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A PREDATOR-PREY MODEL WITH EFFECT OF TOXICITY AND WITH STOCHASTIC PERTURBATION

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Abstract. This paper reports on the behaviors of stochastic predator-prey populations in toxic environment. We show that the model established in this paper possesses non-negative solutions as this is essential in any population dynamics model. We also carry out analysis on the asymptotic behaviour of the model. We show that the model is ultimate bounded under suitable condition. At last, numerical simulations are carried out to support our results.

Keywords: Brownian motion; Stochastic differential equation; Non-negative solutions.

2010 AMS Subject Classification: 34C27, 34D05, 34A37.

1. Introduction

In recent years, the effects of toxicants emitted into the environment from industrial and household resources on biological species have received much attention of researchers [1-6].

In [6], the following model is discussed by Tapasi Das et al.

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$$(1)\qquad \begin{cases} \dfrac{dx_1(t)}{dt} = r_1x_1(t)(1-\dfrac{x_1(t)}{L}) - \alpha x_1(t)x_2(t) - \gamma_1 x_1^3(t), \\ \dfrac{dx_2(t)}{dt} = -r_2x_2(t) + \beta x_1(t)x_2(t) - \gamma_2 x_2^2(t). \end{cases}$$

In this model, $r_1, r_2, \alpha, \beta, \gamma_1, \gamma_2$ are positive real numbers, $x_1(t)$ is the size of the prey population at time t; $x_2(t)$ is the size of the predator population at time t subject to the non-negative initial condition $x_1(0) > 0, x_2(0) > 0$. The interactions between populations and toxicant are modeled by means of a system of two differential equations. In absence of predators, the prey population grows with a relative rate r_1, while in absence of prey, the predators die out exponentially with a relative rate r_2. L is the environmental carrying capacity of the prey population. The prey reproduction is influenced by predators only while the predator reproduction is limited by the amount of prey caught. The amount of the prey consumed by a predator per unit time is given by αx_1. A fraction $\dfrac{\beta}{\alpha}(0 < \beta < \alpha < 1)$ of the energy consumed with this biomass goes into predator reproduction while the rest of the energy is spent to sustain metabolism and hunting activity of predators. The prey is directly infected by some external toxic substance while the predator feeding on this infected prey is indirectly affected by the toxic substance The terms $\gamma_1 x_1^3$ and $\gamma_2 x_2^2$ show these effects.

They considered the bioeconomic harvesting of this model and examined the possibility of existence of a bionomic equilibrium as well as optimal harvesting policy.

In fact, population dynamics is inevitably affected by environmental white noise which is an important component in an ecosystem [7-10].

Taking into account the effect of randomly fluctuating environment, we incorporate white noise in each equations of the system (12). We assume that fluctuations in the environment will manifest themselves mainly as fluctuations in the growth rate of the population.

$$r_1 \to r_1 + adB_1(t), \quad r_2 \to r_2 + bdB_2(t),$$

where $B_1(t)$ and $B_2(t)$ are mutually independent Brownian motions, positive numbers a and b represent the intensities of the white noise. The stochastic system takes the following form:

$$(2) \qquad \begin{cases} dx_1(t) = [r_1x_1(t)(1-\frac{x_1(t)}{L}) - \alpha x_1(t)x_2(t) - \gamma_1 x_1^3(t)]dt + ax_1(t)dB_1(t), \\ dx_2(t) = [-r_2x_2(t) + \beta x_1(t)x_2(t) - \gamma_2 x_2^2(t)]dt - bx_2(t)dB_2(t). \end{cases}$$

The set-up of this paper is as follows. In Section 2, we prove the positivity of the solutions which is a very important property for any model on population dynamics which uses stochastic differential equations. We carry out analysis on the asymptotic behaviour of the model in Section 3. In Section 4, we provide some condition to the ultimate boundedness of the model.

2. Non-negative solutions

We first prove the positivity of the solutions.In this paper,we let $(\Omega, \mathscr{F}, P)$ be a complete probability space with a filteration $\{\mathscr{F}_t\}_{t\geqslant 0}$ satisfying the usual conditions(i.e. it is increasing and right continuous while $\mathscr{F}_0$ contains all P-null sets). Let $B(t)$ be the one-dimensional Brownian motion defined on this probability space. Also let $R^2_{++} = \{x \in R^2 : x_i > 0 \text{ for all } 1 \leqslant i \leqslant 2\}$ and let $x(t) = (x_1(t), x_2(t))$.

Lemma2.1 *Let $a > 0, b > 0, c > 0$. Then the function $f(x) = -ax^3 + bx^2 + cx$ is upper bounded in R^+.*

Theorem 2.2 *Assume that $r_1, r_2, \alpha, \beta, \gamma_1, \gamma_2, a, b$ are positive real numbers and $0 < \beta < \alpha < 1$, then for any initial value $x_0 \in R^2_{++}$,there is a unique solution $x(t)$ to Eqs.(3) on $t \geqslant 0$ and the solution will remain in R^2_{++} with probability 1, namely $x(t) \in R^2_{++}$ for all $t \geqslant 0$ almost surely.*

Proof. For any given initial value $x_0 \in R^2_{++}$, the coefficients of the equation are locally Lipschitz continuous, there is a unique local solution $x(t)$ to $Eqs.(3)$ on $t \in [0, \tau_e)$, where τ_e is the explosion time.If $\tau_e = \infty$ a.s, this solution is global . Let $k_0 \geqslant 0$ be sufficiently large so that $(x_1(0), x_2(0))$ lies within the interval $[1/k_0, k_0]$. For each integer $k \geqslant k_0$, define the stopping time:

$$\tau_k = \inf\{t \in [0, \tau_e], x_i(t) \notin (1/k, k) \text{ for some } i, 1 \leqslant i \leqslant 2\}$$

$\emptyset$ denotes the empty set ,and we set $\inf\emptyset = \infty$.Clearly,τ_k is increasing as $k \to \infty$. Set $\tau_\infty = \lim_{k\to\infty} \tau_k$, where $\tau_\infty \leqslant \tau_e$ a.s. If we can show that $\tau_\infty = \infty$, a.s. for all $t \geqslant 0$, then $\tau_e = \infty$,and

$x(t) \in R^2_{++}$ a.s. for all $t \geqslant 0$. We need to show that $\tau_\infty = \infty$ a.s.to complete the proof. For if this statement is false, then there is a pair of constants $T > 0$ and $\varepsilon > 0$ such that

$$P\{\tau_\infty \leqslant T\} > \varepsilon.$$

Hence there is an integer $k_1 \geqslant k_0$ such that

$$P\{\tau_k \leqslant T\} > \varepsilon \quad \text{for all} \quad k \geqslant k_1. \tag{3}$$

Define a C^2-function $V : R^2_{++} \to R_+$ by

$$V(x) = x_1(t) + 1 - \log x_1(t) + x_2(t) + 1 - \log x_2(t).$$

The non-negative of this function is obviously[7].Using Itô's formula, we get

$$\begin{aligned}
& dV(x(t)) \\
= & [1 - \frac{1}{x_1(t)}]dx_1(t) + [1 - \frac{1}{x_2(t)}]dx_2(t) + \frac{1}{2}\frac{1}{x_1^2(t)}dx_1(t)dx_1(t) \\
& + \frac{1}{2}\frac{1}{x_2^2(t)}dx_2(t)dx_2(t) \\
= & \{[r_1x_1(t)(1 - \frac{x_1(t)}{L}) - \alpha x_1(t)x_2(t) - \gamma_1 x_1^3(t)] - [r_1(1 - \frac{x_1(t)}{L}) - \alpha x_2(t) \\
& - \gamma_1 x_1^2(t)] + [-r_2x_2(t) + \beta x_1(t)x_2(t) - \gamma_2 x_2^2(t)] - [-r_2 + \beta x_1(t) - \gamma_2 x_2(t)] \\
& + \frac{1}{2}a^2 + \frac{1}{2}b^2\}dt + a(x_1(t) - 1)dB_1(t) - b(x_2(t) - 1)dB_2(t) \\
\leqslant & [(r_1 + \frac{r_1}{L})x_1(t) + \gamma_1 x_1^2(t) - \gamma_1 x_1^3(t) + r_2 + \gamma_2 x_2(t) - \gamma_2 x_2^2 + \frac{1}{2}a^2 + \frac{1}{2}b^2]dt \\
& + a(x_1(t) - 1)dB_1(t) - b(x_2(t) - 1)dB_2(t).
\end{aligned}$$

Note that by Lemma 2.1, $(r_1 + \frac{r_1}{L})x_1(t) + \gamma_1 x_1^2(t) - \gamma_1 x_1^3(t)$ is bounded where $x_1(t) > 0$ and $r_2 + \gamma_2 x_2(t) - \gamma_2 x_2^2(t)$ is bounded obviously. There is $M > 0$ such that

$$(r_1 + \frac{r_1}{L})x_1(t) + \gamma_1 x_1^2(t) - \gamma_1 x_1^3(t) + r_2 + \gamma_2 x_2(t) - \gamma_2 x_2^2 + \frac{1}{2}a^2 + \frac{1}{2}b^2 \leqslant M.$$

Therefore,

$$dV(x(t) \leqslant Mdt + a(x_1(t) - 1)dB_1(t) - b(x_2(t) - 1)dB_2(t).$$

It yields if $t_1 \leq T$, then

$$\begin{aligned}
& \int_0^{\tau_k \wedge t_1} dV(t) \leqslant \int_0^{\tau_k \wedge t_1} Mdt \\
& + \int_0^{\tau_k \wedge t_1} a(x_1(t) - 1)dB_1(t) + \int_0^{\tau_k \wedge t_1} -b(x_2(t) - 1)dB_2(t).
\end{aligned}$$

This implies that

$$EV(x(\tau_k \wedge T)) \leqslant V(x_0) + ME(\tau_k \wedge T) \leqslant V(x_0) + MT. \tag{4}$$

Set $\Omega_k = \{\tau_k \leqslant T\}$ for $k \leqslant k_1$ and by (2.1) $P(\Omega_k) \geqslant \varepsilon$, Note that for every $\omega \in \Omega_k$, there is some i such that $x_i(\tau_k, \omega)$ equals either k or $\frac{1}{k}$, and hence $V(x(\tau_k, \omega))$ is no less either

$$k + 1 - \log k$$

or

$$\frac{1}{k} + 1 - \log \frac{1}{k}.$$

Consequently,

$$V(\tau_k, \omega)) \geq (k + 1 - \log k) \wedge (\frac{1}{k} + 1 - \log \frac{1}{k}).$$

It then follows from (2.1) and (2.2) that

$$V(x_0) + KT \geqslant E[1_{\Omega_k(\omega)} V(x(\tau_k, \omega))] \geqslant \varepsilon([(k + 1 - \log k] \wedge [\frac{1}{k} + 1 - \log \frac{1}{k})]),$$

where 1_{Ω_k} is the indicator of Ω_k. Letting $k \to \infty$ lead to the contradiction:

$$\infty > V(x_0) + KT = \infty, \tag{5}$$

so we must therefore have $\tau_\infty = \infty$ a.s.

3. Asymptotic behaviour and stability

Definition 3.1. Suppose that $0 \leqslant t_0 < T < \infty$. Let x_0 be an $\mathscr{F}_0$-measurable R^d-valued random variable such that $E|x_0|^2 < \infty$.Let $f : R^d \times [t_0, T] \to R^d$ and $g : R^d \times [t_0, T] \to R^d$ be both Borel measurable with $f(0,t) = 0$ and $g(0,t) = 0$ for all $t \leqslant t_0$. Consider Itô-type stochastic differential equation

$$dx(t) = f(x(t),t)dt + g(x(t),t)dB(t) \tag{6}$$

on $t_0 < t < T$, with initial value $x(t_0) = x_0$. Write $x(t;t_0,x_0)$ for the value of the solution to this equation at time t. The trivial solution of Eq.(3.1) is said to be almost surely exponentially stable if

$$\limsup_{t \to \infty} \frac{1}{t} \log |x(t;t_0,x_0)| < 0 \quad \text{a.s.}$$

for all $x_0 \in R^d$.

Theorem 3.2. *Under the condition:* $r_1 < \dfrac{a^2b^2}{2(a^2+b^2)}$ *;* $x_1(t)$ *and* $x_2(t)$ *are almost surely exponentially stable in the sense that* $x_1(t)$ *and* $x_2(t)$ *will tend to their equilibrium value* 0 *exponentially with probability* 1.

Proof. Let $V(x) = \log(x_1(t)+x_2(t))$ for $x_1, x_2 \in (0,\infty)$. Using Itô-formula, we get

$$\begin{aligned}
& dV(x(t)) \\
= \; & \frac{1}{x_1(t)+x_2(t)}\{[r_1x_1(t)(1-\frac{x_1(t)}{L})-\alpha x_1(t)x_2(t)-\gamma_1 x_1^3(t)]dt \\
& \qquad +ax_1(t)dB_1(t)\} \\
& +\frac{1}{x_1(t)+x_2(t)}\{-r_2x_1(t)+\beta x_1(t)x_2(t)-\gamma_2 x_2^2(t)]dt-bx_2(t)dB_2(t)\} \\
& -\frac{1}{2}\frac{1}{(x_1(t)+x_2(t))^2}a^2x_1^2(t)dt-\frac{1}{2}\frac{1}{(x_1(t)+x_2(t))^2}b^2x_2^2(t)dt \\
\leqslant \; & \frac{1}{2[x_1(t)+x_2(t)]^2}[2r_1x_1(t)(x_1(t)+x_2(t))-a^2x_1^2(t)-b^2x_2^2(t)]dt \\
& +\frac{ax_1(t)}{x_1(t)+x_2(t)}dB_1(t)-\frac{bx_2(t)}{x_1(t)+x_2(t)}dB_2(t) \\
\leqslant \; & \frac{1}{2[x_1(t)+x_2(t)]^2}[2r_1(x_1(t)+x_2(t))^2-a^2x_1^2(t)-b^2x_2^2(t)]dt \\
& +\frac{ax_1(t)}{x_1(t)+x_2(t)}dB_1(t)-\frac{bx_2(t)}{x_1(t)+x_2(t)}dB_2(t).
\end{aligned}$$

We can write the term $2r_1(x_1(t)+x_2(t))^2 - a^2x_1^2(t) - b^2x_2^2(t)$ in the following way

$$\begin{pmatrix} x_1(t) & x_2(t) \end{pmatrix} \begin{pmatrix} 2r_1-a^2 & 2r_1 \\ 2r_1 & 2r_1-b^2 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}.$$

Now consider the matrix:

$$\begin{pmatrix} 2r_1-a^2 & 2r_1 \\ 2r_1 & 2r_1-b^2 \end{pmatrix}.$$

when the condition $r_1 < \dfrac{a^2b^2}{2(a^2+b^2)}$ is satisfied,and note that $\dfrac{a^2+b^2}{4} \geq \dfrac{a^2b^2}{2(a^2+b^2)}$,the above matrix is negative-define with largest(negative) eigenvalue

$$\lambda_{\max} = \frac{4r_1-a^2-b^2+\sqrt{16r_1^2+(a^2-b^2)^2}}{2} < 0. \tag{7}$$

Then

$$\begin{aligned}(8)\qquad &\begin{pmatrix} x_1(t) & x_2(t) \end{pmatrix} \begin{pmatrix} r_1-a^2 & r_1 \\ r_1 & r_1-b^2 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \\ &\leqslant \lambda_{\max}(x_1^2(t)+x_2^2(t)) = -|\lambda_{\max}|(x_1^2(t)+x_2^2(t)).\end{aligned}$$

Therefore, we have

$$\begin{aligned}(9)\qquad & dV(x(t)) \leqslant \\ & -|\lambda_{\max}|\frac{x_1^2(t)+x_2^2(t)}{2(x_1(t)+x_2(t))^2}dt + \frac{ax_1(t)}{x_1(t)+x_2(t)}dB_1(t) - \frac{bx_2(t)}{x_1(t)+x_2(t)}dB_2(t).\end{aligned}$$

As $-(x_1^2(t)+x_2^2(t)) \leqslant -0.5(x_1(t)+x_2(t))^2$, substituting this in inequality (3.4) we get

$$dV(x(t) \leqslant -\frac{1}{4}|\lambda_{\max}|dt + \frac{ax_1(t)}{x_1(t)+x_2(t)}dB_1(t) - \frac{bx_2(t)}{x_1(t)+x_2(t)}dB_2(t).$$

Integrating the above inequality and using the fact that

$$\lim_{t\to\infty}\sup\frac{1}{t}|B_i(t)| = 0 \quad \text{(Mao [8])},$$

we get

$$\lim_{t\to\infty}\sup\frac{1}{t}\log|x(t;t_0,x_0)| \leqslant -\frac{1}{4}|\lambda_{max}| < 0 \text{ a.s.}$$

which complete the proof.

4. Ultimate boundedness

Definition 4.1. Equation (1.2) is said to be stochastically ultimately bounded if for any $\varepsilon \in (0,1)$,there is a positive constant $H = H(\varepsilon)$ such that for any initial value $x_0 \in R^2_{++}$, the solution $x(t)$ of Eq.(1.2) has the property that

$$\limsup_{t\to\infty}\mathbb{P}\{|x(t)| \leqslant H\} \geqslant 1-\varepsilon$$

Theorem 4.2. *Under assumption:*$r_2 > \dfrac{1+b^2}{2}$ *and* $\gamma_1 > \dfrac{4L\beta^3}{27\gamma_2^3}$, *Eq.(1.2) is stochastically ultimately bounded.*

Proof. Consider

$$V(x) = x_1^2 + x_2^2 \text{ for } x \in R^2_{++}.$$

By the Itô's formula, we have

$$
\begin{aligned}
& dV(x(t)) \\
= \;& 2x_1(t)dx_1(t) + 2x_2(t)dx_2(t) + dx_1(t)dx_1(t) + dx_2(t)dx_2(t) \\
= \;& 2x_1(t)\{[r_1x_1(t)(1-\frac{x_1(t)}{L}) - \alpha x_1(t)x_2(t) - \gamma_1 x_1^3(t)]dt + ax_1(t)dB_1(t)\} \\
& +2x_2(t)\{[-r_2x_2(t) + \beta x_1(t)x_2(t) - \gamma_2 x_2^2(t)]dt - bx_2(t)dB_2(t)\} \\
& +(a^2x_1^2 + b^2x_2^2)dt \\
= \;& [F(x_1(t),x_2(t)) - V(x_1(t),x_2(t))]dt + 2ax_1^2(t)dB_1(t) - 2bx_2^2(t)dB_2(t),
\end{aligned}
\tag{10}
$$

where

$$
\begin{aligned}
F(x_1(t),x_2(t)) \;=\;& (2r_1+1+a^2)x_1^2(t) - 2\frac{r_1}{L}x_1^3(t) - 2\gamma_1 x_1^4(t) \\
& +(1+b^2-2r_2)x_2^2(t) + 2\beta x_1(t)x_2^2(t) - 2\gamma_2 x_2^3(t) \\
\leq\;& (2r_1+1+a^2)x_1^2(t) \\
& +(1+b^2-2r_2)x_2^2(t) + 2\beta x_1(t)x_2^2(t) - 2\gamma_2 x_2^3(t)
\end{aligned}
$$

$F(x_1(t),x_2(t))$ is bounded when $x_1(t) \geq 0, x_2(t) \geq 0$. In fact, to any positive number $u > 0$, let $f(x_2) = (1+b^2-2r_2)x_2^2 + 2\beta ux_2^2 - 2\gamma_2 x_2^3$, for $r_2 > \frac{1+b^2}{2}$, we get $f(x_2) \leqslant 2\beta ux_2^2 - 2\gamma_2 x_2^3 = g(x_2)$. Let $g^{'}(x_2) = 0$ we get $x_2 = 0$ and $x_2 = \frac{2\beta}{3\gamma_2}u > 0$, so, $f_{max}(x_2) \leq g(\frac{2\beta}{3\gamma_2}u) = \frac{8\beta^3}{27\gamma_2^2}u^3$ which implies

$$(1+b^2-2r_2)x_2^2(t) + 2\beta x_1(t)x_2^2(t) - 2\gamma_2 x_2^3(t) \leqslant \frac{8\beta^3}{27\gamma_2^2}x_1^3(t).$$

Substitute this in $F(x_1(t),x_2(t))$, we get

$$F(x_1(t),x_2(t)) \leqslant \frac{8\beta^3}{27\gamma_2^2}x_1^3(t) + (2r_1+1+a^2)x_1^2(t) - 2\frac{\gamma_1}{L}x_1^3(t).$$

When $\gamma_1 > \frac{4L\beta^3}{27\gamma_2^3}$, $F(x_1(t),x_2(t))$ is bounded on R_{++}^2 obviously. There is $H_1 > 0$ such that

$$F(x_1(t),x_2(t)) \leqslant H_1.$$

It yields

$$dV(x(t)) \leqslant [H_1 - V(x(t))]dt + +2ax_1^2(t)dB_1(t) - 2bx_2^2(t)dB_2(t).$$

Now, by the Itô's formula again, we have

$$
\begin{aligned}
d[e^t V(x(t))] &= e^t V(x(t)) + e^t dV(x(t)) \\
&\leq H_1 dt + 2ae^t x_1^2(t) dB_1(t) - 2be^t x_2^2(t) dB_2(t).
\end{aligned} \tag{11}
$$

Let k_0 be sufficiently large for $x_1(t)$ and $x_2(t)$ lying within the interval $[\frac{1}{k_0}, k_0]$, define the stopping time

$$
\tau_k = \inf\{t \in [0, \tau_e], x_i(t) \notin (1/k, k) \text{ for some } i, 1 \leqslant i \leqslant 2\}.
$$

Clearly $\tau_k \to \infty$ almost surely as $k \to \infty$. It then follows from (4.2) that

$$
\mathbb{E}[e^{t\wedge\tau_k} V(x(t\wedge\tau_k))] \leqslant V(x_0) + H_1 \mathbb{E} \int_0^{t\wedge\tau_k} e^s ds.
$$

Let $k \to \infty$ yields

$$
e^t \mathbb{E}[V(x(t)] \leqslant V(x_0) + H_1(e^t - 1).
$$

It implies

$$
\mathbb{E}[V(x(t)] \leqslant e^{-t} V(x_0) + H_1.
$$

Thus

$$
\mathbb{E}|x(t)|^2 \leqslant e^{-t} V(x_0) + H_1.
$$

This implies

$$
\limsup_{t\to\infty} \mathbb{E}|x(t)|^2 \leqslant H_1.
$$

For any $\varepsilon > 0$, let $H = \sqrt{\frac{H_1}{\varepsilon}}$, by Chebyshev's inequality,

$$
\mathbb{P}\{|x(t)|^4 > H^4\} \leqslant \frac{\mathbb{E}\sqrt{|x(t)|^4}}{\sqrt{H^4}} = \frac{\mathbb{E}|x(t)|^2}{H^2}.
$$

Hence,

$$
\begin{aligned}
\limsup_{t\to\infty} \mathbb{P}\{|x(t)| > H\} &= \limsup_{t\to\infty} \mathbb{P}\{|x(t)|^4 > H^4\} \\
&\leqslant \limsup_{t\to\infty} \frac{\mathbb{E}|x(t)|^2}{H^2} \\
&\leqslant \frac{H_1}{H^2} = \varepsilon.
\end{aligned}
$$

5. Numerical simulation

At last, we numerically simulate the solution of Eq. (1.2) to substantiate the analytical findings. Consider the discretization equation:

$$(12)\qquad \begin{cases} x_{n+1} = x_n + (r_1 x_n(1 - x_n/L) - \alpha x_n y_n - \gamma_1 x_n^3)\Delta t + a x_n \sqrt{\Delta t}\xi_n + \frac{a^2}{2} x_n(\Delta t \xi_n^2 - \Delta t); \\ y_{n+1} = y_n + (-r_2 y_n + \beta x_n y_n - \gamma_2 y_n^2)\Delta t - b y_n \sqrt{\Delta t}\eta_n - \frac{b^2}{2} y_n(\Delta t \eta_n^2 - \Delta t)), \end{cases}$$

where ξ_n and η_n ,$n = 1, 2, ..., n$, are the Gaussian random variables $N(0, 1)$.

Using the numerical simulation method given out above and the help of Matlab software, choosing suitable parameters, we get simulations of the stochastic system (1.2).

We choose parameters that condition $r_1 < \dfrac{a^2 b^2}{2(a^2 + b^2)}$ is staisfied,the simulation showed in Figure 1 confirms the situdation that the two species are nearly extincted what we get in Theorem 3.2.And we let $r_2 > \dfrac{1 + b^2}{2}$ and $\gamma_1 > \dfrac{4L\beta^3}{27\gamma_2^3}$,Figure 2 shows that the populations of prey and predator are bounded .

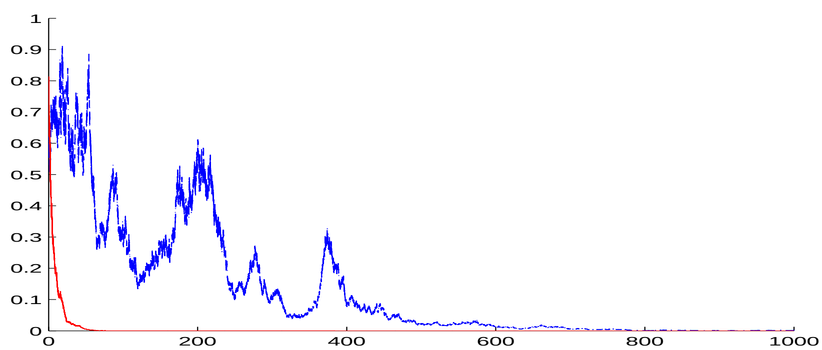

FIGURE 1. Solutions of systems (1.2) with the initial conditions $x_1(0) = 0.6$,$x_2(0) = 0.8$,$r_1 = 0.001, L = 3, \alpha = 0.05, \gamma_1 = 0.03, r_2 = 0.1, \beta = 0.01, \gamma_2 = 0.02, a = 0.1, b = 0.1$ respectively.

Conflict of Interests

The authors declare that there is no conflict of interests.

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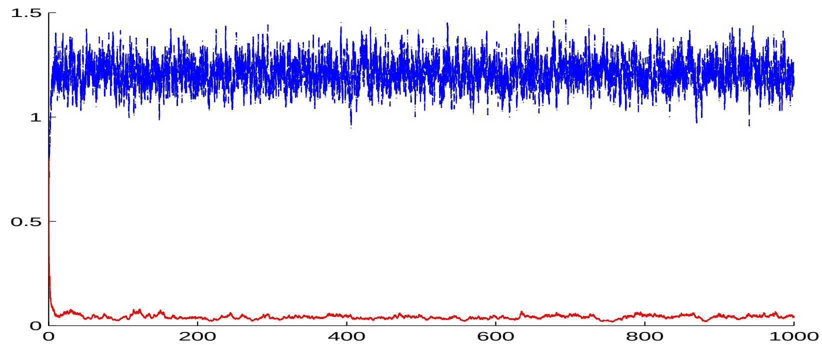

FIGURE 2. Solutions of systems (1.2) with the initial conditions $x_1(0) = 0.6$,$x_2(0) = 0.8, r_1 = 1.2, L = 2, \alpha = 0.8, \gamma_1 = 0.3, r_2 = 0.1, \beta = 0.5, \gamma_2 = 2, a = 0.1, b = 0.1$ respectively.

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