lemma 3.1

and (3.4), we obtain

$$\lim_{t\to\infty}\int_0^1 (u-u^*)^2\,dx = \lim_{t\to\infty}\int_0^1 (v-v^*)^2\,dx = 0. \tag{3.5}$$

It follows from (3.5) and Gagliardo-Nirenberg-type inequality $|u|_\infty \le C|u|_{1,2}^{1/2}|u|_2^{1/2}$ that (u,v) converges uniformly to (u^*,v^*). By the fact that $H(u,v)$ is decreasing for $t\ge 0$, it is obvious that (u^*,v^*) is globally asymptotically stable. So the proof of Theorem 1.2 is completed.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (No. 11161041, 31260098) and the Fundamental Research Funds for the Central Universities (No. 31920130004, zyz2012084).

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