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PERMANENCE OF A GILPIN-AYALA COMPETITION SYSTEM WITH INFINITE DELAYS AND SINGLE FEEDBACK CONTROL

DANHONG WANG

Department of Mathematics, Minjiang University, Fuzhou, Fujian 350108, China

Abstract. In this paper, a two species Gilpin-Ayala competition system with infinite delays and single feedback control variable is studied. The sufficient conditions for permanence are obtained. Our result shows that by choosing suitable feedback control variable, the extinct species in original system could become permanent.

Keywords: Permanence; Gilpin-Ayala competition system; Infinite delay; Feedback control.

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1. Introduction

Traditional Lotka-Volterra competition system can be expressed as follows

$$x_i'(t) = x_i(t)[b_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t)], \;\; i = 1,2,...,n. \tag{1}$$

As we well know,system (1) has been studied extensively,many excellent results of system (1) are obtained; see [1-6,10,13-14] and the references therein. In 1973, Ayala et al. proposed the

*Corresponding author
E-mail address: dingdongwdh@126.com
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following competition system by experiment.

$$
\begin{aligned}
x_1'(t) &= r_1x_1(t)\big[1-(\tfrac{x_1(t)}{k_1})^{\theta_1}-\alpha_{12}\tfrac{x_2(t)}{k_1}\big],\\
x_2'(t) &= r_2x_2(t)\big[1-\alpha_{21}\tfrac{x_1(t)}{k_2}-(\tfrac{x_2(t)}{k_2})^{\theta_2}\big].
\end{aligned}
\tag{2}
$$

Gilpin-Ayala competition system also has been studied from different aspects by many researhers; see [7-12,15-16] and the references therein. Chen [9] proposed the following system

$$
\begin{aligned}
x_1'(t) &= r_1x_1(t)\big[1-(\tfrac{x_1(t)}{k_1})^{\theta_1}-\alpha_{12}\textstyle\int_{-\infty}^{t}\tfrac{x_2(s)}{k_1}dh_2(t-s)\big],\\
x_2'(t) &= r_2x_2(t)\big[1-\alpha_{21}\textstyle\int_{-\infty}^{t}\tfrac{x_1(s)}{k_2}dh_1(t-s)-(\tfrac{x_2(t)}{k_2})^{\theta_2}\big].
\end{aligned}
\tag{3}
$$

and systematically discussed the stability of system (3). Wang [15] studied system (3). We assumed the following inequalities

$$(H_1): k_1>\alpha_{12}k_2, k_2\le\alpha_{21}k_1; \theta_1\ge 1, \theta_2\le 1;$$

hold,and proved the extinction of the system,that is,for any solution $(x_1(t),x_2(t))^T$ of system (3), one has $\lim\limits_{t\to+\infty}x_2(t)=0, \lim\limits_{t\to+\infty}x_1(t)=k_1$.

On the other hand, ecosystem in the real world are continuously disturbed by unpredictable forces which can result in changes in the biological parameters such as survival rates. In the language of control variables,we call the disturbance functions as control variables. In some cases, one may choose a single control strategy,and such a strategy has influence on both species.For instance, in the medical system, when we take chemotherapeutic drugs for cancer patients,cancer cells will decrease rapidly, but at the same time, drugs do harm to normal cells and body's immune function; see [17] and the references therein.

Above analysis motivated us to propose the following two species competitive system with single feedback control variable

$$
\begin{aligned}
&x_1'(t)=r_1x_1(t)\big(1-(\tfrac{x_1(t)}{k_1})^{\theta_1}-\alpha_{12}\textstyle\int_{-\infty}^{t}\tfrac{x_2(s)}{k_1}dh_2(t-s)-c_1\int_{-\infty}^{t}u(s)dk_1(t-s)\big),\\
&x_2'(t)=r_2x_2(t)\big(1-\alpha_{21}\textstyle\int_{-\infty}^{t}\tfrac{x_1(s)}{k_2}dh_1(t-s)-(\tfrac{x_2(t)}{k_2})^{\theta_2}+c_2\int_{-\infty}^{t}u(s)dk_2(t-s)\big),\\
&u'(t)=f-eu(t)+d_1\textstyle\int_{-\infty}^{t}x_1(s)dh_3(t-s)-d_2\int_{-\infty}^{t}x_2(s)dh_4(t-s).
\end{aligned}
\tag{4}
$$

here the coefficients $r_i,k_i,\theta_i,\alpha_{ij},c_i,d_i,f,e,(i,j=1,2,i\neq j)$ are all positive constants. The delay kernels: $h_i(s):[0,\infty)\to R,(i=1,2,3,4),k_i(s):[0,\infty)\to R,(i=1,2)$ are non-increasing functions of bounded variation such that

$$\int_0^\infty dh_i(s) = -1 \text{ and } \int_0^\infty dk_i(s) = -1.$$

we shall consider system (4) together with the initial conditions

$$\begin{aligned} &x_i(s) = \phi_i(s) > 0, s \in (-\infty, 0], i = 1,2. \\ &u(s) = \psi(s) > 0, s \in (-\infty, 0]. \end{aligned} \tag{5}$$

here $\phi_i, \psi \in BC^+ = \big\{\phi \in \big(C(-\infty,0],[0,\infty)\big) : \phi(0) > 0 \text{ and } \phi \text{ is bounded } \big\}, (i = 1,2)$.
Now,we further assume the coefficients of the system (4) satisfy the inequalities:

$$(H_2) : 1 - \frac{\alpha_{12}k_2}{k_1}(1 + \frac{c_2(f + d_1k_1)}{e})^{\frac{1}{\theta_2}} > c_1(\frac{f + d_1k_1}{e});$$

$$\frac{c_2}{e}(f - d_2k_2((1 + \frac{c_2(f + d_1k_1)}{e})^{\frac{1}{\theta_2}})) > \frac{\alpha_{21}k_1}{k_2} - 1.$$

It is well known that by the fundamental theory of functional differential equations [18],system (4) has a unique solution $(x_1(t), x_2(t), u(t))^T$ satisfying the initial condition (5).We can easily prove $x_i(t) > 0, i = 1,2, u(t) > 0$ in maximal interval of existence of the solution.In this paper, the solution of system (4) satisfying the initial conditions (5) is said to be positive.

The aim of this paper is, by further developing the analysis technique of Francisco Montes de Oca and Miguel Vivas and using the differential inequality theory,to obtain a set of sufficient conditions to ensure the permanence of the system (4).Our result will show that (5), (H_1) and (H_2) will ensure the permanence of the system (4).This means that for system (3), feedback control is an effective method to avoid the extinction of the species.

2. Preliminaries

In this section, we shall state several lemmas which will be useful in the proving of main results.

Lemma 2.1. *Let $x : R \to R$ be a bounded nonnegative continuous function,and let $h : [0, +\infty) \to R$ be a non-increasing function of bounded variation such that $\int_0^\infty dh(s) = -1$, then we have*

$$\liminf_{t\to+\infty} x(t) \le \liminf_{t\to+\infty} \int_{-\infty}^t x(t)dh(t-s) \le \limsup_{t\to+\infty} \int_{-\infty}^t x(t)dh(t-s) \le \limsup_{t\to+\infty} x(t).$$

Proof. Lemma2.1 can be regard as a generalized version of Lemma3 of Montes de Oca and Zeeman [4] and can be proved in a similar way.

Lemma 2.2. *If $a > 0, b > 0$ and $x' \geq x(b - ax^{\alpha})$, where α is a positive constant, when $t \geq 0, x(0) > 0$, we have $\liminf_{t\to+\infty} x(t) \geq (\frac{b}{a})^{\frac{1}{\alpha}}$; If $a > 0, b > 0$and $x' \leq x(b - ax^{\alpha})$, where α is a positive constant,when $t \geq 0, x(0) > 0$, we have $\limsup_{t\to+\infty} x(t) \leq (\frac{b}{a})^{\frac{1}{\alpha}}$.*

Lemma 2.3. *If $a > 0, b > 0$ and $x' \geq b - ax$, when $t \geq 0, x(0) > 0$, we have $\liminf_{t\to+\infty} x(t) \geq \frac{b}{a}$; If $a > 0, b > 0$and $x' \leq b - ax$, when $t \geq 0, x(0) > 0$, we have $\limsup_{t\to+\infty} x(t) \leq \frac{b}{a}$.*

Lemma 2.2 and Lemma 2.3 are direct corollary of Lemma 2.2 of Chen [10].

3. Permanence

In this section, we discuss the permanence of system (4).

Theorem 3.1. *Let $(x_1(t), x_2(t), u(t))^T$ is any solution of (4) and (5). Assume that (H_1) and (H_2) hold. Then system (4) is permanent.*

Proof. Let $(x_1(t), x_2(t), u(t))^T$ be any solution of (4) and (5). From the first equation of system (4) it follows that

$$x_1'(t) \leq r_1 x_1(t)\big(1 - (\frac{x_1(t)}{k_1})^{\theta_1}\big). \tag{6}$$

Thus, as a direct corollary of Lemma2.2, one has

$$\limsup_{t\to+\infty} x_1(t) \leq k_1 \overset{\text{def}}{=} \overline{x_1}. \tag{7}$$

From Lemma 2.1, we have

$$\limsup_{t\to+\infty} \int_{-\infty}^{t} x_1(t) dh_3(t-s) \leq \limsup_{t\to+\infty} x_1(t) \leq \overline{x_1}.$$

For any small positive constant $\varepsilon > 0$, it follows from(7) that there exists a $T_1 > 0$ such that for $t > T_1$,

$$\int_{-\infty}^{t} x_1(t) dh_3(t-s) \leq \overline{x_1} + \varepsilon. \tag{8}$$

(8) together with the third equation of system (4) leads to

$$u'(t) \le f - eu(t) + d_1(\overline{x_1} + \varepsilon). \tag{9}$$

From Lemma 2.3, one has

$$\limsup_{t\to+\infty} u(t) \le \frac{f + d_1(\overline{x_1} + \varepsilon)}{e}.$$

Setting $\varepsilon \to 0$ in above inequality leads to

$$\limsup_{t\to+\infty} u(t) \le \frac{f + d_1\overline{x_1}}{e} \stackrel{\text{def}}{=} \overline{u}. \tag{10}$$

From Lemma 2.1, we have

$$\limsup_{t\to+\infty} \int_{-\infty}^{t} u(t)dk_2(t-s) \le \limsup_{t\to+\infty} u(t) \le \overline{u}. \tag{11}$$

For above $\varepsilon > 0$, it follows from (11) that there exists a $T_2 > T_1 > 0$ such that for $t > T_2$,

$$\int_{-\infty}^{t} u(t)dk_2(t-s)u \le \overline{u} + \varepsilon. \tag{12}$$

(12) together with the second equation of system (1.4) leads to

$$x_2'(t) \le r_2 x_2(t)\big(1 - (\frac{x_2(t)}{k_2})^{\theta_2} + c_2(\overline{u} + \varepsilon)\big). \tag{13}$$

From Lemma 2.2, one has

$$\limsup_{t\to+\infty} x_2(t) \le (1 + c_2(\overline{u} + \varepsilon))^{\frac{1}{\theta_2}} \cdot k_2.$$

Setting $\varepsilon \to 0$ in above inequality leads to

$$\limsup_{t\to+\infty} x_2(t) \le (1 + c_2\overline{u})^{\frac{1}{\theta_2}} \cdot k_2 \stackrel{\text{def}}{=} \overline{x_2}. \tag{14}$$

From Lemma 2.1, we have

$$\begin{aligned}
&\limsup_{t\to+\infty} \textstyle\int_{-\infty}^{t} x_2(s)dh_2(t-s) \le \limsup_{t\to+\infty} x_2(t) \le \overline{x_2},\\
&\limsup_{t\to+\infty} \textstyle\int_{-\infty}^{t} x_2(s)dh_4(t-s) \le \limsup_{t\to+\infty} x_2(t) \le \overline{x_2},\\
&\limsup_{t\to+\infty} \textstyle\int_{-\infty}^{t} u(t)dk_1(t-s) \le \limsup_{t\to+\infty} u(t) \le \overline{u}.
\end{aligned} \tag{15}$$

For any small positive constant $\varepsilon > 0$, from (H_1) and (H_2), without loss of generality, we may choose ε small enough such that

$$1 - \frac{\alpha_{12}}{k_1}((1 + \frac{c_2(f + d_1k_1)}{e})^{\frac{1}{\theta_2}} \cdot k_2 + \varepsilon) > c_1(\frac{f + d_1k_1}{e} + \varepsilon),$$

$$f-(d_2((1+\frac{c_2(f+d_1k_1)}{e})^{\frac{1}{\theta_2}}\cdot k_2+\varepsilon))>0.$$

hold together. That is, we can take ε small enough such that

$$\begin{aligned}&1-\tfrac{\alpha_{12}}{k_1}(\overline{x_2}+\varepsilon)-c_1(\overline{u}+\varepsilon)>0,\\&f-d_2(\overline{x_2}+\varepsilon)>0.\end{aligned}\tag{16}$$

hold together. For above $\varepsilon>0$, from (15) it follows that there exists a $T_3>T_2>T_1>0$ such that for $t>T_3$,

$$\begin{aligned}&\textstyle\int_{-\infty}^t x_2(s)dh_2(t-s)\le\overline{x_2}+\varepsilon,\\&\textstyle\int_{-\infty}^t x_2(s)dh_4(t-s)\le\overline{x_2}+\varepsilon,\\&\textstyle\int_{-\infty}^t u(t)dk_1(t-s)\le\overline{u}+\varepsilon.\end{aligned}\tag{17}$$

(17) together with the first and third equation of system (4) leads to

$$\begin{aligned}&x_1'(t)\ge r_1x_1(t)\big(1-(\tfrac{x_1(t)}{k_1})^{\theta_1}-\tfrac{\alpha_{12}}{k_1}(\overline{x_2}+\varepsilon)-c_1(\overline{u}+\varepsilon)\big),\\&u'(t)\ge f-eu(t)-d_2(\overline{x_2}+\varepsilon).\end{aligned}\tag{18}$$

By applying Lemma 2.2 and Lemma 2.3 to (18), one obtains

$$\begin{aligned}&\liminf_{t\to+\infty}x_1(t)\ge(1-\tfrac{\alpha_{12}}{k_1}(\overline{x_2}+\varepsilon)-c_1(\overline{u}+\varepsilon))^{\frac{1}{\theta_1}}\cdot k_1,\\&\liminf_{t\to+\infty}u(t)\ge\tfrac{f-d_2(\overline{x_2}+\varepsilon)}{e}.\end{aligned}\tag{19}$$

Setting $\varepsilon\to0$ in above inequalities leads to

$$\begin{aligned}&\liminf_{t\to+\infty}x_1(t)\ge(1-\tfrac{\alpha_{12}}{k_1}\overline{x_2}-c_1\overline{u})^{\frac{1}{\theta_1}}\cdot k_1\overset{\text{def}}{=}\underline{x_1},\\&\liminf_{t\to+\infty}u(t)\ge\tfrac{f-d_2\overline{x_2}}{e}\overset{\text{def}}{=}\underline{u}.\end{aligned}\tag{20}$$

From Lemma 2.1, we have

$$\begin{aligned}&\limsup_{t\to+\infty}\textstyle\int_{-\infty}^t x_1(s)dh_1(t-s)\le\displaystyle\limsup_{t\to+\infty}x_1(t)\le\overline{x_1},\\&\limsup_{t\to+\infty}\textstyle\int_{-\infty}^t u(t)dk_2(t-s)\ge\displaystyle\liminf_{t\to+\infty}u(t)\ge\underline{u}.\end{aligned}\tag{21}$$

For any small positive constant $\varepsilon>0$, from $(H_1),(H_2)$, without loss of generality, we may choose ε small enough such that

$$\frac{(f-d_2k_2((1+\frac{c_2(f+d_1k_1)}{e})^{\frac{1}{\theta_2}}))}{e}>0,$$

$$1-\frac{\alpha_{21}(k_1+\varepsilon)}{k_2}+c_2(\frac{(f-d_2k_2((1+\frac{c_2(f+d_1k_1)}{e})^{\frac{1}{\theta_2}}))}{e}-\varepsilon)>0.$$

hold together. That is, we can take ε small enough such that

$$\varepsilon < \frac{1}{2}\underline{u},$$

$$1 - \frac{\alpha_{21}(\overline{x_1}+\varepsilon)}{k_2} + c_2(\underline{u}-\varepsilon) > 0.$$

hold together.For above $\varepsilon > 0$, it follows from (21) that there exists a $T_4 > T_3 > T_2 > T_1 > 0$ such that for $t > T_4$,

$$\begin{aligned} &\textstyle\int_{-\infty}^{t} x_1(s)dh_1(t-s) \leq (\overline{x_1}+\varepsilon), \\ &\textstyle\int_{-\infty}^{t} u(t)dk_2(t-s) \geq (\underline{u}-\varepsilon). \end{aligned} \tag{22}$$

(22) together with the second equation of system (1.4) leads to

$$x_2'(t) \geq r_2 x_2(t)\big(1 - \frac{\alpha_{21}}{k_2}(\overline{x_1}+\varepsilon) + c_2(\underline{u}-\varepsilon) - (\frac{x_2(t)}{k_2})^{\theta_2}\big). \tag{23}$$

Again,by applying Lemma 2.2 to (23), one obtains

$$\liminf_{t\to+\infty} x_2(t) \geq (1 - \frac{\alpha_{21}(\overline{x_1}+\varepsilon)}{k_2} + c_2(\underline{u}-\varepsilon))^{\frac{1}{\theta_2}} \cdot k_2 > 0.$$

Setting $\varepsilon \to 0$ in above inequality leads to

$$\liminf_{t\to+\infty} x_2(t) \geq (1 - \frac{\alpha_{21}\overline{x_1}}{k_2} + c_2\underline{u})^{\frac{1}{\theta_2}} \cdot k_2 \stackrel{\text{def}}{=} \underline{x_2}. \tag{24}$$

Now, let $\alpha = \frac{1}{2}min\{\underline{x_i}, \underline{u}, i = 1,2\}, \beta = 2max\{\overline{x_i}, \overline{u}, i = 1,2\}$,then α,β is independent of any positive solution of system (4), also from (7), (10), (14) and (24) it immediately follows that

$$0 < \alpha \leq \liminf_{t\to+\infty} x_i(t) \leq \limsup_{t\to+\infty} x_i(t) \leq \beta < +\infty,$$

$$0 < \alpha \leq \liminf_{t\to+\infty} u(t) \leq \limsup_{t\to+\infty} u(t) \leq \beta < +\infty.$$

Above inequalities shows that system (4) is permanent.The proof of the theorem is complete.

Conflict of Interests

The author declares that there is no conflict of interests.

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