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PERMANENCE FOR A DISCRETE COMPETITIVE SYSTEM WITH FEEDBACK CONTROLS

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Abstract. A nonautonomous discrete competitive system with nonlinear inter-inhibition terms and feedback controls is studied in this paper. By using difference inequality theory, a set of conditions which guarantee the permanence of system is obtained. The results indicate that feedback control variables have no influence on the persistent property of the system. Our results not only supplement but also improve some existing ones. Numerical simulations show the feasibility of our results.

Keywords: Discrete; Permanence; Competitive; Feedback Controls.

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1. Introduction

For any bounded sequence $\{a(n)\}$, $a^L = \inf_{n\in N}\{a(n)\}$, $a^U = \sup_{n\in N}\{a(n)\}$. Recently, many authors pay attention to the following competitive system with nonlinear inter-inhibition terms (see [1-5]):

$$(1)\qquad \begin{cases} \dot{x_1}(t) &= x_1(t)\Big\{r_1(t) - a_1(t)x_1(t) - \dfrac{c_2(t)x_2(t)}{1+x_2(t)}\Big\}, \\ \dot{x_2}(t) &= x_2(t)\Big\{r_2(t) - a_2(t)x_2(t) - \dfrac{c_1(t)x_1(t)}{1+x_1(t)}\Big\}, \end{cases}$$

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where x_i $(i=1,2)$ are the population densities of two competing species; r_i $(i=1,2)$ are the intrinsic growth rates of species; a_i $(i=1,2)$ are the rates of intraspecific competition of the first and second species, respectively; c_i $(i=1,2)$ are the rates of interspecific competition of the first and second species, respectively. For more ecological sense of model (1), one can see [1] and the references cited therein. By using differential inequality, the module containment theorem and the Lyapunov function, the existence and global asymptotic stability of positive almost periodic solutions of system (1) is obtained by Wang *et al.* [2].

As we all know that continuous models can excellently show the dynamic behaviors of those populations who have a long life cycle, overlapping generations, and large quantity; Also, the discrete-time models governed by difference equations are more appropriate than the continuous ones when populations have a short life expectancy, nonoverlapping generations in the real word. Considering discrete-time models can provide efficient computational models of continuous models for numerical simulations, Qin *et al.* [3] study the following system which is the discrete analogue of system (1):

$$(2)\qquad \begin{cases} x_1(n+1) &= x_1(n)\exp\Big\{r_1(n)-a_1(n)x_1(n)-\dfrac{c_2(n)x_2(n)}{1+x_2(n)}\Big\}, \\ x_2(n+1) &= x_2(n)\exp\Big\{r_2(n)-a_2(n)x_2(n)-\dfrac{c_1(n)x_1(n)}{1+x_1(n)}\Big\}, \end{cases}$$

they investigated the permanence and global asymptotic stability of positive periodic solutions of system (2). When all coefficients in system (2) are bounded nonnegative almost periodic sequences, Wang and Liu [4] further investigate the existence, uniqueness and uniformly asymptotic stability of positive almost periodic solution of the above almost periodic system. Qin *et al.* [3] obtained the following result about permanence of system (2).

Theorem A (see [3]). *Suppose that system* (2) *satisfies the following assumptions:*

$$r_1^L - c_2^U > 0,\ \ r_2^L - c_1^U > 0. \qquad (A_1)$$

Then system (2) *is permanent i.e. any positive solution* $(x_1(n),x_2(n))^T$ *of system* (2) *satisfies*

$$0 < x_{i*} \le \liminf_{n\to+\infty} x_i(n) \le \limsup_{n\to+\infty} x_i(n) \le x_i^* < +\infty.$$

Noting that ecosystems in the real world are often disturbed by outside continuous forces, Wang et al. [5] incorporate feedback controls into model (2) and consider the following model:

$$
(3)\qquad \begin{cases} x_1(n+1) &= x_1(n)\exp\Big\{r_1(n)-a_1(n)x_1(n)-\dfrac{c_2(n)x_2(n)}{1+x_2(n)}-e_1(n)u_1(n)\Big\},\\ x_2(n+1) &= x_2(n)\exp\Big\{r_2(n)-a_2(n)x_2(n)-\dfrac{c_1(n)x_1(n)}{1+x_1(n)}-e_2(n)u_2(n)\Big\},\\ \Delta u_1(n) &= -b_1(n)u_1(n)+d_1(n)x_1(n),\ \Delta u_2(n)=-b_2(n)u_2(n)+d_2(n)x_2(n), \end{cases}
$$

where $x_i(n)$ stand for the densities of species x_i $(i=1,2)$ at the nth generation, respectively, for $i=1,2$, $\{a_i(n)\}$, $\{b_i(n)\}$, $\{c_i(n)\}$, $\{d_i(n)\}$, $\{e_i(n)\}$ and $\{r_i(n)\}$ are all bounded nonnegative sequences such that

$$
(4)\qquad \begin{array}{c} 0<a_i^L\le a_i(n)\le a_i^U,\quad 0<c_i^L\le c_i(n)\le c_i^U,\quad 0<d_i^L\le d_i(n)\le d_i^U,\\ 0<e_i^L\le e(n)\le e_i^U,\quad 0<r_i^L\le r_i(n)\le r_i^U,\quad 0<b_i^L\le b_i(n)\le b_i^U\le 1. \end{array}
$$

By using Lyapunov function and some preliminary lemmas, the existence and uniformly asymptotic stability of unique positive almost periodic solution of the system (3) are investigated by Wang *et al.* [5]. More specifically, as for permanence, Wang *et al.* [5] obtained the following result.

Theorem B (see [5]). *If the following inequalities*

$$
r_1^L-c_2^U-e_1^Uu_1^*>0,\ \ r_2^L-c_1^U-e_2^Uu_2^*>0 \qquad (A_2)
$$

hold, then system (3) *is permanent i.e. any positive solution* $(x_1(n),x_2(n),u_1(n),u_2(n))^T$ *of system* (3) *satisfies*

$$
\begin{aligned} 0\le x_{i*}\le \liminf_{n\to+\infty}x_i(n)\le \limsup_{n\to+\infty}x_i(n)\le x_i^*<+\infty,\\ 0\le u_{i*}\le \liminf_{n\to+\infty}u_i(n)\le \limsup_{n\to+\infty}u_i(n)\le u_i^*<+\infty, \end{aligned}
$$

where $x_i^*=\dfrac{exp(r_i^U-1)}{a_i^L}$ *and* $u_i^*=\dfrac{x_i^*d_i^U}{b_i^L}$, *for* $i=1,2$.

Comparing with Theorem A, Theorem B shows that feedback control variables play important roles on the persistent property of the system (3). But the question is whether or not the feedback control variables have influence on the permanence of the system. On the other hand, as was pointed out by Fan and Wang [6], "if we use the method of comparison theorem, then the additional condition, in some extent, is necessary. But for the system itself, this condition may not necessary." In [6] , by establishing a new difference inequality, Fan and Wang showed that feedback control has no influence on the permanence of a single species discrete model. Their success motivated us to consider the persistent property of system (3). Indeed, in this paper, we will apply the analysis technique of Fan and Wang [6] to establish sufficient conditions, which is independent of feedback control variables, to ensure the permanence of the system. We finally obtain the following main results:

Theorem C . *Assume that*

$$r_1^L - c_2^U > 0,\ \ r_2^L - c_1^U > 0 \tag{A_3}$$

hold, then system (3) *is permanent.*

Comparing with Theorem B, it is easy to see that (A_3) in Theorem C are weaker than (A_2) in Theorem B and feedback control variables have no influence on the permanent property of system (3), so our results improve the main results in [5]. For more works on this direction, one could refer to [7-18] and the references cited therein.

By the biological meaning, we will focus our discussion on the positive solutions of system (3). So, we consider (3) together with the following initial conditions:

$$x_i(0) > 0,\ \ u_i(0) > 0,\ \ i = 1,2. \tag{5}$$

It is not difficult to see that the solutions of (3)-(5) are well defined and satisfy

$$x_i(n) > 0,\ \ u_i(n) > 0,\ \ i = 1,2,\ \text{for } n \in N. \tag{6}$$

The remaining part of this paper is organized as follows. In Section 2, we will introduce several lemmas. The permanence of system (3) is then studied in Section 3. In Section 4, a suitable example together with its numerical simulations shows the feasibility of our results.

2. Preliminaries

In this section, we will introduce several useful lemmas.

Lemma 2.1 (see [19]). *Assume that* $\{x(n)\}$ *satisfies*

$$x(n+1) \geq x(n) exp\{a(n) - b(n)x(n)\},\ n \geq N_0,$$

$\limsup_{n\to+\infty} x(n) \le x^*$ *and* $x(N_0) > 0$, *where* $a(n)$ *and* $b(n)$ *are non-negative sequences bounded above and below by positive constants and* $N_0 \in N$. *Then*

$$\liminf_{n\to+\infty} x(n) \ge \min\{\frac{a^L}{b^U} exp\{a^L - b^U x^*\}, \frac{a^L}{b^U}\}.$$

Lemma 2.2 (see [6]). *Assume that* $A > 0$ *and* $y(0) > 0$. *Suppose that*

$$y(n+1) \le Ay(n) + B(n),\ n = 1, 2, ...$$

Then for any integer $k \le n$,

$$y(n) \le A^k y(n-k) + \sum_{i=0}^{k-1} A^i B(n-i-1).$$

Especially, if $A < 1$ *and* B *is bounded above with respect to* M, *then*

$$\limsup_{n\to+\infty} y(n) \le \frac{M}{1-A}.$$

Lemma 2.3 (see [6]). *Assume that* $A > 0$ *and* $y(0) > 0$. *Suppose that*

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Then for any integer $k \le n$,

$$y(n) \ge A^k y(n-k) + \sum_{i=0}^{k-1} A^i B(n-i-1).$$

Especially, if $A < 1$ *and* B *is bounded above with respect to* m^*, *then*

$$\liminf_{n\to+\infty} y(n) \ge \frac{m^*}{1-A}.$$

3. Permanence

In this section, we detail the proof of our main result by several lemmas.

Lemma 3.1 (see [5]**).** *Any positive solution* $(x_1(n), x_2(n), u_1(n), u_2(n))^T$ *of system (3) satisfies*

(7) $$\limsup_{n\to+\infty} x_i(n) \le x_i^* \quad \limsup_{n\to+\infty} u_i(n) \le u_i^*,$$

where x_i^* and u_i^* $(i=1,2)$ are defined in Theorem B.

Lemma 3.2 *Assume*

$$r_1^L - c_2^U > 0 \tag{A_{31}}$$

holds, then there exist two positive constants x_{1*} and u_{1*} such that

$$\liminf_{n\to+\infty} x_1(n) \geq x_{1*},\ \liminf_{n\to+\infty} u_1(n) \geq u_{1*},$$

where x_{1*} and u_{1*} are defined in the proof.

Proof. According to Lemma 3.1, for any $\varepsilon > 0$ small enough, there exists enough large $N_1 > 0$, such that for $n \geq N_1$,

$$x_1(n) \leq x_1^* + \varepsilon,\ u_1(n) \leq u_1^* + \varepsilon. \tag{8}$$

Thus, it follows from (8) and the first equation of system (3) that

$$\begin{aligned} x_1(n+1) &\geq x_1(n)\exp\Big\{r_1^L - a_1^U(x_1^*+\varepsilon) - c_2^U - e_1^U(u_1^*+\varepsilon)\Big\} \\ &\geq x_1(n)\exp\Big\{-a_1^U(x_1^*+\varepsilon) - c_2^U - e_1^U(u_1^*+\varepsilon)\Big\} \\ &\stackrel{\triangle}{=} x_1(n)\exp\{D_{1\varepsilon}\} \end{aligned} \tag{9}$$

for $n \geq N_1$, where $D_{1\varepsilon} = -a_1^U(x_1^*+\varepsilon) - c_2^U - e_1^U(u_1^*+\varepsilon) < 0$. For any integer $k \leq n$, it follows from (9) that

$$\prod_{j=n-k}^{n-1} \frac{x_1(j+1)}{x_1(j)} \geq \prod_{j=n-k}^{n-1} \exp\{D_{1\varepsilon}\} = \exp\{D_{1\varepsilon}k\}.$$

Thus

$$x_1(n-k) \leq x_1(n)\exp\{-D_{1\varepsilon}k\} \tag{10}$$

From the third equation of system (3), we have

$$\begin{aligned} u_1(n+1) &= (1-b_1(n))u_1(n) + d_1(n)x_1(n) \\ &\leq (1-b_1^L)u_1(n) + d_1^U x_1(n) \\ &\stackrel{\triangle}{=} A_1u_1(n) + B_1(n), \end{aligned} \tag{11}$$

where $A_1 = 1 - b_1^L$ and $B_1(n) = d_1^U x_1(n)$. Then, for any $k \le n$, according to Lemma 2.2, (10) and (11) that

$$
\begin{aligned}
u_1(n) &\le A_1^k u_1(n-k) + \sum_{i=0}^{k-1} A_1^i B_1(n-i-1) \\
&= A_1^k u_1(n-k) + \sum_{i=0}^{k-1} A_1^i d_1^U x_1(n-i-1) \\
&\le A_1^k u_1(n-k) + d_1^U x_1(n) \sum_{i=0}^{k-1} A_1^i \exp\{-D_{1\varepsilon}(i+1)\}.
\end{aligned} \tag{12}
$$

Note that $0 < b_1^L < 1$, hence $0 < A_1 < 1$. Therefore,

$$
0 \le A_1^k u_1(n-k) \le A_1^k (u_1^* + \varepsilon) \to 0, \text{ as } k \to \infty. \tag{13}
$$

Then, there exists a positive integer $N_2 > N_1$ such that for any positive solution of system (3), $e_1^U A_1^{N_2}(u_1^* + \varepsilon) < \frac{1}{2}\left(r_1^L - c_2^U\right)$ for all $n \ge N_2$. In fact, we could choose $N_2 = \max\{1, \frac{\ln P_1}{\ln A_1} + 1\}$, where $P_1 = \frac{r_1^L - c_2^U}{2e_1^U(u_1^* + \varepsilon)}$. Fix N_2, for $n \ge N_1 + N_2$, we get

$$
\begin{aligned}
u_1(n) &\le A_1^{N_2} u_1(n-N_2) + d_1^U x_1(n) \sum_{i=0}^{N_2-1} A_1^i \exp\{-D_{1\varepsilon}(i+1)\} \\
&\le A_1^{N_2}(u_1^* + \varepsilon) + d_1^U x_1(n) \sum_{i=0}^{N_2-1} A_1^i \exp\{-D_{1\varepsilon}(i+1)\} \\
&\triangleq A_1^{N_2}(u_1^* + \varepsilon) + G_{1\varepsilon} x_1(n),
\end{aligned} \tag{14}
$$

where $G_{1\varepsilon} = d_1^U \sum_{i=0}^{N_2-1} A_1^i \exp\{-D_{1\varepsilon}(i+1)\}$. Substituting (14) into the first equation of system (3), we can get

$$
\begin{aligned}
x_1(n+1) &\ge x_1(n)\exp\Big\{r_1^L - a_1^U x_1(n) - c_2^U - e_1^U u_1(n)\Big\} \\
&\ge x_1(n)\exp\Big\{r_1^L - a_1^U x_1(n) - c_2^U - e_1^U\left(A_1^{N_2}(u_1^* + \varepsilon) + G_{1\varepsilon} x_1(n)\right)\Big\} \\
&= x_1(n)\exp\Big\{r_1^L - c_2^U - e_1^U A_1^{N_2}(u_1^* + \varepsilon) - \left(a_1^U + e_1^U G_{1\varepsilon}\right) x_1(n)\Big\} \\
&\ge x_1(n)\exp\Big\{\frac{1}{2}\left(r_1^L - c_2^U\right) - \left(a_1^U + e_1^U G_{1\varepsilon}\right) x_1(n)\Big\} \\
&\triangleq x(n)\exp\Big\{E_1 - E_{2\varepsilon} x_1(n)\Big\},
\end{aligned} \tag{15}
$$

where $E_1 = \frac{1}{2}\left(r_1^L - c_2^U\right)$ and $E_{2\varepsilon} = a_1^U + e_1^U G_{1\varepsilon}$. By applying Lemma 2.1 to (15), it immediately follows that

$$
\liminf_{n \to +\infty} x_1(n) \ge \min\{\frac{E_1}{E_{2\varepsilon}} \exp\{E_1 - E_{2\varepsilon} x_1^*\}, \frac{E_1}{E_{2\varepsilon}}\}.
$$

Setting $\varepsilon \to 0$ in the above inequality, we obtain

$$\liminf_{n\to+\infty} x_1(n) \geq \min\{\frac{E_1}{E_2}\exp\{E_1 - E_2 x_1^*\}, \frac{E_1}{E_2}\} \triangleq x_{1*}. \tag{16}$$

It follows from (16) that there exists large enough $N_3 \geq N_1 + N_2$ such that

$$x_1(n) \geq \frac{x_{1*}}{2}, \text{ for all } n \geq N_3. \tag{17}$$

This together with the third equation of system (3) leads to

$$\Delta u_1(n) \geq -b_1(n)u_1(n) + \frac{x_{1*}d_1(n)}{2}, \text{ for all } n \geq N_3.$$

Hence,

$$u_1(n+1) \geq (1 - b_1^U)u_1(n) + \frac{x_{1*}d_1^L}{2}, \text{ for all } n \geq N_3. \tag{18}$$

By applying Lemma 2.3, it follows from (18) that

$$\liminf_{n\to+\infty} u_1(n) \geq \frac{d_1^L x_{1*}}{2b_1^U} \triangleq u_{1*}.$$

This completes the proof the proof of Lemma 3.2.

Lemma 3.3 *Assume*

$$r_2^L - c_1^U > 0 \tag{A_{32}}$$

holds, then there exist two positive constants x_{2*} and u_{2*} such that

$$\liminf_{n\to+\infty} x_2(n) \geq x_{2*}, \ \liminf_{n\to+\infty} u_2(n) \geq u_{2*},$$

where x_{2*} and u_{2*} are defined in the proof.

Proof. The proof of Lemma 3.3 is similar to that of Lemma 3.2. So we omit the detail here.

Lemmas 3.1-3.3 show that the conclusion of Theorem C holds.

4. Example and numeric simulation

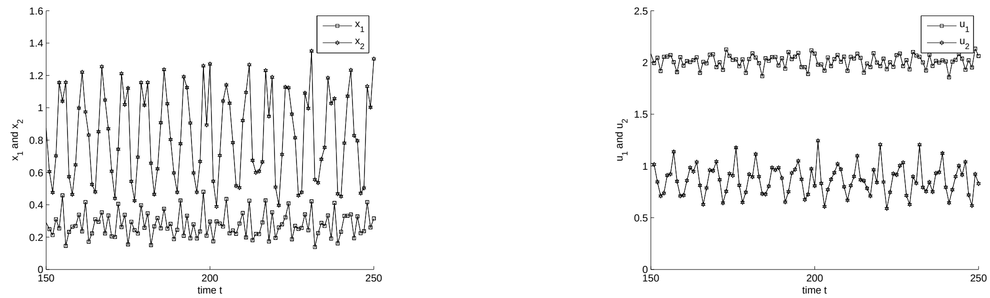

FIGURE 1. Dynamic behavior of the system (19) with the initial condition $(x_1(0),x_2(0),u_1(0),u_2(0)) = (0.1,0.3,0.2,0.04)^T$ and $(0.2,0.1,0.6,0.5)^T$, respectively.

In this section, we give the following example to verify the feasibilities of Theorem C:

$$
(19)\quad \begin{cases}
x_1(n+1) = x_1(n)\exp\Big\{2.5+0.5\sin(\sqrt{7}n)-(1.3+0.2\cos n)x_1(n) \\
\qquad -\dfrac{(0.75+0.25\sin(\sqrt{11}n))x_2(n)}{1+x_2(n)}-(0.9+0.1\cos(\sqrt{3}n))u_1(n)\Big\}, \\
x_2(n+1) = x_2(n)\exp\Big\{2.8-(2.2+0.2\sin n)x_2(n) \\
\qquad -\dfrac{(0.5+0.25\cos(\sqrt{13}n))x_1(n)}{1+x_1(n)}-(1+0.5\sin n)u_1(n)\Big\}, \\
\Delta u_1(n) = -(0.08+0.02\sin(\sqrt{2}n))u_1(n)+(0.6+0.4\cos(\sqrt{7}n))x_1(n), \\
\Delta u_2(n) = -(0.73+0.03\cos(\sqrt{5}n))u_2(n)+(0.8+0.2\sin(n))x_2(n)),
\end{cases}
$$

In this case, we have

$$
(20)\quad r_1^L-c_2^U=1>0,\ \ r_2^L-c_1^U=2.05>0
$$

(4.2) shows that (A_3) holds, so the system (19) is permanent according to Theorem C. Our numerical simulation supports our result (see Fig. 1). However,

$$
(21)\quad r_1^L-c_2^U-e_1^U u_1^*\approx -110.9554<0,\ \ r_2^L-c_1^U-e_2^U u_2^*\approx -2.7518<0,
$$

that is to say (A_2) does not hold and we could not obtain the result of the permanence from Theorem B. Thus our results improve the main results in [5].

Conflict of Interests

The author declares that there is no conflict of interests.

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References

[1] K. Gopalsamy, Stability and Oscillations in Delay Differential Equations of Population Dynamics, vol. 74 of Mathematics and Its Applications, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1992.

[2] Q. Wang, Z. Liu and Z. Li, Existence and global asymptotic stability of positive almost periodic solutions of a two-species competitive system, Int. J. Biomath. 7 (2014), Article ID 1450040.

[3] W. Qin, Z. Liu and Y. Chen, Permanence and global stability of positive periodic solutions of a discrete competitive system, Discrete Dyn. Nat. Soc. 2009 (2009), Article ID 830537.

[4] Q. Wang, Z. Liu, Uniformly asymptotic stability of positive almost periodic solutions for a discrete competitive system, J. Appl. Math. 2013 (2013), Article ID 182158.

[5] Q. Wang, Z. Liu and Z. Li, Positive Almost Periodic Solutions for a Discrete Competitive System Subject to Feedback Controls, J. Appl. Math. 2013 (2013), Article ID 429163.

[6] Y. Fan, L. Wang, Permanence for a discrete model with feedback control and delay, Discrete Dyn. Nat. Soc. 2008 (2008), Article ID 945109.

[7] J. Xu, Z. Teng and H. Jiang, Permanence and global attractivity for discrete nonautonomous two-species Lotka-Volterra competitive system with delays and feedback controls, Periodica Math. Hungarica 63 (2011), 19-45.

[8] Muhammadhaji A, Z. Teng and L. Nie, Permanence in nonautonomous discrete Lotka-Volterra n-species competitive systems with pure-delays and feedback controls, Int. J. Math. 24 (2013) 1350053.

[9] W. Yang, X. Li, Feedback control has no influence on the persistent property of a single species discrete model with time delays, Math. Comput. Modelling 54 (2011), 2007-2013.

[10] S. Yu, J. Zhang, Permanence of a discrete modified Leslie-Gower predator-prey system with delay and feedback controls, Acta Anal. Funct. Appl. 16 (2014), 244-249 (in Chinese).

[11] S. Yu, Global stability of a modified Leslie-Gower model with Beddington-DeAngelis functional response, Adv. Difference Equa. 2014 (2014), Article ID 84.

[12] L. Chen, L. Chen and Z. Li, Permanence of a delayed discrete mutualism model with feedback controls, Math. Comput. Modelling 50 (2009), 1083-1089.

[13] S.Yu, F. Chen, Almost periodic solution of a modified Leslie-Gower predator- prey model with Holling-type II schemes and mutual interference, Int. J. Biomath. 7 (2014) Article ID 1450028.

[14] S. Yu, Almost periodic solution of Nicholson's blowflies model with feedback control, J. Fuzhou Univ. (Natural Science Edition) 38 (2010), 481-485 (in Chinese).

[15] S. Yu, Global asymptotic stability of a predator-prey model with modified Leslie-Gower and Holling-type II schemes, Discrete Dyn. Nat. Soc. 2012 (2012), Article ID 208167.

[16] L. Wang, Y. Fan, Permanence for a discrete Nicholsons blowflies model with feedback control and delay, Int. J. Bio. 1 (2008), 433-442.

[17] J. Chen, S. Yu, Permanence for a Discrete Ratio-Dependent Predator-Prey System with Holling Type III Functional Response and Feedback Controls, Discrete Dyn. Nat. Soc. 2013 (2013), Article ID 326848.

[18] F. Chen, Permanence of a discrete N-species cooperation system with time delays and feedback controls, Appl. Math. Comput. 186 (2007), 23-29.

[19] F. Chen, Permanence for the discrete mutualism model with time delays, Math. Comput. Modelling 47 (2008), 431-435.