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GLOBAL STABILITY ANALYSIS OF A NONAUTONOMOUS STAGE STRUCTURED TWO PREY-ONE PREDATOR SYSTEM WITH INTERSPECIFIC COMPETITION AND MATURATION DELAY

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Abstract. In this paper, we investigate a nonautonomous two prey-one predator systems with stage structure for each species. Interspecific competition between mature dominant prey and mature sub-dominant prey species are considered, and three discrete time delays are incorporated into the model due to maturation time for sub-dominant prey, dominant prey and predator species, respectively. Positivity and boundedness of solutions are analytically studied. By utilizing some comparison arguments, an iterative technique is proposed to discuss permanence of solutions. Furthermore, existence of positive periodic solutions is investigated based on continuation theorem of coincidence degree theory. By constructing some appropriate Lyapunov functionals, sufficient conditions for global stability of the unique positive periodic solution are analyzed. Numerical simulations are carried out to show consistency with theoretical analysis obtained in this paper.

Keywords: Prey predator; Interspecific competition; Maturation delay; Permanence; Global stability analysis.

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1. Introduction

Many species in the natural world have a life history that takes them through immature stage and mature stage. Some vital biological rates (rates of survival and reproduction) of individuals are always identical and dependent on stage structure [25]. In the past several decades, much research efforts have been put into investigating interaction and coexistence mechanism of prey predator system with stage structure [3, 5, 6, 7, 8, 9, 10, 12]. In order to discuss dynamic effect of stage structure and maturation process on population dynamics, constant time delays are introduced to reflect the maturation delay in the prey predator system [5, 11, 13, 14, 15, 17, 19, 22, 32, 34], which have been one of the most important fields of interest. By incorporating a discrete time delay into single species model, a stage structured model is proposed in the pioneering work [5], where the discrete time delay reflects a delayed birth of immature species and a reduced survival of immature species to their maturity. The mathematical model proposed in [5] takes the following form,

$$\begin{cases} \dot{x}_i(t) = \alpha x_m(t) - \gamma x_i(t) - \alpha e^{-\gamma\tau} x_m(t-\tau), \\ \dot{x}_m(t) = \alpha e^{-\gamma\tau} x_m(t-\tau) - \beta x_m^2(t), \end{cases} \tag{1}$$

where $x_i(t)$ and $x_m(t)$ represents the population density of immature and mature species at time t, respectively. $\alpha > 0$ denotes the birth rate of immature species, $\gamma > 0$ stands for the death rate of immature species. $\beta > 0$ is the death and overcrowding rate of mature species. $\tau > 0$ denotes time of immature species to maturity. The term $\alpha e^{-\gamma\tau} x_m(t-\tau)$ represents the immature species who are born at time $t-\tau$ and survive at time t with immature species death rate γ, and therefore represents transformation of immature stage to mature stage. It is found that all ecologically relevant solutions tend to the positive equilibrium solution as time $t \to \infty$, and various aspects of the above proposed model including positivity and boundedness of solutions are discussed in [5].

Competition is an interaction among competing species, in which the fitness of one species is lowered by the presence of another species within ecosystem. Generally, competition is very important in determining the biological characteristics of species, and there are two types of competition, intraspecific competition and interspecific competition [10, 25]. In this paper, we will pay special attention to the interspecific competition between two mature prey species. Interspecific competition refers to the competition between two or more species for some limiting resource. This limiting resource can be food or nutrients, space, mates, nesting sites, anything for which demand is greater than supply. When one species is a better competitor, interspecific competition negatively influences the other species by reducing population sizes and/or growth rates, which in turn affects population dynamics of the competitor [7, 10]. It should be noted that interspecific competition has the potential to alter species, communities and the evolution of interacting species. On an individual organism level, interspecific competition can occur as interference or exploitative competition. Two vivid interspecific competition examples in the natural world are given as follows. If a tree species in a dense forest grows taller than surrounding tree species, it is able to absorb more of the incoming sunlight. However, less sunlight is then available for the trees that are shaded by the taller tree. Cheetahs and lions can also be in interspecific competition, since both species feed on the same prey, and can be negatively impacted by the presence of the other because they will have less food [7, 30].

In the 1920s, the dynamic impacts of competition on population dynamics are systematically discussed in [1, 2]. Under some necessary simplified assumptions that there are not migration and carrying capacities, competition coefficients for both species are constants, Lotka and Volterra propose a mathematical model in [1, 2], which takes the following form,

$$\begin{cases} \dot{x}_1(t) = x_1(t)(b_1 - a_{11}x_1(t) - a_{12}x_2(t)), \\ \dot{x}_2(t) = x_2(t)(b_2 - a_{22}x_2(t) - a_{21}x_1(t)), \end{cases} \tag{2}$$

where $x_i(t)$ $(i = 1,2)$ represents population density of the competing ith species at time t, respectively. b_i $(i = 1,2)$ denotes the birth rate of the corresponding species; a_{ij} $(i, j = 1,2,i \neq j)$ is the corresponding linear reduction of the ith species' rate growth by interspecific competition, the jth species. a_{ii} $(i = 1,2)$ stands for the corresponding linear reduction of the ith species' rate growth by intraspecific competition. It should be noted that model (2) combines the effects of each species on the other and creates a theoretical prediction of interactions that can be used to understand how different factors affect the outcomes of competitive interactions. However, some factors, which may affect the outcome of competitive interactions and dynamics of one or both species, are not considered in model (2).

It is well known that biological or environmental parameters, such as seasonal effects of weather, food supplies and mating habits, are naturally periodically subject to fluctuation in time [7, 10]. Furthermore, effects of a periodically varying environment are important for evolutionary theory as the selective forces on systems in a fluctuating environment differ from those in a stable environment [12, 25]. Hence, it is constructive to assume periodicity of environment and incorporate the periodicity of parameters into mathematical model, which can be found in [14, 15, 16]. Zeng et al. [14] propose a nonautonomous competitive two-species model with stage structure in one species, where conditions of permanence are obtained. Furthermore, existence and asymptotic stability of periodic solution are proved under some assumptions if the proposed model turns out to be a periodic system. A two-species Lotka-Volterra type competition model with stage structure for both species is proposed and investigated in [15], where the individuals of each species are classified as immature and mature. By constructing a suitable Lyapunov function, sufficient conditions are derived for global stability of nonnegative equilibria of the proposed model in the case of constant coefficients. Furthermore, a set of easily verifiable sufficient conditions are obtained for the existence of positive periodic solution when coefficient are assumed to be positively continuous periodic functions. In [16], a time delayed periodic system which describes the competition among mature species. The evolutionary behavior of the proposed model is analyzed and some sufficient conditions for competitive coexistence and exclusion are obtained.

By considering a seasonally fluctuating survival environment, some nonautonomous competitive Lokta-Volterra systems [18, 20, 21, 23, 24, 26, 27, 28, 29, 31, 32, 33, 34, 36] are proposed in recent years. A nonautonomous competitive Lotka-Volterra system is studied in [18], it reveals that a computable necessary and sufficient condition for the system to be totally permanent when the growth rates have averages and the interaction coefficients are nonnegative constants. Along with line of this research, permanence for a class of competitive Lotka-Volterra systems are discussed in [31] which extends the work done in [18], and a computable necessary and sufficient condition is found for the permanence of all subsystems of the system with its small perturbation on the interaction matrix. In [20], a two-species competitive model with stage structure is discussed, and the dynamics of coupled system of semilinear parabolic equations with time delays are investigated, which show that the introduction of diffusion does not affect the permanence and extinction of the species though the introduction of stage structure brings negative effect on it. In [21], sufficient conditions are obtained for the existence of a unique, globally attractive, strictly positive (componentwise), almost periodic solution of a non-autonomous, almost periodic competitive two species model with a stage structure in one species. According to two types of well known periodic single species growth models with time delay, two corresponding periodic competitive systems with multiple delays are proposed in [23], and the same criteria for existence and globally asymptotic stability of positive periodic solutions are derived. In [24], a discrete periodic competitive model with stage structure is established, and some sufficient and realistic conditions are obtained for existence of a positive periodic solution of the proposed system. In [26], a periodic non-autonomous competitive stage-structured system with infinite delay is considered, where the adult members of n-species are in competition. For each of the n-species the model incorporates a time delay which represents the time from birth to maturity of that species. Infinite delay is introduced which denotes the influential effect of the entire past history of the system on the current competition interactions. By using the comparison principle that if the growth rates are sufficiently large then the solutions are uniformly permanent. Then by using Horns fixed point Theorem, the existence of positive periodic solution of the system with finite delay is discussed. Finally, it is proved that even the system with infinite delay admits a positive periodic solution. In [27], a non-autonomous predator-prey system with discrete time-delay is studied, where there is epidemic disease in the predator. By using some techniques of the differential inequalities and delay differential inequalities, the permanence of system is discussed under some appropriate conditions. When all the coefficients of the system is periodic, the existence and global attractivity of the positive periodic solution are studied by Mawhins continuation theorem and constructing a suitable Lyapunov functional. Furthermore, when the coefficients of the system are not absolutely periodic but almost periodic, sufficient conditions are also derived for the existence and asymptotic stability of the almost periodic solution. In [28], general n-species non-autonomous Lotka Volterra competitive systems with pure-delays and feedback controls are discussed. New sufficient conditions for which a part of the n-species remains permanent, are established by applying the method of multiple Lyapunov functionals and introducing a new analysis technique.

It should be noted that there are many species in which only immature individuals are predated by their predators, and one typical example in the natural world is given as follows. Chinese fir-bellied newt, which is unable to feed on the mature Rana chensinensis, can only feed on the immature Rana chensinesis [25]. By considering stage structure and predation habit, Wang et al. [29] study a nonautonomous predator prey model with stage structure and double time delays due to maturation time for both prey and predator, where only immature prey is under predation. The mathematical model proposed in [29] takes the following form,

$$
\begin{cases}
\dot{x}_1(t) = a_1(t)x_2(t) - r_1(t)x_1(t) - a_1(t-\tau_1)e^{-\int_{t-\tau_1}^{t} r_1(s)+k_1(s)y_2(s)\mathrm{d}s}x_2(t-\tau_1) \\
\qquad\qquad -k_1(t)x_1(t)y_2(t), \\
\dot{x}_2(t) = a_1(t-\tau_1)e^{-\int_{t-\tau_1}^{t} r_1(s)+k_1(s)y_2(s)\mathrm{d}s}x_2(t-\tau_1) - \beta_1(t)x_2^2(t), \\
\dot{y}_1(t) = a_2(t)x_1(t)y_2(t) - r_2(t)y_1(t) - a_2(t-\tau_2)e^{-\int_{t-\tau_2}^{t} r_2(s)\mathrm{d}s}x_1(t-\tau_2)y_2(t-\tau_2), \\
\dot{y}_2(t) = a_2(t-\tau_2)e^{-\int_{t-\tau_2}^{t} r_2(s)\mathrm{d}s}x_1(t-\tau_2)y_2(t-\tau_2) - \beta_2(t)y_2^2(t),
\end{cases}
\tag{3}
$$

where $x_1(t)$ and $x_2(t)$ denotes population density of the immature and mature prey species at time t, respectively; $y_1(t)$ and $y_2(t)$ represents population density of the immature and mature predator species at time t, respectively; The birth rate of immature prey/predator species is proportional to the exiting mature prey/predator species with a proportionality $a_1(t) > 0$ and $a_2(t) > 0$, respectively; death rate of immature prey/predator species is proportional to the existing immature prey/predator population with a proportionality $r_1(t) > 0$ and $r_2(t) > 0$, respectively; the death rate of mature prey/predator species is proportional to the square of the existing species with a proportionality $\beta_1(t) > 0$ and $\beta_2(t) > 0$, respectively. Only immature prey species is under predation with a proportionality $k_1(t) > 0$. $a_1(t)$, $a_2(t)$, $r_1(t)$, $r_2(t)$, $\beta_1(t)$, $\beta_2(t)$, $k_1(t)$ are continuously positive periodic functions with period ω. $\tau_1 > 0$ and $\tau_2 > 0$ represents maturation delay for prey and predator species, respectively. Furthermore, sufficient conditions for global stability of unique positive periodic solution are given in [29].

By utilizing Brouwer fixed point theorem and constructing a suitable Lyapunov functional, the periodic solution and global stability for a nonautonomous competitive Lotka-Volterra diffusion system is investigated in [32], it can be found that the system has a unique periodic solution which is globally stable under some appropriate conditions. In [33], a delay differential equation model for the interaction between two species is investigated. The maturation delay for each species is modelled as a distribution, to allow for the possibility that individuals may take different amount of time to maturity. Positivity and boundedness of the solutions are studied, and global stability is analyzed for each equilibrium. A Lotka-Volterra competitive system with infinite delay and feedback controls is proposed in [34]. By using the method of multiple Lyapunov functionals, some sufficient conditions are obtained based on developing a new analysis technique, which guarantee that some of the n species are driven to extinction. By using Mawhins continuation theorem of coincidence degree theory, an impulsive non-autonomous Lotka Volterra predator prey system with harvesting terms is investigated in [35]. Some sufficient conditions for the existence of multiple positive almost periodic solutions for the system under consideration are discussed. Furthermore, existence of multiple positive almost periodic solutions to other types of population systems can be studied by using the same method obtained in this paper. A three dimensional nonautonomous competitive Lotka-Volterra system is considered in [36], it is shown that if growth rates are positive, bounded and continuous functions, and the averages of growth rates satisfy certain inequalities, then any positive solution has the property that one of its components vanishes. In [37], an almost periodic multispecies Lotka Volterra mutualism system with time delays and impulsive effects is investigated. By using the theory of comparison theorem and constructing a suitable Lyapunov functional, sufficient conditions which guarantee the existence and uniqueness and global asymptotical stability of almost periodic solution of this system are obtained.

It may be pointed out that dynamic effect of predation is not investigated in [20, 23, 24], and dynamic effect of interspecific competition among prey species is not investigated in [29]. Furthermore, the work done in [18, 31, 32, 33, 34, 36] are discussed based on competitive prey-predator models, while dynamic effect of stage structure and maturation delay for prey/predator species are not studied. To author's best knowledge, combined dynamic effects of maturation delay and interspecific competition on stage structured prey predator system have not been simultaneously investigated under periodically varying environment.

In the natural world, two competing prey-one predator ecosystem exists extensively. Species within such ecosystem usually live in a fluctuating environment that is periodically affected by seasonal effects. Some biological rates of species are identical in predation habit and interspecific competition due to stage structure. A vivid case in point is herring-capelin-Antarctic minke whale ecosystem in Antarctic area [7, 25], where survival environment is seasonally affected by weather and food supplies. Especially, there are different maturation durations for three species, and each species shows different biological characteristics corresponding to specific stage structure. For predator species, Antarctic minke whale, mature Antarctic minke whale only consume immature herring and capelin species, and immature Antarctic minke whale can not attack prey species; For two prey species, mature herring and capelin species compete for limited life resource, which contribute to growth of the immature species, respectively. Hence, it is necessary to investigate the combined dynamic effect of maturation delay and interspecific competition on population dynamics of two prey-one predator ecosystem in a periodic environment.

The rest sections of this paper are organized as follows. By considering interspecific competition of prey species, this paper extends the work [29]. A nonautonomous stage structured two prey-one predator model is established in the second section. Interspecific competition between mature dominant prey and mature sub-dominant prey species are considered, and three discrete time delays are utilized to reflect the maturation time for sub-dominant prey, dominant prey and predator species, respectively. The positivity and boundedness of solutions are analytically studied in the third section. By utilizing some comparison arguments, an iterative technique is proposed to discuss permanence of solutions in the fourth section. Existence of positive periodic solutions is considered based on continuation theorem of coincidence degree theory in the fifth section. By constructing some appropriate Lyapunov functionals, sufficient conditions for global stability of the unique positive periodic solution are analyzed in the sixth section. Numerical simulations are provided to support the theoretical findings. Finally, this paper ends with a conclusion.

2. Model formulation

In this section, model (3) is extended by incorporating interspecific competition between mature dominant prey and mature sub-dominant prey species, a nonautonomous stage structured two prey-one predator system with competition and maturation delay will be constructed based on the following hypotheses.

(H1): $x_{i1}(t)$ $(i=1,2)$ represents population density of the ith immature prey species at time t, respectively. $x_{i2}(t)$ $(i=1,2)$ represents population density of the ith mature prey species at time t, respectively. The birth rate of the ith immature prey species is proportional to the ith exiting mature prey species with a proportionality $a_{ii}(t)>0$ $(i=1,2)$, respectively; Death rate of the ith immature prey species is proportional to the existing immature prey species with a proportionality $r_i(t)>0$ $(i=1,2)$.

(H2): Generally speaking, two prey species compete each other for the limited life resource within closed environment, but this competition only happens among mature individual and does not involve the immature individual. Hence, $a_{ij}(t)$ $(i,j=1,2$ and $i\neq j)$ denotes the corresponding linear reduction of the ith mature prey species' rate growth by the jth mature prey species due to interspecific competition. Death rate of the ith mature prey species is proportional to the square of the ith mature prey species with a proportionality $\beta_i(t)>0$ $(i=1,2)$, respectively. It is assumed that $a_{12}(t)>a_{21}(t)$ in this paper. Hence, $x_{11}(t)$ and $x_{12}(t)$ represents population density of immature sub-dominant prey and mature sub-dominant prey species, respectively. $x_{21}(t)$ and $x_{22}(t)$ represents population density of immature dominant prey and mature dominant prey species, respectively.

(H3): Discrete time delay $\tau_i>0$ $(i=1,2)$ represents maturation delay for the ith existing immature prey population, respectively. The mathematical term

$$a_{i1}(t-\tau_i)e^{-\int_{t-\tau_i}^{t} r_i(s)+a_{i3}(s)y_2(s)\mathrm{d}s}x_{i2}(t-\tau_i),$$

represents population density of the ith immature prey species that were born at time $t-\tau_i$ $(i=1,2)$, which still survive at time t and are transferred from the immature stage to the mature stage at time t, respectively.

(H4): $y_1(t)$ and $y_2(t)$ represents population density of immature and mature predator species at time t, respectively. It is assumed that immature predator population do not feed on prey and do not have the ability to reproduce. Only immature prey species $(x_{i1}(t), i=1,2)$ are predated by their predator species $y_2(t)$ with proportionality $a_{13}(t)>0$ and $a_{23}>0$, respectively. The birth rate of immature predator species $y_1(t)$ is proportional to predation effect with proportionality $0<a_{31}(t)<a_{13}(t)$ and $0<a_{32}(t)<a_{23}(t)$, which implies that biomass of two immature prey species can not be completely converted to growth of predator species. Such assumptions practically coincides with the biomass conversion law in the real world. Discrete time delay $\tau_3>0$ represents maturation delay for the immature predator population. The mathematical term

$$[a_{31}(t-\tau_3)x_{11}(t-\tau_3)+a_{32}(t-\tau_3)x_{21}(t-\tau_3)]e^{-\int_{t-\tau_3}^{t} r_3(s)\mathrm{d}s}y_2(t-\tau_3),$$

represents population density of the immature predator species that were born at time $t-\tau_3$, which still survive at time t and are transferred from the immature stage to the mature stage at time t. Death rate of the immature predator species is proportional to the existing immature predator species with a proportionality $r_3(t)>0$. Death rate of the mature predator species is proportional to the square of the existing mature predator species with a proportionality $\beta_3(t)>0$.

(H5): In this paper, $a_{ii}(t)>0$ $(i=1,2)$, $a_{ij}(t)$ $(i,j=1,2,3$ and $i\neq j)$, $r_i(t)>0$ $(i=1,2,3)$, $\beta_i(t)>0$ $(i=1,2,3)$ are assumed to be continuously positive periodic functions with period $\omega>0$.

Base on (H1)-(H5), a nonautonomous stage structured two prey-one predator system with competition and maturation delay is constructed as follows,

$$
(4)\qquad \begin{cases}
\dot{x}_{11}(t) = a_{11}(t)x_{12}(t) - r_1(t)x_{11}(t) - a_{11}(t-\tau_1)e^{-\int_{t-\tau_1}^{t} r_1(s)+a_{13}(s)y_2(s)\mathrm{d}s} \\
\qquad x_{12}(t-\tau_1) - a_{13}(t)x_{11}(t)y_2(t), \\
\dot{x}_{12}(t) = a_{11}(t-\tau_1)e^{-\int_{t-\tau_1}^{t} r_1(s)+a_{13}(s)y_2(s)\mathrm{d}s}x_{12}(t-\tau_1) - a_{12}(t)x_{12}(t)x_{22}(t) \\
\qquad -\beta_1(t)x_{12}^2(t), \\
\dot{x}_{21}(t) = a_{22}(t)x_{22}(t) - r_2(t)x_{21}(t) - a_{22}(t-\tau_2)e^{-\int_{t-\tau_2}^{t} r_2(s)+a_{23}(s)y_2(s)\mathrm{d}s}x_{22}(t-\tau_2) \\
-a_{23}(t)x_{21}(t)y_2(t), \\
\dot{x}_{22}(t) = a_{22}(t-\tau_2)e^{-\int_{t-\tau_2}^{t} r_2(s)+a_{23}(s)y_2(s)\mathrm{d}s}x_{22}(t-\tau_2) - a_{21}(t)x_{12}(t)x_{22}(t) \\
\qquad -\beta_2(t)x_{22}^2(t), \\
\dot{y}_1(t) = [a_{31}(t)x_{11}(t) + a_{32}(t)x_{21}(t)]y_2(t) - r_3(t)y_1(t) - a_{31}(t-\tau_3)x_{11}(t-\tau_3) \\
\qquad e^{-\int_{t-\tau_3}^{t} r_3(s)\mathrm{d}s}y_2(t-\tau_3) - a_{32}(t-\tau_3)x_{21}(t-\tau_3)e^{-\int_{t-\tau_3}^{t} r_3(s)\mathrm{d}s}y_2(t-\tau_3), \\
\dot{y}_2(t) = [a_{31}(t-\tau_3)x_{11}(t-\tau_3) + a_{32}(t-\tau_3)x_{21}(t-\tau_3)]e^{-\int_{t-\tau_3}^{t} r_3(s)\mathrm{d}s}y_2(t-\tau_3) \\
\qquad -\beta_3(t)y_2^2(t).
\end{cases}
$$

In the rest sections of this paper, model (4) is investigated with the following initial conditions

$$
(5)\qquad \begin{cases}
x_{1i}(\theta) = \phi_{1i}(\theta) > 0, -\tau_1 \le \theta \le 0, i = 1,2; \\
x_{2i}(\theta) = \phi_{2i}(\theta) > 0, -\tau_2 \le \theta \le 0, i = 1,2; \\
y_i(\theta) = \psi_i(\theta) > 0, -\tau_3 \le \theta \le 0, i = 1,2.
\end{cases}
$$

For continuity of the initial conditions, it is required that

$$
(6)\qquad \begin{cases}
x_{11}(0) = \int_{-\tau_1}^{0} a_{11}(\theta)\phi_{12}(\theta)e^{\int_0^{\theta}[r_1(s)+a_{13}(s)\psi_2(s)]\mathrm{d}s}\mathrm{d}\theta, \\
x_{21}(0) = \int_{-\tau_2}^{0} a_{22}(\theta)\phi_{22}(\theta)e^{\int_0^{\theta}[r_2(s)+a_{23}(s)\psi_2(s)]\mathrm{d}s}\mathrm{d}\theta, \\
y_1(0) = \int_{-\tau_3}^{0}[a_{31}(\theta)\phi_{11}(\theta) + a_{32}(\theta)\phi_{21}(\theta)]e^{\int_0^{\theta} r_3(s)\mathrm{d}s}\psi_2(\theta)\mathrm{d}\theta.
\end{cases}
$$

3. Positivity and boundedness of solutions

In this section, positivity and boundedness of solutions of model (4) with initial conditions (5) and (6) are analytically discussed.

Firstly, some mathematical notations are adopted for convenience of the following statement,

$$
f^L = \min_{t\in[0,\omega]} |f(t)|, f^M = \max_{t\in[0,\omega]} |f(t)|,
$$

where $f(t)$ is a ω-periodic continuous function.

Theorem 3.1 *Solutions of model (4) with initial conditions (5) and (6) are positive for all $t > 0$.*

Proof. Firstly, we show that $x_{12}(t) > 0$ for all $t > 0$. Otherwise, if it is false, since $x_{12}(t) > 0$ for all $t \in [-\tau_1, 0]$, then it can be derived that there exists a $t_1 > 0$ such that $x_{12}(t_1) = 0$.

Define $t_2 = \inf\{t > 0 | x_{12}(t) = 0\}$. According to the definition of t_2, it can be obtained that

$$\dot{x}_{12}(t_2) < 0. \tag{7}$$

It follows from the second equation of model (4) that

$$\dot{x}_{12}(t_2) = \begin{cases} a_{11}(t_2 - \tau_1)e^{\int_{t_2-\tau_1}^{t_2} -r_1(s)-a_{13}(s)y_2(s)\mathrm{d}s}\phi_{12}(t_2 - \tau_1), & 0 \leq t_2 \leq \tau_1, \\ a_{11}(t_2 - \tau_1)e^{\int_{t_2-\tau_1}^{t_2} -r_1(s)-a_{13}(s)y_2(s)\mathrm{d}s}x_{12}(t_2 - \tau_1), & t_2 > \tau_1, \end{cases}$$

and it is easy to show that $\dot{x}_{22}(t_2) > 0$ for all $t > 0$, which is a contradiction to (7). Hence, $x_{12}(t) > 0$ for all $t > 0$.

By using the first equation of model (4) with initial conditions (5) and (6), it can be obtained that

$$\dot{x}_{11}(t) > -(r_1^M + a_{13}^M y_2^M)x_{11}(t) - a_{11}^M e^{-\int_{t-\tau_1}^{t}(r_1^M + a_{13}^M y_2^M)\mathrm{d}s}x_{12}(t - \tau_1).$$

Considering the auxiliary equation,

$$\dot{u}(t) = -(r_1^M + a_{13}^M y_2^M)u(t) - a_{11}^M e^{-\int_{t-\tau_1}^{t}(r_1^M + a_{13}^M y_2^M)\mathrm{d}s}x_{12}(t - \tau_1), \tag{8}$$

with the initial condition $u(0) = a_{11}^M \int_0^{\tau_1} e^{(r_1^M + a_{13}^M y_2^M)(s-\tau_1)}x_{12}(s - \tau_1)\mathrm{d}s$.

Based on Eq. (8), it is easy to show that

$$x_{11}(t) > u(t), \quad \dot{u}(t) < 0, \tag{9}$$

hold for all $t > 0$, which derives that $u(t)$ is strictly decreasing for all $t > 0$.

By solving Eq. (8), it derives that

$$u(t) = e^{-(r_1^M + a_{13}^M y_2^M)t}[u(0) - a_{11}^M \int_0^t e^{(r_1^M + a_{13}^M y_2^M)(s-\tau_1)}x_{12}(s - \tau_1)\mathrm{d}s].$$

It follows from further computation that

$$u(\tau_1) = e^{-(r_1^M + a_{13}^M y_2^M)\tau_1}[u(0) - a_{11}^M \int_0^{\tau_1} e^{(r_1^M + a_{13}^M y_2^M)(s-\tau_1)}x_{12}(s - \tau_1)\mathrm{d}s],$$

and it is easy to show that $u(\tau_1)=0$. Since $u(\tau_1)=0$ and $u(t)$ is strictly decreasing for $t\in[0,\tau_1]$, it can be derived that $u(t)>0$ for $t\in[0,\tau_1]$. By repeating this argument, it derives that $u(t)>0$ for all $t>0$. By virtue of (9), it is easy to show that $x_{11}(t)>0$ for all $t>0$.

Secondly, by utilizing the similar proof, it can be shown that $x_{21}(t)>0$ and $x_{22}(t)>0$ for all $t>0$.

Finally, we show that $y_2(t)>0$ for all $t>0$. Otherwise, if it is false, since $y_2(t)>0$ for all $t\in[-\tau_1,0]$, then it can be derived that there exists a $t_3>0$ such that $y_2(t_3)=0$.

Define $t_4=\inf\{t>0|y_2(t)=0\}$. According to the definition of t_4, it can be obtained that

$$\dot{y}_2(t_4)<0. \tag{10}$$

It follows from positivity of $x_{i1}>0$ $(i=1,2)$ and the sixth equation of model (4) that

$$\dot{y}_2(t_4)=\begin{cases}[a_{31}(t_4-\tau_3)\phi_{11}(t_4-\tau_3)+a_{32}(t_4-\tau_3)\phi_{21}(t_4-\tau_3)]e^{-\int_{t_4-\tau_3}^{t_4}r_3(s)\mathrm{d}s}\psi_2(t_4-\tau_3),\\ 0\le t_4\le\tau_3,\\ [a_{31}(t_4-\tau_3)x_{11}(t_4-\tau_3)+a_{32}(t_4-\tau_3)x_{21}(t_4-\tau_3)]e^{-\int_{t_4-\tau_3}^{t_4}r_3(s)\mathrm{d}s}y_2(t_4-\tau_3),\\ t_4>\tau_3,\end{cases}$$

and it is easy to show that $\dot{y}_2(t_4)>0$ for all $t>0$, which is a contradiction to (10). Hence, $y_2(t)>0$ for all $t>0$.

By using the fifth equation of model (4) with initial conditions (5) and (6), it can be obtained that

$$\dot{y}_1(t)>-r_3^My_1(t)-(a_{31}^Mx_{11}^M+a_{32}^Mx_{21}^M)e^{-\int_{t-\tau_3}^{t}r_3^M\mathrm{d}s}y_2(t-\tau_3).$$

Considering the auxiliary equation,

$$\dot{v}(t)=-r_3^Mv(t)-(a_{31}^Mx_{11}^M+a_{32}^Mx_{21}^M)e^{-\int_{t-\tau_3}^{t}r_3^M\mathrm{d}s}y_2(t-\tau_3), \tag{11}$$

with the initial condition $v(0)=(a_{31}^Mx_{11}^M+a_{32}^Mx_{21}^M)\int_0^{\tau_3}e^{r_3^M(s-\tau_3)}y_2(s-\tau_3)\mathrm{d}s$.

Based on Eq. (11), it is easy to show that

$$y_1(t)>v(t),\quad \dot{v}(t)<0, \tag{12}$$

hold for all $t>0$, which derives that $v(t)$ is strictly decreasing for all $t>0$.

By solving Eq. (11), it derives that

$$v(t)=e^{-r_3^M t}[v(0)-(a_{31}^M x_{11}^M+a_{32}^M x_{21}^M)]\int_0^{\tau_3} e^{r_3^M(s-\tau_3)}y_2(s-\tau_3)\mathrm{d}s].$$

It follows from further computation that

$$v(\tau_3)=e^{-r_3^M \tau_3}[v(0)-(a_{31}^M x_{11}^M+a_{32}^M x_{21}^M)]\int_0^{\tau_3} e^{r_3^M(s-\tau_3)}y_2(s-\tau_3)\mathrm{d}s],$$

and it is easy to show that $v(\tau_3)=0$. Since $v(\tau_3)=0$ and $v(t)$ is strictly decreasing for $t\in[0,\tau_3]$, it can be derived that $v(t)>0$ for $t\in[0,\tau_3]$. By repeating this argument, it derives that $v(t)>0$ for all $t>0$.

By virtue of (12), it is easy to show that $y_1(t)>0$ for all $t>0$.

Theorem 3.2 *Solutions of model (4) with initial conditions (5) and (6) are bounded.*

Proof. Let $a_3^L=\min\{a_{13}^L,a_{23}^L\}$, $r^L=\min\{r_1^L,r_2^L,r_3^L\}$, and define

$$w(t)=a_{31}^M(x_{11}(t)+x_{12}(t))+a_{32}^M(x_{21}(t)+x_{22}(t))+a_3^L(y_1(t)+y_2(t)),$$

where $(x_{11}(t),x_{12}(t),x_{21}(t),x_{22}(t),y_1(t),y_2(t))$ is an arbitrary positive solution of model (4) with the initial conditions (5) and (6).

By calculating the derivative of $w(t)$ along the solution of model (4), it gives that

$$\begin{aligned}
\dot{w}(t) &= a_{31}^M[a_{11}(t)x_{12}(t)-r_1(t)x_{11}(t)-a_{13}(t)x_{11}(t)y_2(t)-a_{12}(t)x_{12}(t)x_{22}(t)-\beta_1(t)x_{12}^2(t)]\\
&\quad +a_{32}^M[a_{22}(t)x_{22}(t)-r_2(t)x_{21}(t)-a_{23}(t)x_{21}(t)y_2(t)-a_{21}(t)x_{12}(t)x_{22}(t)-\beta_2(t)x_{22}^2(t)]\\
&\quad +a_3^L[(a_{31}(t)x_{11}(t)+a_{32}(t)x_{21}(t))y_2(t)-r_3(t)y_1(t)-\beta_3(t)y_2^2(t)]\\
&\le -r^L w(t)+a_{31}^M(a_{11}^M+r^L)x_{12}(t)-\beta_1^L a_{31}^M x_{12}^2(t)\\
&\quad +a_{32}^M(a_{22}^M+r^L)x_{22}(t)-\beta_2^L a_{32}^M x_{22}^2(t)+r^L a_3^L y_2(t)-\beta_3^L a_3^L y_2^2(t),\\
&\le -r^L w(t)+\frac{a_{31}^M\beta_2^L\beta_3^L(a_{11}^M+r^L)^2+a_{32}^M\beta_1^L\beta_3^L(a_{22}^M+r^L)^2+a_3^L\beta_1^L\beta_2^L(r^L)^2}{4\beta_1^L\beta_2^L\beta_3^L}.
\end{aligned}$$

By using the standard comparison principle [12], it follows from the above inequality that

$$w(t)\le\frac{a_{31}^M\beta_2^L\beta_3^L(a_{11}^M+r^L)^2+a_{32}^M\beta_1^L\beta_3^L(a_{22}^M+r^L)^2+a_3^L\beta_1^L\beta_2^L(r^L)^2}{4\beta_1^L\beta_2^L\beta_3^L r^L}:=W. \tag{13}$$

Hence, solutions of model (4) with initial conditions (5) and (6) are bounded. □

4. Permanence of solutions

By utilizing some comparison arguments, an iterative technique is proposed in this section, which is utilized to discuss permanence of solutions of model (4) with initial conditions (5) and (6). Firstly, some lemmas and definitions are introduced in order to facilitate the following proof.

Lemma 4.1. [8] *Consider the following differential equation*

$$\dot{x}(t) = ax(t-\tau) - bx(t) - cx^2(t),$$

where $a,b,c,\tau > 0$ *and* $x(t) > 0$ *for* $-\tau \le t \le 0$*, we have*

- **(i):** *if* $a > b$*, then* $\lim_{t\to+\infty} x(t) = \frac{a-b}{c}$*,*
- **(ii):** *if* $a < b$*, then* $\lim_{t\to+\infty} x(t) = 0$*.*

Lemma 4.2. [8] *Consider the following differential equation* $\dot{x}(t) = dx(t-\sigma) - ex^2(t)$*, where* $d,e,\sigma > 0$ *and* $x(t) > 0$ *for* $-\sigma \le t \le 0$*, we have* $\lim_{t\to+\infty} x(t) = \frac{d}{e}$*.*

Definition 4.1. [9] Consider the following differential equation

$$\dot{X}(t) = f(t, X_t(\theta)), \tag{14}$$

where $t \ge 0, \theta \in [-\tau, 0], X \in \mathbb{R}^n$. Model (14) is said to be permanent if for any solution $X(t,\phi)$, there exists a constant $m > 0$ and $T = T(\phi)$ such that $X(t) > m$ for all $t > T$.

Theorem 4.1. *If the following inequalities hold*

$$a_{11}^L \beta_2^L e^{-(r_1^M + a_{13}^M \bar{W})\tau_1} > a_{12}^M a_{22}^M e^{-r_2^L \tau_2}, a_{22}^L \beta_1^L e^{-(r_2^M + a_{23}^M \bar{W})\tau_2} > a_{21}^M a_{11}^M e^{-r_1^L \tau_1},$$

then solutions of model (4) is permanent with initial conditions (5) and (6), where $\bar{W} = \frac{W}{a_3^L}$ *and* W *is defined in (13).*

Proof. According to the second equation of model (4) and Theorem 3.1, it gives that

$$\dot{x}_{12}(t) \le a_{11}^M e^{-r_1^L \tau_1} x_{12}(t-\tau_1) - \beta_1^L x_{12}^2(t). \tag{15}$$

By virtue of Lemma 4.2 and (15), there exists a positive time T_1 such that for sufficiently small $\varepsilon > 0$ and $t \geq T_1$, it yields

$$x_{12}(t) \leq \frac{a_{11}^M e^{-r_1^L \tau_1}}{\beta_1^L} + \varepsilon := M_2^{(1)}. \tag{16}$$

It follows from the first equation and a direct computation, it can be obtained that

$$\begin{aligned} x_{11}(t) &= \int_{t-\tau_1}^{t} a_{11}(s) e^{\int_t^s r_1(m)+a_{13}(m)y_2(m)\mathrm{d}m} x_{12}(s)\mathrm{d}s. \\ &= e^{-\int_0^t r_1(m)+a_{13}(m)y_2(m)\mathrm{d}m} \int_{t-\tau_1}^{t} a_{11}(s) e^{\int_0^s r_1(m)+a_{13}(m)y_2(m)\mathrm{d}m} x_{12}(s)\mathrm{d}s. \end{aligned} \tag{17}$$

For any $t \geq T_1$, it follows from Theorem 3.1, (16) and (17) that

$$x_{11}(t) \leq \frac{a_{11}^M M_2^{(1)}}{r_1^L} := M_1^{(1)}. \tag{18}$$

Based on the fourth equation of model (4) and Theorem 3.1, it can be obtained that

$$\dot{x}_{22}(t) \leq a_{22}^M e^{-r_2^L \tau_2} x_{22}(t-\tau_2) - \beta_2^L x_{22}^2(t), \tag{19}$$

holds for $t \geq T_1$. By virtue of Lemma 4.2 and (19), there exists $T_2 > T_1$ such that for sufficiently small $\varepsilon > 0$ and $t \geq T_2$, it yields

$$x_{22}(t) \leq \frac{a_{22}^M e^{-r_2^L \tau_2}}{\beta_2^L} + \varepsilon := M_4^{(1)}. \tag{20}$$

By direct computing, it follows from the third equation of model (4) that,

$$\begin{aligned} x_{21}(t) &= \int_{t-\tau_2}^{t} a_{22}(s) e^{\int_t^s r_2(m)+a_{23}(m)y_2(m)\mathrm{d}m} x_{22}(s)\mathrm{d}s. \\ &= e^{-\int_0^t r_2(m)+a_{23}(m)y_2(m)\mathrm{d}m} \int_{t-\tau_2}^{t} a_{22}(s) e^{\int_0^s r_2(m)+a_{23}(m)y_2(m)\mathrm{d}m} x_{22}(s)\mathrm{d}s. \end{aligned} \tag{21}$$

For any $t \geq T_2$, it follows from (20) and (21) that

$$x_{21}(t) \leq \frac{a_{22}^M M_4^{(1)}}{r_2^L} := M_3^{(1)}. \tag{22}$$

Based on (18) and (22), it follows from the sixth equation of model (4) that

$$\dot{y}_2(t) \leq (a_{31}^M M_1^{(1)} + a_{32}^M M_3^{(1)}) e^{-r_3^L \tau_3} y_2(t-\tau_3) - \beta_3^L y_2^2(t), \tag{23}$$

holds for $t \geq T_2$. By virtue of Lemma 4.2 and (23), there exists $T_3 > T_2$ such that for sufficiently small $\varepsilon > 0$ and $t \geq T_3$, it yields

$$y_2(t) \leq \frac{(a_{31}^M M_1^{(1)} + a_{32}^M M_3^{(1)})e^{-r_3^L \tau_3}}{\beta_3^L} + \varepsilon := M_6^{(1)}. \tag{24}$$

By direct computing, it follows from the fifth equation of model (4) that,

$$\begin{aligned} y_1(t) &= \int_{t-\tau_3}^{t} [a_{31}(s)x_{11}(s) + a_{32}x_{21}(s)]e^{\int_t^s r_3(m)\mathrm{d}m} y_2(s)\mathrm{d}s. \\ &= e^{-\int_0^t r_3(m)\mathrm{d}m} \int_{t-\tau_3}^{t} [a_{31}(s)x_{11}(s) + a_{32}x_{21}(s)]e^{\int_0^s r_3(m)\mathrm{d}m} y_2(s)\mathrm{d}s. \end{aligned} \tag{25}$$

For any $t \geq T_3$, it follows from (24) and (25) that

$$y_1(t) \leq \frac{(a_{31}^M M_1^{(1)} + a_{32}^M M_3^{(1)})(1 - e^{-r_3^M \tau_3})M_6^{(1)}}{r_3^L} := M_5^{(1)}. \tag{26}$$

According to the second equation of model (4), (20) and (24), it gives that

$$\dot{x}_{12}(t) \geq a_{11}^L e^{-(r_1^M + a_{13}^M M_6^{(1)})\tau_1} x_{12}(t-\tau_1) - a_{12}^M M_4^{(1)} x_{12}(t) - \beta_1^M x_{12}^2(t), \tag{27}$$

holds for $t \geq T_3$. If $a_{11}^L \beta_2^L e^{-(r_1^M + a_{13}^M \bar{W})\tau_1} > a_{12}^M a_{22}^M e^{-r_2^L \tau_2}$ holds, then it is easy to show that $a_{11}^L e^{-(r_1^M + a_{13}^M M_6^{(1)})\tau_1} > a_{12}^M M_4^{(1)}$. Based on Lemma 4.1, there exists $T_4 > T_3$ such that for sufficiently small $\varepsilon > 0$ and

$$x_{12}(t) \geq \frac{a_{11}^L e^{-(r_1^M + a_{13}^M M_6^{(1)})\tau_1} - a_{12}^M M_4^{(1)}}{\beta_1^M} - \varepsilon := m_2^{(1)}, \tag{28}$$

holds for $t \geq T_4$. For any $t \geq T_4$, it follows from (17) and (28) that

$$x_{11}(t) \geq \frac{a_{11}^L m_2^{(1)}(1 - e^{-r_1^L \tau_1})}{r_1^M + a_{13}^M M_6^{(1)}} := m_1^{(1)}. \tag{29}$$

Based on the fourth equation of model (4), (16) and (24), it can be obtained that

$$\dot{x}_{22}(t) \geq a_{22}^L e^{-(r_2^M + a_{23}^M M_6^{(1)})\tau_2} x_{22}(t-\tau_2) - a_{21}^M M_2^{(1)} x_{22}(t) - \beta_2^M x_{22}^2(t), \tag{30}$$

holds for $t \geq T_4$. If $a_{22}^L \beta_1^L e^{-(r_2^M + a_{23}^M \bar{W})\tau_2} > a_{21}^M a_{11}^M e^{-r_1^L \tau_1}$ holds, then it is easy to show that $a_{22}^L e^{-(r_2^M + a_{23}^M M_6^{(1)})\tau_2} > a_{21}^M M_2^{(1)}$. By using Lemma 4.1, there exists $T_5 > T_4$ such that for sufficiently small $\varepsilon > 0$ and it yields that

$$x_{22}(t) \geq \frac{a_{22}^L e^{-(r_2^M + a_{23}^M M_6^{(1)})\tau_2} - a_{21}^M M_2^{(1)}}{\beta_2^M} - \varepsilon := m_4^{(1)}, \tag{31}$$

holds for $t \geq T_5$. For any $t \geq T_5$, it follows from (21) and (31) that

$$x_{21}(t) \geq \frac{a_{22}^L m_4^{(1)} (1 - e^{-r_2^L \tau_2})}{r_2^M + a_{23}^M M_6^{(1)}} := m_3^{(1)}. \tag{32}$$

Based on (29) and (32), it follows from the sixth equation of model (4) that

$$\dot{y}_2(t) \geq (a_{31}^L m_1^{(1)} + a_{32}^L m_3^{(1)}) e^{-r_3^M \tau_3} y_2(t - \tau_3) - \beta_3^M y_2^2(t), \tag{33}$$

holds for $t \geq T_5$. By using Lemma 4.2, there exists $T_6 > T_5$ such that for sufficiently small $\varepsilon > 0$ and it yields that

$$y_2(t) \geq \frac{(a_{31}^L m_1^{(1)} + a_{32}^L m_3^{(1)}) e^{-r_3^M \tau_3}}{\beta_3^M} - \varepsilon := m_6^{(1)}, \tag{34}$$

holds for $t \geq T_6$. For any $t \geq T_6$, it follows from (25) and (34) that

$$y_1(t) \geq \frac{(a_{31}^L m_1^{(1)} + a_{32}^L m_3^{(1)})(1 - e^{-r_3^M \tau_3}) m_6^{(1)}}{r_3^M} := m_5^{(1)}. \tag{35}$$

According to the second equation of model (4) and (34), it gives that for any $t \geq T_6$

$$\dot{x}_{12}(t) \leq a_{11}^M e^{-(r_1^L + a_{13}^L m_6^{(1)})\tau_1} x_{12}(t - \tau_1) - \beta_1^L x_{12}^2(t). \tag{36}$$

By virtue of Lemma 4.2 and (36), there exists $T_7 > T_6$ such that for sufficiently small $\varepsilon > 0$ and $t \geq T_7$, it yields

$$x_{12}(t) \leq \frac{a_{11}^M e^{-(r_1^L + a_{13}^L m_6^{(1)})\tau_1}}{\beta_1^L} + \varepsilon := M_2^{(2)}. \tag{37}$$

For any $t \geq T_7$, it follows from (17) and (37) that

$$x_{11}(t) \leq \frac{a_{11}^M M_2^{(2)} (1 - e^{-(r_1^M + a_{13}^M M_6^{(1)})\tau_1})}{r_1^L + a_{13}^L m_6^{(1)}} := M_1^{(2)}. \tag{38}$$

Based on the fourth equation of model (4) and (34), it can be obtained that

$$\dot{x}_{22}(t) \leq a_{22}^M e^{-(r_2^L + a_{23}^L m_6^{(1)})\tau_2} x_{22}(t - \tau_2) - \beta_2^L x_{22}^2(t), \tag{39}$$

holds for $t \geq T_7$. By virtue of Lemma 4.2 and (39), there exists $T_8 > T_7$ such that for sufficiently small $\varepsilon > 0$ and $t \geq T_8$, it yields

$$x_{22}(t) \leq \frac{a_{22}^M e^{-(r_2^L + a_{23}^L m_6^{(1)})\tau_2}}{\beta_2^L} + \varepsilon := M_4^{(2)}. \tag{40}$$

For any $t \geq T_8$, it follows from (21) and (40) that

$$x_{21}(t) \leq \frac{a_{22}^M M_4^{(2)}(1 - e^{-(r_2^M + a_{23}^M M_6^{(1)})\tau_2})}{r_2^L + a_{23}^L m_6^{(1)}} := M_3^{(2)}. \tag{41}$$

Based on (38) and (41), it follows from the sixth equation of model (4) that

$$\dot{y}_2(t) \leq (a_{31}^M M_1^{(2)} + a_{32}^M M_3^{(2)})e^{-r_3^L \tau_3} y_2(t - \tau_3) - \beta_3^L y_2^2(t). \tag{42}$$

holds for $t \geq T_8$. By virtue of Lemma 4.2 and (42), there exists $T_9 > T_8$ such that for sufficiently small $\varepsilon > 0$ and $t \geq T_9$, it yields

$$y_2(t) \leq \frac{(a_{31}^M M_1^{(2)} + a_{32}^M M_3^{(2)})e^{-r_3^L \tau_3}}{\beta_3^L} + \varepsilon := M_6^{(2)}, \tag{43}$$

For any $t \geq T_9$, it follows from (25) and (43) that

$$y_1(t) \leq \frac{(a_{31}^M M_1^{(2)} + a_{32}^M M_3^{(2)})(1 - e^{-r_3^M \tau_3})M_6^{(2)}}{r_3^L} := M_5^{(2)}. \tag{44}$$

According to the second equation of model (4), (40) and (43), it gives that

$$\dot{x}_{12}(t) \geq a_{11}^L e^{-(r_1^M + a_{13}^M M_6^{(2)})\tau_1} x_{12}(t - \tau_1) - a_{12}^M M_4^{(2)} x_{12}(t) - \beta_1^M x_{12}^2(t), \tag{45}$$

holds for $t \geq T_9$. If $a_{11}^L \beta_2^L e^{-(r_1^M + a_{13}^M \bar{W})\tau_1} > a_{12}^M a_{22}^M e^{-r_2^L \tau_2}$ holds, then it is easy to show that $a_{11}^L e^{-(r_1^M + a_{13}^M M_6^{(2)})\tau_1} > a_{12}^M M_4^{(2)}$. Based on Lemma 4.1, there exists $T_{10} > T_9$ such that for sufficiently small $\varepsilon > 0$ and

$$x_{12}(t) \geq \frac{a_{11}^L e^{-(r_1^M + a_{13}^M M_6^{(2)})\tau_1} - a_{12}^M M_4^{(2)}}{\beta_1^M} - \varepsilon := m_2^{(2)}, \tag{46}$$

holds for $t \geq T_{10}$. For any $t \geq T_{10}$, it follows from (17), (34) and (46) that

$$x_{11}(t) \geq \frac{a_{11}^L m_2^{(1)}(1 - e^{-(r_1^L + a_{13}^L m_6^{(1)})\tau_1})}{r_1^M + a_{13}^M M_6^{(2)}} := m_1^{(2)}. \tag{47}$$

Based on the fourth equation of model (4), (37) and (43), it can be obtained that

$$\dot{x}_{22}(t) \geq a_{22}^L e^{-(r_2^M + a_{23}^M M_6^{(2)})\tau_2} x_{22}(t - \tau_2) - a_{21}^M M_2^{(2)} x_{22}(t) - \beta_2^M x_{22}^2(t), \tag{48}$$

holds for $t \geq T_{10}$. If $a_{22}^L \beta_1^L e^{-(r_2^M + a_{23}^M \bar{W})\tau_2} > a_{21}^M a_{11}^M e^{-r_1^L \tau_1}$ hold, then it is easy to show that $a_{22}^L e^{-(r_2^M + a_{23}^M M_6^{(2)})\tau_2} > a_{21}^M M_2^{(2)}$. By using Lemma 4.1, there exists $T_{11} > T_{10}$ such that for sufficiently small $\varepsilon > 0$ and it yields that

$$x_{22}(t) \geq \frac{a_{22}^L e^{-(r_2^M + a_{23}^M M_6^{(2)})\tau_2} - a_{21}^M M_2^{(2)}}{\beta_2^M} - \varepsilon := m_4^{(2)}, \tag{49}$$

holds for $t \geq T_{11}$. For any $t \geq T_{11}$, it follows from (21), (34) and (49) that

$$x_{21}(t) \geq \frac{a_{22}^L m_4^{(1)} (1 - e^{-(r_2^L + a_{23}^L m_6^{(1)})\tau_2})}{r_2^M + a_{23}^M M_6^{(2)}} := m_3^{(2)}. \tag{50}$$

Based on (47) and (50), it follows from the sixth equation of model (4) that

$$\dot{y}_2(t) \geq (a_{31}^L m_1^{(2)} + a_{32}^L m_3^{(2)}) e^{-r_3^M \tau_3} y_2(t - \tau_3) - \beta_3^M y_2^2(t), \tag{51}$$

holds for $t \geq T_{11}$. By using Lemma 4.2, there exists $T_{12} > T_{11}$ such that for sufficiently small $\varepsilon > 0$ and it yields that

$$y_2(t) \geq \frac{(a_{31}^L m_1^{(2)} + a_{32}^L m_3^{(2)}) e^{-r_3^M \tau_3}}{\beta_3^M} - \varepsilon := m_6^{(2)}, \tag{52}$$

holds for $t \geq T_{12}$. For any $t \geq T_{12}$, it follows from (25) and (52) that

$$y_1(t) \geq \frac{(a_{31}^L m_1^{(2)} + a_{32}^L m_3^{(2)})(1 - e^{-r_3^M \tau_3}) m_6^{(2)}}{r_3^M} := m_5^{(2)}. \tag{53}$$

By using simple computation, twelve sequences will be obtained by repeating the discussion in this manner, which are given as follows:

$$
\begin{cases}
M_1^{(n+1)} = \frac{a_{11}^M(1-e^{-(r_1^M+a_{13}^M M_6^{(n)})\tau_1})M_2^{(n+1)}}{r_1^L+a_{13}^L m_6^{(n)}}, \\
M_2^{(n+1)} = \frac{a_{11}^M e^{-(r_1^L+a_{13}^L m_6^{(n)})\tau_1}}{\beta_1^L}+\varepsilon, \\
M_3^{(n+1)} = \frac{a_{22}^M(1-e^{-(r_2^M+a_{23}^M M_6^{(n)})\tau_2})M_4^{(n+1)}}{r_2^L+a_{23}^L m_6^{(n)}}, \\
M_4^{(n+1)} = \frac{a_{22}^M e^{-(r_2^L+a_{23}^L m_6^{(n)})\tau_2}}{\beta_2^L}+\varepsilon, \\
M_5^{(n+1)} = \frac{(a_{31}^M M_1^{(n+1)}+a_{32}^M M_3^{(n+1)})(1-e^{-r_3^M\tau_3})M_6^{(n+1)}}{r_3^L}, \\
M_6^{(n+1)} = \frac{(a_{31}^M M_1^{(n+1)}+a_{32}^M M_3^{(n+1)})e^{-r_3^L\tau_3}}{\beta_3^L}+\varepsilon, \\
m_1^{(n+1)} = \frac{a_{11}^L(1-e^{-(r_1^L+a_{13}^L m_6^{(n)})\tau_1})m_2^{(n+1)}}{r_1^M+a_{13}^M M_6^{(n+1)}}, \\
m_2^{(n+1)} = \frac{a_{11}^L e^{-(r_1^M+a_{13}^M M_6^{(n+1)})\tau_1}-a_{12}^M M_4^{(n+1)}}{\beta_1^M}-\varepsilon, \\
m_3^{(n+1)} = \frac{a_{22}^L(1-e^{-(r_2^L+a_{23}^L m_6^{(n)})\tau_2})m_4^{(n+1)}}{r_2^M+a_{23}^M M_6^{(n+1)}}, \\
m_4^{(n+1)} = \frac{a_{22}^L e^{-(r_2^M+a_{23}^M M_6^{(n+1)})\tau_2}-a_{21}^M M_2^{(n+1)}}{\beta_2^M}-\varepsilon, \\
m_5^{(n+1)} = \frac{(a_{31}^L m_1^{(n+1)}+a_{32}^L m_3^{(n+1)})(1-e^{-r_3^M\tau_3})m_6^{(n+1)}}{r_3^M}, \\
m_6^{(n+1)} = \frac{(a_{31}^L m_1^{(n+1)}+a_{32}^L m_3^{(n+1)})e^{-r_3^M\tau_3}}{\beta_3^M}-\varepsilon.
\end{cases}
\tag{54}
$$

It is easy to show that $M_i^{(n)} > 0$ and the sequences $\{M_i^{(n)}\}$ $(i = 1,2,\cdots,6)$ are decreasing as n increases, which derives $\lim_{n\to\infty} M_i^{(n)} = M_i^*$ exists. Furthermore, it is easy to show that $m_i^{(n)} < M_i^{(n)}$ and the sequences $\{m_i^{(n)}\}$ $(i = 1,2,\cdots,6)$ are increasing as n increases, which derives

$\lim_{n\to\infty} m_i^{(n)} = m_i^*$ exists. Consequently, it follows from (54) that

$$
\begin{cases}
M_1^* = \frac{a_{11}^M(1-e^{-(r_1^M+a_{13}^M M_6^*)\tau_1})M_2^*}{r_1^L+a_{13}^L m_6^*}, M_2^* = \frac{a_{11}^M e^{-(r_1^L+a_{13}^L m_6^*)\tau_1}}{\beta_1^L}, \\
M_3^* = \frac{a_{22}^M(1-e^{-(r_2^M+a_{23}^M M_6^*)\tau_2})M_4^*}{r_2^L+a_{23}^L m_6^*}, M_4^* = \frac{a_{22}^M e^{-(r_2^L+a_{23}^L m_6^{(*)})\tau_2}}{\beta_2^L}, \\
M_5^* = \frac{(a_{31}^M M_1^*+a_{32}^M M_3^*)(1-e^{-r_3^M\tau_3})M_6^*}{r_3^L}, M_6^* = \frac{(a_{31}^M M_1^*+a_{32}^M M_3^*)e^{-r_3^L\tau_3}}{\beta_3^L}, \\
m_1^* = \frac{a_{11}^L(1-e^{-(r_1^L+a_{13}^L m_6^*)\tau_1})m_2^*}{r_1^M+a_{13}^M M_6^*}, m_2^* = \frac{a_{11}^L e^{-(r_1^M+a_{13}^M M_6^*)\tau_1}-a_{12}^M M_4^*}{\beta_1^M}, \\
m_3^* = \frac{a_{22}^L(1-e^{-(r_2^L+a_{23}^L m_6^*)\tau_2})m_4^*}{r_2^M+a_{23}^M M_6^*}, m_4^* = \frac{a_{22}^L e^{-(r_2^M+a_{23}^M M_6^*)\tau_2}-a_{21}^M M_2^*}{\beta_2^M}, \\
m_5^* = \frac{(a_{31}^L m_1^*+a_{32}^L m_3^*)(1-e^{-r_3^L\tau_3})m_6^*}{r_3^M}, m_6^* = \frac{(a_{31}^L m_1^*+a_{32}^L m_3^*)e^{-r_3^M\tau_3}}{\beta_3^M}.
\end{cases}
\tag{55}
$$

Based on Definition 4.1 and (55), it can be concluded that solutions of model (4) are persistent provided that the following inequalities hold

$$
a_{11}^L\beta_2^L e^{-(r_1^M+a_{13}^M\bar{W})\tau_1} > a_{12}^M a_{22}^M e^{-r_2^L\tau_2}, a_{22}^L\beta_1^L e^{-(r_2^M+a_{23}^M\bar{W})\tau_2} > a_{21}^M a_{11}^M e^{-r_1^L\tau_1}.
$$

5. Existence of positive periodic solution

In this section, existence of positive periodic solutions is investigated based on continuation theorem of coincidence degree theory. Firstly, some definitions and lemmas are introduced in the following part.

Definition 5.1 [4] Let $L: \mathrm{Dom}L \subset X \to Y$ be a linear mapping, and $N: X \to Y$ be a continuous mapping, where X and Y are real Banach spaces. If $\mathrm{dimKer}L = \mathrm{codimIm}L < +\infty$ and $\mathrm{Im}L$ is closed in Y, then L is called a Fredholm mapping of index zero.

If L is Fredholm mapping of index zero and there exist continuous projectors $P: X \to X$ and $Q: Y \to Y$ such that $\mathrm{Im}P = \mathrm{Ker}L$, $\mathrm{Im}L = \mathrm{Ker}Q = \mathrm{Im}(I-Q)$, then restriction L_p of L to $\mathrm{Dom}L \cap \mathrm{Ker}P: (I-P)X \to \mathrm{Im}L$ is invertible.

Definition 5.2 [4] Denote the inverse of L_p by K_p. Supposing Ω is an open bounded subset of X, if $QN(\bar{\Omega})$ is bounded and $K_p(I-Q)N: \Omega \to X$ is compact, then the mapping N is called L-compact on $\bar{\Omega}$. Since $\mathrm{Im}Q$ is isomorphic to $\mathrm{Ker}L$, there exists an isomorphism $J: \mathrm{Im}Q \to \mathrm{Ker}L$.

Lemma 5.1 [4] *Let $\Omega \subset X$ be an open bounded set, L be a Fredholm mapping of index zero and N be L-compact on $\bar{\Omega}$. If the following three conditions hold*

(i): *$Lx \neq \lambda Nx$ for any $\lambda \in (0,1)$ and $x \in \partial\Omega \cap DomL$,*

(ii): *$QNx \neq 0$ for any $x \in \partial\Omega \cap KerL$,*

(iii): *$deg\{JQN, \Omega \cap KerL, 0\} \neq 0$,*

then $Lx = Nx$ has at least one solution in $\bar{\Omega} \cap DomL$.

The existence of positive periodic solution of model (4) with initial conditions (5) and (6) will be discussed in the following part. Consider the subsystem of model (4),

$$(56)\qquad \begin{cases} \dot{x}_{12}(t) = a_{11}(t-\tau_1)e^{-\int_{t-\tau_1}^{t} r_1(s)+a_{13}(s)y_2(s)\mathrm{d}s}x_{12}(t-\tau_1) - a_{12}(t)x_{12}(t)x_{22}(t) \\ \qquad\qquad -\beta_1(t)x_{12}^2(t), \\ \dot{x}_{22}(t) = a_{22}(t-\tau_2)e^{-\int_{t-\tau_2}^{t} r_2(s)+a_{23}(s)y_2(s)\mathrm{d}s}x_{22}(t-\tau_2) - a_{21}(t)x_{12}(t)x_{22}(t) \\ \qquad\qquad -\beta_2(t)x_{22}^2(t), \\ \dot{y}_2(t) = [a_{31}(t-\tau_3)x_{11}(t-\tau_3) + a_{32}(t-\tau_3)x_{21}(t-\tau_3)]e^{-\int_{t-\tau_3}^{t} r_3(s)\mathrm{d}s}y_2(t-\tau_3) \\ \qquad\qquad -\beta_3(t)y_2^2(t). \end{cases}$$

Let $u_1(t) = \ln[x_{12}(t)], u_2(t) = \ln[x_{22}(t)], u_3(t) = \ln[y_2(t)]$. By substituting $u_1(t)$, $u_2(t)$ and $u_3(t)$ into (56), it can be obtained that

(57)
$$\begin{cases} \dot{u}_1(t) = a_{11}(t-\tau_1)e^{-\int_{t-\tau_1}^{t} r_1(s)+a_{13}(s)e^{u_3(s)}\mathrm{d}s}e^{u_1(t-\tau_1)-u_1(t)} - a_{12}(t)e^{u_2(t)} \\ \qquad\qquad -\beta_1(t)e^{u_1(t)}, \\ \dot{u}_2(t) = a_{22}(t-\tau_2)e^{-\int_{t-\tau_2}^{t} r_2(s)+a_{23}(s)e^{u_3(s)}\mathrm{d}s}e^{u_2(t-\tau_2)-u_2(t)} - a_{21}(t)e^{u_1(t)} \\ \qquad\qquad -\beta_2(t)e^{u_2(t)}, \\ \dot{u}_3(t) = a_{31}(t-\tau_3)e^{-\int_{t-\tau_3}^{t} r_3(s)\mathrm{d}s}e^{u_3(t-\tau_3)-u_3(t)}\int_{t-\tau_1-\tau_3}^{t-\tau_3} a_{11}(s)e^{\int_{t-\tau_3}^{s} r_1(m)+a_{13}(m)e^{u_3(m)}\mathrm{d}m}e^{u_1(s)}\mathrm{d}s \\ \qquad\qquad +a_{32}(t-\tau_3)e^{-\int_{t-\tau_3}^{t} r_3(s)\mathrm{d}s}e^{u_3(t-\tau_3)-u_3(t)}\int_{t-\tau_2-\tau_3}^{t-\tau_3} a_{22}(s)e^{\int_{t-\tau_3}^{s} r_2(m)+a_{23}(m)e^{u_3(m)}\mathrm{d}m}e^{u_2(s)}\mathrm{d}s \\ \qquad\qquad -\beta_3(t)e^{u_3(t)}. \end{cases}$$

It should be noted that if model (57) has an ω-periodic solution $(u_1^*(t), u_2^*(t), u_3^*(t))^T$, then $(x_{12}^*(t), x_{22}^*(t), y_2^*(t))^T = (e^{u_1^*(t)}, e^{u_2^*(t)}, e^{u_3^*(t)})^T$ is a positive ω-periodic solution of model (56). In order to utilize Lemma 5.1 in a straightforward manner, we define

$$X = Y = \{(u_1(t), u_2(t), u_3(t))^T \in C(\mathbb{R}, \mathbb{R}^3) : u_i(t+\omega) = u_i(t), i = 1,2,3\},$$

and $\| (u_1(t), u_2(t), u_3(t))^T \| = \max_{t\in[0,\omega]} |u_1(t)| + \max_{t\in[0,\omega]} |u_2(t)| + \max_{t\in[0,\omega]} |u_3(t)|$, where $|\cdot|$ denotes the Euclidean norm, it is easy to show that both X and Y are Banach spaces with the norm $\|\cdot\|$, then define $\mathrm{Dom}L \cap X \to X, L(u_1(t), u_2(t), u_3(t))^T = (\frac{\mathrm{d}u_1(t)}{\mathrm{d}t}, \frac{\mathrm{d}u_2(t)}{\mathrm{d}t}, \frac{\mathrm{d}u_3(t)}{\mathrm{d}t})^T$, where $\mathrm{Dom}L = \{(u_1(t), u_2(t), u_3(t))^T \in C(\mathbb{R}, \mathbb{R}^3)\}$, $N \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix}$ and

(58)

$$\begin{cases} f_1(t) = a_{11}(t-\tau_1)e^{-\int_{t-\tau_1}^t r_1(s)+a_{13}(s)e^{u_3(s)}\mathrm{d}s} e^{u_1(t-\tau_1)-u_1(t)} - a_{12}(t)e^{u_2(t)} - \beta_1(t)e^{u_1(t)}, \\ f_2(t) = a_{22}(t-\tau_2)e^{-\int_{t-\tau_2}^t r_2(s)+a_{23}(s)e^{u_3(s)}\mathrm{d}s} e^{u_2(t-\tau_2)-u_2(t)} - a_{21}(t)e^{u_1(t)} - \beta_2(t)e^{u_2(t)}, \\ f_3(t) = a_{31}(t-\tau_3)e^{-\int_{t-\tau_3}^t r_3(s)\mathrm{d}s} e^{u_3(t-\tau_3)-u_3(t)} \int_{t-\tau_1-\tau_3}^{t-\tau_3} a_{11}(s)e^{\int_{t-\tau_3}^s r_1(m)+a_{13}(m)e^{u_3(m)}\mathrm{d}m} e^{u_1(s)}\mathrm{d}s \\ \qquad +a_{32}(t-\tau_3)e^{-\int_{t-\tau_3}^t r_3(s)\mathrm{d}s} e^{u_3(t-\tau_3)-u_3(t)} \int_{t-\tau_2-\tau_3}^{t-\tau_3} a_{22}(s)e^{\int_{t-\tau_3}^s r_2(m)+a_{23}(m)e^{u_3(m)}\mathrm{d}m} e^{u_2(s)}\mathrm{d}s \\ \qquad -\beta_3(t)e^{u_3(t)}. \end{cases}$$

Furthermore, we define

$$P \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = Q \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\omega}\int_0^\omega u_1(t)\mathrm{d}t \\ \frac{1}{\omega}\int_0^\omega u_2(t)\mathrm{d}t \\ \frac{1}{\omega}\int_0^\omega u_3(t)\mathrm{d}t \end{bmatrix}, \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \in X = Y.$$

According to the above definitions, it is not difficult to verify that

$$\mathrm{Ker}L = \{x | x \in X, x = h, h \in \mathbb{R}^3\}, \mathrm{Im}L = \{y \in Y | \int_0^\omega y(t)\mathrm{d}t = 0\}$$

are closed in Y, $\mathrm{dimKer}L = \mathrm{codimIm}L = 3$, and both P and Q are continuous projectors such that $\mathrm{Im}P = \mathrm{Ker}L$ and $\mathrm{Ker}Q = \mathrm{Im}L = \mathrm{Im}(I-Q)$. Based on the above analysis, it can be obtained that L is a Fredholm mapping of index zero. Furthermore, the inverse $K_p : \mathrm{Im}L \to \mathrm{Dom}L \cap \mathrm{Ker}P$ of L_p exists and takes the following form $K_p(y) = \int_0^t y(s)\mathrm{d}s - \frac{1}{\omega}\int_0^\omega \int_0^t y(s)\mathrm{d}s\mathrm{d}t$. Hence, $QN : X \to Y$ and $K_p(I-Q)N : X \to X$ can be defined as follows, respectively,

$$QNx = \begin{bmatrix} \frac{1}{\omega}\int_0^\omega f_1(t)\mathrm{d}t, \\ \frac{1}{\omega}\int_0^\omega f_2(t)\mathrm{d}t, \\ \frac{1}{\omega}\int_0^\omega f_3(t)\mathrm{d}t \end{bmatrix},$$

$$K_p(I-Q)Nx = \int_0^t Nx(s)\mathrm{d}s - \frac{1}{\omega}\int_0^\omega \int_0^t Nx(s)\mathrm{d}s\mathrm{d}t - (\frac{t}{\omega} - \frac{1}{2})\int_0^\omega Nx(s)\mathrm{d}s.$$

It is easy to show that QN and $K_p(I-Q)N$ are continuous. In order to facilitate the proof based on Lemma 5.1, we also need to find an appropriate open and bounded subset Ω, which can be found by the following two steps:

Step 1: According to the operator equation $Lx = \lambda Nx$ for $\lambda \in (0,1)$, the upper and lower bound of $u_1(t)$, $u_2(t)$ and $u_3(t)$ will be estimated as follows:

$$\begin{cases} \frac{\mathrm{d}u_1(t)}{\mathrm{d}t} = \lambda f_1(t), \\ \frac{\mathrm{d}u_2(t)}{\mathrm{d}t} = \lambda f_2(t), \\ \frac{\mathrm{d}u_3(t)}{\mathrm{d}t} = \lambda f_3(t). \end{cases} \tag{59}$$

where $f_1(t)$, $f_2(t)$ and $f_3(t)$ have been defined in (58). Supposing that $(u_1(t),u_2(t),u_3(t))^T \in X$ is a solution of model (59) for some $\lambda \in (0,1)$. Based on definition $(u_1(t),u_2(t),u_3(t))^T \in X$, there exist $\xi_i,\eta_i \in [0,\omega]$ such that $u_i(\xi_i) = \min_{t\in[0,\omega]} u_i(t), u_i(\eta_i) = \max_{t\in[0,\omega]} u_i(t), i = 1,2,3$. Multiplying the first equation of model (59) by $e^{u_1(t)}$ and integrating it over $[0,\omega]$, it gives that

$$\begin{aligned} &\int_0^\omega a_{11}(t-\tau_1)e^{u_1(t-\tau_1)}e^{-\int_{t-\tau_1}^t r_1(s)+a_{13}(s)e^{u_3(s)}\mathrm{d}s}\mathrm{d}t \\ &= \int_0^\omega a_{12}(t)e^{u_1(t)+u_2(t)} + \beta_1(t)e^{2u_1(t)}\mathrm{d}t. \end{aligned} \tag{60}$$

It follows from (60) that

$$\beta_1^L \int_0^\omega e^{2u_1(t)}\mathrm{d}t \le \int_0^\omega a_{11}^M e^{-r_1^L\tau_1} e^{u_1(t)}\mathrm{d}t. \tag{61}$$

On the other hand, by using the inequality

$$(\int_0^\omega e^{u_1(t)}\mathrm{d}t)^2 \le \omega \int_0^\omega e^{2u_1(t)}\mathrm{d}t. \tag{62}$$

Based on (61) and (62), it can be obtained that $\beta_1^L(\int_0^\omega e^{u_1(t)}\mathrm{d}t)^2 \le a_{11}^M \omega e^{-r_1^L\tau_1} \int_0^\omega e^{u_1(t)}\mathrm{d}t$, which derives that

$$\int_0^\omega e^{u_1(t)}\mathrm{d}t \le \frac{a_{11}^M \omega e^{-r_1^L\tau_1}}{\beta_1^L}, u_1(\xi_1) \le \ln \frac{a_{11}^M e^{-r_1^L\tau_1}}{\beta_1^L}. \tag{63}$$

By multiplying the second equation of model (59) by $e^{u_2(t)}$ and integrating it over $[0,\omega]$, it gives that

$$\begin{aligned} &\int_0^\omega a_{22}(t-\tau_1)e^{u_2(t-\tau_2)}e^{-\int_{t-\tau_2}^t r_2(s)+a_{23}(s)e^{u_3(s)}\mathrm{d}s}\mathrm{d}t \\ &= \int_0^\omega a_{21}(t)e^{u_1(t)+u_2(t)} + \beta_1(t)e^{2u_2(t)}\mathrm{d}t. \end{aligned} \tag{64}$$

It follows from (64) that

$$\beta_2^L \int_0^\omega e^{2u_2(t)}\mathrm{d}t \leq \int_0^\omega a_{22}^M e^{-r_2^L\tau_2} e^{u_2(t)}\mathrm{d}t. \tag{65}$$

On the other hand, by using the inequality

$$(\int_0^\omega e^{u_2(t)}\mathrm{d}t)^2 \leq \omega \int_0^\omega e^{2u_2(t)}\mathrm{d}t. \tag{66}$$

Based on (65) and (66), it can be obtained that $\beta_2^L(\int_0^\omega e^{u_2(t)}\mathrm{d}t)^2 \leq a_{22}^M \omega e^{-r_2^L\tau_2}\int_0^\omega e^{u_2(t)}\mathrm{d}t$, which derives that

$$\int_0^\omega e^{u_2(t)}\mathrm{d}t \leq \frac{a_{22}^M \omega e^{-r_2^L\tau_2}}{\beta_2^L}, u_2(\xi_2) \leq \ln\frac{a_{22}^M e^{-r_2^L\tau_2}}{\beta_2^L}. \tag{67}$$

It follows from the first equation of model (59), (63) and (67) that

$$\begin{aligned}\int_0^\omega |u_1'(t)|\mathrm{d}t &< \int_0^\omega a_{11}(t-\tau_1)e^{u_1(t-\tau_1)-u_1(t)}e^{-\int_{t-\tau_1}^t r_1(s)+a_{13}(s)e^{u_3(s)}\mathrm{d}s}\mathrm{d}t \\ &\quad + \int_0^\omega a_{12}(t)e^{u_2(t)} + \beta_1(t)e^{u_1(t)}\mathrm{d}t \\ &\leq 2\omega(\frac{a_{11}^M\beta_1^M e^{-r_1^L\tau_1}}{\beta_1^L} + \frac{a_{12}^M a_{22}^M e^{-r_2^L\tau_2}}{\beta_2^L}).\end{aligned} \tag{68}$$

According to (63) and (68), it can be obtained that

$$\begin{aligned}u_1(t) &\leq u_1(\xi_1) + \int_0^\omega |u_1'(t)|\mathrm{d}t \\ &\leq \ln\frac{a_{11}^M e^{-r_1^L\tau_1}}{\beta_1^L} + 2\omega(\frac{a_{11}^M\beta_1^M e^{-r_1^L\tau_1}}{\beta_1^L} + \frac{a_{12}^M a_{22}^M e^{-r_2^L\tau_2}}{\beta_2^L}) := A_1.\end{aligned} \tag{69}$$

It follows from the second equation of model (59), (63) and (67) that

$$\begin{aligned}\int_0^\omega |u_2'(t)|\mathrm{d}t &< \int_0^\omega a_{22}(t-\tau_1)e^{u_2(t-\tau_2)-u_2(t)}e^{-\int_{t-\tau_2}^t r_2(s)+a_{23}(s)e^{u_3(s)}\mathrm{d}s}\mathrm{d}t \\ &\quad + \int_0^\omega a_{21}(t)e^{u_1(t)} + \beta_2(t)e^{u_2(t)}\mathrm{d}t \\ &\leq 2\omega(\frac{a_{22}^M\beta_2^M e^{-r_2^L\tau_2}}{\beta_2^L} + \frac{a_{11}^M a_{21}^M e^{-r_1^L\tau_1}}{\beta_1^L}).\end{aligned} \tag{70}$$

According to (67) and (70), it can be obtained that

$$\begin{aligned}u_2(t) &\leq u_2(\xi_2) + \int_0^\omega |u_2'(t)|\mathrm{d}t \\ &\leq \ln\frac{a_{22}^M e^{-r_2^L\tau_2}}{\beta_2^L} + 2\omega(\frac{a_{22}^M\beta_2^M e^{-r_2^L\tau_2}}{\beta_2^L} + \frac{a_{11}^M a_{21}^M e^{-r_1^L\tau_1}}{\beta_1^L}) := A_2.\end{aligned} \tag{71}$$

By multiplying the third equation of model (59) by $e^{u_3(t)}$ and integrating it over $[0,\omega]$, it gives that

$$\begin{aligned}&\int_0^\omega a_{31}(t-\tau_3)\mathrm{d}t\int_{t-\tau_1-\tau_3}^{t-\tau_3} a_{11}(s)e^{u_1(s)+u_3(t-\tau_3)}e^{\int_{t-\tau_3}^{s} r_1(m)+a_{13}(m)e^{u_3(m)}\mathrm{d}m-\int_{t-\tau_3}^{t} r_3(m)\mathrm{d}m}\mathrm{d}s\\ &+\int_0^\omega a_{32}(t-\tau_3)\mathrm{d}t\int_{t-\tau_2-\tau_3}^{t-\tau_3} a_{22}(s)e^{u_2(s)+u_3(t-\tau_3)}e^{\int_{t-\tau_3}^{s} r_2(m)+a_{23}(m)e^{u_3(m)}\mathrm{d}m-\int_{t-\tau_3}^{t} r_3(m)\mathrm{d}m}\mathrm{d}s\\ &=\int_0^\omega \beta_3(t)e^{2u_3(t)}\mathrm{d}t. \qquad (72)\end{aligned}$$

It follows from (72) that

$$\begin{aligned}\beta_3^L\int_0^\omega e^{2u_3(t)}\mathrm{d}t \;\; \leq \;\; & \frac{a_{31}^M(a_{11}^M)^2e^{-r_1^L\tau_1-r_3^L\tau_3}e^{\frac{2a_{11}^M\beta_1^M\omega e^{-r_1^L\tau_1}}{\beta_1^L}}}{\beta_1^Lr_1^L}\int_0^\omega e^{u_3(t)}\mathrm{d}t\\ &+\frac{a_{32}^M(a_{22}^M)^2e^{-r_2^L\tau_2-r_3^L\tau_3}e^{\frac{2a_{22}^M\beta_2^M\omega e^{-r_2^L\tau_2}}{\beta_1^L}}}{\beta_2^Lr_2^L}\int_0^\omega e^{u_3(t)}\mathrm{d}t. \qquad (73)\end{aligned}$$

On the other hand, by using the inequality

$$(\int_0^\omega e^{u_3(t)}\mathrm{d}t)^2\leq\omega\int_0^\omega e^{2u_3(t)}\mathrm{d}t. \qquad (74)$$

Based on (73) and (74), it can be obtained that

$$\begin{aligned}\beta_3^L(\int_0^\omega e^{u_3(t)}\mathrm{d}t)^2 \;\; \leq \;\; & \frac{a_{31}^M(a_{11}^M)^2\omega e^{-r_1^L\tau_1-r_3^L\tau_3}e^{\frac{2a_{11}^M\beta_1^M\omega e^{-r_1^L\tau_1}}{\beta_1^L}}}{\beta_1^Lr_1^L}\int_0^\omega e^{u_3(t)}\mathrm{d}t\\ &+\frac{a_{32}^M(a_{22}^M)^2\omega e^{-r_2^L\tau_2-r_3^L\tau_3}e^{\frac{2a_{22}^M\beta_2^M\omega e^{-r_2^L\tau_2}}{\beta_1^L}}}{\beta_2^Lr_2^L}\int_0^\omega e^{u_3(t)}\mathrm{d}t,\end{aligned}$$

which derives that

$$u_3(\xi_3)\leq\ln[\frac{a_{31}^M(a_{11}^M)^2e^{-r_1^L\tau_1-r_3^L\tau_3}e^{\frac{2a_{11}^M\beta_1^Me^{-r_1^L\tau_1}}{\beta_1^L}}}{\beta_1^L\beta_3^Lr_1^L}+\frac{a_{32}^M(a_{22}^M)^2e^{-r_2^L\tau_2-r_3^L\tau_3}e^{\frac{2a_{22}^M\beta_2^Me^{-r_2^L\tau_2}}{\beta_1^L}}}{\beta_2^L\beta_3^Lr_2^L}]. \qquad (75)$$

It follows from the third equation of model (59) and (75) that

$$\begin{aligned}
&\int_0^\omega |u_3'(t)|\mathrm{d}t \\
< \ & \int_0^\omega a_{31}(t-\tau_3)\mathrm{d}t \int_{t-\tau_1-\tau_3}^{t-\tau_3} a_{11}(s)e^{u_1(s)+u_3(t-\tau_3)-u_3(t)} e^{\int_{t-\tau_3}^{s} r_1(m)+a_{13}(m)e^{u_3(m)}\mathrm{d}m-\int_{t-\tau_3}^{t} r_3(m)\mathrm{d}m}\mathrm{d}s \\
&+\int_0^\omega a_{32}(t-\tau_3)\mathrm{d}t \int_{t-\tau_2-\tau_3}^{t-\tau_3} a_{22}(s)e^{u_2(s)+u_3(t-\tau_3)-u_3(t)} e^{\int_{t-\tau_3}^{s} r_2(m)+a_{23}(m)e^{u_3(m)}\mathrm{d}m-\int_{t-\tau_3}^{t} r_3(m)\mathrm{d}m}\mathrm{d}s \\
&+\int_0^\omega \beta_3(t)e^{u_3(t)}\mathrm{d}t \\
\leq \ & 2\omega\beta_3^M\Big[\frac{a_{31}^M(a_{11}^M)^2e^{-r_1^L\tau_1-r_3^L\tau_3}}{\beta_1^L\beta_3^L r_1^L}e^{\frac{2a_{11}^M\beta_1^M\omega e^{-r_1^L\tau_1}}{\beta_1^L}}+\frac{a_{32}^M(a_{22}^M)^2e^{-r_2^L\tau_2-r_3^L\tau_3}}{\beta_2^L\beta_3^L r_2^L}e^{\frac{2a_{22}^M\beta_2^M\omega e^{-r_2^L\tau_2}}{\beta_1^L}}\Big].
\end{aligned}$$

(76)

According to (75) and (76), it can be obtained that

$$\begin{aligned}
(77)\quad & u_3(t) \\
\leq \ & u_3(\xi_3)+\int_0^\omega |u_3'(t)|\mathrm{d}t \\
\leq \ & \ln\Big[\frac{a_{31}^M(a_{11}^M)^2e^{-r_1^L\tau_1-r_3^L\tau_3}}{\beta_1^L\beta_3^L r_1^L}e^{\frac{2a_{11}^M\beta_1^M\omega e^{-r_1^L\tau_1}}{\beta_1^L}}+\frac{a_{32}^M(a_{22}^M)^2e^{-r_2^L\tau_2-r_3^L\tau_3}}{\beta_2^L\beta_3^L r_2^L}e^{\frac{2a_{22}^M\beta_2^M\omega e^{-r_2^L\tau_2}}{\beta_1^L}}\Big] \\
&+2\omega\beta_3^M\Big[\frac{a_{31}^M(a_{11}^M)^2e^{-r_1^L\tau_1-r_3^L\tau_3}}{\beta_1^L\beta_3^L r_1^L}e^{\frac{2a_{11}^M\beta_1^M\omega e^{-r_1^L\tau_1}}{\beta_1^L}}+\frac{a_{32}^M(a_{22}^M)^2e^{-r_2^L\tau_2-r_3^L\tau_3}}{\beta_2^L\beta_3^L r_2^L}e^{\frac{2a_{22}^M\beta_2^M\omega e^{-r_2^L\tau_2}}{\beta_1^L}}\Big]:=A_3.
\end{aligned}$$

(78)

It should be noted that

$$\int_0^\omega a_{11}(t-\tau_1)e^{-\int_{t-\tau_1}^{t} r_1(s)+a_{13}(s)e^{u_3(s)}\mathrm{d}s}e^{u_1(t-\tau_1)}\mathrm{d}t=\int_0^\omega a_{11}(t)e^{-\int_{t}^{t+\tau_1} r_1(s)+a_{13}(s)e^{u_3(s)}\mathrm{d}s}e^{u_1(t)}\mathrm{d}t.$$

Based on (60), it can be obtained that

$$\int_0^\omega \beta_1(t)e^{2u_1(t)}\mathrm{d}t \geq (a_{11}^L e^{-(r_1^M+a_{13}^M e^{A_3})\tau_1}-a_{12}^M e^{A_3})\int_0^\omega e^{u_1(t)}\mathrm{d}t,$$

which derives that

$$u_1(\eta_1)\geq \ln\frac{a_{11}^L e^{-(r_1^M+a_{13}^M e^{A_3})\tau_1}-a_{12}^M e^{A_3}}{\omega\beta_1^M} \tag{79}$$

holds provided that $a_{11}^L e^{-(r_1^M+a_{13}^M e^{A_3})\tau_1} > a_{12}^M e^{A_3}$. According to (68) and (78), it can be obtained that

$$\begin{aligned} u_1(t) &\geq u_1(\eta_1) - \int_0^\omega |u_1'(t)|\mathrm{d}t \\ &\geq \ln \frac{a_{11}^L e^{-(r_1^M+a_{13}^M e^{A_3})\tau_1} - a_{12}^M e^{A_3}}{\omega\beta_1^M} \\ &\quad -2\omega\Big(\frac{a_{11}^M \beta_1^M e^{-r_1^L\tau_1}}{\beta_1^L} + \frac{a_{12}^M a_{22}^M e^{-r_2^L\tau_2}}{\beta_2^L}\Big) := B_1. \end{aligned} \tag{80}$$

By virtue of (69) and (79), if $a_{11}^L e^{-(r_1^M+a_{13}^M e^{A_3})\tau_1} > a_{12}^M e^{A_3}$, then

$$\max_{t\in[0,\omega]} |u_1(t)| < \max \left\{ \begin{array}{l} |\ln \frac{a_{11}^M e^{-r_1^L\tau_1}}{\beta_1^L}| \\ +2\omega(\frac{a_{11}^M \beta_1^M e^{-r_1^L\tau_1}}{\beta_1^L} + \frac{a_{12}^M a_{22}^M e^{-r_2^L\tau_2}}{\beta_2^L}), \\ |\ln \frac{a_{11}^L e^{-(r_1^M+a_{13}^M e^{A_3})\tau_1} - a_{12}^M e^{A_3}}{\omega\beta_1^M}| \\ +2\omega(\frac{a_{11}^M \beta_1^M e^{-r_1^L\tau_1}}{\beta_1^L} + \frac{a_{12}^M a_{22}^M e^{-r_2^L\tau_2}}{\beta_2^L}) \end{array} \right\} := C_1. \tag{81}$$

Similarly, it is easy to show that

$$\int_0^\omega a_{22}(t-\tau_2) e^{-\int_{t-\tau_2}^t r_2(s)+a_{23}(s)e^{u_3(s)}\mathrm{d}s} e^{u_2(t-\tau_2)}\mathrm{d}t = \int_0^\omega a_{22}(t) e^{-\int_t^{t+\tau_2} r_2(s)+a_{23}(s)e^{u_3(s)}\mathrm{d}s} e^{u_2(t)}\mathrm{d}t.$$

Based on (64), it can be obtained that $\int_0^\omega \beta_2(t) e^{2u_2(t)}\mathrm{d}t \geq (a_{22}^L e^{-(r_2^M+a_{23}^M e^{A_3})\tau_2} - a_{21}^M e^{A_3}) \int_0^\omega e^{u_2(t)}\mathrm{d}t$, which derives that

$$u_2(\eta_2) \geq \ln \frac{a_{22}^L e^{-(r_2^M+a_{23}^M e^{A_3})\tau_2} - a_{21}^M e^{A_3}}{\omega\beta_2^M}, \tag{82}$$

holds provided that $a_{22}^L e^{-(r_2^M+a_{23}^M e^{A_3})\tau_2} > a_{21}^M e^{A_3}$. According to (70) and (81), it can be obtained that

$$\begin{aligned} u_2(t) &\geq u_2(\eta_2) - \int_0^\omega |u_2'(t)|\mathrm{d}t \\ &\geq \ln \frac{a_{22}^L e^{-(r_2^M+a_{23}^M e^{A_3})\tau_2} - a_{21}^M e^{A_3}}{\omega\beta_2^M} \\ &\quad -2\omega\Big(\frac{a_{22}^M \beta_2^M e^{-r_2^L\tau_2}}{\beta_2^L} + \frac{a_{21}^M a_{11}^M e^{-r_1^L\tau_1}}{\beta_1^L}\Big) := B_2. \end{aligned} \tag{83}$$

By virtue of (71) and (82), if $a_{22}^L e^{-(r_2^M+a_{23}^M e^{A_3})\tau_2} > a_{21}^M e^{A_3}$, then

$$\max_{t\in[0,\omega]}|u_2(t)| < \max\left\{\begin{array}{l} |\ln\frac{a_{22}^M e^{-r_2^L\tau_2}}{\beta_2^L}| \\ +2\omega(\frac{a_{22}^M\beta_2^M e^{-r_2^L\tau_2}}{\beta_2^L}+\frac{a_{21}^M a_{11}^M e^{-r_1^L\tau_1}}{\beta_1^L}), \\ |\ln\frac{a_{22}^L e^{-(r_2^M+a_{23}^M e^{A_3})\tau_2}-a_{21}^M e^{A_3}}{\omega\beta_2^M}| \\ +2\omega(\frac{a_{22}^M\beta_2^M e^{-r_2^L\tau_2}}{\beta_2^L}+\frac{a_{21}^M a_{11}^M e^{-r_1^L\tau_1}}{\beta_1^L}) \end{array}\right\} := C_2. \tag{84}$$

Furthermore, it is easy to show that

$$\begin{aligned}&\int_0^\omega a_{31}(t-\tau_3)\mathrm{d}t\int_{t-\tau_1-\tau_3}^{t-\tau_3} a_{11}(s)e^{u_1(s)+u_3(t-\tau_3)}e^{\int_{t-\tau_3}^s r_1(m)+a_{13}(m)e^{u_3(m)}\mathrm{d}m-\int_{t-\tau_3}^t r_3(m)\mathrm{d}m}\mathrm{d}s\\ &=\int_0^\omega a_{31}(t)\mathrm{d}t\int_{t-\tau_1}^{t} a_{11}(s)e^{u_1(s)+u_3(t)}e^{\int_t^s r_1(m)+a_{13}(m)e^{u_3(m)}\mathrm{d}m-\int_t^{t+\tau_3} r_3(m)\mathrm{d}m}\mathrm{d}s,\end{aligned}$$

$$\begin{aligned}&\int_0^\omega a_{32}(t-\tau_3)\mathrm{d}t\int_{t-\tau_2-\tau_3}^{t-\tau_3} a_{22}(s)e^{u_2(s)+u_3(t-\tau_3)}e^{\int_{t-\tau_3}^s r_2(m)+a_{23}(m)e^{u_3(m)}\mathrm{d}m-\int_{t-\tau_3}^t r_3(m)\mathrm{d}m}\mathrm{d}s\\ &=\int_0^\omega a_{32}(t)\mathrm{d}t\int_{t-\tau_2}^{t} a_{22}(s)e^{u_2(s)+u_3(t)}e^{\int_t^s r_2(m)+a_{23}(m)e^{u_3(m)}\mathrm{d}m-\int_t^{t+\tau_3} r_3(m)\mathrm{d}m}\mathrm{d}s.\end{aligned}$$

Based on (72), it can be obtained that

$$\int_0^\omega \beta_3(t)e^{2u_3(t)}\mathrm{d}t \geq [\frac{a_{11}^L a_{31}^L e^{B_1-r_3^M\tau_3}(1-e^{-r_1^L\tau_1})}{r_1^M+a_{13}^M e^{A_3}}+\frac{a_{22}^L a_{32}^L e^{B_2-r_3^M\tau_3}(1-e^{-r_2^L\tau_2})}{r_2^M+a_{23}^M e^{A_3}}]\int_0^\omega e^{u_3(t)}\mathrm{d}t,$$

which yields that

$$u_3(\eta_3) \geq \ln[\frac{a_{11}^L a_{31}^L e^{B_1-r_3^M\tau_3}(1-e^{-r_1^L\tau_1})}{\omega\beta_3^M(r_1^M+a_{13}^M e^{A_3})}+\frac{a_{22}^L a_{32}^L e^{B_2-r_3^M\tau_3}(1-e^{-r_2^L\tau_2})}{\omega\beta_3^M(r_2^M+a_{23}^M e^{A_3})}]. \tag{85}$$

According to (76) and (84), it derives that

$$\begin{aligned} (86)\quad & u_3(t)\\ \geq\ & u_3(\eta_3)-\int_0^\omega |u_3'(t)|\mathrm{d}t\\ \geq\ & \ln[\frac{a_{11}^L a_{31}^L e^{B_1-r_3^M\tau_3}(1-e^{-r_1^L\tau_1})}{\beta_3^M(r_1^M+a_{13}^M e^{A_3})}+\frac{a_{22}^L a_{32}^L e^{B_2-r_3^M\tau_3}(1-e^{-r_2^L\tau_2})}{\beta_3^M(r_2^M+a_{23}^M e^{A_3})}]\\ & -2\omega\beta_3^M[\frac{a_{31}^M(a_{11}^M)^2e^{-r_1^L\tau_1-r_3^L\tau_3}}{\beta_1^L\beta_3^L r_1^L}e^{\frac{2a_{11}^M\beta_1^M\omega e^{-r_1^L\tau_1}}{\beta_1^L}}+\frac{a_{32}^M(a_{22}^M)^2e^{-r_2^L\tau_2-r_3^L\tau_3}}{\beta_2^L\beta_3^L r_2^L}e^{\frac{2a_{22}^M\beta_2^M\omega e^{-r_2^L\tau_2}}{\beta_1^L}}] := B_3.\end{aligned}$$

(87)

By virtue of (77) and (85), it follows that

$$
\begin{aligned}
&\max_{t\in[0,\omega]}|u_3(t)| \\
&< \max\left\{\begin{array}{l}
|\ln[\frac{a_{31}^M(a_{11}^M)^2e^{-r_1^L\tau_1-r_3^L\tau_3}}{\beta_1^L\beta_3^Lr_1^L}e^{\frac{2a_{11}^M\beta_1^M\omega e^{-r_1^L\tau_1}}{\beta_1^L}}+\frac{a_{32}^M(a_{22}^M)^2e^{-r_2^L\tau_2-r_3^L\tau_3}}{\beta_2^L\beta_3^Lr_2^L}e^{\frac{2a_{22}^M\beta_2^M\omega e^{-r_2^L\tau_2}}{\beta_1^L}}]| \\
+2\omega\beta_3^M[\frac{a_{31}^M(a_{11}^M)^2e^{-r_1^L\tau_1-r_3^L\tau_3}}{\beta_1^L\beta_3^Lr_1^L}e^{\frac{2a_{11}^M\beta_1^M\omega e^{-r_1^L\tau_1}}{\beta_1^L}}+\frac{a_{32}^M(a_{22}^M)^2e^{-r_2^L\tau_2-r_3^L\tau_3}}{\beta_2^L\beta_3^Lr_2^L}e^{\frac{2a_{22}^M\beta_2^M\omega e^{-r_2^L\tau_2}}{\beta_1^L}}], \\
|\ln[\frac{a_{11}^La_{31}^Le^{B_1-r_3^M\tau_3}(1-e^{-r_1^L\tau_1})}{\omega\beta_3^M(r_1^M+a_{13}^Me^{A_3})}+\frac{a_{22}^La_{32}^Le^{B_2-r_3^M\tau_3}(1-e^{-r_2^L\tau_2})}{\omega\beta_3^M(r_2^M+a_{23}^Me^{A_3})}]| \\
+2\omega\beta_3^M[\frac{a_{31}^M(a_{11}^M)^2e^{-r_1^L\tau_1-r_3^L\tau_3}}{\beta_1^L\beta_3^Lr_1^L}e^{\frac{2a_{11}^M\beta_1^M\omega e^{-r_1^L\tau_1}}{\beta_1^L}}+\frac{a_{32}^M(a_{22}^M)^2e^{-r_2^L\tau_2-r_3^L\tau_3}}{\beta_2^L\beta_3^Lr_2^L}e^{\frac{2a_{22}^M\beta_2^M\omega e^{-r_2^L\tau_2}}{\beta_1^L}}]
\end{array}\right\} := C_3.
\end{aligned}
\tag{88}
$$

Step 2: It is obvious that C_i $(i=1,2,3)$ defined in (80), (83) and (86) are independent of λ. In order to construct an appropriate open and bounded subset Ω, denote $C=C_1+C_2+C_3+C_0$, where C_0 is sufficiently large such that the unique solution $(u_1^*,u_2^*,u_3^*)^T$ of the following algebraic equations

$$
\left\{\begin{array}{l}
\frac{1}{\omega}\int_0^\omega f_1(t)\mathrm{d}t=0, \\
\frac{1}{\omega}\int_0^\omega f_2(t)\mathrm{d}t=0, \\
\frac{1}{\omega}\int_0^\omega f_3(t)\mathrm{d}t=0.
\end{array}\right.
\tag{89}
$$

satisfies $\|\ (u_1^*,u_2^*,u_3^*)^T\ \|=|u_1^*|+|u_2^*|+|u_3^*|<C$, where $f_i(t)$ (i=1, 2, 3) have been defined in (58). Select $\Omega=\{(u_1(t),u_2(t),u_3(t))^T\in X:\|\ (u_1,u_2,u_3)^T\ \|<C\}$, which implies that condition (i) of Lemma 5.1 holds. When $(u_1(t),u_2(t),u_3(t))^T\in\partial\Omega\cap\mathrm{Ker}L=\partial\Omega\cap\mathbb{R}^3$, $(u_1,u_2,u_3)^T$ is a constant vector in $\mathbb{R}^3$ with $|u_1|+|u_2|+|u_3|=C$. Consequently, it can be concluded that

$$
QN\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix}=\begin{bmatrix}\frac{1}{\omega}\int_0^\omega f_1(t)\mathrm{d}t,\\ \frac{1}{\omega}\int_0^\omega f_2(t)\mathrm{d}t\\ \frac{1}{\omega}\int_0^\omega f_3(t)\mathrm{d}t\end{bmatrix}\neq\begin{bmatrix}0\\0\\0\end{bmatrix},
$$

which implies that condtion (ii) of Lemma 5.1 is satisfied. Take $J=I:\mathrm{Im}Q\to\mathrm{Ker}L$, $(u_1,u_2,u_3)^T\to(u_1,u_2,u_3)^T$. It follows from straightforward computation that

$$
\deg(JQN(u_1,u_2,u_3)^T,\Omega\cap\mathrm{Ker}L,(0,0,0)^T)=1,
$$

where (u_1^*, u_2^*, u_3^*) is the unique solution of model (59). Hence, the condition (iii) of Lemma 5.1 holds. Furthermore, it is easy to see that the set $\{K_p(I-Q)Nx | x \in \bar{\Omega}\}$ is equicontinuous and uniformly bounded. By using the Arzela-Ascoli theorem [4], it can be shown that $K_p(I-Q)N : \bar{\Omega} \to X$ is compact and N is L-compact. Consequently, all conditions (i)-(iii) of Lemma 5.1 hold for Ω. It follows from Lemma 5.1 that model (57) has at least one ω-periodic solution $(u_1^*(t), u_2^*(t), u_3^*(t))^T$, and model (56) has at least one ω-periodic solution $(x_{12}^*(t), x_{22}^*(t), y_2^*(t))^T = (e^{u_1^*(t)}, e^{u_2^*(t)}, e^{u_3^*(t)})^T$. Let $(x_{12}^*(t), x_{22}^*(t), y_2^*(t))^T$ be a positive ω-periodic solution of model (56), it follows from (17) and (21) that

$$x_{11}^*(t) = e^{-\int_0^t r_1(m)+a_{13}(m)y_2^*(m)\mathrm{d}m} \int_{t-\tau_1}^{t} a_{11}(s) e^{\int_0^s r_1(m)+a_{13}(m)y_2^*(m)\mathrm{d}m} x_{12}^*(s)\mathrm{d}s,$$

$$x_{21}^*(t) = e^{-\int_0^t r_1(m)+a_{23}(m)y_2^*(m)\mathrm{d}m} \int_{t-\tau_2}^{t} a_{22}(s) e^{\int_0^s r_2(m)+a_{23}(m)y_2^*(m)\mathrm{d}m} x_{22}(s)\mathrm{d}s,$$

are ω-periodic continuous function. Based on (25), it follows from further computation that

$$y_1^*(t) = e^{-\int_0^t r_3(m)\mathrm{d}m} \int_{t-\tau_3}^{t} [a_{31}(s)x_{11}^*(s) + a_{32}x_{21}^*(s)](s) e^{\int_0^s r_3(m)\mathrm{d}m} y_2^*(s)\mathrm{d}s,$$

are ω-periodic continuous function. Based on the above analysis, the existence of positive ω-periodic solution of model (4) is concluded in the following theorem.

Theorem 5.1 *If* $a_{11}^L e^{-(r_1^M + a_{13}^M e^{A_3})\tau_1} > a_{12}^M e^{A_3}$ *and* $a_{22}^L e^{-(r_2^M + a_{23}^M e^{A_3})\tau_2} > a_{21}^M e^{A_3}$ *hold, where* A_3 *has been defined in (77), then model (4) with initial conditions (5) and (6) has at least one positive* ω*-periodic solution.*

6. Permanence of solutions

By constructing appropriate Lyapunov functionals, sufficient conditions for global stability of the unique positive periodic solution are analyzed.

Theorem 6.1 *If* $\liminf_{t\to+\infty} E_i(t) > 0$, $i = 1,2,3$, *then model (4) with initial conditions (5) and (6) has a unique positive* ω*-periodic globally stable solution, where* $q > 0$ *is a constant and* m_i^*, M_i^*, $i = 1,2,3,4,5,6$ *are defined in (55),*

$$E_1 = 2\beta_1(t)m_2^* + (1+q)a_{12}(t)(m_2^* + m_4^*) - a_{11}(t)e^{-\int_t^{t+\tau_1} r_1(m)+a_{13}(m)m_6^* dm}$$
$$- qa_{11}(t)M_6^* \int_{t+\tau_3}^{t+\tau_1+\tau_3} a_{31}(s-\tau_3) e^{\int_{s-\tau_3}^{t} r_1(m)+a_{31}(m)M_6^* dm - \int_{s-\tau_3}^{s} r_3(m)dm} ds,$$

$$
\begin{aligned}
E_2 = {} & 2\beta_2(t)m_4^* + (1+q)a_{21}(t)(m_2^* + m_4^*) \\
& - a_{22}(t)e^{-\int_t^{t+\tau_2} r_2(m)+a_{23}(m)m_6^* dm} \\
& - qa_{22}(t)M_6^* \int_{t+\tau_3}^{t+\tau_2+\tau_3} a_{32}(s-\tau_3)e^{\int_{s-\tau_3}^{t} r_2(m)+a_{32}(m)M_6^* dm - \int_{s-\tau_3}^{s} r_3(m)dm} ds,
\end{aligned}
$$

and

$$
\begin{aligned}
E_3 = {} & 2q\beta_2(t)m_6^* - \int_t^{t+\tau_1} a_{11}(s-\tau_1)a_{13}(t)M_2^* ds \\
& - \int_t^{t+\tau_2} a_{22}(s-\tau_2)a_{23}(t)M_4^* ds \\
& - qa_{31}(t)e^{-\int_t^{t+\tau_3} r_3(m)dm} \int_{t-\tau_1}^{t} a_{11}(m)M_2^* e^{\int_t^m r_1(\theta)+a_{13}(\theta)M_6^* d\theta} dm \\
& - qa_{13}(t)\int_{t+\tau_3}^{t+\tau_1+\tau_3} a_{31}(s-\tau_3)M_6^* e^{-\int_{s-\tau_3}^{s} r_3(m)dm} \int_{s-\tau_1-\tau_3}^{s-\tau_3} a_{11}(m)M_2^* dm ds \\
& - qa_{32}(t)e^{-\int_t^{t+\tau_3} r_3(m)dm} \int_{t-\tau_2}^{t} a_{22}(m)M_4^* e^{\int_t^m r_2(\theta)+a_{23}(\theta)M_6^* d\theta} dm \\
& - qa_{23}(t)\int_{t+\tau_3}^{t+\tau_2+\tau_3} a_{32}(s-\tau_3)M_6^* e^{-\int_{s-\tau_3}^{s} r_3(m)dm} \int_{s-\tau_2-\tau_3}^{s-\tau_3} a_{22}(m)M_4^* dm ds.
\end{aligned}
$$

Proof. Constructing a Lyapunov functional as follows,

$$
\begin{aligned}
V_1(t) \quad = \quad & |x_{12}(t) - x_{12}^*(t)| + \int_t^{t+\tau_1}\int_{s-\tau_1}^{t} a_{11}(s-\tau_1)x_{12}^*(s-\tau_1)a_{13}(u)|y_2(u) - y_2^*(u)| \mathrm{d}u\mathrm{d}s \\
& + \int_{t-\tau_1}^{t} a_{11}(s)e^{\int_s^{s+\tau_1} -r_1(m)-a_{13}(m)y_2(m)\mathrm{d}m}|x_{12}(s) - x_{12}^*(s)|\mathrm{d}s \\
& + |x_{22}(t) - x_{22}^*(t)| + \int_t^{t+\tau_2}\int_{s-\tau_2}^{t} a_{22}(s-\tau_1)x_{22}^*(s-\tau_2)a_{23}(u)|y_2(u) - y_2^*(u)|\mathrm{d}u\mathrm{d}s \\
& + \int_{t-\tau_2}^{t} a_{22}(s)e^{\int_s^{s+\tau_2} -r_2(m)-a_{23}(m)y_2(m)\mathrm{d}m}|x_{22}(s) - x_{22}^*(s)|\mathrm{d}s. \qquad (90)
\end{aligned}
$$

By calculating the upper right derivative of $V_1(t)$ along the positive ω-periodic solutions of model (4), it can be obtained that

$$
\begin{aligned}
(91)\qquad & D^+V_1(t) \\
= \; & a_{11}(t-\tau_1)\mathrm{sgn}[x_{12}(t)-x_{12}^*(t)]\{e^{\int_{t-\tau_1}^{t}-r_1(s)-a_{13}(s)y_2(s)\mathrm{d}s}(x_{12}(t-\tau_1) \\
& -x_{12}^*(t-\tau_1)) \\
& +x_{12}^*(t-\tau_1)[e^{\int_{t-\tau_1}^{t}-r_1(s)-a_{23}(s)y_2(s)\mathrm{d}s}-e^{\int_{t-\tau_1}^{t}-r_1(s)-a_{23}(s)y_2^*(s)\mathrm{d}s}]\} \\
& -\mathrm{sgn}[x_{12}(t)-x_{12}^*(t)]\{a_{12}(t)x_{22}(t)(x_{12}(t)-x_{12}^*(t)) \\
& +a_{12}(t)x_{12}^*(t)(x_{22}(t)-x_{22}^*(t)) \\
& +\beta_1(t)(x_{12}(t)+x_{12}^*(t))(x_{12}(t)-x_{12}^*(t))\} \\
& +a_{22}(t-\tau_2)\mathrm{sgn}[x_{22}(t)-x_{22}^*(t)]\{e^{\int_{t-\tau_2}^{t}-r_2(s)-a_{23}(s)y_2(s)\mathrm{d}s}(x_{22}(t-\tau_2) \\
& -x_{22}^*(t-\tau_2)) \\
& +x_{12}^*(t-\tau_1)[e^{\int_{t-\tau_1}^{t}-r_1(s)-a_{23}(s)y_2(s)\mathrm{d}s}-e^{\int_{t-\tau_1}^{t}-r_1(s)-a_{23}(s)y_2^*(s)\mathrm{d}s}]\} \\
& -\mathrm{sgn}[x_{22}(t)-x_{22}^*(t)]\{a_{21}(t)x_{12}(t)(x_{22}(t)-x_{22}^*(t)) \\
& +a_{21}(t)x_{22}^*(t)(x_{12}(t)-x_{12}^*(t)) \\
& +\beta_2(t)(x_{22}(t)+x_{22}^*(t))(x_{22}(t)-x_{22}^*(t))\} \\
& +\int_t^{t+\tau_1} a_{11}(s-\tau_1)x_{11}^*(s-\tau_1)a_{13}(t)|y_2(t)-y_2^*(t)|\mathrm{d}s \\
& -\int_{t-\tau_1}^{t} a_{11}(t-\tau_1)x_{11}^*(t-\tau_1)a_{13}(u)|y_2(u)-y_2^*(u)|\mathrm{d}u \\
& +\int_t^{t+\tau_2} a_{22}(s-\tau_2)x_{22}^*(s-\tau_2)a_{23}(t)|y_2(t)-y_2^*(t)|\mathrm{d}s \\
& -\int_{t-\tau_2}^{t} a_{22}(t-\tau_2)x_{22}^*(t-\tau_2)a_{23}(u)|y_2(u)-y_2^*(u)|\mathrm{d}u \\
& +a_{11}(t)e^{\int_t^{t+\tau_1}-r_1(m)-a_{13}y_2(m)\mathrm{d}m}|x_{12}(t)-x_{12}^*(t)| \\
& +a_{22}(t)e^{\int_t^{t+\tau_2}-r_2(m)-a_{23}y_2(m)\mathrm{d}m}|x_{22}(t)-x_{22}^*(t)| \\
& -a_{11}(t-\tau_1)e^{\int_{t-\tau_1}^{t}-r_1(m)-a_{13}(m)y_2(m)\mathrm{d}m}|x_{12}(t-\tau_1)-x_{12}^*(t-\tau_1)| \\
(92)\qquad & -a_{22}(t-\tau_2)e^{\int_{t-\tau_2}^{t}-r_2(m)-a_{23}(m)y_2(m)\mathrm{d}m}|x_{22}(t-\tau_2)-x_{22}^*(t-\tau_2)|.
\end{aligned}
$$

Further computations show that

$$
\begin{aligned}
& D^+V_1(t) \\
\leq\ & a_{11}(t-\tau_1)x_{12}^*(t-\tau_1)|e^{\int_{t-\tau_1}^t -r_1(s)-a_{13}(s)y_2(s)\mathrm{d}s} - e^{\int_{t-\tau_1}^t -r_1(s)-a_{13}(s)y_2^*(s)\mathrm{d}s}| \\
& +a_{22}(t-\tau_2)x_{22}^*(t-\tau_2)|e^{\int_{t-\tau_2}^t -r_2(s)-a_{23}(s)y_2(s)\mathrm{d}s} - e^{\int_{t-\tau_2}^t -r_2(s)-a_{23}(s)y_2^*(s)\mathrm{d}s}| \\
& -\beta_1(t)(x_{12}(t)+x_{12}^*(t))|x_{12}(t)-x_{12}^*(t)| - \beta_2(t)(x_{22}(t)+x_{22}^*(t))|x_{22}(t)-x_{22}^*(t)| \\
& -a_{12}(t)(x_{22}(t)+x_{12}^*(t))|x_{12}(t)-x_{12}^*(t)| - a_{21}(t)(x_{12}(t)+x_{22}^*(t))|x_{22}(t)-x_{22}^*(t)| \\
& +\int_t^{t+\tau_1} a_{11}(s-\tau_1)x_{11}^*(s-\tau_1)a_{13}(t)|y_2(t)-y_2^*(t)|\mathrm{d}s \\
& -\int_{t-\tau_1}^t a_{11}(t-\tau_1)x_{11}^*(t-\tau_1)a_{13}(u)|y_2(u)-y_2^*(u)|\mathrm{d}u \\
& +\int_t^{t+\tau_2} a_{22}(s-\tau_2)x_{22}^*(s-\tau_2)a_{23}(t)|y_2(t)-y_2^*(t)|\mathrm{d}s \\
& -\int_{t-\tau_2}^t a_{22}(t-\tau_2)x_{22}^*(t-\tau_2)a_{23}(u)|y_2(u)-y_2^*(u)|\mathrm{d}u \\
& +a_{11}(t)e^{\int_t^{t+\tau_1} -r_1(m)-a_{13}y_2(m)\mathrm{d}m}|x_{12}(t)-x_{12}^*(t)| \\
& +a_{22}(t)e^{\int_t^{t+\tau_2} -r_2(m)-a_{23}y_2(m)\mathrm{d}m}|x_{22}(t)-x_{22}^*(t)|. \qquad (93)
\end{aligned}
$$

Since $|e^{-x}-e^{-y}| \leq |x-y|$ holds for arbitrary $x \geq 0$ and $y \geq 0$, it follows from (90) that

$$
\begin{aligned}
D^+V_1(t) \leq\ & -[\beta_1(t)(x_{12}(t)+x_{12}^*(t)) + a_{12}(t)(x_{22}(t)+x_{22}^*(t))]|x_{22}(t)-x_{12}^*(t)| \\
& -[\beta_2(t)(x_{22}(t)+x_{22}^*(t)) + a_{21}(t)(x_{12}(t)+x_{22}^*(t))]|x_{22}(t)-x_{22}^*(t)| \\
& +\int_t^{t+\tau_1} a_{11}(s-\tau_1)x_{12}^*(s-\tau_1)a_{13}(t)|y_2(t)-y_2^*(t)|\mathrm{d}s \\
& +\int_t^{t+\tau_2} a_{22}(s-\tau_2)x_{22}^*(s-\tau_2)a_{23}(t)|y_2(t)-y_2^*(t)|\mathrm{d}s \\
& +a_{11}(t)e^{\int_t^{t+\tau_1} -r_1(m)-a_{13}(m)y_2(m)\mathrm{d}m}|x_{12}(t)-x_{12}^*(t)| \\
& +a_{22}(t)e^{\int_t^{t+\tau_2} -r_2(m)-a_{23}(m)y_2(m)\mathrm{d}m}|x_{22}(t)-x_{22}^*(t)|. \qquad (94)
\end{aligned}
$$

Similarly, constructing another Lyapunov functional as follows,

$$
\begin{aligned}
V_2(t) \;=\; & \int_{t-\tau_3}^{t}\int_{s-\tau_1}^{s} a_{31}(s)e^{-\int_s^{s+\tau_3} r_3(m)\mathrm{d}m}|y_2(s)-y_2^*(s)| \\
& \times a_{11}(m)e^{\int_s^m r_1(\theta)+a_{31}(\theta)y_2(\theta)\mathrm{d}\theta}x_{12}(m)\mathrm{d}m\mathrm{d}s \\
& +\int_t^{t+\tau_1}\int_{s-\tau_1-\tau_3}^{t-\tau_3} a_{31}(s-\tau_3)e^{-\int_{s-\tau_3}^{s} r_3(m)\mathrm{d}m}y_2^*(s-\tau_3)a_{11}(u)|x_{12}(u)-x_{12}^*(u)| \\
& \quad\times e^{\int_{s-\tau_3}^{s} r_1(m)+a_{13}(m)y_2(m)\mathrm{d}m}\mathrm{d}u\mathrm{d}s \\
& +\int_t^{t+\tau_1}\int_{s-\tau_1-\tau_3}^{t-\tau_3} a_{31}(s-\tau_3)e^{-\int_{s-\tau_3}^{s} r_3(m)\mathrm{d}m}y_2^*(s-\tau_3)a_{13}(u)|y_2(u)-y_2^*(u)| \\
& \quad\times\int_{s-\tau_1-\tau_3}^{s-\tau_3} a_{11}(m)x_{12}^*(m)\mathrm{d}m\mathrm{d}u\mathrm{d}s \\
& +\int_t^{t+\tau_3}\int_u^{u+\tau_1} a_{11}(u-\tau_3)a_{31}(s-\tau_3)y_2^*(s-\tau_3)|x_{12}(u-\tau_3)-x_{12}^*(u-\tau_3)| \\
& \quad\times e^{-\int_{s-\tau_3}^{s} r_3(m)\mathrm{d}m}e^{\int_{s-\tau_3}^{u-\tau_s} r_1(\theta)+a_{13}(\theta)y_2(\theta)\mathrm{d}\theta}\mathrm{d}s\mathrm{d}u \\
& +\int_t^{t+\tau_3}\int_u^{u+\tau_1} a_{31}(s-\tau_3)y_2^*(s-\tau_3)a_{13}(u-\tau_3)|y_2(u-\tau_3)-y_2^*(u-\tau_3)| \\
& \quad\times e^{-\int_{s-\tau_3}^{s} r_3(m)\mathrm{d}m}\int_{s-\tau_1-\tau_3}^{s-\tau_3} a_{11}(m)x_{12}^*(m)\mathrm{d}m\mathrm{d}s\mathrm{d}u \\
& +\int_{t-\tau_3}^{t}\int_{s-\tau_2}^{s} a_{32}(s)e^{-\int_s^{s+\tau_3} r_3(m)\mathrm{d}m}|y_2(s)-y_2^*(s)|a_{22}(m)e^{\int_s^m r_2(\theta)+a_{32}(\theta)y_2(\theta)\mathrm{d}\theta} \\
& x_{22}(m)\mathrm{d}m\mathrm{d}s \\
& +\int_t^{t+\tau_2}\int_{s-\tau_2-\tau_3}^{t-\tau_3} a_{32}(s-\tau_3)e^{-\int_{s-\tau_3}^{s} r_3(m)\mathrm{d}m}y_2^*(s-\tau_3)a_{22}(u)|x_{22}(u)-x_{22}^*(u)| \\
& \quad\times e^{\int_{s-\tau_3}^{s} r_2(m)+a_{23}(m)y_2(m)\mathrm{d}m}\mathrm{d}u\mathrm{d}s \\
& +\int_t^{t+\tau_2}\int_{s-\tau_2-\tau_3}^{t-\tau_3} a_{32}(s-\tau_3)e^{-\int_{s-\tau_3}^{s} r_3(m)\mathrm{d}m}y_2^*(s-\tau_3)a_{23}(u)|y_2(u)-y_2^*(u)| \\
& \quad\times\int_{s-\tau_2-\tau_3}^{s-\tau_3} a_{22}(m)x_{22}^*(m)\mathrm{d}m\mathrm{d}u\mathrm{d}s \\
& +\int_t^{t+\tau_3}\int_u^{u+\tau_2} a_{22}(u-\tau_3)a_{32}(s-\tau_3)y_2^*(s-\tau_3)|x_{22}(u-\tau_3)-x_{22}^*(u-\tau_3)| \\
& \quad\times e^{-\int_{s-\tau_3}^{s} r_3(m)\mathrm{d}m}e^{\int_{s-\tau_3}^{u-\tau_s} r_2(\theta)+a_{23}(\theta)y_2(\theta)\mathrm{d}\theta}\mathrm{d}s\mathrm{d}u \\
& +\int_t^{t+\tau_3}\int_u^{u+\tau_2} a_{32}(s-\tau_3)y_2^*(s-\tau_3)a_{23}(u-\tau_3)|y_2(u-\tau_3)-y_2^*(u-\tau_3)| \\
& \quad\times e^{-\int_{s-\tau_3}^{s} r_3(m)\mathrm{d}m}\int_{s-\tau_2-\tau_3}^{s-\tau_3} a_{22}(m)x_{22}^*(m)\mathrm{d}m\mathrm{d}s\mathrm{d}u+|y_2(t)-y_2^*(t)|. \qquad (95)
\end{aligned}
$$

By calculating the upper right derivative of $V_2(t)$ along the positive ω-periodic solutions of model (4), it can be obtained that

$$
\begin{aligned}
D^+V_2(t) \leq\ & a_{31}(t)e^{-\int_t^{t+\tau_3} r_3(m)\mathrm{d}m}\int_{t-\tau_1}^{t} a_{11}(m)x_{12}(m)e^{\int_t^m r_1(\theta)+a_{13}(\theta)y_2(\theta)\mathrm{d}\theta}\mathrm{d}m \\
& \times|y_2(t)-y_2^*(t)| \\
& +\int_{t+\tau_3}^{t+\tau_1+\tau_3} a_{31}(s-\tau_3)e^{-\int_{s-\tau_3}^{s} r_3(m)\mathrm{d}m}y_2^*(s-\tau_3)e^{\int_{s-\tau_3}^{t} r_1(m)+a_{31}(m)y_2(m)\mathrm{d}m}\mathrm{d}s \\
& \quad\times a_{11}(t)|x_{12}(t)-x_{12}^*(t)| \\
& +\int_{t+\tau_3}^{t+\tau_1+\tau_3} a_{31}(s-\tau_3)y_2^*(s-\tau_3)e^{-\int_{s-\tau_3}^{s} r_3(m)\mathrm{d}m}\mathrm{d}s \\
& \quad\times\int_{s-\tau_1-\tau_3}^{s-\tau_3} a_{11}(m)a_{13}(t)x_{12}^*(m)\mathrm{d}m|y_2(t)-y_2^*(t)| \\
& +a_{32}(t)e^{-\int_t^{t+\tau_3} r_3(m)\mathrm{d}m}\int_{t-\tau_2}^{t} a_{22}(m)x_{22}(m)e^{\int_t^m r_2(\theta)+a_{23}(\theta)y_2(\theta)\mathrm{d}\theta}\mathrm{d}m \\
& |y_2(t)-y_2^*(t)| \\
& +\int_{t+\tau_3}^{t+\tau_2+\tau_3} a_{32}(s-\tau_3)e^{-\int_{s-\tau_3}^{s} r_3(m)\mathrm{d}m}y_2^*(s-\tau_3)e^{\int_{s-\tau_3}^{t} r_2(m)+a_{32}(m)y_2(m)\mathrm{d}m}\mathrm{d}s \\
& \quad\times a_{22}(t)|x_{22}(t)-x_{22}^*(t)| \\
& +\int_{t+\tau_3}^{t+\tau_2+\tau_3} a_{32}(s-\tau_3)y_2^*(s-\tau_3)e^{-\int_{s-\tau_3}^{s} r_3(m)\mathrm{d}m}\mathrm{d}s \\
& \quad\times\int_{s-\tau_2-\tau_3}^{s-\tau_3} a_{22}(m)a_{23}(t)x_{22}^*(m)\mathrm{d}m|y_2(t)-y_2^*(t)| \\
& -a_{12}(t)(x_{22}(t)+x_{12}^*(t))|x_{12}(t)-x_{12}^*(t)| \\
& -a_{21}(t)(x_{12}(t)+x_{22}^*(t))|x_{22}(t)-x_{22}^*(t)| \\
& -\beta_2(t)(y_2(t)+y_2^*(t))|y_2(t)-y_2^*(t)|. \qquad (96)
\end{aligned}
$$

Let $V(t)=V_1(t)+qV_2(t)$, where $q>0$ is a constant. By calculating the upper right derivative of $V(t)$ along the positive ω-periodic solution of model (4) based on (91) and (93), it can be obtained as follows:

$$
\begin{aligned}
D^+V(t) \leq\; & -|x_{12}(t)-x_{12}^*(t)|\{2\beta_1(t)m_2^*+(1+q)a_{12}(t)(m_2^*+m_4^*) \\
& -a_{11}(t)e^{\int_t^{t+\tau_1}-r_1(m)-a_{13}(m)m_6^*\mathrm{d}m} \\
& -qa_{11}(t)M_6^*\int_{t+\tau_3}^{t+\tau_1+\tau_3}a_{31}(s-\tau_3)e^{\int_{s-\tau_3}^{t}r_1(m)+a_{31}(m)M_6^*\mathrm{d}m-\int_{s-\tau_3}^{s}r_3(m)\mathrm{d}m}\mathrm{d}s\} \\
& -|x_{22}(t)-x_{22}^*(t)|\{2\beta_2(t)m_4^*+(1+q)a_{21}(t)(m_2^*+m_4^*) \\
& -a_{22}(t)e^{\int_t^{t+\tau_2}-r_2(m)-a_{23}(m)m_6^*\mathrm{d}m} \\
& -qa_{22}(t)M_6^*\int_{t+\tau_3}^{t+\tau_2+\tau_3}a_{32}(s-\tau_3)e^{\int_{s-\tau_3}^{t}r_2(m)+a_{32}(m)M_6^*\mathrm{d}m-\int_{s-\tau_3}^{s}r_3(m)\mathrm{d}m}\mathrm{d}s\} \\
& -|y_2(t)-y_2^*(t)|\{2q\beta_2(t)m_6^*-\int_t^{t+\tau_1}a_{11}(s-\tau_1)a_{13}(t)M_2^*\mathrm{d}s \\
& -\int_t^{t+\tau_2}a_{22}(s-\tau_2)a_{23}(t)M_4^*\mathrm{d}s \\
& -qa_{31}(t)e^{-\int_t^{t+\tau_3}r_3(m)\mathrm{d}m}\int_{t-\tau_1}^{t}a_{11}(m)M_2^*e^{\int_t^m r_1(\theta)+a_{13}(\theta)M_6^*\mathrm{d}\theta}\mathrm{d}m \\
& -q\int_{t+\tau_3}^{t+\tau_1+\tau_3}a_{31}(s-\tau_3)M_6^*e^{-\int_{s-\tau_3}^{s}r_3(m)\mathrm{d}m}\int_{s-\tau_1-\tau_3}^{s-\tau_3}a_{11}(m)a_{13}(t)M_2^*\mathrm{d}m\mathrm{d}s \\
& -qa_{32}(t)e^{-\int_t^{t+\tau_3}r_3(m)\mathrm{d}m}\int_{t-\tau_2}^{t}a_{22}(m)M_4^*e^{\int_t^m r_2(\theta)+a_{23}(\theta)M_6^*\mathrm{d}\theta}\mathrm{d}m \\
& -q\int_{t+\tau_3}^{t+\tau_2+\tau_3}a_{32}(s-\tau_3)M_6^*e^{-\int_{s-\tau_3}^{s}r_3(m)\mathrm{d}m}\int_{s-\tau_2-\tau_3}^{s-\tau_3}a_{22}(m)a_{23}(t)M_4^*\mathrm{d}m\mathrm{d}s\}.
\end{aligned}\tag{97}
$$

According to Theorem 4.1, there exists a positive value $T>0$, when $t \geq T$ it gives that

$$
\begin{cases}
m_2^*-\varepsilon<x_{12}(t)<M_2^*+\varepsilon, m_2^*-\varepsilon<x_{12}^*(t)<M_2^*+\varepsilon, \\
m_4^*-\varepsilon<x_{22}(t)<M_4^*+\varepsilon, m_4^*-\varepsilon<x_{22}^*(t)<M_4^*+\varepsilon, \\
m_6^*-\varepsilon<y_2(t)<M_6^*+\varepsilon, m_6^*-\varepsilon<y_2^*(t)<M_6^*+\varepsilon,
\end{cases}
$$

holds for sufficiently small $\varepsilon>0$. Based on (94), when $t>T+\max\{\tau_1,\tau_2,\tau_3\}$, it derives that

$$
D^+V(t) \leq -(E_1(t)-\varepsilon)|x_{12}(t)-x_{12}^*(t)|-(E_2(t)-\varepsilon)|x_{22}(t)-x_{22}^*(t)|-(E_3(t)-\varepsilon)|y_2(t)-y_2^*(t)|,
$$

where $E_1(t)$, $E_2(t)$ and $E_3(t)$ have been given in Theorem 6.1. If $\liminf_{t\to+\infty}E_i(t)>0$ for $i=1,2,3$, then there exists three constants $\delta_i>0$ $(i=1,2,3)$ such that for $t \geq T^* := T+2\max\{\tau_1,\tau_2,\tau_3\}$

$$
E_1(t) \geq \delta_1, E_2(t) \geq \delta_2, E_3(t) \geq \delta_3.
$$

Consequently, for $t \geq T^*$ we have

$$D^+V(t) \leq -\frac{\delta_1}{2}|x_{12}(t)-x_{12}^*(t)| - \frac{\delta_2}{2}|x_{22}(t)-x_{22}^*(t)| - \frac{\delta_3}{2}|y_2(t)-y_2^*(t)|. \tag{98}$$

By integrating both sides of (95) on the interval $[T^*,t]$, it can be obtained that for $t \geq T^*$,

$$V(t)+\frac{\delta_1}{2}\int_{T^*}^t |x_{12}(s)-x_{12}^*(s)|\mathrm{d}s+\frac{\delta_2}{2}\int_{T^*}^t |x_{22}(s)-x_{22}^*(s)|\mathrm{d}s+\frac{\delta_3}{2}\int_{T^*}^t |y_2(s)-y_2^*(s)|\mathrm{d}s \leq V(T^*).$$

Hence, $V(t)$ is bounded on the interval $[T^*,+\infty)$ and

$$\int_{T^*}^t |x_{12}(s)-x_{12}^*(s)|\mathrm{d}s < +\infty,$$
$$\int_{T^*}^t |x_{22}(s)-x_{22}^*(s)|\mathrm{d}s < +\infty,$$
$$\int_{T^*}^t |y_2(s)-y_2^*(s)|\mathrm{d}s < +\infty.$$

According to Barbalat's Lemma [4], it can be concluded that

$$\lim_{t\to\infty}|x_{12}(t)-x_{12}^*(t)| = 0, \tag{99}$$

$$\lim_{t\to\infty}|x_{22}(t)-x_{22}^*(t)| = 0, \tag{100}$$

$$\lim_{t\to\infty}|y_2(t)-y_2^*(t)| = 0. \tag{101}$$

It follows from (17) and (21) that

$$\begin{aligned}
|x_{11}(t)-x_{11}^*(t)| &\leq \int_{t-\tau_1}^t a_{11}(s)e^{\int_t^s r_1(m)+a_{13}(m)y_2(m)\mathrm{d}m}|x_{12}(s)-x_{12}^*(s)|\mathrm{d}s \\
&\quad + \int_{t-\tau_1}^t a_{11}(s)x_{12}^*(s)|e^{\int_t^s r_1(m)+a_{13}y_2(m)\mathrm{d}m} - e^{\int_t^s r_1(m)+a_{13}y_2^*(m)\mathrm{d}m}|\mathrm{d}s \\
&\leq \int_{t-\tau_1}^t a_{11}^M|x_{12}(s)-x_{12}^*(s)|\mathrm{d}s + \int_{t-\tau_1}^t a_{11}^M M_2 \int_s^t a_{13}^M|y_2(m)-y_2^*(m)|\mathrm{d}m\mathrm{d}s,
\end{aligned} \tag{102}$$

$$\begin{aligned}
|x_{21}(t)-x_{21}^*(t)| &\leq \int_{t-\tau_2}^t a_{22}(s)e^{\int_t^s r_2(m)+a_{23}(m)y_2(m)\mathrm{d}m}|x_{22}(s)-x_{22}^*(s)|\mathrm{d}s \\
&\quad + \int_{t-\tau_2}^t a_{22}(s)x_{22}^*(s)|e^{\int_t^s r_2(m)+a_{23}y_2(m)\mathrm{d}m} - e^{\int_t^s r_2(m)+a_{23}y_2^*(m)\mathrm{d}m}|\mathrm{d}s \\
&\leq \int_{t-\tau_2}^t a_{22}^M|x_{22}(s)-x_{22}^*(s)|\mathrm{d}s + \int_{t-\tau_2}^t a_{22}^M M_4 \int_s^t a_{23}^M|y_2(m)-y_2^*(m)|\mathrm{d}m\mathrm{d}s.
\end{aligned} \tag{103}$$

Based on (96), (97), (99) and (100), it can be concluded that

$$\lim_{t\to\infty}|x_{11}(t)-x_{11}^*(t)|=0,\lim_{t\to\infty}|x_{21}(t)-x_{21}^*(t)|=0. \tag{104}$$

It follows from (25) that

$$\begin{aligned}|y_1(t)-y_1^*(t)| &\le \int_{t-\tau_3}^t [a_{31}(s)x_{11}(s)+a_{32}(s)x_{21}(s)]e^{\int_t^s r_3(m)\mathrm{d}m}|y_2(t)-y_2^*(s)|\mathrm{d}s \\ &\le \int_{t-\tau_3}^t (a_{31}^M M_1+a_{32}^M M_3)|y_2(t)-y_2^*(s)|\mathrm{d}s.\end{aligned} \tag{105}$$

Based on (98) and (102), it can be obtained that

$$\lim_{t\to\infty}|y_1(t)-y_1^*(t)|=0. \tag{106}$$

Therefore, it follows from (96), (97), (98), (101) and (103) that model (4) with initial conditions (5) and (6) has a unique positive ω-periodic globally stable solution □

Numerical simulations are carried out to substantiate the analytical findings obtained in this paper. In order to facilitate the numerical simulations, 2π-periodic continuous functions introduced in model (4) are selected as follows: $a_{11}(t)=2.1+\frac{\sin(t)}{10}$, $r_1(t)=0.2+\frac{\sin(t)}{200}$, $a_{13}(t)=\frac{1}{30}+\frac{\sin(t)}{20}$, $a_{12}(t)=0.2+\frac{\sin(t)}{300}$, $\beta_1(t)=1+\frac{\sin(t)}{300}$, $a_{22}=1.1+\frac{\sin(t)}{18}$, $r_2(t)=0.01+\frac{\sin(t)}{50}$, $a_{23}(t)=\frac{1}{50}+\frac{\sin(t)}{15}$, $a_{21}(t)=0.15+\frac{\sin(t)}{270}$, $a_{31}(t)=\frac{3}{100}+\frac{9\sin(t)}{200}$, $a_{32}(t)=\frac{1577}{10000}+\frac{1577\sin(t)}{30000}$, $r_3(t)=0.032+\frac{\sin(t)}{180}$ and $\beta_3(t)=0.3+\frac{\sin(t)}{60}$.

The maturation delay for sub-dominant prey, dominant prey species and predator species are given as follows: $\tau_1=0.1$, $\tau_2=0.12$ and $\tau_3=0.25$, respectively. By using straightforward computations, it can be found that $a_{11}^L\beta_2^L e^{-(r_1^M+a_{13}^M\bar{W})\tau_1}>a_{12}^M a_{22}^M e^{-r_2^L\tau_2}$ and $a_{22}^L\beta_1^L e^{-(r_2^M+a_{23}^M\bar{W})\tau_2}>a_{21}^M a_{11}^M e^{-r_1^L\tau_1}$ hold, then solutions of model (4) are persistent. It follows from complicated computation that $a_{11}^L e^{-(r_1^M+a_{13}^M e^{A_3})\tau_1}>a_{12}^M e^{A_3}$ and $a_{22}^L e^{-(r_2^M+a_{23}^M e^{A_3})\tau_2}>a_{21}^M e^{A_3}$, which derives that model (4) has at least one positive ω-periodic solution based on Theorem 5.1. Further computations show that $E_1(t)\ge 0.3428$, $E_2(t)\ge 0.7006$ and $E_3(t)\ge 1.2951$. Consequently, it follows from Theorem 6.1 that model (4) has a unique positive 2π-periodic globally stable solution $(x_{11}^*(t),\ x_{12}^*(t),\ x_{21}^*(t),\ x_{22}^*(t),\ y_1^*(t),\ y_2^*(t))^T$, whose dynamical responses are plotted in Figure 1. Corresponding limit cycle of the unique positive 2π-periodic globally stable solution $(x_{11}^*(t),x_{12}^*(t),x_{21}^*(t),x_{22}^*(t),y_1^*(t),y_2^*(t))^T$ is plotted in the $x_{11}-x_{12}$ plane, $x_{21}-x_{22}$ and y_1-y_2 plane, which can be found in Figure 2, Figure 3 and Figure 4, respectively.

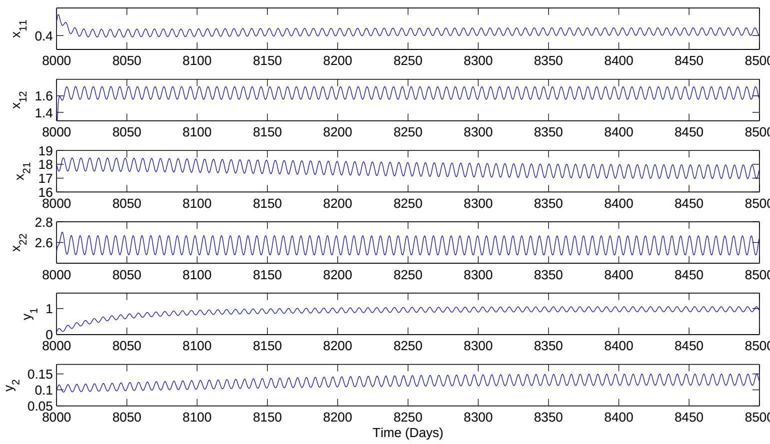

FIGURE 1. Dynamical responses of the unique positive 2π-periodic globally stable solution of model (4), and the stable solution are plotted with initial value $(0.15, 1.52, 12.1, 2.5, 0.05, 0.1)$ from Time 8000 to 8500, where parameters of model (4) are given as follows: $a_{11}(t) = 2.1 + \frac{\sin(t)}{10}$, $r_1(t) = 0.2 + \frac{\sin(t)}{200}$, $a_{13}(t) = \frac{1}{30} + \frac{\sin(t)}{20}$, $a_{12}(t) = 0.2 + \frac{\sin(t)}{300}$, $\beta_1(t) = 1 + \frac{\sin(t)}{300}$, $a_{22} = 1.1 + \frac{\sin(t)}{18}$, $r_2(t) = 0.01 + \frac{\sin(t)}{50}$, $a_{23}(t) = \frac{1}{50} + \frac{\sin(t)}{15}$, $a_{21}(t) = 0.15 + \frac{\sin(t)}{270}$, $a_{31}(t) = \frac{3}{100} + \frac{9\sin(t)}{200}$, $a_{32}(t) = \frac{1577}{10000} + \frac{1577\sin(t)}{30000}$, $r_3(t) = 0.032 + \frac{\sin(t)}{180}$ and $\beta_3(t) = 0.3 + \frac{\sin(t)}{60}$.

7. Conclusion

Generally speaking, it takes some time for species to reach maturity and the species compete each other for the limited life resource within closed environment, but this competition only happens among the mature individual and does not involve the immature individual. When one species is a better competitor, interspecific competition negatively influences the other species by reducing population sizes and/or growth rates, which in turn affects population dynamics of the competitor [7, 10]. Consequently, it is necessary to investigate the dynamic effect of interspecific competition and maturation delay on population dynamics of two competing prey and predator system, which are important issues from mathematical and experimental points of view. In this paper, a nonautonomous dynamical model is proposed, where interspecific

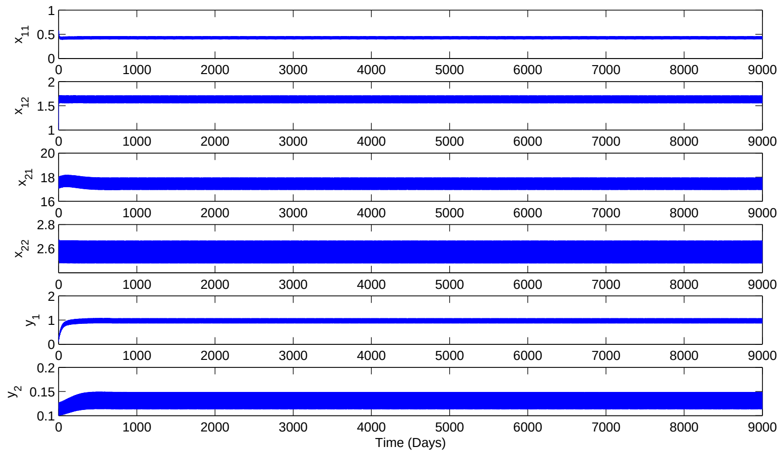

FIGURE 2. Dynamical responses of the unique positive 2π-periodic globally stable solution of model (4), and the stable solution are plotted with initial value $(0.15, 1.52, 12.1, 2.5, 0.05, 0.1)$ from Time 0 to 9000, where parameters of model (4) are given as follows: $a_{11}(t) = 2.1 + \frac{\sin(t)}{10}$, $r_1(t) = 0.2 + \frac{\sin(t)}{200}$, $a_{13}(t) = \frac{1}{30} + \frac{\sin(t)}{20}$, $a_{12}(t) = 0.2 + \frac{\sin(t)}{300}$, $\beta_1(t) = 1 + \frac{\sin(t)}{300}$, $a_{22} = 1.1 + \frac{\sin(t)}{18}$, $r_2(t) = 0.01 + \frac{\sin(t)}{50}$, $a_{23}(t) = \frac{1}{50} + \frac{\sin(t)}{15}$, $a_{21}(t) = 0.15 + \frac{\sin(t)}{270}$, $a_{31}(t) = \frac{3}{100} + \frac{9\sin(t)}{200}$, $a_{32}(t) = \frac{1577}{10000} + \frac{1577\sin(t)}{30000}$, $r_3(t) = 0.032 + \frac{\sin(t)}{180}$ and $\beta_3(t) = 0.3 + \frac{\sin(t)}{60}$.

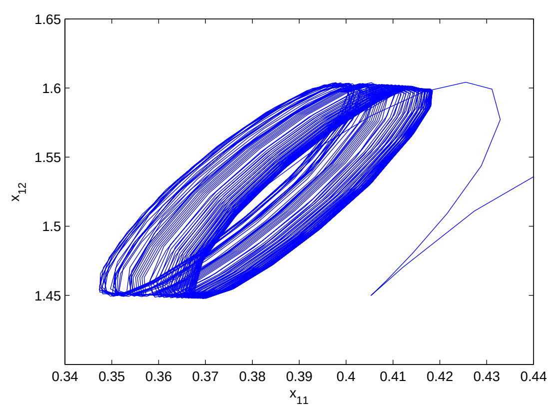

FIGURE 3. A limit cycle corresponding to dynamical responses of model (4) shown in Figure 1, which is plotted in the $x_{11} - x_{12}$ plane.

competition between mature dominant prey species and sub-dominant prey species are considered, and three discrete time delays are incorporated into the model due to maturation time for sub-dominant prey, dominant prey and predator species, respectively.

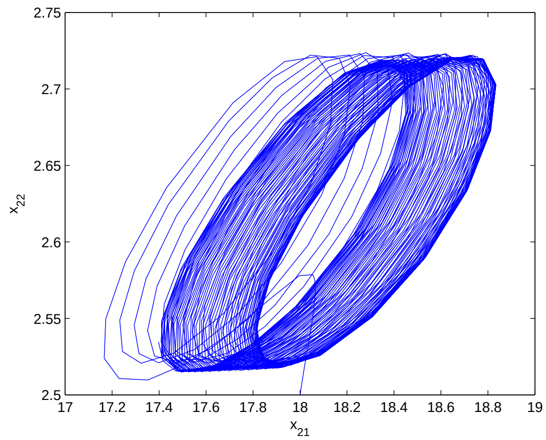

FIGURE 4. A limit cycle corresponding to dynamical responses of model (4) shown in Figure 1, which is plotted in the $x_{21}-x_{22}$ plane.

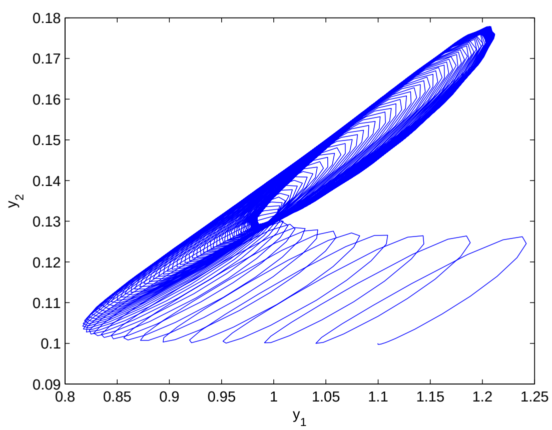

FIGURE 5. A limit cycle corresponding to dynamical responses of model (4) shown in Figure 1, which is plotted in the y_1-y_2 plane.

Qualitative analyses of the proposed model are discussed in this paper. It follows from Theorem 3.1 and Theorem 3.2 that solutions of model (4) with given initial conditions are positive and ultimately bounded. By utilizing some comparison arguments, an iterative technique is proposed to discuss permanence of solutions of model (4), which can be found in Theorem 4.1. Furthermore, existence of positive periodic solutions is considered in Theorem 5.1 based on continuation theorem of coincidence degree theory, which shows that model (4) has at least one positive periodic solution. By constructing some appropriate Lyapunov functionals, sufficient conditions for global stability of the unique positive periodic solution are analyzed, i.e.,

$\liminf_{t\to+\infty} E_i(t) > 0, i = 1,2,3$, which can be found in Theorem 6.1. Finally, numerical simulations are provided to show dynamical responses of the unique positive 2π-periodic globally stable solution, which are plotted in Figure 1. Furthermore, the phase portrait of model system (4), corresponding limit cycle is plotted in the $x_{11} - x_{12}$, $x_{21} - x_{22}$ and $y_1 - y_2$ plane, which can be found Figure 2, Figure 3 and Figure 4, respectively. Since biological phenomenon associated with interspecific competition and maturation delay extensively exists within prey predator ecosystem in the natural world, theoretical results obtained in this paper are theoretically beneficial to discuss dynamic effect of maturation delay and interspecific competition on population dynamics as well as interaction and coexistence mechanism of two competing prey and one predator system, it makes this work made in this paper has some positive and new features.

Conflict of Interests

The authors declare that there is no conflict of interests.

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