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# FOUR POSITIVE ALMOST PERIODIC SOLUTIONS TO AN IMPULSIVE NON-AUTONOMOUS LOTKA-VOLTERRA PREDATOR-PREY SYSTEM WITH HARVESTING TERMS

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**Abstract.** In this paper, by using Mawhins continuation theorem of coincidence degree theory and some analytical approaches, we establish the existence of four positive almost periodic solutions for an impulsive non-autonomous two species almost periodic Lotka-Volterra predator-prey system with time delay and harvesting terms. Furthermore, our results improve the main results of paper [1]. An example is given to illustrate the effectiveness of our results.

**Keywords:** Four positive almost periodic solutions; Lotka-Volterra predator-prey system; Coincidence degree; Harvesting term.

**2010 AMS Subject Classification:** 34C27, 34K14, 34K45, 92D25.

## 1. Introduction

In [1], the authors proposed a non-autonomous two species Lotka-Volterra predator-prey system with harvesting terms model:

$$\begin{aligned} \frac{dx(t)}{dt} &= x(t)\Big(a_1(t) - b_1(t)x(t) - c_1(t)y(t)\Big) - h_1(t), \\ \frac{dy(t)}{dt} &= y(t)\Big(a_2(t) - b_2(t)y(t) + c_2(t)x(t)\Big) - h_2(t). \end{aligned} \tag{1}$$

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Considering the periodicity of the environment(e.g, seasonal effects of weather, food supplies, mating habits,etc), under the assumptions of periodicity of the parameters of (1), using Mawhin's continuation theorem of coincidence degree theory, the authors of [1] established the four positive periodic solutions to (1). However, it is more realistic to consider almost periodic systems than periodic systems. Also, to the best of our knowledge, there are very few published letters considering the almost periodic solutions for non-autonomous Lotka-Volterra predator-prey system with impulse and time delay by using Mawhins continuation theorem of coincidence degree theory.

In fact, the ecological system is often deeply perturbed by activities of human explortation such as planting and harvesting. To accurately describe the system, one needs to use the impulsive differential equations. There are many papers to investigate the impulsive systems and many excellent results are obtained. See, for example, [2-7] and the references cited therein). Also, time delay is an important factor of mathematical models in ecology. The models with delay and impulsive effect have been investigated, see the papers [8-12] and the references cited therein.

Motivated by above, in this paper, we are concerned with the following impulsive non-autonomous two species Lotka-Volterra predator-prey system with time delay and harvesting terms model:

$$\begin{aligned}
&\tfrac{dx(t)}{dt} = x(t)\Big(a_1(t) - b_1(t)x(t) - c_1(t)y(t-\tau_2(t))\Big) - h_1(t), t \neq t_k,\\
&\tfrac{dy(t)}{dt} = y(t)\Big(a_2(t) - b_2(t)y(t) + c_2(t)x(t-\tau_1(t))\Big) - h_2(t), t \neq t_k,\\
&x(t_k^+) = (1+\Gamma_{1k})x(t_k), t = t_k,\\
&y(t_k^+) = (1+\Gamma_{2k})y(t_k), t = t_k,
\end{aligned} \tag{2}$$

where $x(t)$ and $y(t)$ denote the densities of prey and predator species respectively; $a_1(t)$, $b_1(t)$, $h_1(t)$ stand for the prey species birth rate, death rate and harvesting rate, respectively; $a_2(t)$, $b_2(t)$, $h_2(t)$ stand for the predator species birth rate, death rate and harvesting rate, respectively; $c_1(t)$ represent the predator species predation rate on the prey species; $c_2(t)$ stands for the transformation rate between the prey species and the predator species; $a_i(t)$, $b_i(t)$, $c_i(t)$, $h_i(t)(i = 1,2)$ are all bounded and positive continuous almost periodic functions, the time delay

$\tau_1(t)$ and $\tau_2(t)$ are all nonnegative continuous almost periodic functions; $\Gamma_{ik} > -1 (i = 1, 2)$ are all constants and $0 = t_0 < t_1 < t_2 < ...t_k < t_{k+1} < ...$, are impulse points with $\lim_{k\to+\infty} t_k = +\infty$.

Our main purpose of this paper is by using Mawhins continuation theorem of coincidence degree theory to establish the existence of four positive almost periodic solutions for system (2). For the work concerning the existence of positive almost periodic solutions of almost periodic population models which was done by using coincidence degree theory, we refer the reader to [13-15].

The organization of this paper is as follows. In Section2, we state some definitions lemmas which are useful in later sections and make some preparations. In Section3, using Mawhins continuation theorem of coincidence degree theory and some analytical approaches, we establish sufficient conditions for the existence of four positive almost periodic solutions to system(2).

## 2. Preliminaries

In this section, we give a short introduction to some referred definitions and lemmas that will come into play later on.

$AP(R) = \{f(t) : f(t)$ is a continuous, real valued, almost periodic function on $R\}$. Suppose that $f(t, \phi)$ is almost periodic in $t$, uniformly with respect to $\phi \in C([-\sigma, 0], R)$. $T(f, \varepsilon, S)$ will denote the set of $\varepsilon-$ almost periods with respect to $S \subset C([-\sigma, 0], R)$, $l(\varepsilon, S)$ the inclusion interval, $\Lambda(f)$ the set of Fourier exponents, mod(f) the module of f, and $m(f)$ the mean value.

Let $PC(R, R^n)$=$\{\varphi : R \to R^n, \varphi$ is a piecewise continuous function with points of discontinuity of the first kind at $t_k, k = 1, 2, ...$, at which $\varphi(t_k^-)$ and $\varphi(t_k^+)$ exist and $\varphi(t_k^-) = \varphi(t_k)\}$.

**Definition 2.1.** [16] The family of sequences $\{t_k^j = t_{k+j} - t_k, k, j \in Z\}$ is said to be equipotentially almost periodic if for arbitrarye $\varepsilon > 0$, there exists a relatively dense set $\varepsilon-$ almost periods, that are common for any sequences.

**Definition 2.2.** [16] The function $\varphi \in PC(R, R)$ is said to be almost periodic, if the following conditions hold:

(1) the set of sequences $\{t_k^j = t_{k+j} - t_k, k, j \in Z\}$ is equipotentially almost periodic;

(2) for any $\varepsilon > 0$ there exists a real number $\delta = \delta(\varepsilon) > 0$ such that if the points $t_1$ and $t_2$ belong to the same interval of continuity of $\varphi(t)$ and $|t_1 - t_2| < \delta$, then $|\varphi(t_1) - \varphi(t_2)| < \varepsilon$;

(3) for any $\varepsilon > 0$ there exists a relatively dense set $T$ of $\varepsilon-$ almost periodic such that if $\tau \in T$, then $|\varphi(t+\tau) - \varphi(t)| < \varepsilon$ for all $t \in R$ which satisfy the condition $|t - t_k| > \varepsilon, k \in Z$.

**Lemma 2.1.** [13] *If $f(t) \in AP(R)$, then there exists $t_0 \in R$ such that $f(t_0) = m(f)$.*

**Lemma 2.2.** [17] *Assume that $f(t) \in AP(R)$. Then $f(t)$ is bounded on $R$.*

**Lemma 2.3.** [13] *Assume that $x(t) \in AP(R) \cap C^1(R,R)$, then there exist two point sequences $\{\xi_k\}_{k=1}^{\infty}$, $\{\eta_k\}_{k=1}^{\infty}$, such that $x'(\xi_k) = x'(\eta_k) = 0$, $\lim\limits_{k\to+\infty} \xi_k = +\infty$ and $\lim\limits_{k\to+\infty} \eta_k = -\infty$.*

**Lemma 2.4.** [13] *Assume that $x(t) \in AP(R) \cap C^1(R,R)$, then $x(t)$ falls into one of the following four cases:*

*(i) there are $\xi, \eta \in R$ such that $x(\xi) = \sup\limits_{t\in R} x(t)$ and $x(\eta) = \inf\limits_{t\in R} x(t)$. In this case, $x'(\xi) = x'(\eta) = 0$.*

*(ii) there are no $\xi, \eta \in R$ such that $x(\xi) = \sup\limits_{t\in R} x(t)$ and $x(\eta) = \inf\limits_{t\in R} x(t)$. In this case, for any $\varepsilon > 0$, there exist two points $\xi, \eta \in R$ such that $x'(\xi) = x'(\eta) = 0$, $x(\xi) > \sup\limits_{t\in R} x(t) - \varepsilon$ and $x(\eta) < \inf\limits_{t\in R} x(t) + \varepsilon$.*

*(iii) there is a $\xi \in R$ such that $x(\xi) = \sup\limits_{t\in R} x(t)$ and there is no $\eta \in R$ such that $x(\eta) = \inf\limits_{t\in R} x(t)$. In this case, $x'(\xi) = 0$ and for any $\varepsilon > 0$, there exist an $\eta$ such that $x'(\eta) = 0$ and $x(\eta) < \inf\limits_{t\in R} x(t) + \varepsilon$.*

*(iv) there is an $\eta \in R$ such that $x(\eta) = \inf\limits_{t\in R} x(t)$ and there is no $\xi \in R$ such that $x(\xi) = \sup\limits_{t\in R} x(t)$. In this case, $x'(\eta) = 0$ and for any $\varepsilon > 0$, there exist a $\xi$ such that $x'(\xi) = 0$ and $x(\xi) > \sup\limits_{t\in R} x(t) - \varepsilon$.*

*Consider the following system*

$$\begin{aligned} \frac{dx(t)}{dt} &= x(t)\Big(a_1(t) - \overline{b}_1(t)x(t) - \overline{c}_1(t)y(t-\tau_2(t))\Big) - \overline{h}_1(t), \\ \frac{dy(t)}{dt} &= y(t)\Big(a_2(t) - \overline{b}_2(t)y(t) + \overline{c}_2(t)x(t-\tau_1(t))\Big) - \overline{h}_2(t), \end{aligned} \tag{3}$$

*where*

$$\overline{b}_1(t) = b_1(t)\prod_{0<t_k<t}(1+\Gamma_{1k}), \qquad \overline{c}_1(t) = c_1(t)\prod_{0<t_k<t}(1+\Gamma_{2k}),$$

$$\overline{h}_1(t)=h_1(t)\prod_{0<t_k<t}(1+\Gamma_{1k})^{-1},\quad \overline{b}_2(t)=b_2(t)\prod_{0<t_k<t}(1+\Gamma_{2k}),$$
$$\overline{c}_2(t)=c_2(t)\prod_{0<t_k<t}(1+\Gamma_{1k}),\qquad \overline{h}_2(t)=h_2(t)\prod_{0<t_k<t}(1+\Gamma_{2k})^{-1}.$$

**Lemma 2.5.** *For systems* (2) *and* (3), *the following results hold:*

*(1) if* $(x(t),y(t))^T$ *is a solution of (2), then*

$$(\overline{x}(t),\overline{y}(t))^T=\Big(\prod_{0<t_k<t}(1+\Gamma_{1k})^{-1}x(t),\ \prod_{0<t_k<t}(1+\Gamma_{2k})^{-1}y(t)\Big)^T$$

*is a solution of (3).*

*(2) if* $(\overline{x}(t),\overline{y}(t))^T$ *is a solution of (3), then*

$$(x(t),y(t))^T=\Big(\prod_{0<t_k<t}(1+\Gamma_{1k})\overline{x}(t),\ \prod_{0<t_k<t}(1+\Gamma_{2k})\overline{y}(t)\Big)^T$$

*is a solution of (2).*

**Proof.** (1) Suppose that $(x(t),y(t))^T$ is a solution of (2). Let

$$\overline{x}(t)=\prod_{0<t_k<t}(1+\Gamma_{1k})^{-1}x(t),\ \ \overline{y}(t)=\prod_{0<t_k<t}(1+\Gamma_{2k})^{-1}y(t),$$

we first show that $\overline{x}(t),\overline{y}(t)$ are continuous. Since $\overline{x}(t),\overline{y}(t)$ are continuous on each interval $(t_k,t_{k+1}]$, it is sufficient to check the continuity of $\overline{x}(t),\overline{y}(t)$ at the impulse points $t_k,k\in Z^+$. Since

$$\overline{x}(t_k^+)=\prod_{0<t_s\le t_k}(1+\Gamma_{1s})^{-1}x(t_k^+)=(1+\Gamma_{1k})^{-1}\prod_{0<t_s<t_k}((1+\Gamma_{1s})^{-1}(1+\Gamma_{1k})x(t_k)=\overline{x}(t_k)$$

and

$$\overline{x}(t_k^-)=\prod_{0<t_s<t_k^-}(1+\Gamma_{1s})^{-1}x(t_k^-)=\prod_{0<t_s<t_k}(1+\Gamma_{1s})^{-1}x(t_k)=\overline{x}(t_k),$$

thus $\overline{x}(t_k)$ is continuous on $[0,+\infty)$. Using the same method, we get $\overline{y}(t_k)$ is continuous on $[0,+\infty)$. By substituting

$$x(t)=\prod_{0<t_k<t}(1+\Gamma_{1k})\overline{x}(t),\ \ y(t)=\prod_{0<t_k<t}(1+\Gamma_{2k})\overline{y}(t)$$

into the equation of system (2), we obtain

$$\frac{d\overline{x}(t)}{dt}=\overline{x}(t)\Big(a_1(t)-\overline{b}_1(t)\overline{x}(t)-\overline{c}_1(t)\overline{y}(t-\tau_2(t))\Big)-\overline{h}_1(t),$$

$$\frac{d\bar{y}(t)}{dt} = \bar{y}(t)\Big(a_2(t) - \bar{b}_2(t)\bar{y}(t) + \bar{c}_2(t)\bar{x}(t-\tau_1(t))\Big) - \bar{h}_2(t).$$

Therefore, $(\bar{x}(t), \bar{y}(t))^T$ is a solution of (3).

(2) Suppose that $(\bar{x}(t), \bar{y}(t))^T$ is a solution of (3). Let

$$x(t) = \prod_{0<t_k<t}(1+\Gamma_{1k})\bar{x}(t), \;\; y(t) = \prod_{0<t_k<t}(1+\Gamma_{2k})\bar{y}(t),$$

then for any $t \neq t_k, k \in Z^+$, by substituting

$$\bar{x}(t) = \prod_{0<t_k<t}(1+\Gamma_{1k})^{-1}x(t), \;\; \bar{y}(t) = \prod_{0<t_k<t}(1+\Gamma_{2k})^{-1}y(t)$$

into the equation of system (2.1), we obtain

$$\frac{dx(t)}{dt} = x(t)\Big(a_1(t) - b_1(t)x(t) - c_1(t)y(t-\tau_2(t))\Big) - h_1(t),$$

$$\frac{dy(t)}{dt} = y(t)\Big(a_2(t) - b_2(t)y(t) + c_2(t)x(t-\tau_1(t))\Big) - h_2(t),$$

and for $t = t_k, k \in Z^+$, we obtain

$$\begin{aligned} x(t_k^+) &= \lim_{t\to t_{k+}} \prod_{0<t_k<t}(1+\Gamma_{1k})\bar{x}(t) = \prod_{0<t_s\leq t_k}(1+\Gamma_{1s})\bar{x}(t_k) \\ &= (1+\Gamma_{1k})\prod_{0<t_s<t_k}(1+\Gamma_{1s})\bar{x}(t_k) = (1+\Gamma_{1k})x(t_k). \end{aligned}$$

Similarly, we have

$$y(t_k^+) = (1+\Gamma_{2k})y(t_k).$$

Therefore, $(x(t), y(t))^T$ is a solution of (2).

**Lemma 2.6.** [1] *Let $x > 0, y > 0, z > 0$ and $x > 2\sqrt{yz}$, for the functions $f(x,y,z) = \frac{x+\sqrt{x^2-4yz}}{2z}$ and $g(x,y,z) = \frac{x-\sqrt{x^2-4yz}}{2z}$, the following assertions hold.*

*(1) $f(x,y,z)$ and $g(x,y,z)$ are monotonically increasing and monotonically decreasing on the variable $x \in (0,+\infty)$, respectively.*

*(2) $f(x,y,z)$ and $g(x,y,z)$ are monotonically decreasing and monotonically increasing on the variable $y \in (0,+\infty)$, respectively.*

*(3) $f(x,y,z)$ and $g(x,y,z)$ are monotonically decreasing and monotonically increasing on the variable $z \in (0,+\infty)$, respectively.*

For the sake of convenience, we denote $f^l = \inf_{t\in[0,\omega]} f(t)$, $f^M = \sup_{t\in[0,\omega]} f(t)$ here $f(t)$ is a continuous almost periodic function.

Throughout this paper, we need the following assumptions:

$(H_1)$ $a_1^l - \overline{c}_1^M H_1 > 2\sqrt{\overline{b}_1^M \overline{h}_1^M}$, $\quad a_2^l > 2\sqrt{\overline{b}_2^M \overline{h}_2^M}$,

where

$$H_1 = \frac{a_2^M + \overline{c}_2^M l_1^+}{\overline{b}_2^l}.$$

$(H_2)$ The set of sequences $\{t_k^j = t_{k+j} - t_k, k, j \in Z\}$ is uniformly almost periodic.

$(H_3)$ $\prod_{0<t_k<t}(1+\Gamma_{ik}), (i = 1,2)$ is almost periodic.

For simplicity, we need to introduce some notations as follows:

$$l_1^{\pm} = \frac{a_1^M \pm \sqrt{(a_1^M)^2 - 4\overline{b}_1^l \overline{h}_1^l}}{2\overline{b}_1^l},\ A^{\pm} = \frac{(a_1^l - \overline{c}_1^M H_1) \pm \sqrt{(a_1^l - \overline{c}_1^M H_1)^2 - 4\overline{b}_1^M \overline{h}_1^M}}{2\overline{b}_1^M},$$
$$l_2^{\pm} = \frac{a_2^l \pm \sqrt{(a_2^l)^2 - 4\overline{b}_2^M \overline{h}_2^M}}{2\overline{b}_2^M},\ H_2 = \frac{h_2^l}{a_2^M + \overline{c}_2^M l_1^+}.$$

**Lemma 2.7.** *For the following equation*

$$a_1(t) - \overline{b}_1(t)e^{u_1(t)} - \overline{h}_1(t)e^{-u_1(t)} = 0,$$

$$a_2(t) - \overline{b}_2(t)e^{u_2(t)} - \overline{h}_2(t)e^{-u_2(t)} = 0,$$

*by the assumption $H_1$ and lemma 2.6, we have the following inequalitie*

$$\ln l_1^- < \ln u_1^- < \ln A^- < \ln A^+ < \ln u_1^+ < \ln l_1^+.$$

*Since*

$$l_2^+ = \frac{a_2^l + \sqrt{(a_2^l)^2 - 4\overline{b}_2^M \overline{h}_2^M}}{2\overline{b}_2^M} < \frac{a_2^l}{\overline{b}_2^M} < \frac{a_2^M}{\overline{b}_2^l} < \frac{a_2^M + \overline{c}_2^M l_1^+}{\overline{b}_2^l} = H_1,$$
$$l_2^- = \frac{a_2^l - \sqrt{(a_2^l)^2 - 4\overline{b}_2^M \overline{h}_2^M}}{2\overline{b}_2^M} = \frac{2h_2^M}{a_2^l + \sqrt{(a_2^l)^2 - 4\overline{b}_2^M \overline{h}_2^M}} > \frac{h_2^M}{a_2^l} > \frac{h_2^l}{a_2^M} > \frac{h_2^l}{a_2^M + \overline{c}_2^M l_1^+} = H_2,$$

*similarly, we have $u_2^+ < H_1$ and $u_2^- < l_2^-$.*

*Then, we have*

$$\ln H_2 < \ln u_2^- < \ln l_2^- < \ln l_2^+ < \ln u_2^+ < \ln H_1,$$

*where*

$$u_1^{\pm}=\frac{a_1(t)\pm\sqrt{(a_1(t))^2-4\overline{b}_1(t)\overline{h}_1(t)}}{2\overline{b}_1(t)},u_2^{\pm}=\frac{a_2(t)\pm\sqrt{(a_2(t))^2-4\overline{b}_2(t)\overline{h}_2(t)}}{2\overline{b}_2(t)}.$$

## 3. Main results

In this section, by using Mawhins continuation theorem, we will show a theorem about four positive almost periodic solutions for system (2).

Let $X$ and $Z$ be real normed vector spaces. Let $L$: $DomL\subset X\to Z$ be a linear mapping and $N:X\times[0,1]\to Z$ be a continuous mapping. The mapping $L$ will be called a Fredholm mapping of index zero if dim $KerL=codim ImL<+\infty$ and $ImL$ is closed in $Z$. If $L$ is a Fredholm mapping of index zero, then there exist continuous projectors $P:X\to X$ and $Q:Z\to Z$ such that $ImP=KerL$ and $KerQ=ImL=Im(I-Q)$, and $X=KerL\oplus KerP$, $Z=ImL\oplus ImQ$. It follows that $L|_{DomL\bigcap kerP}:(I-P)X\to ImL$ is invertible and its inverse is denoted by $K_p$. If $\Omega$ is a bounded open subset of $X$, the mapping $N$ is called $L-$ compact on $\overline{\Omega}\times[0,1]$, if $QN(\overline{\Omega}\times[0,1])$ is bounded and $K_p(I-Q)N:\overline{\Omega}\times[0,1]\to X$ is compact. Because $ImQ$ is isomorphic to $KerL$, there exists an isomorphism$J$:$ImQ\to KerL$.

**Lemma 3.1.** [18] *Let $L$ be a Fredholm mapping of index zero and let $N$ be $L-$compact on $\overline{\Omega}\times[0,1]$ Assume that:*

*(a) for each $\lambda\in(0,1)$, every solution $x$ of $Lx=\lambda N(x,\lambda)$ is such that $x\notin\partial\Omega\cap DomL$*

*(b) $QN(x,0)x\neq 0$ for each $x\in\partial\Omega\cap KerL$;*

*(c) $deg(JQN(x,0),\Omega\cap KerL,0)\neq 0$.*

*Then $Lx=N(x,1)$ has at least one solution in $\overline{\Omega}\bigcap DomL$.*

In what follows, we always assume that $(H_3)$ holds. Consider $X=Z=V_1\bigoplus V_2$, $V_1=\{z(t)=(z_1(t),z_2(t))^T:z_i(t)\in AP(R),mod(z_i(t))\subseteq mod(F_i),\forall\mu\in\Lambda(z_i(t))$ satisfies $|\mu|\geq\alpha,(i=1,2)\}$, satisfies that $V_1\cup\{a_i(t),\overline{b}_i(t),\overline{c}_i(t),\tau_i(t),\ \overline{h}_i(t),(i=1,2)\}$ is equipotentially almost periodic. $V_2=\{z(t)\equiv(c_1,c_2)\in R^2\}$ where

$$F_1(t,\varphi_1,\varphi_2)=a_1(t)-\overline{b}_1(t)e^{\varphi_1(0)}-\overline{c}_1(t)e^{\varphi_2(-\tau_2(t))}-\overline{h}_1(t)e^{-\varphi_1(0)},$$

$$F_2(t,\varphi_1,\varphi_2) = a_2(t) - \overline{b}_2(t)e^{\varphi_2(0)} + \overline{c}_2(t)e^{\varphi_1(-\tau_1(t))} - \overline{h}_2(t)e^{-\varphi_2(0)},$$

in which $\varphi_i \in C([-\tau,0),R), i=1,2, \tau = max\sup_{t\in R}\{\tau_1(t),\tau_2(t)\}$ and $\alpha$ is given positive constant. Define

$$\|z\| = \sup_{t\in R}|z_1(t)| + \sup_{t\in R}|z_2(t)| \quad for\,all\, z\in X = Z.$$

Similar to the proofs of Lemma 3.1, Lemma 3.2 in [14] and Lemma 3.3 in [15], one can easily prove the following three lemmas, respectively.

**Lemma 3.2.** *X and Z are Banach spaces equipped with the norm* $\|.\|$.

**Lemma 3.3.** *Let* $L: X\to Z, Lx = x' = (x_1', x_2')^T$, *then L is a Fredholm mapping of index zero.*

**Lemma 3.4.** *Let* $N: X\times[0,1]\to Z$, $N(x(t),\lambda) = (N(x_1(t),\lambda), N(x_2(t),\lambda))^T = (G_1^x, G_2^x)^T$, *where*

$$G_1^x = N(x_1(t),\lambda) = a_1(t) - \overline{b}_1(t)e^{x_1(t)} - \lambda\overline{c}_1(t)e^{x_2(t-\tau_2(t))} - \overline{h}_1(t)e^{-x_1(t)},$$

$$G_2^x = N(x_2(t),\lambda) = a_2(t) - \overline{b}_2(t)e^{x_2(t)} + \lambda\overline{c}_2(t)e^{x_1(t-\tau_1(t))} - \overline{h}_2(t)e^{-x_2(t)}$$

*and*

$$P: X\to X, Px = \Big(m(x_1), m(x_2)\Big)^T,\ \ Q: Z\to Z, Qz = \Big(m(z_1), m(z_2)\Big)^T.$$

*Then N is L−compact on* $\overline{\Omega}$, *where* $\Omega$ *is an open bounded subset of X.*

**Theorem 3.1.** *Assume that* $(H_1)-(H_3)$ *hold, then system* $(2)$ *has at least four positive almost periodic solutions.*

**Proof.** By making the substitutions $x(t) = exp(u_1(t)), y(t) = exp(u_2(t))$, then system (3) is reformulated as

$$\begin{aligned} u_1'(t) &= a_1(t) - \overline{b}_1(t)e^{u_1(t)} - \overline{c}_1(t)e^{u_2(t-\tau_2(t))} - \overline{h}_1(t)e^{-u_1(t)},\\ u_2'(t) &= a_2(t) - \overline{b}_2(t)e^{u_2(t)} + \overline{c}_2(t)e^{u_1(t-\tau_1(t))} - \overline{h}_2(t)e^{-u_2(t)}. \end{aligned} \tag{4}$$

Then if there exists almost periodic solution $(u_1(t),u_2(t))^T$ of $(4)$, We can get at least one positive almost periodic solutions $(x(t),y(t))^T$ of $(3)$.

In order to use Lemma 3.1, we have to find at least four appropriate open bounded subsets $X$. Corresponding to the operator equation $Lx = \lambda N(x,\lambda), \lambda \in (0,1)$, we have

$$\begin{aligned} u_1'(t) &= \lambda\left(a_1(t) - \overline{b}_1(t)e^{u_1(t)} - \lambda\overline{c}_1(t)e^{u_2(t-\tau_2(t))} - \overline{h}_1(t)e^{-u_1(t)}\right), \\ u_2'(t) &= \lambda\left(a_2(t) - \overline{b}_2(t)e^{u_2(t)} + \lambda\overline{c}_2(t)e^{u_1(t-\tau_1(t))} - \overline{h}_2(t)e^{-u_2(t)}\right). \end{aligned} \tag{5}$$

Assume that $u = (u_1,u_2)^T \in X$ is an almost periodic solution of system (5) for some $\lambda \in (0,1)$. Then by Lemma 2.4, for any $\varepsilon > 0$ and $a \in R$ there exist $\xi_i, \eta_i \in [a, a+l(\varepsilon)] \cap T(u,\varepsilon), i = 1,2$ such that $u_i(\xi_i) > u_i^M - \varepsilon, u_i(\eta_i) < u_i^l + \varepsilon$ and $u_i'(\xi_i) = u_i'(\eta_i) = 0$. From this and (5), we have

$$\begin{aligned} a_1(\xi_1) - \overline{b}_1(\xi_1)e^{u_1(\xi_1)} - \lambda\overline{c}_1(\xi_1)e^{u_2(\xi_1-\tau_2(\xi_1))} - \overline{h}_1(\xi_1)e^{-u_1(\xi_1)} &= 0, (a), \\ a_2(\xi_2) - \overline{b}_2(\xi_2)e^{u_2(\xi_2)} + \lambda\overline{c}_2(\xi_2)e^{u_1(\xi_2-\tau_1(\xi_2))} - \overline{h}_2(\xi_2)e^{-u_2(\xi_2)} &= 0, (b) \end{aligned} \tag{6}$$

and

$$\begin{aligned} a_1(\eta_1) - \overline{b}_1(\eta_1)e^{u_1(\eta_1)} - \lambda\overline{c}_1(\eta_1)e^{u_2(\eta_1-\tau_2(\eta_1))} - \overline{h}_1(\eta_1)e^{-u_1(\eta_1)} &= 0, (a), \\ a_2(\eta_2) - \overline{b}_2(\eta_2)e^{u_2(\eta_2)} + \lambda\overline{c}_2(\eta_2)e^{u_1(\eta_2-\tau_1(\eta_2))} - \overline{h}_2(\eta_2)e^{-u_2(\eta_2)} &= 0. (b). \end{aligned} \tag{7}$$

According to equation $(a)$ of (6), we have

$$a_1(\xi_1) - \overline{b}_1(\xi_1)e^{u_1(\xi_1)} - \overline{h}_1(\xi_1)e^{-u_1(\xi_1)} = \lambda\overline{c}_1(\xi_1)e^{u_2(\xi_1-\tau_2(\xi_1))} > 0.$$

It follows that

$$\overline{b}_1^l e^{2u_1(\xi_1)} - a_1^M e^{u_1(\xi_1)} + \overline{h}_1^l \le \overline{b}_1(\xi_1)e^{2u_1(\xi_1)} - a_1(\xi_1)e^{u_1(\xi_1)} + \overline{h}_1(\xi_1) < 0,$$

namely,

$$\overline{b}_1^l e^{2u_1(\xi_1)} - a_1^M e^{u_1(\xi_1)} + \overline{h}_1^l < 0,$$

which implies that

$$\ln\frac{a_1^M - \sqrt{(a_1^M)^2 - 4\overline{b}_1^l\overline{h}_1^l}}{2\overline{b}_1^l} < u_1(\xi_1) < \ln\frac{a_1^M + \sqrt{(a_1^M)^2 - 4\overline{b}_1^l\overline{h}_1^l}}{2\overline{b}_1^l},$$

namely,

$$\ln l_1^- < u_1(\xi_1) < \ln l_1^+. \tag{8}$$

Similarly, from the equation $(a)$ of (7), we obtain

$$\ln l_1^- < u_1(\eta_1) < \ln l_1^+. \tag{9}$$

From the equation $(b)$ of $(6)$, we obtain

$$\begin{aligned}
\overline{b}_2^M e^{2u_2(\xi_2)} + \overline{h}_2^M &\geq \overline{b}_2(\xi_2)e^{2u_2(\xi_2)} + \overline{h}_2(\xi_2) \\
&= [a_2(\xi_2) + \lambda \overline{c}_2(\xi_2)e^{u_1(\xi_2 - \tau_1(\xi_2))}]e^{u_2(\xi_2)} \\
&> a_2(\xi_2)e^{u_2(\xi_2)} \\
&\geq a_2^l e^{u_2(\xi_2)}.
\end{aligned}$$

It follows that

$$\overline{b}_2^M e^{2u_2(\xi_2)} - a_2^l e^{u_2(\xi_2)} + \overline{h}_2^M > 0,$$

which implies that

$$u_2(\xi_2) > \frac{a_2^l + \sqrt{(a_2^l)^2 - 4\overline{b}_2^M \overline{h}_2^M}}{2\overline{b}_2^M}, \quad u_2(\xi_2) < \frac{a_2^l - \sqrt{(a_2^l)^2 - 4\overline{b}_2^M \overline{h}_2^M}}{2\overline{b}_2^M},$$

namely,

$$u_2(\xi_2) > \ln l_2^+ \ \ or \ \ u_2(\xi_2) < \ln l_2^-. \tag{10}$$

Similarly, from the equation $(b)$ of $(7)$, we obtain

$$u_2(\eta_2) > \ln l_2^+ \ \ or \ \ u_2(\eta_2) < \ln l_2^-. \tag{11}$$

Moreover, from the equation $(b)$ of $(6)$, we have

$$\begin{aligned}
\overline{b}_2^l e^{u_2(\xi_2)} &\leq \overline{b}_2(\xi_2)e^{u_2(\xi_2)} \\
&< \overline{b}_2(\xi_2)e^{u_2(\xi_2)} + \overline{h}_2(\xi_2)e^{-u_2(\xi_2)} = a_2(\xi_2) + \lambda \overline{c}_2(\xi_2)e^{u_1(\xi_2 - \tau_1(\xi_2))} \\
&< a_2^M + \overline{c}_2^M l_1^+,
\end{aligned}$$

which implies that

$$u_2(\xi_2) < \ln \frac{a_2^M + \overline{c}_2^M l_1^+}{b_2^l} = \ln H_1. \tag{12}$$

Similarly, from the equation $(b)$ of $(7)$, we obtain

$$\begin{aligned}\overline{h}_2^l e^{-u_2(\eta_2)} &\leq \overline{h}_2(\eta_2)e^{-u_2(\eta_2)} \\ &< \overline{b}_2(\eta_2)e^{u_2(\eta_2)} + \overline{h}_2(\eta_2)e^{-u_2(\eta_2)} = a_2(\eta_2) + \lambda\overline{c}_2(\eta_2)e^{u_1(\eta_2-\tau_1(\eta_2))} \\ &< a_2^M + \overline{c}_2^M l_1^+,\end{aligned}$$

which implies that

$$u_2(\eta_2) > \ln \frac{h_2^l}{a_2^M + \overline{c}_2^M l_1^+} = \ln H_2. \quad (13)$$

It follows from $(10) - (13)$ and Lemma 2.7, we get

$$\ln H_2 < u_2(\eta_2) < u_2(\xi_2) < \ln l_2^- \ \ or\ \ \ln l_2^+ < u_2(\eta_2) < u_2(\xi_2) < \ln H_1. \quad (14)$$

According to equation $(a)$ of $(6)$, we have

$$\begin{aligned}\overline{b}_1^M e^{u_1(\xi_1)} + \overline{h}_1^M e^{-u_1(\xi_1)} + c_1^M H_1 &> \overline{b}_1(\xi_1)e^{u_1(\xi_1)} + \overline{h}_1(\xi_1)e^{-u_1(\xi_1)} + \lambda\overline{c}_1(\xi_1)e^{u_2(\xi_1-\tau_2(\xi_1))} \\ &= a_1(\xi_1) \geq a_1^l.\end{aligned}$$

Hence, we have $\overline{b}_1^M e^{2u_1(\xi_1)} - (a_1^l - \overline{c}_1^M H_1)e^{u_1(\xi_1)} + \overline{h}_1^M > 0$, which implies that

$$u_1(\xi_1) > \frac{(a_1^l - \overline{c}_1^M H_1) + \sqrt{(a_1^l - \overline{c}_1^M H_1)^2 - 4\overline{b}_1^M \overline{h}_1^M}}{2\overline{b}_1^M},$$

or

$$u_1(\xi_1) < \frac{(a_1^l - \overline{c}_1^M H_1) - \sqrt{(a_1^l - \overline{c}_1^M H_1)^2 - 4\overline{b}_1^M \overline{h}_1^M}}{2\overline{b}_1^M},$$

namely,

$$u_1(\xi_1) > \ln A^+ \ or\ u_1(\xi_1) < \ln A^-. \quad (15)$$

Similarly, from the equation $(a)$ of $(7)$, we obtain

$$u_1(\eta_1) > \ln A^+ \ or\ u_1(\eta_1) < \ln A^-. \quad (16)$$

It follows from $(8), (9), (15), (16)$ and Lemma 2.7, we get

$$\ln l_1^- < u_1(\eta_1) < u_1(\xi_1) < \ln A^- \ \ or\ \ \ln A^+ < u_1(\eta_1) < u_1(\xi_1) < \ln l_1^+. \quad (17)$$

By (14) and (17), we have for all $t \in R$

$$\ln l_1^- < u_1(t) < \ln A^- \ or \ \ln A^+ < u_1(t) < \ln l_1^+, \tag{18}$$

and

$$\ln H_2 < u_2(t) < \ln l_2^- \ or \ \ln l_2^+ < u_2(t) < \ln H_1. \tag{19}$$

Clearly, $\ln l_1^{\pm}, \ln l_2^{\pm}, \ln A^{\pm}, \ln H_1$ and $\ln H_2$ are independent of $\lambda$. We denote

$$\Omega_1 = \{u = (u_1, u_2)^T \in X \ \ |u_1(t) \in (\ln l_1^-, \ln A^-), u_2(t) \in (\ln H_2, \ln l_2^-)\},$$

$$\Omega_2 = \{u = (u_1, u_2)^T \in X \ \ |u_1(t) \in (\ln l_1^-, \ln A^-), u_2(t) \in (\ln l_2^+, \ln H_1)\},$$

$$\Omega_3 = \{u = (u_1, u_2)^T \in X \ \ |u_1(t) \in (\ln A^+, \ln l_1^+), u_2(t) \in (\ln H_2, \ln l_2^-)\},$$

$$\Omega_4 = \{u = (u_1, u_2)^T \in X \ \ |u_1(t) \in (\ln A^+, \ln l_1^+), u_2(t) \in (\ln l_2^+, \ln H_1)\}.$$

Thus $\Omega_k, k = 1,2,3,4$ are bounded open subsets of $X$, $\Omega_i \cap \Omega_j = \emptyset, i \neq j$. Thus $\Omega_k$ satisfies the requirement $(a)$ in Lemma 3.1.

Now we show that $(b)$ of Lemma 3.1 holds, i.e., we prove when $u \in \partial\Omega_i \cap KerL = \partial\Omega_i \cap R^2, QN(u,0) \neq (0,0)^T, i = 1,2,3,4$. If it is not true, then when $u \in \partial\Omega_i \cap KerL = \partial\Omega_i \cap R^2, i = 1,2,3,4$. constant vector $u = (u_1, u_2)^T$ with $u \in \partial\Omega_i, i = 1,2,3,4$ satisfies

$$m(a_1(t) - \overline{b}_1(t)e^{u_1} - \overline{h}_1(t)e^{-u_1}) = 0$$

and

$$m(a_2(t) - \overline{b}_2(t)e^{u_2} - \overline{h}_2(t)e^{-u_2}) = 0.$$

In view of the mean value theorem of calculous, there exist two points $\zeta_1, \zeta_2$ such that

$$\begin{aligned} &a_1(\zeta_1) - \overline{b}_1(\zeta_1)e^{u_1} - \overline{h}_1(\zeta_1)e^{-u_1} = 0, \\ &a_2(\zeta_2) - \overline{b}_2(\zeta_2)e^{u_2} - \overline{h}_2(\zeta_2)e^{-u_2} = 0. \end{aligned} \tag{20}$$

From (20), we have

$$u_1^{\pm} = \ln \frac{a_1(\zeta_1) \pm \sqrt{(a_1(\zeta_1))^2 - 4\overline{b}_1(\zeta_1)\overline{h}_1(\zeta_1)}}{2\overline{b}_1(\zeta_1)},$$
$$u_2^{\pm} = \ln \frac{a_2(\zeta_2) \pm \sqrt{(a_2(\zeta_2))^2 - 4\overline{b}_2(\zeta_2)\overline{h}_2(\zeta_2)}}{2\overline{b}_2(\zeta_2)}. \tag{21}$$

According to Lemma 2.7, we obtain

$$\ln l_1^- < \ln u_1^- < \ln A^- < \ln A^+ < \ln u_1^+ < \ln l_1^+,$$
$$\ln H_2 < \ln u_2^- < \ln l_2^- < \ln l_2^+ < \ln u_2^+ < \ln H_1. \tag{22}$$

Then $u$ belongs to one of $\Omega_i \cap R^2, i = 1,2,3,4$. This contradicts the fact that $u \in \partial\Omega_i \cap R^2, i = 1,2,3,4$. This proves $(b)$ in Lemma 3.1 holds.

Finally, we show that $(c)$ in Lemma 3.1 holds. Note that the system of algebraic equations

$$a_1(\zeta_1) - \overline{b}_1(\zeta_1)e^x - \overline{h}_1(\zeta_1)e^{-x} = 0,$$
$$a_2(\zeta_2) - \overline{b}_2(\zeta_2)e^y - \overline{h}_2(\zeta_2)e^{-y} = 0,$$

has four distinct solutions since $H_1$ holds:

$$(x_1^*, y_1^*) = (\ln x_-, \ln y_-), \quad (x_2^*, y_2^*) = (\ln x_-, \ln y_+),$$
$$(x_3^*, y_3^*) = (\ln x_+, \ln y_-), \quad (x_4^*, y_4^*) = (\ln x_+, \ln y_+),$$

where

$$x_{\pm} = \frac{a_1(\zeta_1) \pm \sqrt{(a_1(\zeta_1))^2 - 4\overline{b}_1(\zeta_1)\overline{h}_1(\zeta_1)}}{2\overline{b}_1(\zeta_1)},$$
$$y_{\pm} = \frac{a_2(\zeta_2) \pm \sqrt{(a_2(\zeta_2))^2 - 4\overline{b}_2(\zeta_2)\overline{h}_2(\zeta_2)}}{2\overline{b}_2(\zeta_2)}.$$

From $(21), (22)$, we have

$$(x_1^*, y_1^*) \in \Omega_1, \quad (x_2^*, y_2^*) \in \Omega_2, \quad (x_3^*, y_3^*) \in \Omega_3, \quad (x_4^*, y_4^*) \in \Omega_4.$$

Since $KerL = ImQ$, we can take $J = I$. A direct computation gives, we get

$$deg\{JQN(u,0),\Omega_i \cap KerL,(0,0)^T\} = sign\begin{vmatrix} -\overline{b}_1(\zeta_1)x^* + \dfrac{\overline{h}_1(\zeta_1)}{x^*} & 0 \\ 0 & -\overline{b}_2(\zeta_2)y^* + \dfrac{\overline{h}_2(\zeta_2)}{y^*} \end{vmatrix}.$$

Since

$$a_1(\zeta_1) - \overline{b}_1(\zeta_1)x^* - \frac{\overline{h}_1(\zeta_1)}{x^*} = 0,$$

and

$$a_2(\zeta_2) - \overline{b}_2(\zeta_2)y^* - \frac{\overline{h}_2(\zeta_2)}{y^*} = 0,$$

we find that

$$deg\{JQN(u,0),\Omega_i \cap KerL,(0,0)^T\} = sign[(a_1(\zeta_1) - 2\overline{b}_1(\zeta_1)x^*)(a_2(\zeta_2) - 2\overline{b}_2(\zeta_2)y^*)],$$

$i = 1,2,3,4$. Thus

$$deg\{JQN(u,0),\Omega_1 \cap KerL,(0,0)^T\} = sign[(a_1(\zeta_1) - 2\overline{b}_1(\zeta_1)x_-)(a_2(\zeta_2) - 2\overline{b}_2(\zeta_2)y_-)] = 1,$$

$$deg\{JQN(u,0),\Omega_2 \cap KerL,(0,0)^T\} = sign[(a_1(\zeta_1) - 2\overline{b}_1(\zeta_1)x_-)(a_2(\zeta_2) - 2\overline{b}_2(\zeta_2)y_+)] = -1,$$

$$deg\{JQN(u,0),\Omega_3 \cap KerL,(0,0)^T\} = sign[(a_1(\zeta_1) - 2\overline{b}_1(\zeta_1)x_+)(a_2(\zeta_2) - 2\overline{b}_2(\zeta_2)y_-)] = -1,$$

$$deg\{JQN(u,0),\Omega_4 \cap KerL,(0,0)^T\} = sign[(a_1(\zeta_1) - 2\overline{b}_1(\zeta_1)x_+)(a_2(\zeta_2) - 2\overline{b}_2(\zeta_2)y_+)] = 1,$$

namely,

$$deg\{JQN(u,0),\Omega_i \cap KerL,(0,0)^T\} \neq 0, i = 1,2,3,4.$$

So far, we have proved that $\Omega_k, k = 1,2,3,4$ satisfies all the assumptions in Lemma 3.1. Hence, system (4) has at least 4 different almost periodic solutions. So, system (3) has at least 4 different positive almost periodic solutions. If $(\overline{x}(t),\overline{y}(t))^T$ is an almost periodic solution of system (3), by using Lemma 2.5, we know that

$$\Big(x(t) = \prod_{0<t_k<t}(1+\Gamma_{1k})\overline{x}(t), y(t) = \prod_{0<t_k<t}(1+\Gamma_{2k})\overline{y}(t)\Big)^T$$

is a solution of system (2). Therefore, system (2) has at least 4 different positive almost periodic solutions. This completes the proof of Theorem 3.1.

Consider the following non-autonomous Lotka-Volterra predator-prey system with harvesting terms

$$\begin{aligned} \tfrac{dx(t)}{dt} &= x(t)\Big(a_1(t)-b_1(t)x(t)-c_1(t)y(t-\tau_2(t))\Big)-h_1(t),\\ \tfrac{dy(t)}{dt} &= y(t)\Big(a_2(t)-b_2(t)y(t)+c_2(t)x(t-\tau_1(t))\Big)-h_2(t). \end{aligned} \tag{23}$$

Similar to the proof of Theorem 3.1, one can easily obtain

**Corollary 3.1.** *Assume that the following condition holds* $(H_1^{'})$ $a_1^l - c_1^M H_1 > 2\sqrt{b_1^M h_1^M}, a_2^l > 2\sqrt{b_2^M h_2^M}$. *Then system(23) has at least* 4 *different positive almost periodic solutions. Since condition unrelated to delays, thus, if* $\tau_i(t)\equiv 0(i=1,2,)$, *our results also supplement the results of Zhao and Ye (see[17]).*

## 4. An example

Consider the following two species non-autonomous Lotka-Volterra predator-prey with with impulsive and harvesting terms:

$$\begin{aligned} &x^{'}(t)=x(t)\Big(3+\sin\sqrt{2}t-\frac{8+2\cos t}{17}x(t)-\frac{2+\sin t}{90}y(t-5|\sin t|)\Big)-\frac{153+17\cos\sqrt{5}t}{400}, t\neq t_k,\\ &y^{'}(t)=x(t)\Big(3+\cos\sqrt{3}t-\frac{5+\sin t}{9}y(t)+\frac{4+2\sin t}{17}x(t-2|\sin t|)\Big)-\frac{18+9\cos\sqrt{5}t}{50}, t\neq t_k,\\ &x(t_k^+)=(1+(-0.15))x_1(t_k), t=t_k,\\ &x(t_k^+)=(1+(-0.1))x_2(t_k), t=t_k. \end{aligned} \tag{24}$$

In this case, $a_1(t)=3+\sin\sqrt{2}t$, $b_1(t)=\dfrac{8+2\cos t}{17}$, $c_1(t)=\dfrac{2+\sin t}{90}$, $h_1(t)=\dfrac{153+17\cos\sqrt{5}t}{400}$, $a_2(t)=3+\cos\sqrt{3}t$, $b_2(t)=\dfrac{5+\sin t}{9}$, $c_2(t)=\dfrac{4+2\sin t}{17}$, $h_2(t)=\dfrac{18+9\cos\sqrt{5}t}{50}$, $\tau_1(t)=2|\sin t|$, $\tau_2(t)=5|\sin t|$. Then, we have $\overline{b}_1(t)=b_1(t)\prod\limits_{0<t_k<t}(1+\Gamma_{1k})=\dfrac{8+2\cos t}{17}\times(1+(-0.15))=\dfrac{4+\cos t}{10}$, $\overline{c}_1(t)=c_1(t)\prod\limits_{0<t_k<t}(1+\Gamma_{2k})=\dfrac{2+\sin t}{90}\times(1+(-0.1))=\dfrac{2+\sin t}{100}$, $\overline{h}_1(t)=h_1(t)\prod_{0<t_k<t}(1+\Gamma_{1k})^{-1}=\dfrac{153+17\cos\sqrt{5}t}{400}(1+(-0.15))^{-1}=\dfrac{9+\cos\sqrt{5}t}{20}$, $\overline{b}_2(t)=b_2(t)\prod_{0<t_k<t}(1+\Gamma_{2k})=\dfrac{5+\sin t}{9}\times(1+(-0.1))=\dfrac{5+\sin t}{10}$, $\overline{c}_2(t)=c_2(t)\prod\limits_{0<t_k<t}(1+\Gamma_{1k})=\dfrac{4+2\sin t}{17}(1+(-0.15))$

$= \dfrac{2+\sin t}{10}$, $\overline{h}_2(t) = h_2(t) \displaystyle\prod_{0<t_k<t} (1+\Gamma_{2k})^{-1} = \dfrac{18+9\cos\sqrt{5}t}{50}(1+(-0.1))^{-1} = \dfrac{2+\cos\sqrt{5}t}{5}$.

Since

$$l_1^+ = \frac{a_1^M + \sqrt{(a_1^M)^2 - 4\overline{b}_1^l \overline{h}_1^l}}{2\overline{b}_1^l} = \frac{4+\sqrt{4^2 - 4\times\frac{3}{10}\times\frac{8}{20}}}{2\times\frac{3}{10}} = \frac{20+2\sqrt{97}}{3},$$

$$H_1 = \frac{a_2^M + \overline{c}_2^M l_1^+}{\overline{b}_2^l} = \frac{4+\frac{3}{10}\times\frac{20+2\sqrt{97}}{3}}{\frac{4}{10}} = 15 + \frac{\sqrt{97}}{2},$$

$$2\sqrt{\overline{b}_1^M \overline{h}_1^M} = 2\sqrt{\frac{5}{10}\times\frac{10}{20}} = 1, \quad 2\sqrt{\overline{b}_2^M \overline{h}_2^M} = 2\sqrt{\frac{6}{10}\times\frac{3}{5}} = \frac{6}{5},$$

we have

$$a_1^l - \overline{c}_1^M H_1 = 2 - \frac{3}{100}(15+\frac{\sqrt{97}}{2}) > \frac{7}{5} > 1 = 2\sqrt{\overline{b}_1^M \overline{h}_1^M},$$

$$2 = a_2^l > 2\sqrt{\overline{b}_2^M \overline{h}_2^M} = \frac{6}{5}.$$

Hence, all conditions of Theorem 3.1 are satisfied, then, system (24) has at least four positive almost periodic solutions.

## 5. Discussions and conclusion

On the existence of four positive almost periodic solutions for an impulsive Lotka-Volterra predator-prey system with time delay and harvesting terms, to the best of our knowledge, the aspect results have not yet appeared in the related literature. Since both delay and harvesting systems are very important in implementations and applications, while it is troublesome to study the existence of positive almost periodic solutions for impulsive system , respectively, it is meaningful to study almost periodic solutions for an impulsive Lotka-Volterra predator-prey system with time delay and harvesting terms. In this paper, some sufficient conditions are derived to guarantee the existence of four positive almost periodic solutions for an impulsive Lotka-Volterra predator-prey system with time delay and harvesting terms.

**Conflict of Interests**

The authors declare that there is no conflict of interests.

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