Available online at http://scik.org
Commun. Math. Biol. Neurosci. 2015, 2015:10
ISSN: 2052-2541

ALMOST PERIODIC SOLUTION OF A DELAYED NICHOLSON'S BLOWFLIES MODEL WITH FEEDBACK CONTROL

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Abstract. In this paper, we study the problem of positive almost periodic solutions for the Nicholson's blowflies mode with feedback control and multiple time-varying delays. By applying the properties of almost periodic function and exponential dichotomy of linear system as well as fixed point theorem, we establish the conditions for the existence uniqueness and exponential convergence of the positive almost periodic solution of the equations. Moreover, an example and its numerical simulation are given to illustrate our main results.

Keywords: Nicholson's blowflies model; Positive almost periodic solution; Delay; Feedback control.

2010 AMS Subject Classification: 34K14, 92D25.

1. Introduction

It is well known that the theory of Nicholson's blowflies model has made a remarkable progress in the past forty years with main results scattered in numerous research papers; see, for example, [1-7] and the references cited therein.

In the real world, the delays in differential equations of population and ecology problems are usually time-varying. Recently, Chen and Liu [8] considered a class of the generalized Nicholson's blowflies mode with multiple

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Received December 15, 2014

time-varying delay as follows:

$$x'(t) - \alpha(t)x(t) + \sum_{j=1}^{m} \beta_j(t)x(t-\tau_j(t))e^{-\gamma_j(t)x(t-\tau_j(t))}, \tag{1.1}$$

where $t \in R$, $\alpha, \beta_j, \gamma_j, \tau_j (j = 1, \cdots, m) : R \to (0, +\infty)$ are almost periodic functions. By constructing suitable Lyapunov functional, they showed that under a set of algebraic conditions, system (1.1) has a unique positive almost periodic solution. The solutions of this model converge exponentially to a positive almost periodic solution.

On the other hand, ecosystem in the real world is continuously disturbed by unpredictable forces such as survival rates. Practical interest in ecology is the question of whether or not an ecosystem can withstand those unpredictable disturbances which persist for a finite period of time. In the language of control variable, we call the disturbance functions as control variables. Recently, some excellent results [9-12] which are concerned with existence and the stability of almost periodic solution of the single species or multi-species competition system with feedback control are obtained. However, to the best of the author's knowledge, to this day, few work has dealt with the almost periodic solution of Nicholson's blowflies models with feedback control and time-varying delays.

Motivated by the above, we consider the following generalized Nicholson's blowflies model with feedback control and multiple time-varying delays:

$$\begin{aligned} x'(t) &= -\alpha(t)x(t) + \sum_{j=1}^{m} \beta_j(t)x(t-\tau_j(t))e^{-\gamma_j(t)x(t-\tau_j(t))} - c(t)x(t)u(t-\zeta(t)), \\ u'(t) &= -\lambda(t)u(t) + g(t)x(t-\delta(t)), \end{aligned} \tag{1.2}$$

where $x(t)$ is a population size at time t, $u(t)$ is the indirect control variable, and $c(t)$, λ, $g(t)$ are almost periodic functions. For convenience, we introduce the notations

$$f^- = \inf_{t\in R} f(t), \quad f^+ = \sup_{t\in R} f(t),$$

where f is a continuous bounded function defined on $[0, +\infty)$. It will be assumed that

$$\alpha^- > 0, \beta_j^- > 0, \gamma_j^- > 0, c^- > 0, \lambda^- > 0, g^- > 0, (j = 1, \cdots, m)$$

and

$$\tau^+ = \max_{1\le j\le m} \{\sup_{t\in R} \tau_j(t)\} > 0, (j = 1, \cdots, m), \ \tau = \max\{\tau^+, \zeta^+, \delta^+\}.$$

Let $\mathbb{R}^2(\mathbb{R}^2_{+0})$ be the set of all (nonnegative) real vectors. Denote $\mathbb{C} = C([-\tau, 0], \mathbb{R}^2)$ and $\mathbb{C}_+ = C([-\tau, 0], R^2_{+0})$ as the Banach space of continuous functions. If $x(t)$, $u(t)$ are defined on $[t_0 - \tau, \sigma)$ with $t_0, \sigma \in R^1$, then we defined $X_t \in \mathbb{C}$ as $X_t = (x(t), u(t))$ where $x_t(\theta) = x(t+\theta)$, $u_t(\theta) = u(t+\theta)$ for all $\theta \in [-\tau, 0]$. From the viewpoint of mathematical biology, we consider (1.2) together with the following initial conditions

$$x_{t_0} = \varphi_1, u_{t_0} = \varphi_2, \varphi = (\varphi_1, \varphi_2)^T \in \mathbb{C}_+, \varphi_i(0) > 0, i = 1, 2, \tag{1.3}$$

where $\varphi_i(\theta), (i = 1, 2), \theta \in [-\tau, 0]$ is continuous and positive.

We take $X_t(t_0,\varphi)=X(t,t_0,\varphi)$ as a solution of the initial value problem (1.2) and (1.3) with $X_{t_0}(t_0,\varphi)=\varphi(t_0\in \mathbb{R})$. Also, let $[t_0,\eta(\varphi))$ be the maximal right-interval of existence of $X_t(t_0,\varphi)$.

2. Preliminaries

Definition 2.1 (see [13]) Let $x\in R^n$ and $Q(t)$ be a $n\times n$ continuous matrix defined on R. The linear system

$$x'(t)=Q(t)x(t). \tag{2.1}$$

is said to admit an exponential dichotomy on R if there exist positive constants k, α , projection P and the fundamental solution matrix $X(t)$ of (2.1) satisfying

$$\| X(t)PX^{-1}(s) \|\le ke^{-\alpha(t-s)} \text{ for all } t\ge s,$$

$$\| X(t)(I-P)X^{-1}(s) \|\le ke^{-\alpha(s-t)} \text{ for all } t\le s.$$

Set

$$B=\{\varphi|\varphi=(\varphi_1(t),\varphi_2(t))^T \text{is an almost periodic function on } R\}.$$

For any $\varphi\in B$, we define an induced module $\| \varphi \|_B=\sup_{t\in R}\| \varphi(t) \|$, the B is a Banach space.

Lemma 2.1 (see [13]) *If the linear system (2.1) admits an exponential dichotomy, the almost periodic system*

$$x'(t)=Q(t)x(t)+g(t). \tag{2.2}$$

has an unique almost periodic solution $x(t)$, and

$$x(t)=\int_{-\infty}^{t}X(t)PX^{-1}(s)g(s)ds-\int_{t}^{+\infty}X(t)(I-P)X^{-1}(s)g(s)ds. \tag{2.3}$$

Lemma 2.2 (see [13]) *Let $c_i(t)$ be an almost periodic function on R and*

$$M[c_i]=\lim_{T\to+\infty}\frac{1}{T}\int_{T}^{t+T}c_i(s)ds>0,\ i=1,2,\cdots,n.$$

Then the linear system

$$x'(t)=diag(-c_1(t),-c_2(t),\cdots,-c_n(t))x(t),$$

admits an exponential dichotomy on R.

Set $B^*=\{\varphi|\varphi\in B,k_1\le\varphi_1\le K_1,\ k_2\le\varphi_2\le K_2\}$.

Lemma 2.3 (see [14]) *If $u(t),\ g(t):R\to R$ are almost periodic, then $u(t-g(t))$ is almost periodic.*

We also suppose the following condition (H_1) hold.

(H_1)there exist four constants $K_1, K_2, k_1,$ and k_2 such that

$$K_1>k_1,\ K_2>k_2,\ K_1>\sum_{j=1}^{m}(\frac{\beta_j}{\gamma_j})^+\frac{1}{\alpha^-e},\ \frac{1}{\min\limits_{1\le j\le m}\gamma_j^-}<k_1<\sum_{j=1}^{m}\frac{\beta_j^-}{\alpha^+}K_1e^{-\gamma_j^+K_1}-\frac{c^+K_1K_2}{\alpha^+}$$

Lemma 2.4 *Let* (H_1) *hold, and* $B^* = \{\varphi | \varphi \in B, k_1 \le \varphi_1 \le K_1,\ k_2 \le \varphi_2 \le K_2\}$. *Then, for* $\varphi \in B^*$, *the solution* $X(t,t_0,\varphi)$ *of (1.2) and (1.3) satisfies*

$$k_1 < x(t,t_0,\varphi_1) < K_1, k_2 < u(t,t_0,\varphi_2) < K_2, \text{ for all } t \in [t_0, \eta(\varphi)) \tag{2.4}$$

and $\eta(\varphi) = +\infty$.

Proof. Set $x(t) = x(t,t_0,\varphi_1)$. Let $[t_0,T) \subseteq [t_0,\eta(\varphi)]$ be a interval such that

$$0 < x(t) \ \text{ for all } t \in [t_0,T). \tag{2.5}$$

We claim that

$$0 < x(t) < K_1 \text{ for all } t \in [t_0,T). \tag{2.6}$$

Assume, by way of contradiction, that (2.6) does not hold. Then, it exist $t_1 \in [t_0,T)$ such that

$$x(t_1) = K_1 \text{ and } 0 < x(t) < K_1 \text{ for all } t \in [t_0 - \tau, t_1). \tag{2.7}$$

Calculating the derivative of $x(t)$, from (H_1) and the fact that $\sup\limits_{u\ge 0} ue^{-u} = \frac{1}{e}$, the first equation of system (1.2) and (2.7) yield that

$$\begin{aligned}
0 \le x'(t_1) &\le -\alpha(t_1)x(t_1) + \sum_{j=1}^{m} \beta_j(t_1)x(t_1 - \tau_j(t_1))e^{-\gamma_j(t_1)x(t_1-\tau_j(t_1))} \\
&\le -\alpha^- x(t_1) + \sum_{j=1}^{m} \frac{\beta_j(t_1)}{\gamma_j(t_1)}\gamma_j(t_1)x(t_1-\tau_j(t_1))e^{-\gamma_j(t_1)x(t_1-\tau_j(t_1))} \\
&\le -\alpha^- x(t_1) + \sum_{j=1}^{m} \big(\frac{\beta_j}{\gamma_j}\big)^+ \frac{1}{e} \\
&= \alpha^- \big[-K_1 + \sum_{j=1}^{m} \big(\frac{\beta_j}{\gamma_j}\big)^+ \frac{1}{\alpha^- e}\big] < 0,
\end{aligned}$$

which is a contradiction and implies that (2.6) holds. In view of $u(t_0) = \varphi_2(0) > 0$, integrating the second equation of (1.2) from t_0 to t, we have

$$\begin{aligned}
u(t) &= e^{-\int_{t_0}^{t}\lambda(s)ds}u(t_0) + e^{-\int_{t_0}^{t}\lambda(s)ds}\int_{t_0}^{t} e^{\int_{t_0}^{s}\lambda(\omega)d\omega} g(s)x(s-\delta(s))ds \\
&> 0, \ \text{ for all } t \in [t_0,\eta(\varphi)).
\end{aligned} \tag{2.8}$$

From (2.6) and (2.8), we obtain that $u(t)$ is bounded and there exist positive constants K_2 such that

$$0 < u(t) \le K_2, \ \text{ for all } t \in [t_0,\eta(\varphi)). \tag{2.9}$$

We next show that

$$x(t) > k_1, \ \text{ for all } t \in [t_0,\eta(\varphi)). \tag{2.10}$$

Otherwise, there exists $t_2 \in (t_0,\eta(\varphi))$ such that

$$x(t_2) = k_1 \text{ and } x(t) > k_1 \text{ for all } t \in [t_0 - \tau, t_2). \tag{2.11}$$

Then, from (H_1) and (2.6), we get

$$k_1 < x(t) < K_1,\ \gamma_j^+ x(t) \geq \gamma_j^+ \frac{1}{\min\limits_{1\leq j\leq m} \gamma_j^-},\ \text{for all } t \in [t_0-\tau, t_2),\ j = 1,2,\cdots,m. \tag{2.12}$$

Calculating the derivative of $x(t)$, together with (H_1) and the fact that $\min\limits_{1\leq u\leq \omega} ue^{-u} = \omega e^{-\omega}$, the first equation of system (1.2), (2.11) and (2.12) imply that

$$\begin{aligned}
0 \geq x'(t_2) &= -\alpha(t_2)x(t_2) + \sum_{j=1}^{m} \beta_j(t_2)x(t_2-\tau_j(t_2))e^{-\gamma_j(t_2)x(t_2-\tau_j(t_2))} - c(t_2)x(t_2)u(t_2-\zeta(t_2)) \\
&\geq -\alpha^+ x(t_2) + \sum_{j=1}^{m} \frac{\beta_j(t_2)}{\gamma_j^+}\gamma_j^+ x(t_2-\tau_j(t_2))e^{-\gamma_j^+ x(t_2-\tau_j(t_2))} - c^+K_1K_2 \\
&\geq -\alpha^+ x(t_2) + \sum_{j=1}^{m} \frac{\beta_j(t_2)}{\gamma_j^+}\gamma_j^+ K_1 e^{-\gamma_j^+ K_1} - c^+K_1K_2 \\
&= \alpha^+\big[-k_1 + \sum_{j=1}^{m} \frac{\beta_j^-}{\alpha^+}K_1e^{-\gamma_j^+ K_1} - \frac{c^+K_1K_2}{\alpha^+}\big] > 0,
\end{aligned}$$

which is a contradiction and yield that (2.10) holds. From (2.8) and (2.10), we obtain that $u(t)$ is bounded and there exist positive constants k_2 such that

$$u(t) \geq k_2,\ \text{for all } t \in [t_0, \eta(\varphi)). \tag{2.13}$$

It follows from (2.6) (2.9) (2.10) and (2.13) that (2.4) is true. From Theorem 2.3.1 in [15], we easily obtain $\eta(\varphi) = +\infty$. This end the proof of Lemma 2.1.

3. Main results

Let

$$K_2 > \frac{g^+K_1}{\lambda^-},\ \frac{g^-k_1}{\lambda^+} > k_2,\ \max\left\{\frac{\sum\limits_{j=1}^{m}\beta_j^+}{\alpha^- e^2} + \frac{c^+K_2}{\alpha^-} + \frac{c^+K_1}{\alpha^-}, \frac{g^+}{\lambda^-}\right\} < 1. \tag{3.1}$$

Then, there exists a unique positive almost periodic solution of system (1.2) in the region B^*.

Proof. For any $\phi \in B$, we consider an auxiliary equation

$$\begin{cases} x'(t) = -\alpha(t)x(t) + \sum\limits_{j=1}^{m} \beta_j(t)\phi_1(t-\tau_j(t))e^{-\gamma_j(t)\phi_1(t-\tau_j(t))} - c(t)\phi_1(t)\phi_2(t-\zeta(t)), \\ u'(t) = -\lambda(t)u(t) + g(t)\phi_1(t-\delta(t)). \end{cases} \tag{3.2}$$

It follows from Lemma 2.3 that $\phi_1(t-\tau_j(t))$, $\phi_1(t-\delta(t))$, $\phi_2(t-\zeta(t))$, are almost periodic. Notice that $M[\alpha] > 0$, $M[\lambda] > 0$, it follows from Lemma 2.2 that the linear equation

$$\begin{cases} x'(t) = -\alpha(t)x(t), \\ u'(t) = -\lambda(t)u(t), \end{cases} \tag{3.3}$$

admits an exponential dichotomy on R. Thus, by Lemma 2.1, we obtain that the system (3.2) has exactly one almost periodic solution:

$$\begin{aligned} X^{\phi}(t) &= \{x^{\phi}(t), u^{\phi}(t)\} \\ &= \Big\{ \int_{-\infty}^{t} e^{-\int_s^t \alpha(u)du} \Big(\sum_{j=1}^{m} \beta_j(s)\phi_1(s-\tau_j(s)) e^{-\gamma_j(s)\phi_1(s-\tau_j(s))} - c(s)\phi_1(s)\phi_2(s-\zeta(s)) \Big) ds, \\ & \quad \int_{-\infty}^{t} e^{-\int_s^t \lambda(u)du} \Big(g(s)\phi_1(s-\delta(s)) \Big) ds \Big\}. \end{aligned} \tag{3.4}$$

Define a mapping $T : B \to B$ by setting

$$T(\phi(t)) = X^{\phi}(t), \ \forall \phi \in B.$$

It is easy to see that B^* is a closed subset of B. For any $\phi \in B^*$, from (3.4) and $\sup\limits_{u\geq 0} ue^{u} = \frac{1}{e}$, we have

$$\begin{aligned} x^{\phi}(t) &\leq \int_{-\infty}^{t} e^{-\int_s^t \alpha(u)du} \Big(\sum_{j=1}^{m} \big(\frac{\beta_j}{\gamma_j}\big)^{+} \frac{1}{e} \Big) ds \\ &\leq \sum_{j=1}^{m} \big(\frac{\beta_j}{\gamma_j}\big)^{+} \frac{1}{\alpha^{-} e} < K_1, \\ u^{\phi}(t) &\leq \int_{-\infty}^{t} e^{-\int_s^t \lambda(u)du} g^{+} K_1 ds = \frac{g^{+}\cdot K_1}{\lambda^{-}} < K_2. \end{aligned}$$

Noting that $k_1 > \frac{1}{\min\limits_{1\leq j\leq m} \gamma_j^{-}}$ and $\min\limits_{1\leq u\leq m} ue^{-u} = me^{-m}$, we have

$$\begin{aligned} x^{\phi}(t) &\geq \int_{-\infty}^{t} e^{-\int_s^t \alpha(u)du} \big(\sum_{j=1}^{m} \beta_j^{-} K_1 e^{-\gamma_j^{+} K_1} - c^{+} K_1 K_2 \big) ds \\ &\geq \sum_{j=1}^{m} \frac{\beta_j^{-} K_1 e^{-\gamma_j^{+} K_1}}{\alpha^{-}} - \frac{c^{+} K_1 K_2}{\alpha^{+}} > k_1, \\ u^{\phi}(t) &\geq \int_{-\infty}^{t} e^{-\int_s^t \lambda(u)du} g^{-} k_1 ds = \frac{g^{-}\cdot k_1}{\lambda^{+}} > k_2. \end{aligned}$$

This implies that the mapping T is a self-mapping from B^* to B^* .

Now, we prove that the mapping T is a contraction mapping on B^*. In fact, for $\phi,\ \psi \in B^*$, we get

$$\| T(\phi) - T(\psi) \|_{B} = \Big(\sup_{t\in R} \mid \big(T(\phi)(t) - T(\psi)(t)\big)_1 \mid, \sup_{t\in R} \mid \big(T(\phi)(t) - T(\psi)(t)\big)_2 \mid \Big)$$

$$\begin{aligned} \sup_{t\in R} \big|\big(T(\phi)(t) - T(\psi)(t)\big)_1\big| &= \sup_{t\in R} \Big| \int_{-\infty}^{t} e^{-\int_s^t \alpha(u)du} \Big\{ \sum_{j=1}^{m} \beta_j(s) \Big(\phi_1(s-\tau_j(s)) e^{-\gamma_j(s)\phi_1(s-\tau_j(s))} \\ & \quad - \psi_1(s-\tau_j(s)) e^{-\gamma_j(s)\psi_1(s-\tau_j(s))} \Big) \\ & \quad - c(s) \Big(\phi_1(s)\phi_2(s-\zeta(s)) - \psi_1(s)\psi_2(s-\zeta(s)) \Big) \Big\} ds \Big| \end{aligned}$$

Since $\sup\limits_{u\geq 1} \mid \frac{1-u}{e^{u}} \mid = \frac{1}{e^2}$, we obtain

$$\begin{aligned} \mid xe^{-x} - ye^{-y} \mid &= \Big| \frac{1-(x+\theta(y-x))}{e^{x+\theta(y-x)}} \Big| |x-y| \\ &\leq \frac{1}{e^2} |x-y|, \ \text{where} x, y \in [1, +\infty), 0 < \theta < 1. \end{aligned} \tag{3.5}$$

(3.5) combine with $\frac{1}{\min\limits_{1\le j\le m}\gamma_j^+}<k_1$, we get

$$
\begin{aligned}
\sup_{t\in R}\left|\big(T(\phi)(t)-T(\psi)(t)\big)_1\right| &\le \frac{\sum\limits_{j=1}^{m}\beta_j^+}{\alpha^- e^2}\|\phi-\psi\|_B \\
&\quad+\sup_{t\in R}\int_{-\infty}^{t}e^{-\int_s^t\alpha(u)du}c(t)\Big(\big|\phi_1(s)\phi_2(s-\zeta(s))-\psi_1(s)\phi_2(s-\zeta(s))\big| \\
&\quad+\big|\psi_1(s)\phi_2(t-\zeta(t))-\psi_1(s)\psi_2(s-\zeta(s))\big|\Big)ds \\
&\le \frac{\sum\limits_{j=1}^{m}\beta_j^+}{\alpha^- e^2}\|\phi-\psi\|_B+\sup_{t\in R}\int_{-\infty}^{t}e^{-\int_s^t\alpha(u)du}c(t)\Big(K_2\big|\phi_1(s)-\psi_1(s)\big| \\
&\quad+K_1\big|\phi_2(s-\zeta(s))-\psi_2(s-\zeta(s))\big|\Big)ds \\
&\le \frac{\sum\limits_{j=1}^{m}\beta_j^+}{\alpha^- e^2}\|\phi-\psi\|_B+\frac{c^+K_2}{\alpha^-}\|\phi-\psi\|_B+\frac{c^+K_1}{\alpha^-}\|\phi-\psi\|_B \\
&= \Big(\frac{\sum\limits_{j=1}^{m}\beta_j^+}{\alpha^- e^2}+\frac{c^+K_2}{\alpha^-}+\frac{c^+K_1}{\alpha^-}\Big)\|\phi-\psi\|_B.
\end{aligned}
$$

$$
\begin{aligned}
\sup_{t\in R}\left|\big(T(\phi)(t)-T(\psi)(t)\big)_2\right| &= \sup_{t\in R}\left|\int_{-\infty}^{t}e^{-\int_s^t\lambda(u)du}\Big(g(s)\phi_2(s-\delta(s))-g(s)\psi_2(s-\delta(s))\Big)ds\right| \\
&\le \sup_{t\in R}\int_{-\infty}^{t}e^{-\int_s^t\lambda(u)du}g^+\big|\phi_2(s-\delta(s))-\psi_2(s-\delta(s))\big|ds \\
&\le \frac{g^+}{\lambda^-}\|\phi-\psi\|_B.
\end{aligned}
$$

Hence

$$\| T(\phi)-T(\psi)\|_B\le \max\left\{\frac{\sum\limits_{j=1}^{m}\beta_j^+}{\alpha^- e^2}+\frac{c^+K_2}{\alpha^-}+\frac{c^+K_1}{\alpha^-},\frac{g^+}{\lambda^-}\right\}\|\phi-\psi\|_B.$$

Noting that

$$\max\left\{\frac{\sum\limits_{j=1}^{m}\beta_j^+}{\alpha^- e^2}+\frac{c^+K_2}{\alpha^-}+\frac{c^+K_1}{\alpha^-},\frac{g^+}{\lambda^-}\right\}<1,$$

it is clear that the mapping T is a contraction on B^*. By the fixed point theorem of Banach space, T possesses a unique fixed point $\phi^*\in B^*$ such that $T\phi^*=\phi^*$. By (3.2), ϕ^* satisfies (1.2). So ϕ^* is an almost periodic solution of (1.2) in B^*. The proof of Theorem 3.1 is now complete.

Theorem 3.2. *Let $X^*(t)$ be the positive almost periodic solution of system (1.2) in the region B^*. Suppose that (3.1) holds. Then, the solution $X(t;t_0,\varphi)$ of (1.2) with $\varphi\in C$ converges exponentially to $X^*(t)$ as $t\to+\infty$.*

Proof. Set $X(t)=X(t;t_0,\varphi)$, $z_1(t)=x(t)-x^*(t)$ and $z_2(t)=u(t)-u^*(t)$, where $t\in[t_0-\tau,+\infty)$. Then

$$
\begin{cases}
z_1'(t) = -\alpha(t)z_1(t)+\sum\limits_{j=1}^{m}\beta_j(t)\big(x(t-\tau_j(t))e^{-\gamma_j(t)x(t-\tau_j(t))}-x^*(t-\tau_j(t))e^{-\gamma_j(t)x^*(t-\tau_j(t))}\big) \\
\qquad\quad -c(t)\big(x(t)u(t-\zeta(t))-x^*(t)u^*(t-\zeta(t))\big), \\
z_2'(t) = -\lambda(t)z_2(t)+g(t)z_1(t-\delta(t)).
\end{cases}
\tag{3.6}
$$

Define a continuous function $\Gamma(\mu)$ by setting

$$\Gamma(\mu) = -(\alpha^- - \mu) + \sum_{j=1}^{m} \beta_j^+ \frac{1}{e^2} e^{\mu\tau}, \ \ \mu \in [0,1].$$

Then, we have

$$\Gamma(0) = -\alpha^- + \sum_{j=1}^{m} \beta_j^+ \frac{1}{e^2} < 0, \ \Gamma(\mu) \to +\infty(\mu \to +\infty),$$

which implies that there exist two constants $\eta > 0$ and $\sigma \in (0,\lambda^-) \cap (0,1]$ such that

$$\Gamma(\sigma) = -(\alpha^- - \sigma) + \sum_{j=1}^{m} \beta_j^+ \frac{1}{e^2} e^{\sigma\tau} < -\eta < 0. \tag{3.7}$$

We consider the Lyapunov functional

$$V(t) = z_1(t)e^{\sigma t}.$$

Calculating the upper right derivative of $V(t)$ along the solution $z_t(t)$ of (3.6), we have

$$\begin{aligned} D^+(V(t)) &= -\alpha(t)z_1(t)e^{\sigma t} + \sum_{j=1}^{m} \beta_j(t)\big(x(t-\tau_j(t))e^{-\gamma_j(t)x(t-\tau_j(t))} - x^*(t-\tau_j(t))e^{-\gamma_j(t)x^*(t-\tau_j(t))}\big)e^{\sigma t} \\ &\quad -c(t)\big(x(t)u(t-\zeta(t)) - x^*(t)u^*(t-\zeta(t))\big)e^{\sigma t} + \sigma z_1(t)e^{\sigma t} \\ &\le \Big[(\sigma - \alpha(t))z_1(t) + \sum_{j=1}^{m} \beta_j(t)\big(x(t-\tau_j(t))e^{-\gamma_j(t)x(t-\tau_j(t))} \\ &\quad -x^*(t-\tau_j(t))e^{-\gamma_j(t)x^*(t-\tau_j(t))}\big)\Big]e^{\sigma t}, \ \ \text{for all } t > t_0. \end{aligned} \tag{3.8}$$

We claim that

$$V(t) = z_1(t)e^{\sigma t} < e^{\sigma t_0}\Big(\max_{t\in[t_0-\tau,t_0]} |\varphi_1(t) - x^*(t)| + K_1\Big) := M_1, \ \text{for all } t > t_0 \tag{3.9}.$$

Contrarily, there must exist $T_1 > t_0$ such that

$$V(T_1) = M_1 \text{ and } V(t) < M_1 \text{ for all } t \in [t_0 - \tau, T_1), \tag{3.10}$$

which implies that

$$V(T_1) - M_1 = 0 \text{ and } V(t) - M_1 < 0 \text{ for all } t \in [t_0 - \tau, T_1). \tag{3.11}$$

Together with (3.5), (3.8) and (3.11), we obtain

$$\begin{aligned} 0 &\le D^+(V(T_1 - M_1)) \\ &= D^+(V(T_1)) \\ &\le \Big[(\sigma - \alpha(T_1))z_1(T_1) + \sum_{j=1}^{m} \beta_j(T_1)\big(x(T_1-\tau_j(T_1))e^{-\gamma_j(T_1)x(T_1-\tau_j(T_1))} \\ &\quad -x^*(T_1-\tau_j(T_1))e^{-\gamma_j(T_1)x^*(T_1-\tau_j(T_1))}\big)\Big]e^{\sigma T_1} \\ &\le (\sigma - \alpha(T_1))z_1(T_1)e^{\sigma T_1} + \sum_{j=1}^{m} \beta_j(T_1)\frac{1}{e^2}|z_1(T_1-\tau_j(T_1))|e^{\sigma(T_1-\tau_j(T_1))}e^{\sigma\tau_j(T_1)} \\ &\le \Big[-(\alpha^- - \sigma) + \sum_{j=1}^{m} \beta_j^+ \frac{1}{e^2}e^{\sigma\tau}\Big]M_1. \end{aligned}$$

Thus,

$$0 \le -(\alpha^- - \sigma) + \sum_{j=1}^{m} \beta_j^+ \frac{1}{e^2} e^{\sigma\tau},$$

which contradicts with (3.7). Hence, (3.9) holds. It follows that

$$z_1(t) < M_1 e^{-\sigma t} \quad \text{for all } t > t_0. \tag{3.12}$$

Integrating the second equation of (3.6) from T_0 to $t(\geq T_0+\tau)$, by (3.12), we get

$$\begin{aligned}
z_2(t) &= e^{-\int_{T_0}^t \lambda(s)ds} z_2(T_0) + \int_{T_0}^t e^{-\int_s^t \lambda(v)dv} g(s) z_1(s-\delta(s))ds \\
&\leq z_2(T_0)e^{-\lambda^-(t-T_0)} + g^+ M_1 \int_{T_0}^t e^{\lambda^-(s-t)} e^{-\sigma(s-\delta(s))}ds \\
&= z_2(T_0)e^{\lambda^- T_0} e^{-\lambda^- t} + g^+ M_1 e^{-\lambda^- t} \int_{T_0}^t e^{(\lambda^- -\sigma)s} e^{-\sigma\delta(s)}ds \\
&\leq z_2(T_0)e^{\lambda^- T_0} e^{-\lambda^- t} + \frac{g^+ M_1 e^{-\lambda^- t} e^{\sigma\tau}}{\lambda^- - \sigma}\left(e^{(\lambda^- -\sigma)t} - e^{(\lambda^- -\sigma)T_0}\right) \\
&\leq z_2(T_0)e^{\lambda^- T_0} e^{-\lambda^- t} + \frac{g^+ M_1 e^{\sigma\tau}}{\lambda^- - \sigma} e^{-\sigma t} \\
&\leq \left(z_2(T_0)e^{\lambda^- T_0} e^{-(\lambda^- -\sigma)t} + \frac{g^+ M_1 e^{\sigma\tau}}{\lambda^- - \sigma}\right)e^{-\sigma t} \\
&\leq \left(z_2(T_0)e^{\lambda^- T_0} + \frac{g^+ M_1 e^{\sigma\tau}}{\lambda^- - \sigma}\right)e^{-\sigma t}.
\end{aligned}$$

Let $M_2 = z_2(T_0)e^{\lambda^- T_0} + \dfrac{g^+ M_1 e^{\sigma\tau}}{\lambda^- - \sigma}$, we have

$$z_2(t) \leq M_2 e^{-\sigma t} \quad \text{for all } t > t_0. \tag{3.13}$$

It follows from (3.12) and (3.13) that the solution $X(t;t_0,\varphi)$ of (1.3) converges exponentially to $X^*(t)$ as $t \to +\infty$. This completes the proof of Theorem 3.2.

4. An example

The following example shows the feasibility of our main results.

Example 4.1 Consider Nicholson's blowflies model with feedback control:

$$\begin{cases}
x'(t) = -(19+\cos^2 t)x(t) + e^{e-1}(11+0.01|\sin(\sqrt{2}t)|)x(t-e)e^{-x(t-e)} \\
\qquad\quad +e^{e-1}(11+0.01|\cos(\sqrt{3}t)|)x(t-e)e^{-x(t-e)} - \frac{1+t^2}{10+t^2}x(t)u(t-e^{-1}), \\
u'(t) = -(1+0.1\cos^4 t)u(t) + (0.8+0.1|\sin t|)x(t-e^{-1}).
\end{cases} \tag{4.1}$$

Here corresponding to the system (1.2), we assume that

$$\alpha^- = 19,\ \alpha^+ = 20,\ \beta_j^- = 11e^{-1},\ \beta_j^+ = 11.01e^{-1},\ \gamma_j^- = \gamma_j^+ = 1,$$

$$c^- = 0,\ c^+ = 0.1,\ \tau = e > 0,\ \lambda^- = 1,\ \lambda^+ = 1.1,\ g^- = 0.9,\ g^+ = 0.8,$$

and

$$\sum_{j=1}^{2}\left(\frac{\beta_j}{\gamma_j}\right)^+ \frac{1}{\alpha^- e} = 2\times 11.01 e^{e-1}\frac{1}{193} = 2.377 < e,$$

$$\sum_{j=1}^{2}\frac{\beta_j^-}{\alpha^+}K_1 e^{-\gamma_j^+ K_1} - \frac{c^+ K_1 K_2}{\alpha^+} > 2\frac{11e^{e-1}}{20}ee^{-e} - \frac{0.1e^2}{20} = 1.0631 > 1,$$

$$\frac{\sum_{j=1}^{2}\beta_j^+}{\alpha^- e^2}+\frac{c^+K_2}{\alpha^-}+\frac{c^+K_1}{\alpha^-}=0.903<1,\ \frac{g^+}{\lambda^-}=0.9<1.$$

This implies that Nicholson's blowflies model (4.1) satisfies the condition (H_1) and (3.1) with $K_1=K_2=e,\ k_1=1,\ ,k_2=0.5$. Hence, from Theorem 3.1 and 3.2, system (4.1) has a positive almost periodic solution. Numeric simulation (Fig. 1) strongly imply the above conclusion.

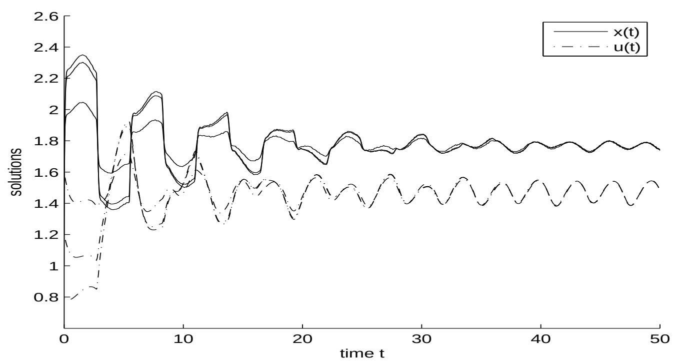

Fig. 1 Dynamic behavior of the solution $(x(t),u(t))^T$ of system (4.1) with the initial value $(\varphi_1(\theta),\varphi_1(\theta))^T=(1,0.8)^T,(1.2,1.2)^T$ *and* $(1.6,1.6)^T$ for $\theta\in[-\tau,0]$, respectively.

Conflict of Interests

The author declares that there is no conflict of interests.

Acknowledgements

This work is supported by the Foundation of Fujian Education Bureau (JA13361) and the National Natural Science Foundation of Fujian Province (2013J01010).

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