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GLOBAL STABILITY OF A PREDATOR-PREY SYSTEM WITH STAGE STRUCTURE OF DISTRIBUTED-DELAY TYPE

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Abstract. In this paper, an autonomous predator-prey system with stage structure of distributed-delay type is studied in this paper. By using an iterative method, the global stability of the interior equilibrium point of the system is investigated. Our result extends the main result in [Global stability of a stage-structured predator-prey system, Applied Mathematics and Computation, 223 (2013), 45-53].

Keywords: Global stability; Predator; Stage structure; Distributed-delay; Equilibrium; Iterative method.

2010 AMS Subject Classification: 34D23, 92B05, 34D40.

1. Introduction

During the last two decades, the study of dynamic behaviors of stage-structured ecosystem become one of the most important research topic, many excellent results have been obtained, see [1]-[25] and the references cited therein.

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In their series paper, Chen *et al.* [2, 3, 4] studied the dynamic behaviors of the following stage-structured predator-prey system (stage structure for both predator and prey).

$$
\begin{aligned}
\dot{x}_1(t) &= r_1(t)x_2(t)-d_{11}x_1(t)-r_1(t-\tau_1)e^{-d_{11}\tau_1}x_2(t-\tau_1),\\
\dot{x}_2(t) &= r_1(t-\tau_1)e^{-d_{11}\tau_1}x_2(t-\tau_1)-d_{12}x_2(t)-b_1(t)x_2^2(t)-c_1(t)x_2(t)y_2(t),\\
\dot{y}_1(t) &= r_2(t)y_2(t)-d_{22}y_1(t)-r_2(t-\tau_2)e^{-d_{22}\tau_2}y_2(t-\tau_2),\\
\dot{y}_2(t) &= r_2(t-\tau_2)e^{-d_{22}\tau_2}y_2(t-\tau_2)-d_{21}y_2(t)-b_2(t)y_2^2(t)+c_2(t)y_2(t)x_2(t),
\end{aligned}
\tag{1}
$$

where $x_1(t)$ and $x_2(t)$ denote the densities of the immature and mature prey species at time t, respectively; $y_1(t)$ and $y_2(t)$ represent the immature and mature population densities of predator species at time t, respectively; $r_i(t)$, $b_i(t)$, $c_i(t) (i=1,2)$ are all continuous functions bounded above and below by positive constants for all $t\geq 0$. $d_{ij}, \tau_i, i, j=1,2$ are all positive constants. By using the comparison theorem of differential equation, they investigated the partial survival and extinction property of the system [2]; By introducing a new lemma and applying the standard comparison theorem, they investigated the persistent property of the system [3]; For the autonomous case, by using an iterative method, they investigated the global stability of the interior equilibrium point of the system [4]. Their result shows that conditions which ensure the permanence of the system is enough to ensure the global stability of the system.

An important assumption behind the work of Chen *et al.* [2, 3, 4] is that all individuals take the identic amount of time to become mature, which seems biologically unreasonable since individuals in a population do not necessarily always mature at the same age [18]. With the aim of overcome this defect, recently, Chen *et al.* [5, 6] proposed the following non-autonomous predator-prey model with stage structure of distributed-delay type:

(2)

$$
\begin{aligned}
\dot{x}_1(t) &= r_1(t)x_2(t)-d_{11}(t)x_1(t)-\int_0^\infty r_1(t-s)f_1(s)\exp\{\int_0^s -d_{11}(m)dm\}x_2(t-s)ds,\\
\dot{x}_2(t) &= \int_0^\infty r_1(t-s)f_1(s)\exp\{\int_0^s -d_{11}(m)dm\}x_2(t-s)ds\\
&\quad -d_{12}(t)x_2(t)-b_1(t)x_2^2(t)-c_1(t)x_2(t)y_2(t),\\
\dot{y}_1(t) &= r_2(t)y_2(t)-d_{22}(t)y_1(t)-\int_0^\infty r_2(t-s)f_2(s)\exp\{\int_0^s -d_{22}(m)dm\}y_2(t-s)ds,\\
\dot{y}_2(t) &= \int_0^\infty r_2(t-s)f_2(s)\exp\{\int_0^s -d_{22}(m)dm\}y_2(t-s)ds\\
&\quad -d_{21}(t)y_2(t)-b_2(t)y_2^2(t)+c_2(t)y_2(t)x_2(t),
\end{aligned}
$$

where $x_1(t)$ and $x_2(t)$ denote the densities of the immature and mature prey species at time t, respectively; $y_1(t)$ and $y_2(t)$ represent the immature and mature population densities of predator species at time t, respectively; $r_i(t), d_{ij}$ $b_i(t)$, $c_i(t)(i,j=1,2)$ are all continuous functions bounded above and below by positive constants for all $t \geq 0$. $f_i(s), i=1,2$ is the probability density function of species i that the maturation time is between s and $s+ds$ with ds being infinitesimal and $\int_0^{\infty} f_i(s)ds = 1$. In [5], they obtained sufficient conditions which concern with the extinction of the system and partial survival of the predator (prey) species, respectively. In [6], they obtained a set of sufficient conditions which ensure the permanence of the system. However, comparing the results of [5, 6] and the results of [2, 3], one could see that the conditions about the permanence and the partial survival of the system with distributed-delay are complex than that of the discrete delay ones, the reason is that in system (1.1), the first and third equation could be expressed in an integral form and consequently, the dynamic behaviors of x_1 and y_1 are determined by x_2 and y_2. While for the distributed-delay case, with the introducing of probability density function $f_i(s)$, the first and third equation in system (2) could no longer be expressed in integral form.

To the best of the authors knowledge, to this day, still no scholars investigate the stability property of the system (2), which is one of the most important topic in the study of population dynamics. As far as the non-autonomous ecosystem with stage-structure of distributed-delay is concerned, only [19] investigated the stability property of a non-autonomous nonlinear stage-structured competition system, however, their conditions are very complicated and not easy to verify. This motivated us to consider a slightly more simple system, *i.e.*, the autonomous case of system (2).

$$
\begin{aligned}
\dot{x}_1(t) &= r_1 x_2(t) - d_{11}x_1(t) - r_1 \int_0^{\infty} f_1(s)\exp\{-d_{11}s\}x_2(t-s)ds, \\
\dot{x}_2(t) &= r_1 \int_0^{\infty} f_1(s)\exp\{-d_{11}s\}x_2(t-s)ds \\
&\quad -d_{12}x_2(t) - b_1 x_2^2(t) - c_1 x_2(t) y_2(t), \\
\dot{y}_1(t) &= r_2 y_2(t) - d_{22}y_1(t) - r_2 \int_0^{\infty} f_2(s)\exp\{-d_{22}s\}y_2(t-s)ds, \\
\dot{y}_2(t) &= r_2 \int_0^{\infty} f_2(s)\exp\{-d_{22}s\}y_2(t-s)ds \\
&\quad -d_{21}y_2(t) - b_2 y_2^2(t) + c_2 y_2(t) x_2(t),
\end{aligned} \tag{3}
$$

where $x_i(t)$ and $y_i(t), i = 1,2$ have the same meaning as that of system (2); r_i, b_i, $c_i (i = 1,2)$, $d_{ij}, i, j = 1,2$ are all positive constants.

The initial conditions for system (3) take the form of

$$x_i(\theta) = \phi_i(\theta), y_i(\theta) = \psi_i(\theta) > 0, \\ \phi_i(0) > 0, \psi_i(0) > 0, i = 1,2, \theta \in (-\infty, 0], \tag{4}$$

where $\phi(t) = (\phi_1(t), \phi_2(t), \psi_1(t), \psi_2(t)) \in UC_g$, which is referred to as the fading memory space [26, p. 46]. Set

$$F_i = \int_0^{\infty} f_i(s) \exp\{-d_{ii}s\} ds, i = 1,2.$$

From [5] we know that $x_2(t) \to 0$ as $t \to +\infty$ if $r_1F_1 \leq d_{12}$ holds. Since we are focus on the stability property of the positive equilibrium, for the rest of the paper, we assum that $r_1F_1 > d_{12}$ holds.

The interior positive equilibrium $E^*(x_1^*, x_2^*, y_1^*, y_2^*)$ of system (3) satisfies the following equations

$$\begin{cases} r_1x_2 - d_{11}x_1 - r_1F_1x_2 = 0, \\ r_1F_1x_2 - d_{12}x_2 - b_1x_2^2 - c_1x_2y_2 = 0, \\ r_2y_2 - d_{22}y_1 - r_2F_2y_2 = 0, \\ r_2F_2y_2 - d_{21}y_2 - b_2y_2^2 + c_2x_2y_2 = 0, \end{cases} \tag{5}$$

and so, if

$$b_2\big(r_1F_1 - d_{12}\big) - c_1\big(r_2F_2 - d_{21}\big) > 0 \tag{6}$$

and

$$c_2\big(r_1F_1 - d_{12}\big) + b_1\big(r_2F_2 - d_{21}\big) > 0 \tag{7}$$

hold, then system (3) admits an unique positive equilibrium $E^*(x_1^*, x_2^*, y_1^*, y_2^*)$, where

$$x_1^* = \frac{r_1(1-F_1)}{d_{11}} x_2^*, \ x_2^* = \frac{b_2\big(r_1F_1 - d_{12}\big) - c_1\big(r_2F_2 - d_{21}\big)}{b_2b_1 + c_2c_1}, \tag{8}$$

$$y_1^* = \frac{r_2(1-F_2)}{d_{22}} y_2^*, \ y_2^* = \frac{c_2\big(r_1F_1 - d_{12}\big) + b_1\big(r_2F_2 - d_{21}\big)}{b_2b_1 + c_2c_1}. \tag{9}$$

Following is the main result of this paper:

Theorem 1.1. *In addition to (6) and (7), further assume that*

(A_1)

$$c_1c_2 < b_1b_2, \tag{10}$$

(A_2)

$$\Big(1-\frac{c_1c_2}{b_1b_2}\Big)\big(r_1F_1-d_{12}\big) > \frac{c_1}{b_2}\big(r_2F_2-d_{21}\big) \tag{11}$$

hold. Then the unique interior equilibrium $E^*(x_1^*, x_2^*, y_1^*, y_2^*)$ of system (3) is globally attractive, that is,

$$\lim_{t\to+\infty} x_i(t) = x_i^*, \quad \lim_{t\to+\infty} y_i(t) = y_i^*, i = 1,2.$$

Remark 1.1. Comparing the corresponding Theorem 3.4 in [1] for system (1) with Theorem 1.1 for (2), we find out that the term

$$F_i = \int_0^\infty f_i(s)\exp\{-d_{ii}s\}ds, i = 1,2.$$

in our result are corresponding to the $e^{-d_{ii}\tau_i}, i = 1,2$ in [4]. That is, we extend the result of finite discrete delay case to the distributed delay case.

2. Some lemmas

Lemma 2.1. [20] *Consider the following equation:*

$$\dot{u}(t) = a\int_0^\infty f(s)\exp\{-ds\}u(t-s)ds - bu(t) - cu^2(t),$$

where $a > 0, b \geq 0, c > 0; d > 0$, and

$$A = a\int_0^\infty f(s)\exp\{-ds\}ds.$$

(1) If $A > b$, then $\lim_{t\to+\infty} u(t) = (A-b)c^{-1}$.
(2) If $A \leq b$, then $\lim_{t\to+\infty} u(t) = 0$.

Lemma 2.2. [6] *Consider the following equation:*

$$\dot{u}(t) = a\int_0^\infty f(s)\exp\{-ds\}u(t-s)ds + bu(t) - cu^2(t), \tag{12}$$

where $a,c>0, b\geq 0; u(t)=\phi(t)>0$ *for* $-\infty\leq t\leq 0$, *and*

$$A=a\int_0^{\infty} f(s)\exp\{-ds\}ds.$$

we have $\lim_{t\to+\infty} u(t)=(A+b)c^{-1}$.

Similarly to the proof of Theorem 2.1 in [1], one could easily obtain the following Lemma.

Lemma 2.3. *Solutions of system (3) with initial conditions (4) are positive for all* $t\geq 0$.

Lemma 2.4. [27] If $a>0, b>0$ and $\dfrac{dx}{dt}\geq b-ax$, when $t\geq 0$ and $x(0)>0$, we have

$$\liminf_{t\to+\infty} x(t)\geq \frac{b}{a}.$$

If $a>0, b>0$ and $\dfrac{dx}{dt}\leq b-ax$, when $t\geq 0$ and $x(0)>0$, we have

$$\limsup_{t\to+\infty} x(t)\leq \frac{b}{a}.$$

Lemma 2.5 *Consider the following equation:*

$$\dot{u}(t)=av(t)-bu(t)-a\int_0^{\infty} f(s)\exp\{-ds\}v(t-s)ds, \tag{13}$$

where $a>0, b\geq 0, c>0; d>0$, *and*

$$F=\int_0^{\infty} f(s)\exp\{-ds\}ds. \tag{14}$$

Assume that $\lim_{t\to+\infty} v(t)=v^*$, *where* v^* *is some constant, then*

$$\lim_{t\to+\infty} u(t)=\frac{a(1-F)}{b}.$$

Proof. Setting $M>\sup\{v(t), t\in R\}$, it follows from $\lim_{t\to+\infty} v(t)=v^*$ and (14) that for any enough small $\varepsilon>0$ ($\varepsilon<\frac{1}{2}\frac{a(1-F)v^*}{a(1+F+MF)}$), there exists a positive number T such that for all $t\geq T$,

$$v^*-\varepsilon<v(t)<v^*+\varepsilon,\ \int_0^{T} f(s)\exp\{-ds\}ds>(1-\varepsilon)F. \tag{15}$$

Now, for $t\geq 2T$, from (13), we have

$$\begin{aligned}\dot{u}(t) &= av(t)-bu(t)-a\int_0^{\infty} f(s)\exp\{-ds\}v(t-s)ds\\ &\leq av(t)-bu(t)-a\int_0^{T} f(s)\exp\{-ds\}v(t-s)ds\\ &\leq a(v^*+\varepsilon)-bu(t)-a(v^*-\varepsilon)(1-\varepsilon)F.\end{aligned} \tag{16}$$

Applying Lemma 2.4 to (16) leads to

$$\limsup_{t\to+\infty} u(t) \le \frac{a(v^*+\varepsilon)-a(v^*-\varepsilon)(1-\varepsilon)F}{b}. \tag{17}$$

Setting $\varepsilon \to 0$ in (17), we obtain

$$\limsup_{t\to+\infty} u(t) \le \frac{a(1-F)v^*}{b}. \tag{18}$$

Also, for $t \ge 2T$, from (13), we have

$$\begin{aligned} \dot{u}(t) &= av(t)-bu(t)-a\int_0^T f(s)\exp\{-ds\}v(t-s)ds \\ &\quad -a\int_T^\infty f(s)\exp\{-ds\}v(t-s)ds \\ &\ge a(v^*-\varepsilon)-bu(t)-a(v^*+\varepsilon)F-aM\varepsilon F. \end{aligned} \tag{19}$$

From the definition of ε,

$$a(v^*-\varepsilon)-a(v^*+\varepsilon)F-aM\varepsilon F > \frac{1}{2}a(1-F)v^* > 0. \tag{20}$$

And so, applying Lemma 2.4 to (19) leads to

$$\liminf_{t\to+\infty} u(t) \ge \frac{a(v^*-\varepsilon)-a(v^*+\varepsilon)F-aM\varepsilon F}{b}. \tag{21}$$

Setting $\varepsilon \to 0$ in (21), we obtain

$$\liminf_{t\to+\infty} u(t) \ge \frac{a(1-F)v^*}{b}. \tag{22}$$

(18) combing with (22) implies that

$$\lim_{t\to+\infty} u(t) = \frac{a(1-F)v^*}{b}. \tag{23}$$

This completes the proof of Lemma 2.5.

3. Proof of the main result

Proof of Theorem 1.1. Conditions (A_1) and (A_2) in Theorem 1.1 is equivalent to

$$\big(r_1F_1-d_{12}\big) > \frac{c_1}{b_2}\big(r_2F_2-d_{21}\big)+\frac{c_1c_2}{b_1b_2}\big(r_1F_1-d_{12}\big), \tag{24}$$

which is equivalent to

$$r_1F_1 > d_{12}+\frac{c_1}{b_2}\Big(r_2F_2-d_{21}+\frac{c_2}{b_1}\big(r_1F_1-d_{12}\big)\Big). \tag{25}$$

Condition (A_1) and (7) in Theorem 1.1 is equivalent to

$$\Big(1-\frac{c_1c_2}{b_1b_2}\Big)\big(r_2F_2-d_{21}\big)+\frac{c_2}{b_1}\Big(1-\frac{c_1c_2}{b_1b_2}\Big)\big(r_1F_1-d_{12}\big)>0, \tag{26}$$

and so,

$$r_2F_2-d_{21}+\frac{c_2}{b_1}\left[r_1F_1-d_{12}-\frac{c_1}{b_2}\Big(r_2F_2-d_{21}+\frac{c_2}{b_1}\big(r_1F_1-d_{12}\big)\Big)\right]>0. \tag{27}$$

From (25) and (27), one could choose $\varepsilon>0$ small enough such that

$$m_1^{(1)}\overset{\text{def}}{=}\frac{r_1F_1-d_{12}-\frac{c_1}{b_2}\Big(r_2F_2-d_{21}+c_2\big(\frac{r_1F_1-d_{12}}{b_1}+\varepsilon\big)\Big)}{b_1}-\varepsilon>0. \tag{28}$$

and

$$r_2F_2-d_{21}+\frac{c_2}{b_1}\left[r_1F_1-d_{12}-\frac{c_1}{b_2}\Big(r_2F_2-d_{21}+c_2\big(\frac{r_1F_1-d_{12}}{b_1}+\varepsilon\big)\Big)\right]>b_2\varepsilon. \tag{29}$$

From the definition of $m_1^{(1)}$ and (29), it follows that

$$m_2^{(1)}\overset{\text{def}}{=}\frac{r_2F_2+c_2m_1^{(1)}-d_{21}}{b_2}-\varepsilon>0. \tag{30}$$

Let $(x_1(t),x_2(t),y_1(t),y_2(t))^T$ be any positive solution of system (3) for $t\geq 0$. From the second equation of system (3) and Lemma 2.3, we have

$$\dot{x}_2(t)<r_1\int_0^{\infty}f_1(s)\exp\{-d_{11}s\}x_2(t-s)ds-d_{12}x_2(t)-b_1x_2^2(t). \tag{31}$$

By applying Lemma 2.1 (1) and standard comparison theorem, it follows that

$$\limsup_{t\to+\infty}x_2(t)\leq\frac{r_1F_1-d_{12}}{b_1}. \tag{32}$$

For $\varepsilon>0$ be defined by (28) and (29), it follows from (32) that there exists a $T_1^{'}>0$ such that

$$x_2(t)<\frac{r_1F_1-d_{12}}{b_1}+\varepsilon\overset{\text{def}}{=}M_1^{(1)}\ \text{for all}\ t>T_1^{'}. \tag{33}$$

For $t>T_1^{'}$, from the forth equation of system (3) and (33), we have

$$\begin{aligned}\dot{y}_2(t)&<r_2\int_0^{\infty}f_2(s)\exp\{-d_{22}s\}y_2(t-s)ds-d_{21}y_2(t)-b_2y_2^2(t)+c_2M_1^{(1)}y_2(t)\\&=r_2\int_0^{\infty}f_2(s)\exp\{-d_{22}s\}y_2(t-s)ds-\big(d_{21}-c_2M_1^{(1)}\big)y_2(t)-b_2y_2^2(t).\end{aligned} \tag{34}$$

By applying Lemma 2.1 (1) or Lemma 2.2 to (34), we can obtain

$$\limsup_{t\to+\infty}y_2(t)\leq\frac{r_2F_2+c_2M_1^{(1)}-d_{21}}{b_2}. \tag{35}$$

For above $\varepsilon > 0$, it follows from (35) that there exists a $T_1 > T_1^{'}$ such that

$$y_2(t) < \frac{r_2F_2 + c_2M_1^{(1)} - d_{21}}{b_2} + \varepsilon \overset{\text{def}}{=} M_2^{(1)} \text{ for all } t > T_1. \tag{36}$$

Thus, for $t > T_1$, from the second equation of system (3) and (36), we have

$$\begin{aligned} \dot{x}_2(t) &\geq r_1\int_0^{\infty} f_1(s)\exp\{-d_{11}s\}x_2(t-s)ds - d_{12}x_2(t) - b_1x_2^2(t) - c_1M_2^{(1)}x_2(t) \\ &= r_1\int_0^{\infty} f_1(s)\exp\{-d_{11}s\}x_2(t-s)ds - \big(d_{12}+c_1M_2^{(1)}\big)x_2(t) - b_1x_2^2(t). \end{aligned} \tag{37}$$

By applying Lemma 2.1 (1) and standard comparison theorem, it follows from (37) that

$$\liminf_{t\to+\infty} x_2(t) \geq \frac{r_1F_1 - d_{12} - c_1M_2^{(1)}}{b_1}, \tag{38}$$

and so, from (28) there exists a $T_2^{'} > T_1$ such that

$$x_2(t) > \frac{r_1F_1 - d_{12} - c_1M_2^{(1)}}{b_1} - \varepsilon = m_1^{(1)} > 0 \text{ for all } t > T_2^{'}. \tag{39}$$

Above inequality together with the forth equation of system (3) leads to

$$\dot{y}_2(t) > r_2\int_0^{\infty} f_2(s)\exp\{-d_{22}s\}y_2(t-s)ds - \big(d_{21} - c_2m_1^{(1)}\big)y_2(t) - b_2y_2^2(t). \tag{40}$$

From this differential inequality, by applying Lemma 2.1 (1) or Lemma 2.2, we have

$$\liminf_{t\to+\infty} y_2(t) \geq \frac{r_2F_2 + c_2m_1^{(1)} - d_{21}}{b_2},$$

and so, from (30) there exists a $T_2 > T_2^{'}$ such that

$$y_2(t) > \frac{r_2F_2 + c_2m_1^{(1)} - d_{21}}{b_2} - \varepsilon = m_2^{(1)} > 0 \text{ for all } t > T_2. \tag{41}$$

From the second equation of system (3) and (41), for $t > T_2$, we have

$$\dot{x}_2(t) < r_1\int_0^{\infty} f_1(s)\exp\{-d_{11}s\}x_2(t-s)ds - d_{12}x_2(t) - b_1x_2^2(t) - c_1m_2^{(1)}x_2(t). \tag{42}$$

By applying Lemma 2.1 (1) and standard comparison theorem, it follows that

$$\limsup_{t\to+\infty} x_2(t) \leq \frac{r_1F_1 - d_{12} - c_1m_2^{(1)}}{b_1}. \tag{43}$$

For $\varepsilon > 0$ be defined by (28) and (29), it follows from (43) that there exists a $T_3^{'} > T_2$ such that

$$x_2(t) < \frac{r_1F_1 - d_{12} - c_1m_2^{(1)}}{b_1} + \frac{\varepsilon}{2} \overset{\text{def}}{=} M_1^{(2)} \text{ for all } t > T_3^{'}. \tag{44}$$

For $t > T_3^{'}$, from the forth equation of system (3) and (44), we have

$$\dot{y}_2(t) \quad < \quad r_2\int_0^{\infty} f_2(s)\exp\{-d_{22}s\}y_2(t-s)ds - \big(d_{21} - c_2M_1^{(2)}\big)y_2(t) - b_2y_2^2(t). \tag{45}$$

By applying Lemma 2.1 (1) or Lemma 2.2 to (45), we can obtain

$$\limsup_{t\to+\infty} y_2(t) \le \frac{r_2F_2 + c_2M_1^{(2)} - d_{21}}{b_2}. \tag{46}$$

For above $\varepsilon > 0$, it follows from (46) that there exists a $T_3 > T_3^{'}$ such that

$$y_2(t) < \frac{r_2F_2 + c_2M_1^{(2)} - d_{21}}{b_2} + \frac{\varepsilon}{2} \overset{\text{def}}{=} M_2^{(2)} \quad \text{for all } t > T_3. \tag{47}$$

Thus, for $t > T_3$, from the second equation of system (3) and (47), we have

$$\dot{x}_2(t) \quad \ge \quad r_1\int_0^{\infty} f_1(s)\exp\{-d_{11}s\}x_2(t-s)ds - \big(d_{12} + c_1M_2^{(2)}\big)x_2(t) - b_1x_2^2(t). \tag{48}$$

By applying Lemma 2.1 (1) and standard comparison theorem, it follows from (48) that

$$\liminf_{t\to+\infty} x_2(t) \ge \frac{r_1F_1 - d_{12} - c_1M_2^{(2)}}{b_1}, \tag{49}$$

and so, from (49) there exists a $T_4^{'} > T_3$ such that

$$x_2(t) > \frac{r_1F_1 - d_{12} - c_1M_2^{(2)}}{b_1} - \frac{\varepsilon}{2} \overset{\text{def}}{=} m_1^{(2)} > 0 \text{ for all } t > T_4^{'}. \tag{50}$$

Above inequality together with the second equation of system (3) leads to

$$\dot{y}_2(t) \quad > \quad r_2\int_0^{\infty} f_2(s)\exp\{-d_{22}s\}y_2(t-s)ds - \big(d_{21} - c_2m_1^{(2)}\big)y_2(t) - b_2y_2^2(t). \tag{51}$$

From this differential inequality, by applying Lemma 2.1 (1) or Lemma 2.2, we have

$$\liminf_{t\to+\infty} y_2(t) \ge \frac{r_2F_2 + c_2m_1^{(2)} - d_{21}}{b_2},$$

and so, there exists a $T_4 > T_4^{'}$ such that

$$y_2(t) \ge \frac{r_2F_2 + c_2m_1^{(2)} - d_{21}}{b_2} - \frac{\varepsilon}{2} \overset{\text{def}}{=} m_2^{(2)} > 0 \text{ for all } t > T_4. \tag{52}$$

Obviously,

$$
\begin{aligned}
M_1^{(2)} &= \frac{r_1F_1 - d_{12} - c_1m_2^{(1)}}{b_1} + \frac{\varepsilon}{2} < \frac{r_1F_1 - d_{12}}{b_1} + \varepsilon = M_1^{(1)};\\
M_2^{(2)} &= \frac{r_2F_2 + c_2M_1^{(2)} - d_{21}}{b_2} + \frac{\varepsilon}{2} < \frac{r_2F_2 + c_2M_1^{(1)} - d_{21}}{b_2} + \varepsilon = M_2^{(1)};\\
m_1^{(2)} &= \frac{r_1F_1 - d_{12} - c_1M_2^{(2)}}{b_1} - \frac{\varepsilon}{2} > \frac{r_1F_1 - d_{12} - c_1M_2^{(1)}}{b_1} - \varepsilon = m_1^{(1)};\\
m_2^{(2)} &= \frac{r_2F_2 + c_2m_1^{(2)} - d_{21}}{b_2} - \frac{\varepsilon}{2} > \frac{r_2F_2 + c_2m_1^{(1)} - d_{21}}{b_2} - \varepsilon = m_2^{(1)}.
\end{aligned}
\tag{53}
$$

Repeating the above procedure, we get four sequences $M_i^{(n)}, m_i^{(n)}, i = 1,2, n = 1,2,\cdots$, such that for $n \geq 2$

$$
\begin{aligned}
M_1^{(n)} &= \frac{r_1F_1 - d_{12} - c_1m_2^{(n-1)}}{b_1} + \frac{\varepsilon}{n};\\
M_2^{(n)} &= \frac{r_2F_2 + c_2M_1^{(n)} - d_{21}}{b_2} + \frac{\varepsilon}{n};\\
m_1^{(n)} &= \frac{r_1F_1 - d_{12} - c_1M_2^{(n)}}{b_1} - \frac{\varepsilon}{n};\\
m_2^{(n)} &= \frac{r_2F_2 + c_2m_1^{(n)} - d_{21}}{b_2} - \frac{\varepsilon}{n}.
\end{aligned}
\tag{54}
$$

Obviously,

$$m_1^{(n)} < x_2(t) < M_1^{(n)}, \;\; m_2^{(n)} < y_2(t) < M_2^{(n)}, \;\; \text{for} \;\; t \geq T_{2n}.$$

We claim that sequences $M_i^{(n)}, i = 1,2$ are strictly decreasing, and sequences $m_i^{(n)}, i = 1,2$ are strictly increasing. To proof this claim, we will carry out by induction. Firstly, from (53) we have

$$M_i^{(2)} < M_i^{(1)}, \;\; m_i^{(2)} > m_i^{(1)}, \;\; i = 1,2.$$

Let us assume now that our claim is true for n, that is,

$$M_i^{(n)} < M_i^{(n-1)}, \;\; m_i^{(n)} > m_i^{(n-1)}, \;\; i = 1,2. \tag{55}$$

By using the second inequality in (55), one could easily see that

$$M_1^{(n+1)} = \frac{r_1F_1 - d_{12} - c_1 m_2^{(n)}}{b_1} + \frac{\varepsilon}{n+1} < \frac{r_1F_1 - d_{12} - c_1 m_2^{(n-1)}}{b_1} + \frac{\varepsilon}{n} = M_1^{(n)}. \tag{56}$$

Similarly, by a straightforward computation, one could easily see that

$$M_2^{(n+1)} < M_2^{(n)}, m_1^{(n+1)} > m_1^{(n)}, m_2^{(n+1)} > m_2^{(n)}. \tag{57}$$

and we have

$$\begin{aligned} 0 < m_1^{(1)} < m_1^{(2)} < \cdots < m_1^{(n)} < x_2(t) < M_1^{(n)} < \cdots < M_1^{(2)} < M_1^{(1)}, \\ 0 < m_2^{(1)} < m_2^{(2)} < \cdots < m_2^{(n)} < y_2(t) < M_2^{(n)} < \cdots < M_2^{(2)} < M_2^{(1)}. \end{aligned} \tag{58}$$

Therefore, the limits of $M_i^{(n)}, m_i^{(n)}, i = 1,2; n = 1,2,\cdots$ exist. Denote that

$$\begin{aligned} \lim_{t\to+\infty} M_1^{(n)} = \overline{x}_2, \quad \lim_{t\to+\infty} M_2^{(n)} = \overline{y}_2, \\ \lim_{t\to+\infty} m_1^{(n)} = \underline{x}_2, \quad \lim_{t\to+\infty} m_2^{(n)} = \underline{y}_2, \end{aligned} \tag{59}$$

Letting $n \to +\infty$ in (54), we obtain

$$\begin{aligned} r_1F_1 - d_{12} - c_1\underline{y}_2 - b_1\overline{x}_2 = 0; \\ r_2F_2 - d_{21} + c_2\overline{x}_2 - b_2\overline{y}_2 = 0; \\ r_1F_1 - d_{12} - c_1\overline{y}_2 - b_1\underline{x}_2 = 0; \\ r_2F_2 - d_{21} + c_2\underline{x}_2 - b_2\underline{y}_2 = 0. \end{aligned} \tag{60}$$

Solving equation (60), one could obtain

$$\overline{x}_2 = \underline{x}_2 = x_2^* = \frac{b_2\big(r_1F_1 - d_{12}\big) - c_1\big(r_2F_2 - d_{21}\big)}{b_2b_1 + c_2c_1}, \tag{61}$$

$$\overline{y}_2 = \underline{y}_2 = y_2^* = \frac{c_2\big(r_1F_1 - d_{12}\big) + b_1\big(r_2F_2 - d_{21}\big)}{b_2b_1 + c_2c_1}, \tag{62}$$

that is

$$\lim_{t\to+\infty} x_2(t) = x_2^* \quad \lim_{t\to+\infty} y_2(t) = y_2^*. \tag{63}$$

By using (63), applying Lemma 2.5 to the first and third equations of system (3), it immediately follows that

$$\lim_{t\to+\infty} x_1(t) = \frac{r_1(1-F_1)x_2^*}{d_{11}} \quad \lim_{t\to+\infty} y_1(t) = \frac{r_2(1-F_2)y_2^*}{d_{22}}. \tag{64}$$

(63) and (64) shows that the unique interior equilibrium $E^*(x_1^*, x_2^*, y_1^*, y_2^*)$ of system (3) is globally attractive. This completes the proof of Theorem 1.1.

Conflict of Interests

The authors declare that there is no conflict of interests.

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