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ALMOST PERIODIC SOLUTIONS OF IMPULSIVE LASOTA-WAZEWSKA MODEL WITH MULTIPLE TIME-VARYING DELAYS ON TIME SCALES

YUYING ZHANG, CHUNHUA ZHANG, XIAOXING CHEN*

College of Mathematics and Computer, Fuzhou University, Fuzhou, Fujian 350002, China

Abstract. In this paper, we study an impulsive Lasota-Wazewska model with multiple time-varying delays on time scales. Based on some basic results about almost periodic dynamic equations on time scales, we obtain some sufficient conditions for the existence and exponential stability of the almost periodic solution to the model. Finally, we give an example to illustrate our main results.

Keywords: Positive almost periodic solutions; Impulsive Lasota-Wazewska model; Exponential stability; Time-varying delays; Time scales.

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1. Introduction

In order to describe the survival of red blood cells in animal, Wazewska-Czyzewska and Lasota [1] proposed the following autonomous functional differential equation

$$x'(t) = -\alpha x(t) + \beta e^{-\gamma x(t-\omega)}, \tag{1}$$

*Corresponding author

E-mail address: cxxing79@163.com

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where $x(t)$ denotes the number of red blood cells at time t, α is the rate of the red blood cells, β and γ describe the production of red blood cells per unite time and ω is the time required to produce a red blood cell. The model and its modifications have been extensively and intensively studied about its oscillation, global attractivity, periodic solutions, almost periodic solutions and so on (see [2-10] and the references cited therein) have been obtained.

In real world phenomenon, the environment varies due to the factors such as seasonal effects of weather, food supplies, mating habits, harvesting. So it is usual to assume the periodicity of parameters in the systems. However, if the various constituent components of the temporally nonuniform environment is with incommensurable (nonintegral multiples) periods, then one has to consider the environment to be almost periodic since there is no a priori reason to expect the existence of periodic solutions. For this reason, the assumption of almost periodicity is more realistic, more important and more general.

It is well known that both continuous time and discrete time are of importance in applications. People usually study the continuous time and the discrete time separately, because it is troublesome to study them together. The theory of measure chains, was introduced by Hilger in his Ph.D. thesis in 1988 [11] in order to unify continuous and discrete analysis. Therefore, the theory has recently received a lot of attention (see [12-17] and the references cited therein).

On the other hand, the theory of impulsive differential equations is now being recognized to be richer than the corresponding theory of differential equations without impulses. Meanwhile it represents a more natural framework for mathematical modelling of many real-world phenomena, such as population dynamic models and Lasota-Wazewska model since many dynamical processes are characterized by the fact that at certain moments of time they undergo abrupt changes of state (see [18-23] and the references cited therein). However, the study on the existence and exponential stability of the almost periodic solution to impulsive Lasota-Wazewska model with multiple time-varying delays on time scales is rare, so it is meaningful to discuss it.

Motivated by the above works, we are concerned with the existence and exponential stability of almost periodic solutions to impulsive Lasota-Wazewska model with multiple time-varying delays described by the following differential equations on time scales

$$\begin{cases} x^{\Delta}(t) = -\alpha(t)x(t) + \sum_{j=1}^{m} \beta_j(t) e^{-\gamma_j(t)x(t-\omega_j(t))}, \ t \neq t_k, \\ \Delta x(t_k) = h_k x(t_k) + \delta_k, \ t \in \mathbb{Z}, \end{cases} \tag{2}$$

where $t \in \mathbb{T}, \ t - \omega_j(t) \in \mathbb{T}, \ j = 1, 2, \cdots, m.$ $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$ is impulses at moments t_k and $t_1 < t_2 < \cdots$ is a strictly increasing sequence such that $\lim_{t \to +\infty} t_k = +\infty, \ h_k, \ \delta_k \in \mathbb{R}$. Due to the biological interpretation of (2), only positive solutions are meaningful and therefore admissible.

For any bounded function $a(t)$ on $\mathbb{T}$, we introduce the following notations:

$$\bar{a} = \sup_{t \in \mathbb{T}} a(t), \ \underline{a} = \inf_{t \in \mathbb{T}} a(t).$$

$PC_{rd}(\mathbb{T}, \mathbb{X})$ denote the set of all rd-piecewise continuous functions from the time scale $\mathbb{T}$ to a Banach space $\mathbb{X}$ which will be introduced in Section 2.

We introduce the following conditions:

(H_1) The functions $\alpha(t), \ \beta_j(t), \ \gamma_j(t), \ \omega_j(t) \in PC_{rd}(\mathbb{T}, \mathbb{R}^+)$ are almost periodic functions on $\mathbb{T}$, $j = 1, 2, \cdots, m$. There exist positive constants λ such that $\alpha(t) \leq \lambda$.

(H_2) $\{h_k\}$, $\{\delta_k\}$ are almost periodic sequences.

(H_3) The set of sequences $\{t_k^j\}$ are equipotentially almost periodic and $\inf_k t_k^1 = \theta > 0$, where $t_k^j = t_{k+j} - t_k, k \in \mathbb{Z}, \ j \in \mathbb{Z}$.

Let $t_0 \in \mathbb{T}$, introduce the notation: $PC_{rd}(t_0)$ is the space of all functions $\phi : [t_0 - \bar{\omega}, t_0] \to \Omega_0$ with respect to a sequence $\theta_1, \theta_2, \cdots, \theta_s \in (t_0 - \bar{\omega}, t_0)_{\mathbb{T}}$.

Let ϕ_0 be an element of $PC_{rd}(t_0)$. Denote by $\eta(t) = \eta(t; t_0, \phi_0), \phi_0 \in \Omega_0$ the solution of system (2), satisfying the initial conditions:

$$\begin{cases} \eta(t; t_0, \phi_0) = \phi_0(t), \ t \in (t_0 - \bar{\omega}, t_0)_{\mathbb{T}}, \\ \eta(t_0^+; t_0, \phi_0) = \phi_0(t). \end{cases}$$

2. Preliminaries

In this section, we will introduce some basic definitions, lemmas which are used in the proof of our main results.

Let $\mathbb{T}$ be a time scale, i.e.,$\mathbb{T}$ is a nonempty closed subset of $\mathbb{R}$.

Definition 2.1. [13] Let $\mathbb{T}$ be a time scale. For $t \in \mathbb{T}$, we define the forward jump operator $\sigma : \mathbb{T} \to \mathbb{T}$ by $\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}$, and the backward jump operator $\rho : \mathbb{T} \to \mathbb{T}$ by $\rho(t) = \sup\{s \in \mathbb{T} : s < t\}$, while the graininess function $\mu : \mathbb{T} \to [0,+\infty)$ is defined by $\mu(t) = \sigma(t) - t$. If $\sigma(t) > t$, we say that t is right-scattered, while if $\rho(t) < t$ we say that t is left-scattered. Also, if $t < \sup\mathbb{T}$ and $\sigma(t) = t$, then t is called right-dense, and if $t > \inf\mathbb{T}$ and $\rho(t) = t$, then t is called left-dense. If $\mathbb{T}$ has a left-scattered maximum m_1, defined $\mathbb{T}^{\kappa} = \mathbb{T} - \{m_1\}$; otherwise, set $\mathbb{T}^{\kappa} = \mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m_2, defined $\mathbb{T}_{\kappa} = \mathbb{T} - \{m_2\}$; otherwise, set $\mathbb{T}_{\kappa} = \mathbb{T}$.

Definition 2.2. [17] Let $f : \mathbb{T} \to \mathbb{R}$ be a function and let $t = (t_1, t_2, \cdots, t_n) \in \mathbb{T}^{\kappa}$. Then, we define $f^{\Delta}(t)$ to be the number (provided it exists) with the property that for any given $\varepsilon > 0$, there is a neighborhood U of t_i with $U = (t_i - \delta, t_i + \delta) \cap \mathbb{T}_i$ for some $\delta > 0$ such that

$$|[f^{\sigma_i}(t) - f_i^s(t)] - f^{\Delta_i}(t)[\sigma_i(t) - s]| \leq \varepsilon|\sigma_i(t) - s| \; for\; all\; s \in U.$$

$f^{\Delta_i}(t)$ is called the partial delta derivative of f at t with respect to the variable t_i.

Definition 2.3. [13] Define the set $\mathscr{R}^+$ of all positive regressive elements of $\mathscr{R}$ by

$$\mathscr{R}^+ = \mathscr{R}^+(\mathbb{T},\mathbb{R}) = \{\mathrm{p} : 1 + \mu(t)p(t) > 0 \; for all\; t \in \mathbb{T}\}.$$

Definition 2.4. [14,16,19] A time scale $\mathbb{T}$ is called an almost periodic time scale if

$$\Pi := \{\tau \in \mathbb{R} : t + \tau \in \mathbb{T}, for\; all\; t \in \mathbb{T}\} \neq \{0\}.$$

Definition 2.5. [19] We say $\varphi : \mathbb{T} \to \mathbb{R}^n$ is rd-piecewise continuous with respect to a sequence $\{\tau_i\} \subset \mathbb{T}$ which satisfy $\tau_i < \tau_{i+1}$, $i \in \mathbb{Z}$, if $\varphi(t)$ is continuous on $[\tau_i, \tau_{i+1})_{\mathbb{T}}$ and rd-continuous on $\mathbb{T}\backslash\{\tau_i\}$. Furthermore, $\tau_i < \tau_{i+1}$, $i \in \mathbb{Z}$, are called intervals of continuity of the function $\varphi(t)$.

For convenience, $PC_{rd}(\mathbb{T},\mathbb{R}^n)$ denotes the set of all piecewise continuous functions with respect to a sequence $\{\tau_i\}$, $i \in \mathbb{Z}$. For any integers i and j, denote $\tau_i^j = \tau_{i+j} - \tau_i$ and consider the sequence $\{\tau_i^j\}$, i, $j \in \mathbb{Z}$. It is easy to verify that the number τ_i^j, i, $j \in \mathbb{Z}$ satisfy

$$\tau_{i+k}^j - \tau_i^j = \tau_{i+j}^k - \tau_i^k,\; \tau_i^j - \tau_i^k = \tau_{i+k}^{j-k}.$$

Definition 2.6. [19] For any $\varepsilon > 0$, let $\Gamma_\varepsilon \subset \Pi$ be a set of real numbers and $\{\tau_i\} \subset \mathbb{T}$. We say $\{\tau_i^j\}$, $i, j \in \mathbb{Z}$ is equipotentially almost periodic on an almost periodic time scale $\mathbb{T}$ if for $r \in \Gamma_\varepsilon \subset \Pi$, there exists at least one integer k such that

$$|\tau_i^j - r| < \varepsilon, \text{ } for\, all\; i \in \mathbb{Z}.$$

Definition 2.7. [19] Let $\mathbb{T}$ be an almost periodic time scale and assume that $\{\tau_i\} \subset \mathbb{T}$ satisfying the derived sequence $\{\tau_i^j\}$, $i, j \in \mathbb{Z}$, is equipotentially almost periodic. We call a function $\varphi \in PC_{rd}(\mathbb{T}, \mathbb{R}^n)$ almost periodic if:

(i) for any $\varepsilon > 0$, there is a positive number $\delta = \delta(\varepsilon)$ such that if the points $t^{'}$ and $t^{''}$ belong to the same interval of continuity and $|t^{'} - t^{''}| < \delta$, then $\|\varphi(t^{'}) - \varphi(t^{''})\| < \varepsilon$;
(ii) for any $\varepsilon > 0$, there is relative dense set $\Gamma \subset \Pi$ of ε-almost periods such that if $\tau \in \Gamma$, then $\|\varphi(t+\tau) - \varphi(t)\| < \varepsilon$ for all $t \in \mathbb{T}$ which satisfy the condition $|t - \tau_i| > \varepsilon$, $i \in \mathbb{Z}$.

Definition 2.8. [19] A subset S of $\mathbb{R}$ is called relatively dense if there exists a positive number L such that $[a, a+L] \cap S \neq \varnothing$ for all $a \in \mathbb{R}$. The number L is called the density index (or the inclusion length) of S.

Lemma 2.1. [19] *An almost periodic function $\varphi \in PC_{rd}(\mathbb{T}, \mathbb{R}^n)$ is bounded on $\mathbb{T}$.*

Lemma 2.2.[19] *If $\varphi \in PC_{rd}(\mathbb{T}, \mathbb{R}^n)$ is an almost periodic function, then for any $\varepsilon > 0$, there exists a relative dense set of intervals of a fixed length γ, $0 < \gamma < \varepsilon$, which consist of ε-almost periods of the function $\varphi(t)$.*

Lemma 2.3. [19] *Let $\varphi \in PC_{rd}(\mathbb{T}, \mathbb{R}^n)$ be an almost periodic function with values in the set $E \subset \mathbb{R}^n$. If $F(y)$ is an uniformly continuous function defined on the set E, then the function $F(\varphi(t))$ is almost periodic.*

Lemma 2.4. [19] *For any two piecewise continuous almost periodic functions with respect to the same sequence $\{\tau_i\} \subset \mathbb{T}$, for any $\varepsilon > 0$ there exists a relative dense set of their common ε-almost periods.*

Consider the following system

$$w^\Delta = A(t)w, t \neq t_i, \; \Delta w|_{t=t_i} = B_i w, \tag{3}$$

where $A \in PC_{rd}(\mathbb{T}, \mathbb{R}^{n\times n})$ is an $n \times n$-matrix function, B_i are constant matrices, $\{\tau_i\} \subset \mathbb{T}$ are fixed times such that $t_i < t_{i+1}$, $i \in \mathbb{Z}$.

Denote by $W(t)$ matrix, the columns of which are the solutions of system (3) that form a fundamental system. The matrix $W(t)$ will be called an $n \times n$ fundamental matrix of system (3) ($n \geq 2$). It follows from the definition of the matrix $W(t)$ that it satisfies the matrix impulsive equation

$$W^{\Delta} = A(t)W,\ t \neq t_i,\ \Delta W|_{t=t_i} = B_i W. \tag{4}$$

A nondegenerate solution of system (4) $W(t)$, which satisfies the condition $W(t_0 = E)$ will be called the Cauchy matrix of system (3) and denoted by $W(t, t_0)$.

Lemma 2.5. [19] *If the following conditions are satisfied:*
(a) any compact interval $[a,b]_{\mathbb{T}}$ contains only a finite number of points t_i;
(b) for all $i \in \mathbb{Z}$, the matrices $E + B_i$ are nonsingular.
Then $W(t,t_0)W^{-1}(\kappa,t_0) = W(t,\kappa)$, where $t_j \leq t_{j+s-1} < \kappa < t_{j+s} < t_{j+k} < t \leq t_{j+s+1}$, $j, k, s \in \mathbb{Z}$. Furthermore, any solution of system (2.1) with initial condition $x(t_0,x_0) = x_0$ can be written as $x(t;t_0,x_0) = W(t,t_0)x_0$.

In the following, consider the system

$$w^{\Delta} = A(t)w + f(t),\ t \neq t_i,\ \Delta w|_{t=t_i} = B_i w + I_i, \tag{5}$$

where the matrix $A \in PC_{rd}(\mathbb{T}, \mathbb{R}^{n\times n})$ and $f(t) \in PC_{rd}(\mathbb{T}, \mathbb{R}^n)$, B_i and the times t_i are the same as in system (3), I_i are constants, (2.3) will be called linear nonhomogeneous impulsive differential system.

The relationship between nonhomogeneous system (5) and the corresponding homogeneous system (3) is given by the following lemma.

Lemma 2.6. [19] *If (a), (b) in Lemma 2.5 are satisfied and let $W(t)$ be a fundamental matrix of system (3). Then for $t > t_0$, every solution of system (3) is given by the formula*

$$w(t) = W(t)[c + \int_{t_0}^{t} W^{-1}(\sigma(\tau))f(\tau)\Delta\tau + \sum_{t_0<t_i<t} W^{-1}(t_i)(E+B_i)^{-1}I_i]. \tag{6}$$

In particular, if $W(t) = W(t,t_0)$ is the Cauchy matrix of system (3), then for $t > t_0$, any solution of (5) with initial condition $x(t_0,x_0) = x_0$ can be written as

$$w(t,x_0) = W(t,x_0)x_0 + \int_{t_0}^{t} W(t,\sigma(\tau))f(\tau)\Delta\tau + \sum_{t_0<t_i<t} W(t,t_i+0)I_i. \tag{7}$$

Lemma 2.7. [19] *If $p \in \mathscr{R}^+$ and $\sup_{t\in\mathbb{T}}|p(t)| = \bar{p} > 0$, then*

$$e_p(t,s) \le e^{\bar{p}(t-s)} \le e^{\bar{p}|t-s|}.$$

Lemma 2.8. [19] *Let $\alpha \in \mathbb{R}^+$. Then $e_{\ominus\alpha}(t,s) \ge e^{-\alpha(t-s)}$, $t > s$, s, $t \in \mathbb{T}$.*

3. Main results

By Lemmas 2.4 and 2.7, similar to the proof of Lemma 35 in Ref.[20], one can easily show the following:

Lemma 3.1. *For system (1.1), let $(H_1)-(H_3)$ hold. Then for each $\varepsilon > 0$ there exist ε_1, $0 < \varepsilon_1 < \varepsilon$, a relative dense set $\overline{T} \subset \Pi$ of real numbers, and a set P of integer numbers, such that the following relations are fulfilled:*

(a) *the following hold:*

$$|\alpha(t+\tau)-\alpha(t)| < \varepsilon, |\beta_j(t+\tau)-\beta_j(t)| < \varepsilon, |\gamma_j(t+\tau)-\gamma_j(t)| < \varepsilon,$$

$$|\tau_j(t+\tau)-\tau_j(t)| < \varepsilon, |t-t_k| > \varepsilon, t \in \mathbb{T}, \tau \in \overline{T}.$$

(b) *the following hold:*

$$|\gamma_{k+q}-\gamma_k| < \varepsilon, |\delta_{k+q}-\delta_k| < \varepsilon, q \in P, k \in \mathbb{Z}.$$

(c) *the following hold:*

$$|t_k^q-\tau| < \varepsilon_1, q \in P, \tau \in \overline{T}, k \in \mathbb{Z}.$$

It follows from Lemmas 2.7 and 3.1, similar to the proof of Lemma 36 in Ref.[20], one can easily show the following lemma:

Lemma 3.2. *For system (2), let $(H_1)-(H_3)$ hold. Then $W(t,s)$ of (2) satisfies the inequality*

$$|W(t,s)| \le e^{-\lambda(t-s)},\ t \ge s,$$

where K is a positive constant, and $W(t,s)$ is almost periodic, i.e. for any $\varepsilon > 0, t, s \in \mathbb{T}$, $|t-t_k| > \varepsilon$, $|s-t_k| > \varepsilon$, $k \in \mathbb{Z}$, there exists a relatively dense set of almost periods $\overline{T} \subset \Pi$ such that for $r \in \overline{T}$, we have

$$|W(t+r,s+r) - W(t,s)| < \varepsilon \Gamma e^{-\frac{\lambda}{2}(t-s)},$$

where Γ is a positive constant.

Lemma 3.3. *If $\varphi \in PC_{rd}(\mathbb{T},\mathbb{R}^n)$ is almost periodic and $\inf_k \tau_k^q = \theta > 0$, $q \in \mathbb{Z}$, then $\{\varphi(\tau_k)\}$ is an almost periodic sequence.*

Theorem 3.1. *Assume that conditions $(H_1)-(H_3)$ hold and the following conditions are satisfied:*

(H_4) $r = \dfrac{1+\lambda\bar{\mu}}{\lambda}(\sum_{j=1}^{m} \overline{\beta_j}\overline{\gamma_j}) < 1.$

(H_5) $(\ominus\lambda) \oplus p < 0$, *where*

$$p = (1+\lambda\bar{\mu}) \sum_{j=1}^{m} \overline{\beta_j}\overline{\gamma_j},$$

where $\bar{\mu} = \sup\limits_{t\in\mathbb{T}} \mu(t)$. Then (1.2) has a unique almost periodic solution $\eta(t)$ which is exponentially stable.

Proof. We denote by AP, $AP \subset PC_{rd}(\mathbb{T},\mathbb{R})$ the set of all almost periodic functions $\phi(t)$ satisfying the inequality $\|\phi\| < K$, where

$$\|\phi\| = \sup_{t\in\mathbb{T}} |\phi(t)|, K = \frac{(1+\lambda\bar{\mu})}{\lambda}\Big(\sum_{j=1}^{m} \overline{\beta_j}\Big) + \frac{C_0}{1-e^{-\lambda\theta}},$$

and $\max\{|\delta_k|\}=C_0$. We define in AP an operator S as

$$S\phi=\int_{-\infty}^{t}W(t,\sigma(s))\Big(\sum_{j=1}^{m}\beta_j(t)e^{-\gamma_j(t)\phi(t-\tau_j(t))}\Big)\Delta s+\sum_{t_k<t}W(t,t_k)\delta_k. \tag{8}$$

Now, we will show that S is self-mapping from AP to AP. For $\phi\in AP$ it follows

$$\begin{aligned}\|S\phi\| &=\sup_{t\in\mathbb{T}}\Big\{\int_{-\infty}^{t}|W(t,\sigma(s))|\Big|\sum_{j=1}^{m}\beta_j(t)e^{-\gamma_j(t)\phi(t-\tau_j(t))}\Big|\Delta s+\sum_{t_k<t}|W(t,t_k)||\delta_k|\Big\}\\ &\le\sup_{t\in\mathbb{T}}\Big\{\int_{-\infty}^{t}e^{-\lambda(t-\sigma(s))}\sum_{j=1}^{m}\overline{\beta_j}\Delta s+\sum_{t_k<t}e^{-\lambda(t-t_k)}C_0\Big\}\\ &\le\sup_{t\in\mathbb{T}}\Big\{\int_{-\infty}^{t}e_{\ominus\lambda}(t,\sigma(s))\sum_{j=1}^{m}\overline{\beta_j}\Delta s+\frac{C_0}{1-e^{\lambda\theta}}\Big\}\\ &\le\frac{(1+\lambda\bar{\mu})}{\lambda}\sum_{j=1}^{m}\overline{\beta_j}+\frac{C_0}{1-e^{-\lambda\theta}}=K.\end{aligned} \tag{9}$$

Let $\tau\in\overline{T}$, $q\in P$, where the sets $\overline{T}$ and P are determined in Lemma 3.1. Then

$$\begin{aligned}&\|S\phi(t+\tau)-S\phi(t)\|\\ &\le\sup_{t\in\mathbb{T}}\Big\{\int_{-\infty}^{t}|W(t+\tau,\sigma(s+\tau))-W(t,\sigma(s))|\sum_{j=1}^{m}|\beta_j(s+\tau)e^{-\gamma_j(s+\tau)\phi(s+\tau-\tau_j(s+\tau))}|\Delta s\\ &+\int_{-\infty}^{t}|W(t,\sigma(s)|\Big|\sum_{j=1}^{m}|\beta_j(s+\tau)e^{-\gamma_j(s+\tau)\phi(s+\tau-\tau_j(s+\tau))}-\sum_{j=1}^{m}\beta_j(s)e^{-\gamma_j(s)\phi(s-\tau_j(s))}\Big|\Delta s\\ &+\sum_{t_k<t}|W(t+\tau,t_{k+q})-W(t,t_k)||\delta_{k+q}|+\sum_{t_k<t}|W(t,t_k)||\delta_{k+q}-\delta_k|\Big\}\\ &\le\sup_{t\in\mathbb{T}}\Big\{\int_{-\infty}^{t}\varepsilon\Gamma e^{-\frac{\lambda}{2}(t-\sigma(s))}\sum_{j=1}^{m}\overline{\beta_j}\Delta s+\int_{-\infty}^{t}e^{-\lambda(t-\sigma(s))}\varepsilon\Delta s\\ &+\sum_{t_k<t}\varepsilon\Gamma e^{-\frac{\lambda}{2}(t-t_k)}C_0+\sum_{t_k<t}e^{-\lambda(t-t_k)}\varepsilon\Big\}\\ &\le\varepsilon\sup_{t\in\mathbb{T}}\Big\{\int_{-\infty}^{t}\Gamma e_{\ominus\frac{\lambda}{2}}(t-\sigma(s)\sum_{j=1}^{m}\overline{\beta_j}\Delta s+\int_{-\infty}^{t}e_{\ominus\lambda}(t-\sigma(s)\Delta s+\frac{\Gamma C_0}{1-e^{-\frac{\lambda}{2}\theta}}+\frac{1}{1-e^{-\lambda\theta}}\Big\}\\ &\le\varepsilon\Big[\Gamma\frac{2+\lambda\bar{\mu}}{\lambda}\sum_{j=1}^{m}\overline{\beta_j}+\frac{(1+\lambda\bar{\mu})}{\lambda}+\frac{\Gamma C_0}{1-e^{-\frac{\lambda}{2}\theta}}+\frac{1}{1-e^{-\lambda\theta}}\Big].\end{aligned} \tag{10}$$

Consequently, by (9) and (10), we obtain that $S\phi \in AP$. Let ϕ, $\psi \in AP$, then

$$\begin{aligned}&\|S\phi - S\psi\|\\ &\le \sup_{t\in\mathbb{T}}\Big\{\int_{-\infty}^{t}|W(t,\sigma(s))|\Big|\sum_{j=1}^{m}\beta_j(t)e^{-\gamma_j(t)\phi(t-\tau_j(t))} - \sum_{j=1}^{m}\beta_j(t)e^{-\gamma_j(t)\psi(t-\tau_j(t))}\Big|\Delta s\Big\}.\end{aligned} \tag{11}$$

In view of $(H_1 - H_6)$ and (8)-(11), from the inequality

$$|e^{-x} - e^{-y}| \le |x-y|,\ x, y \in [0,+\infty),$$

we have

$$\begin{aligned}&\|S\phi - S\psi\|\\ &\le \sup_{t\in\mathbb{T}}\Big\{\int_{-\infty}^{t}e_{\ominus\lambda}(t-\sigma(s)\sum_{j=1}^{m}\beta_j(t)\gamma_j(t)|\phi(t-\tau_j(t)) - \psi(t-\tau_j(t))|\Delta s\Big\}\\ &\le \frac{(1+\lambda\bar{\mu})}{\lambda}\sum_{j=1}^{m}\overline{\beta_j}\overline{\gamma_j}\|\phi-\psi\|\\ &= r\|\phi-\psi\|.\end{aligned} \tag{12}$$

Then from (12), it follows that S is a contracting operator in AP, and there exists a unique almost periodic solution of (2).

Let $x(t)$ be an arbitrary solution of (2), and $\eta(t)$ be almost periodic solution of (2). Then from (8), we obtain

$$\begin{aligned}&x(t)-\eta(t) = W(t,t_0)(x(t_0)-\eta(t_0))\\ &+\int_{t_0}^{t}W(t,\sigma(s))\Big|\sum_{j=1}^{m}\beta_j(t)e^{-\gamma_j(t)x(t-\tau_j(t))} - \sum_{j=1}^{m}\beta_j(t)e^{-\gamma_j(t)\eta(t-\tau_j(t))}\Big|\Delta s.\end{aligned}$$

Hence

$$\begin{aligned}&\|x(t)-\eta(t)\|\\ &\le e^{-\lambda(t-t_0)}\|x(t_0)-\eta(t_0)\| + \int_{t_0}^{t}e^{-\lambda(t-\sigma(s)}\sum_{j=1}^{m}|\beta_j(t)e^{-\gamma_j(t)x(t-\tau_j(t))} - \beta_j(t)e^{-\gamma_j(t)\eta(t-\tau_j(t))}|\Delta s\\ &\le e_{\ominus\lambda}(t,t_0)\|x(t_0)-\eta(t_0)\| + \int_{t_0}^{t}e_{\ominus\lambda}(t,\sigma(s))\sum_{j=1}^{m}\beta_j(t)\gamma_j(t)|x(t-\tau_j(t))-\eta(t-\tau_j(t))|\Delta s\\ &\le e_{\ominus\lambda}(t,t_0)\|x(t_0)-\eta(t_0)\| + (1+\bar{\mu}\lambda)\sum_{j=1}^{m}\overline{\beta_j}\overline{\gamma_j}\int_{t_0}^{t}e_{\ominus\lambda}(t,s)\|x(s)-\eta(s)\|\Delta s.\end{aligned}$$

Set $u(t) = \|x(t) - \eta(t)\| e_{\lambda}(t, t_0)$ and it follows

$$u(t) \le u(t_0) + (1 + \bar{\mu}\lambda) \sum_{j=1}^{m} \overline{\beta_j}\,\overline{\gamma_j} \int_{t_0}^{t} u(s)\Delta s,$$

from Gronwall-Bellmans inequality on time scales (see Ref.[18]), one has

$$u(t) \le u(t_0) e_p(t, t_0),$$

or

$$\|x(t) - \eta(t)\| \le \|x(t_0) - \eta(t_0)\| e_{(\ominus\lambda)\oplus p}(t, t_0),$$

where

$$p = (1 + \bar{\mu}\lambda) \sum_{j=1}^{m} \overline{\beta_j}\,\overline{\gamma_j}.$$

Thus, by condition (H_5) of the theorem, solutions of system (2) are exponentially stable. This completes the proof.

4. An example

Consider the following impulsive Lasota-Wazewska model with multiple time-varying delays on time scale T.

$$\begin{cases} x^{\Delta}(t) = -(1 + \cos^2(t))x(t) + \frac{1+\sin\sqrt{2}t}{10} e^{-\frac{1+\cos\sqrt{3}t}{10} x(t - e^{2|\sin(t)|})} \\ + \frac{1+\cos\sqrt{5}t}{10} e^{-\frac{1+\sin\sqrt{7}t}{10} x(t - e^{3|\cos(t)|})},\ t \ne 2k, \\ \Delta x(2k) = -\frac{1}{20} x(2k) + \frac{1}{20}. \end{cases} \tag{13}$$

Let $\mathbb{T} = \mathbb{R}$, then $\mu(t) = 0$. Obviously,

$$\lambda = \underline{\alpha} = 1,\ \overline{\beta_j} = \overline{\gamma_j} = \frac{1}{5},\ j = 1, 2.$$

So we have

$$r = \frac{1+\lambda\bar{\mu}}{\lambda}(\sum_{j=1}^{m}\overline{\beta_j}\overline{\gamma_j}) = 0.08 < 1,$$

and

$$p-\lambda \leq (1+\lambda\bar{\mu})\sum_{j=1}^{m}\overline{\beta_j}\overline{\gamma_j} - 1 = -0.92 < 0.$$

Thus, $(\ominus\lambda)\oplus p < 0$, by Theorem 3.1, (13) has a unique almost periodic solution $\eta(t)$ which is exponentially stable.

5. Discussions and conclusion

On the existence and stability of almost periodic solutions for an impulsive Lasota-Wazewska model with multiple time-varying delays on time scales, to the best of our knowledge, the aspect results have not yet appeared in the related literature. Since both continuous and discrete systems are very important in implementations and applications, while it is troublesome to study the existence and stability of almost periodic solutions for continuous system and discrete systems, respectively, it is meaningful to study that on time scales which can unify the continuous and discrete situations. In this paper, some sufficient conditions are derived to guarantee the existence and exponential stability of the almost periodic solution for an impulsive Lasota-Wazewska model with multiple time-varying delays on time scales.

Conflict of Interests

The authors declare that there is no conflict of interests.

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REFERENCES

[1] M. Wazewska-Czyzewska, A. Lasota, Mathematical problems of the dynamics of the dynamics of the red blood cells system, Mat. Stos. 6 (1976), 23-40.

[2] K. Gopalsamy, S.I. Trofimchuk, Almost periodic solutions of Lasota-Wazewska-type delay differential equation, J. Math. Anal. Appl. 237 (1999), 106-127.

[3] J.W. Li, S.S. Cheng, Global attractivity in an RBC survival model of Wazewska and Lasota, Quart. Appl. Math. 60 (2002), 477-483.

[4] K. Gopalsamy, Almost periodic solutions of Lasota-Wazewska-type delay differential equation, J. Math. Anal. Appl. 237 (1999), 106-127.

[5] G. Liu, A. Zhao, J. Yan, Existence and global attractivity of unique positive periodic solution for a Lasota-Wazewska model, Nonlinear Anal. 64 (2006), 1737-1746.

[6] H. Zhou, Z. Zhou, Q. Wang, Positive almost periodic solution for a class of LasotaWazewska model with infinite delays, Appl. Math. Comput. 218 (2011), 4501-4506.

[7] Z. Huang, S. Gong, L. Wang, Positive almost periodic solution for a class of LasotaWazewska model with multiple time-varying delays, Comput. Appl. Math. 61 (2011), 755-760.

[8] M.P. Chen, J. S. Yu, Oscillation and global attractivity in the discrete Lasota-Wazewska model, Soochow J. Math. 25 (1999), 1-9.

[9] Z. Zhou, Z. Zhang, Existence and global attractivity positive periodic solution for a discrete model, Electron. J. Differ. Equations, 2006 (95), (2006) 1-8.

[10] X. Wang, Z. Li, Globally dynamical behaviors for discrete Lasota-Wazewska model with several delays and almost periodic coefficients, Int. J. Biomath. 1 (2008), 95-105.

[11] S. Hilger, Analysis on measure chains-a unified approach to continuous and discrete calculus, Results Math. 18 (1990), 18-59.

[12] X.X. Chen, H.J. Guo, Four periodic solutions of a generalized delayed predator-prey system on time scales, Rocky Mountain J. Math. 38 (2008), 1307-1322.

[13] M. Bohner, A. Peterson, Dynamic equations on time scales, an introduction with applications, Boston, Birkhäuser, 2001.

[14] Y.K. Li, C. Wang, Uniformly almost periodic functions and almost periodic solutions to dynamic equations on time scales, Abstr. Appl. Anal. 2011 (2011), 341520.

[15] V. Lakshmikantham, S. Sivasundaram, B. Kaymakcalan, Dynamical systems on measure chains, Dordrecht, Kluwer Acadamic Publishiers, 1996.

[16] Y.K. Li, C. Wang, Almost periodic solutions of shunting inhibitory cellular neural networks on time scales, Commun. Nonlinear Sci. Numer. Simul. 17 (2012), 3258-3266.

[17] B. Jackson, Partial dynamic equations on time scales, J. Comput. Appl. Math. 186 (2006), 391-415.

[18] Y. Peng, Y. Kang, M.X Yuan, et al. Gronwall-type integral inequalities with impulses on time scales, Adv. Differ. Equ. 2011 (2011), 1-26.

[19] C. Wang, Almost periodic solutions of impulsive BAM neural networks with variable delays on time scales, Commun. Nonlinear Sci. Numer. Simulat. 19 (2014), 2828-2842.

[20] Samoilenko,A. M., Perestyuk, N. A., Impulsive differential equations, Singapore: World Scientific Publishing Corporation, 1995.

[21] A.M. Fink, G. Seifert, Liapunov function and almost periodic solutions for almost periodic systems, J. Differ. Equ. 5 (1969), 307-313.

[22] J. Yan, Existence and global attractivity of positive periodic solution for an impulsive Lasota-Wazewska model, J. Math. Anal. Appl. 279 (2003), 111-120.

[23] Gani Tr. Stamov, On the existence of almost periodic solutions for the impulsive LasotaWazewska model, Appl. Math. Lett. 22 (2009), 516-520.