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STABILITY AND HOPF BIFURCATION ANALYSIS OF AN EPIDEMIOLOGICAL MODEL INCORPORATING DELAY AND MEDIA COVERAGE

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Abstract: In this paper, an epidemiological model with delay and media coverage is proposed and analyzed. Both the disease-free and endemic equilibria are found and their stability are studied by using the theory of differential equations. It is proved that the time delay may cause a stable equilibrium to become unstable, and Hopf bifurcation about endemic equilibrium can occur under certain conditions. The report ability of the media coverage plays an important role in the spreading of the diseases.

Keywords: SIRS model; Delay; Media coverage; Stability; Hopf bifurcation

2010 AMS Subject Classification: 34D20, 34K18.

1. Introduction

Infectious disease is a serious problem not only in public health but also in social life. S-ince Kermack and Makendrick constructed a SIRS mathematical model to study epidemiology in 1927, more and more scientists have begun to investigate epidemiological models, such as cholera [1-2], SARS[3-4], chickungunya epidemic, bird flu, HIV infection, and so on.

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When a severely infectious disease takes place in a region, it will cause a large number of illness. If the suspected people know nothing about the disease, they will be lack of protecting measures, and so the disease will spread quickly. In fact, media coverage and education can help people to take preventive measures in time, and so reduce the contact rate of human beings as we have observed during the spreading of severe acute respiratory syndrome(SARS) during 2002 and 2004. How does the media coverage affect the prevalence and control of the epidemic like SARS? Recently, this subject has attracted the attentions of many researchers [3-9]. Liu et al.first emphasized media impact in an EIH model, where H denotes hospitalized individuals, and assumed a transmission coefficient of the exponential form in [3]. Cui et al. constructed an SEI model with logistic growth, in which another contact transmission rate with exponential form μe^{-mI} was proposed to describe the media impact on the infectious diseases. When the basic reproduction number $R_0 > 1$, it was shown that there exists a unique endemic equilibrium and a Hopf bifurcation can occur under the less media impact ($m > 0$ is sufficiently small) while the model may have up to three endemic equilibria if the media impact is stronger enough in [6]. It is well known that time delays are inevitable in population interactions and tend to be destabilizing in the sense that longer delays may destroy the stability of positive equilibria. One often introduces time delays in the variables being modeled, this often yields delay differential models [10-14, 17-18].

In this paper, our main concern is the effect of delay on the transmission of infectious diseases. For this purpose, we consider a time delay mathematical model with a constant period of temporary immunity according to the occurrence and spread law of epidemic as follows:

$$\begin{cases} \frac{dS}{dt} = A - dS - (\beta_1 - \dfrac{\beta_2 I}{m+I})SI + \gamma e^{-d\tau} I(t-\tau), \\ \frac{dI}{dt} = (\beta_1 - \dfrac{\beta_2 I}{m+I})SI - (d+\gamma+\varepsilon)I, \\ \frac{dR}{dt} = \gamma I - dR - \gamma e^{-d\tau} I(t-\tau), \end{cases} \tag{1}$$

where the density of the total population denoted by $N(t)$ is divided into three disjoint classes of individuals, namely the susceptible, infected and temporarily recovered ones, with densities denoted by $S(t)$, $I(t)$ and $R(t)$, respectively. The time delay τ means that a recovered individual is immunized against the disease for this fixed time period, but the model allows death to occur for other reasons. Immunity wanes and individuals still alive return to the susceptible class after time τ.

Here, A is the recruitment rate of susceptible population, d is the natural death rate and ε is the disease related death rate, γ is constant recovery rate. We use β_1 and $\beta(I) = \beta_1 - \frac{\beta_2 I}{m+I}$ to denote the contact rate before and after media alert, respectively. The term $\frac{\beta_2 I}{m+I}$ reflects the decrease of the transmission rate due to the media coverage after infectious individuals appear and are reported. When $I \to \infty$, the decrease of the transmission rate approaches its maximum β_2, and the decrease of the transmission rate equals half of the maximum when the reported infective number arrives at m. Because the coverage report cannot prevent disease from spreading completely, we assume $\beta_1 > \beta_2$. The parameter m reflects the reactive velocity of people and media coverage to the disease [9]. All parameters are assumed to be positive except ε and τ, which are nonnegative.

If $\beta_2 = 0$ and $\tau = 0$, the transmission rate is constant β_1, (1) has been discussed. If $\tau = 0$, the system (1) reduces to an *SIS* model, and if $\tau \to \infty$, the system (1) reduces to an *SIRS* model.

Model (1) is a system of functional differential equations(FDES), the associated initial condition take the form:

$$
\begin{gathered}
S(\theta) = \varphi_1(\theta), I(\theta) = \varphi_2(\theta), R(\theta) = \varphi_3(\theta), \\
\varphi_1(\theta) \geq 0, \varphi_2(\theta) \geq 0, \varphi_3(\theta) \geq 0, \theta \in [-\tau, 0], \\
\varphi_1(0) > 0, \varphi_2(0) > 0, \varphi_3(0) \geq 0,
\end{gathered}
\tag{2}
$$

where $(\varphi_1(\theta), \varphi_2(\theta), \varphi_3(\theta)) \in C([-\tau,0], R^3_{+0})$, and $C([-\tau,0], R^3_{+0})$ is the Banach space of continuous functions mapping from the interval $[-\tau, 0]$ into $R^3_{+0} = \{(x_1, x_2, x_3) : x_i \geq 0, i = 1, 2, 3\}$. By the fundamental theory of FDES [15], we know that there exists a unique solution $(S(t), I(t), R(t))$ to system (1) satisfying initial conditions (2).

The remainder of this paper is organized as follows: In section 1, we prove the positivity and the boundedness of solutions. We will investigate the local stability by analyzing the associated characteristic equation in section 2. In the third section, the Hopf bifurcation is studied. Finally, an ecological significance will be discussed.

2. Positivity and boundedness of solutions

Since $N(t) = S(t) + I(t) + R(t)$, we get the following system from (1)

$$\begin{cases} \frac{dI}{dt} = (\beta_1 - \dfrac{\beta_2 I}{m+I})(N-R-I)I - (d+\gamma+\varepsilon)I, \\ \frac{dR}{dt} = \gamma I - dR - \gamma e^{-d\tau} I(t-\tau), \\ \frac{dN}{dt} = A - dN - \varepsilon I. \end{cases} \tag{3}$$

Theorem 2.1. *Let $(I(t), R(t), N(t))$ be the solution of system (3) satisfying conditions(2), then $I(t), R(t)$ and $N(t)$ are all nonnegative.*

Proof. From the first equation of system (3), we have $I'(t) \geq -(d+\gamma+\varepsilon)I$, which yields $I(t) \geq I(0)e^{-(d+\gamma+\varepsilon)}$. And the third equation of system(1) gives $R = \gamma \int_{t-\tau}^{t} I(s)e^{-(d-s)ds}$. Hence $I(t)$ and $R(t)$ are positive.

By the third equation of system (3), we obtain $\frac{A}{d+\varepsilon} \leq \liminf N(t) \leq \frac{A}{d}$, which implies that $S(t) > 0$ for all t. This completes the proof.

Next, we will prove the boundedness of solutions.

Theorem 2.2. *Any positive solutions of system (3) with initial conditions (2) are ultimately bounded.*

Proof. From the third equation of system (3), we obtain

$$\frac{A}{d+\varepsilon} \leq \liminf N(t) \leq \frac{A}{d}.$$

Since $N(t) > 0$ for all $t \geq 0$, $N(t)$ can not blow up to infinity in finite time, and so any solutions are bounded. Hence, the solution exists for all $t > 0$ in the invariant and compact set

$$\Omega = \{(I,R,N) \in R^3_{+0} : 0 \leq I, R \leq N \leq \frac{A}{d} + \eta\},$$

which is a region of attraction for any arbitrary small constant $\eta > 0$. This completes the proof.

3. Analysis of equilibria

In this section, we study the existence and stability of equilibria of system (3). Setting the right hand side of the model (3) to zero, we get the following biologically relevant equilibria.

(a) Disease-Free Equilibrium $E_0 = (0, 0, \frac{A}{d})$,

(b) Interior Equilibrium $E^*(I^*, R^*, N^*)$,

where

$$(\beta_1 - \frac{\beta_2 I^*}{m + I^*})(N^* - R^* - I^*)I^* - (d + \gamma + \varepsilon)I^* = 0, \tag{4}$$

$$I^* = \frac{A - dN^*}{\varepsilon}, R^* = \frac{\gamma(A - dN^*)(1 - e^{-d\tau})}{d\varepsilon}. \tag{5}$$

The interior equilibrium E^* is positive if and only if $0 < N^* < \frac{A}{d}$. Substituting (5) into (4), we obtain

$$F(N) = [m\beta_1 + (\beta_1 - \beta_2)\frac{A - dN}{\varepsilon}][N - \frac{A - dN}{\varepsilon}(\frac{\gamma(1 - e^{-d\tau})}{d} - 1)] - (d + \gamma + \varepsilon)(m + \frac{A - dN}{\varepsilon}). \tag{6}$$

If $\beta_1 A > d(d + \gamma + \varepsilon)$, from equation (6), we note that $F(\frac{A}{d+\varepsilon}) < 0$ and $F(\frac{A}{d}) = \frac{m\beta_1 A}{d} - m(d + \gamma + \varepsilon) > 0$. This implies that there exists at least a positive root N^* of $F(N) = 0$ in $\frac{A}{d+\varepsilon} < N < \frac{A}{d}$. Knowing the value of N^*, the value of I^* and R^* can be calculated from equation (5). Hence E^* exists and remains positive if $\beta_1 A > d(d + \gamma + \varepsilon)$.

Theorem 3.1. *The disease-free equilibrium E_0 is locally asymptotically stable if $\beta_1 A < d(d + \gamma + \varepsilon)$ holds. Again E_0 is unstable if $\beta_1 A > d(d + \gamma + \varepsilon)$ holds.*

Proof. The variational matrix V_0 for the system (3) corresponding to the equilibrium $E_0(0, 0, \frac{A}{d})$ is the following:

$$V_0 = \begin{pmatrix} \frac{\beta_1 A}{d} - (d + \gamma + \varepsilon) & 0 & 0 \\ \gamma(1 - e^{-(\lambda + d)\tau}) & -d & 0 \\ -\varepsilon & 0 & -d \end{pmatrix}.$$

The characteristic equation of equilibrium E_0 is

$$(\lambda + d)^2(\lambda - \frac{\beta_1 A}{d} + d + \gamma + \varepsilon) = 0.$$

The eigenvalues of V_0 are $\lambda_1 = -d, \lambda_2 = -d, \lambda_3 = \frac{\beta_1 A}{d} - (d + \gamma + \varepsilon)$. Since all the model parameters are assumed to be nonnegative, it follows that the disease-free equilibrium E_0 is locally asymptotically stable if $\beta_1 A < d(d+\gamma+\varepsilon)$ holds, and the disease-free equilibrium E_0 is unstable if $\beta_1 A > d(d+\gamma+\varepsilon)$ holds. This completes the proof.

To show the locally asymptotically stability of the equilibrium E^* for all $\tau \geq 0$, we introduce the following lemma.

Lemma 3.1. [16] *A set of necessary and sufficient conditions for the positive equilibrium E^* to be asymptotically stable for all $\tau \geq 0$ is the following:*

(I) E^* is stable in absence of time delay τ.

(II) There is no purely imaginary root of the characteristic equation (7).

Lemma 3.2. [17] *Let $g(z) = z^3 + \nu_1 z^2 + \nu_2 z + \nu_3, \triangle = \nu_1^2 - 3\nu_2, z_1 = \frac{-\nu_1+\sqrt{\triangle}}{3}$, then*

(1) Equation $g(z) = 0$ has at least one positive root if $\nu_3 < 0$.

(2) Equation $g(z) = 0$ has no positive root if both $\nu_3 \geq 0$ and $\triangle < 0$ hold.

(3) If $\nu_3 \geq 0$, then equation $g(z) = 0$ has a unique positive root if and only if both $z_1 > 0$ and $g(z_1) \leq 0$ hold.

Theorem 3.2. *suppose that both $\beta_1 A > d(d+\gamma+\varepsilon)$ and $(d^2+B_1^2-2B_2)^2 < 3(d^2B_1^2+B_2^2-2d^2B_2-B_3^2)$ hold, then the positive equilibrium E^* exists and is locally asymptotically stable for all $\tau \geq 0$.*

Proof. The characteristic equation for the system (4) corresponding to E^* is given by

$$\phi(\lambda,\tau) = (\lambda+d)(\lambda^2+B_1\lambda+B_2-B_3e^{-\lambda\tau}) = 0, \tag{7}$$

where

$$B_1 = \frac{\beta_2 mI^*(N^*-R^*-I^*)}{(m+I^*)^2} + \frac{(d+\gamma+\varepsilon)I^*}{N^*-R^*-I^*} + d,$$

$$B_2 = d[\frac{\beta_2 mI^*(N^*-R^*-I^*)}{(m+I^*)^2} + \frac{(d+\gamma+\varepsilon)I^*}{N^*-R^*-I^*}] + \frac{I^*(\varepsilon+\gamma)(d+\gamma+\varepsilon)}{N^*-R^*-I^*},$$

$$B_3 = \frac{\gamma e^{-d\tau}I^*(d+\gamma+\varepsilon)}{N^*-R^*-I^*}.$$

The assumption (I) and (II) of Lemma 3.1 require real parts of roots of $\phi(\lambda,0) = 0$ to be negative and $\phi(i\omega,\tau) \neq 0$ (where $i^2 = -1$) for any real ω and τ. When $\tau = 0, \phi(\lambda,0) = (\lambda+d)(\lambda^2+B_1\lambda+B_2-B_3) = 0$, since $B_1 > 0, B_2-B_3 > 0$, all the roots of $\phi(\lambda,0) = 0$ have negative real parts, and so E^* is locally asymptotically stable in the absence of delay, the assumption (I) of Lemma 3.1 is satisfied.

Now, we verify the assumption (II) of Lemma 3.1. Firstly, when $\omega_0 = 0$, we have $\phi(0,\tau) = d(B_2 - B_3) > 0$. Secondly, when $\omega_0 \neq 0$, we have

$$\phi(i\omega_0,\tau) = -i\omega_0^3 - B_1\omega_0^2 + iB_2\omega_0 - iB_3\omega_0 e^{-i\omega_0\tau} - d\omega_0^2 + dB_1\omega_0 i + dB_2 - dB_3\omega_0 e^{-i\omega_0\tau} = 0. \tag{8}$$

Separating the real and imaginary parts of equation (8), we obtain

$$-d\omega_0^2 - B_1\omega_0^2 + dB_2 = dB_3 cos\omega_0\tau + \omega_0 B_3 sin\omega_0\tau, \tag{9}$$

$$-\omega_0^3 + B_1 d\omega_0 + B_2\omega_0 = \omega_0 B_3 cos\omega_0\tau - dB_3 sin\omega_0\tau. \tag{10}$$

Squaring and adding equation (9) and (10), we have

$$\omega_0^6 + v_1\omega_0^4 + v_2\omega_0^2 + v_3 = 0, \tag{11}$$

where

$$v_1 = d^2 + B_1^2 - 2B_2, v_2 = d^2B_1^2 + B_2^2 - 2d^2B_2 - B_3^2, v_3 = d^2(B_2^2 - B_3^2) > 0.$$

Let $z = \omega_0^2$. Equation (11) becomes

$$g(z) = z^3 + v_1 z^2 + v_2 z + v_3 = 0. \qquad (12).$$

Obviously, $\phi(i\omega,\tau) \neq 0$ if and only if equation (12) has no positive root. From the Lemma 3.2, we can conclude that if

$$v_1^2 - 3v_2 = (d^2 + B_1^2 - 2B_2)^2 - 3(d^2B_1^2 + B_2^2 - 2d^2B_2 - B_3^2) < 0$$

holds, the assumption (II) of Lemma 3.1 is satisfied, and so the positive equilibrium E^* is locally asymptotically stable for all $\tau \geq 0$. This completes the proof.

4. Hopf bifurcation analysis

In the following, by using the time delay τ as the bifurcation parameter, the criteria for Hopf bifurcation are given.

Substituting $\lambda = a(\tau) + ib(\tau)$ into the characteristic equation (7) and separating real and imaginary parts, we obtain the following equations:

$$(a^3 - 3ab^2 + a^2d - db^2) + (a^2 + ad - b^2)B_1 + (a+d)B_2 - e^{-a\tau}B_3(a+d)cosb\tau - e^{-a\tau}bB_3sinb\tau = 0, \tag{13}$$

$$(2abd + 3a^2d - b^3) + (2ab + bd)B_1 + bB_2 + e^{-a\tau}B_3(a+d)sinb\tau - e^{-a\tau}bB_3cosb\tau = 0, \tag{14}$$

where a and b are functions of τ. Since the change of stability of E^* will occur at any values of τ for which $a = 0$ and $b \neq 0$. Let $\hat{\tau}$ satisfy $a(\hat{\tau}) = 0$, and $b(\hat{\tau}) = \hat{b} \neq 0$. Then equations (13) and (14) reduce to

$$-\hat{b}^2(d + B_1) + dB_2 - dB_3cos\hat{b}\hat{\tau} - bB_3sin\hat{b}\hat{\tau} = 0, \tag{15}$$

$$-\hat{b}^3 + \hat{b}(dB_1 + B_2) + dB_3sin\hat{b}\hat{\tau} - bB_3cos\hat{b}\hat{\tau} = 0. \tag{16}$$

Squaring and adding the both sides of equations (15) and (16), we have

$$\hat{b}^6 + \nu_1\hat{b}^4 + \nu_2\hat{b}^2 + \nu_3 = 0, \tag{17}$$

where ν_1, ν_2, ν_3 are the same as in equation (11). In order to establish Hopf bifurcation at $\tau = \hat{\tau}$, we need to show that $\frac{da}{d\tau}|_{\tau=\hat{\tau}} \neq 0$. Differentiating (13) and (14) with respect to τ , and then setting $\tau = \hat{\tau}, a = 0$ and $b = \hat{b}$, we obtain

$$L_1\frac{da(\hat{\tau})}{d\tau} + L_2\frac{db(\hat{\tau})}{d\tau} = Y_1, \tag{18}$$

$$-L_2\frac{da(\hat{\tau})}{d\tau} + L_1\frac{db(\hat{\tau})}{d\tau} = Y_2, \tag{19}$$

where

$$L_1 = -3\hat{b}^2 + dB_1 + B_2 + dB_3\hat{\tau}cos\hat{b}\hat{\tau} - B_3cos\hat{b}\hat{\tau} + B_3\hat{b}\hat{\tau}sin\hat{b}\hat{\tau},$$

$$L_2 = -2d\hat{b} - 2B_1\hat{b} + dB_3\hat{\tau}sin\hat{b}\hat{\tau} - B_3\hat{b}\hat{\tau}cos\hat{b}\hat{\tau} - B_3sin\hat{b}\hat{\tau}, \tag{20}$$

$$Y_1 = B_3\hat{b}^2cos\hat{b}\hat{\tau} - dB_3\hat{b}sin\hat{b}\hat{\tau},$$

$$Y_2 = -dB_3\hat{b}cos\hat{b}\hat{\tau} - B_3\hat{b}^2sin\hat{b}\hat{\tau}.$$

Solving (18) and (19), we get

$$\frac{da(\hat{\tau})}{d\tau} = \frac{L_1Y_1 - L_2Y_2}{L_1^2 + L_2{}^2}. \tag{21}$$

Clearly, $\frac{da(\hat{\tau})}{d\tau}$ has the same sign as $L_1Y_1 - L_2Y_2$. From (20), we obtain

$$L_1Y_1 - L_2Y_2 = \hat{b}^2[3\hat{b}^4 + 2\hat{b}^2(-2B_2 + d^2 + B_1^2) + (d^2B_1^2 + B_2^2 - 2d^2B_2 - B_3^2)]. \tag{22}$$

Let

$$G(z) = z^3 + \nu_1 z^2 + \nu_2 z + \nu_3, \tag{23}$$

where ν_1, ν_2, ν_3 are defined in equation (11). Then, from (17) and (23), we have $G(\hat{b}^2) = 0$ and

$$\frac{da(\hat{\tau})}{d\tau} = \frac{\hat{b}^2}{L_1^2 + L_{2^2}} \frac{dG(\hat{b}^2)}{dz}. \tag{24}$$

Hence, we can describe the criterion for instability (stability) as follows:

(H1) If the polynomial $G(z)$ has no positive roots, there can be no change of stability.

(H2) If $G(z)$ is increasing (decreasing) at all of its positive roots, instability (stability) is preserved. Here, $G(0) = \nu_3 > 0$. If $\nu_1 > 0$, then $G(z)$ has either two positive roots or no positive root.

(H3) If $\nu_1 > 0$ and $\nu_2 \geq 0$, then $G(z) \geq G(0) = \nu_3 > 0$ for all $z > 0$, and so the assumption (H1) holds, the stability or instability of the positive equilibrium E^* will be preserved in this case.

(H4) If $\nu_1 > 0$ and $\nu_2 < 0$, since $\nu_3 > 0$, then the minimum of $G(z)$ will exist at $z_{min} = \frac{-\nu_1 + \sqrt{\nu_1^2 - 3\nu_2}}{3}$ and (H1) will hold if $G(z_{min}) > 0$, i.e.,

$$2\nu_1^3 - 9\nu_1\nu_2 + 27\nu_3 > 2(\nu_1^2 - 3\nu_2)^{\frac{3}{2}}. \tag{25}$$

Therefore, we have the following theorem.

Theorem 4.1. *suppose that one of the following conditions holds:*

(i) $\nu_1 > 0$ *and* $\nu_2 \geq 0$*;*

(ii) $\nu_1 > 0, \nu_2 < 0$*, and the inequality (25) is satisfied,*

then if the positive equilibrium E^* *is stable (unstable) at* $\tau = 0$*, it will remain stable (unstable) for all* $\tau > 0$*.*

Theorem 4.2. *Assume that the inequality (25) unsatisfied, and one of the following conditions holds:*

(1) $\nu_1 \leq 0$ *and* $\nu_2 \leq \frac{\nu_1^2}{3}$*;*

(2) $\nu_1 > 0, \nu_2 < 0$,

if E^* is asymptotically stable for $\tau = 0$, and $\hat{b_0}^2$ is the first positive root of equation (23), then a Hopf bifurcation occurs as τ passes through $\hat{\tau}_0$.

Proof. Since $\nu_3 > 0$, if one of the conditions (1) and (2) holds, we can conclude that $z_1 = \frac{-\nu_1+\sqrt{\nu_1^2-3\nu_2}}{3} > 0$. Furthermore, (25) is unsatisfied that means $G(z_1) \leq 0$, from the Lemma 3.1, we obtain that $G(z)$ has a unique positive root denoted by $\hat{b_0}^2$.

On the other hand, since $G(z)$ is a cubic in z and $G(z) \to \infty$ as $t \to \infty$, $G(z)$ must increase at the positive root $\hat{b_0}^2$, from (H2), we know it impossible for E^* to remain stable. Hence, there exists a $\hat{\tau}_0$ such that E^* is asymptotically stable for $\tau < \hat{\tau}_0$, and is unstable for $\tau > \hat{\tau}_0$. As τ passes through $\hat{\tau}_0$, E^* bifurcates into small amplitude periodic solutions of Hopf type. From (15) and (16), we can determine $\hat{\tau}_0$, which is of the form

$$\hat{\tau}_0 = \frac{1}{\hat{b_0}} \arcsin \frac{-\hat{b_0} B_1}{B_3}.$$

It follows from (24) that

$$\frac{da(\hat{\tau}_0)}{d\tau} = \frac{\hat{b_0}^2}{L_1^2 + L_{2^2}} \frac{dG(\hat{b_0}^2)}{dz} > 0,$$

and from the Hopf bifurcation theorem [19], we obtain the conclusion. This completes the proof.

5. Numerical simulation

In this section, based on the theoretical study of system (3), selecting parameters and initial value which meet the conditions:

$$A = 5, \beta_1 = 0.002, \gamma = 0.005, \varepsilon = 0.1, m = 30, d = 0.02,$$

$$E = (S, I, R) = (130, 25, 30).$$

We used MATLAB to obtain the numerical simulation of the system (3), which illustrates the correctness and feasibility of the theoretical study. In Figure 1, we mainly study the impaction from media reports, so we assume that $\tau = 0$. The value of endemic equilibrium clearly reduces when we change the value of β_2 from 0 to 0.0018 . In Figure 2, we fix the value of $\beta_2 = 0$ to study the impaction of time-delay. By changing the value of τ, we further validate the correctness of $\dfrac{dI^*}{d\tau} < 0$.

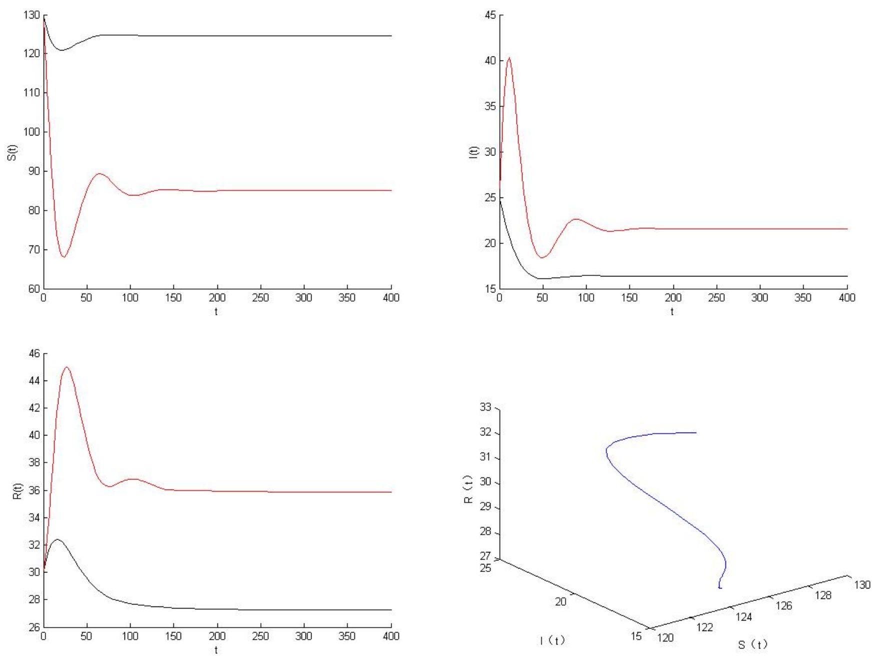

Figure 1: The time-series curve compared $\beta_2 = 0$ with $\beta_2 = 0.0018$, where red line is a curve when $\beta_2 = 0$ and black line is a curve when $\beta_2 = 0.0018$. And τ is 0.

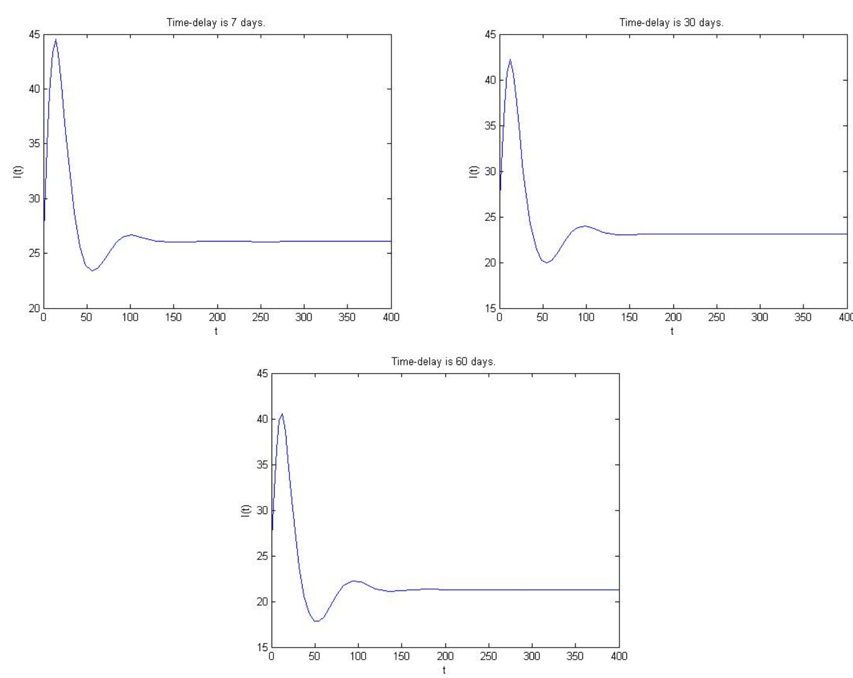

Figure 2: The time-series curve of $I(t)$ based on the value of τ. And β is 0.

6. Discussion

In this paper, we have considered an epidemiological model with delay and media coverage. Firstly, stability and Hopf bifurcation for system (3) with delay are investigated by using the theory of characteristic value. Theorem 4.1 showed that the delay τ is locally harmless. Furthermore, regarding the delay τ as a parameter, the stable equilibrium E^* may lose its stability due to large time delay, which implies that there exists Hopf bifurcations. Conditions of the existence of Hopf bifurcation and bifurcation value are obtained.

Secondly, the effective media coverage greatly influences how people perceive the threat of infectious diseases, which can lower infection and postpone the arrival of the infection peak. In fact, from $\beta(I) = \beta_1 - \dfrac{\beta_2 I}{m+I}$, we have $\frac{\partial \beta(I)}{\partial m} = \frac{\beta_2 I}{(m+I)^2} > 0$, we know that the transmission rate will become smaller as m decreases. That suggests the department of media need to track and report the latest situation as soon as possible, and tell people how to protect themselves from infection when the disease begins to spread. On the other hand, from $\frac{\partial \beta(I)}{\partial \beta_2} = -\frac{I}{m+I} < 0$, we have that the transmission rate will decrease as β_2 increases. It is really critical for the media coverage to give people the facts and so objective reporting about diseases.

Thirdly, the time delay τ is a fixed time period, of which the recovered individual is in susceptible people again. Time delay really impacts the peak value of infection. In Figure 2, we know that the higher value of τ will yield a lower peak value of infection. So a measure with extended time delay is better than others to control the spreading of infectious diseases by medical intervention. In a word, time delay plays an important role in the prevention and control of infectious diseases.

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Conflict of Interests

The authors declare that there is no conflict of interests.

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