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THE IMPACT OF MATURATION DELAY OF MOSQUITOES ON MOSQUITO-BORNE DISEASE TRANSMISSION

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Abstract. In this paper, a mosquito-borne disease model, which address the impacts of the aquatic stage of mosquitoes on mosquito-borne disease transmission, is proposed and analyzed. The dynamical behaviors of the model are analyzed theoretically and numerically. The basic reproduction number which is a monotone decreasing function with respect to the delay has been figured out. Our results imply that the increasing of temperature may exacerbates mosquito-borne disease transmission.

Keywords: Mosquito; Mosquito-borne disease; Maturation delay; Stability analysis.

2010 AMS Subject Classification: 92D30, 34C60.

1. Introduction

Many diseases are transmitted by mosquitoes, like malaria, dengue, West Nile virus, etc. World-wide, mosquito-borne diseases kill more people than any other single factor.

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Mosquito life includes four stages: egg, larva, pupa, and adult. Each of these stages can be easily recognized by their special appearance. The duration of the whole cycle, from egg laying to an adult mosquito eclosion, varies between 7 and 20 days, depending on the ambient temperature of the swamp and the mosquito species involved [8]. Only the adult female mosquitos bite human and animals in order to take blood meals, and the male mosquitoes feed only on plant juices. Female mosquitoes feed on man, domestic animals, such as cattle, horses, goats, etc; all species of birds including chickens and ducks; all types of wild animals including deer, rabbits; and they also feed on snakes, lizards, frogs, and toads. Female mosquitoes will not lay viable eggs without blood meals. Therefore, the abundance of mosquito in a region is closely related to the blood meal resources available in the region ([11]).

In order to control these diseases, it is essential to understand the mosquito population dynamics first and then to consider how resource and temperature can affect mosquito population, and how they in turn can affect mosquito-borne disease transmission. Since only the adult female mosquitoes are responsible for transmitting diseases, therefore in general, models focus only on describing the dynamics of adult female mosquitoes. There have been extensive dynamical modeling studies of the mosquito population and mosquito-borne diseases. In [1, 5, 12, 19] West Nile virus transmission dynamics has been considered. In [3, 13, 14, 17, 18], malaria transmission dynamics has been considered. In [6, 7, 15, 16], dengue transmission dynamics have been studied. In [2], Bolzoni *et al.* studied the role of heterogeneity on the invasion probability of mosquito-borne diseases in multi-host models.

It is our aim to modify the mosquito growth equation to include the factor of maturation delay related to air temperature and to investigate the impact of maturation delay on the transmission dynamics of mosquito-borne diseases.

The rest of the paper is organized as follows. In Section 2, the derivation of a new model is given. We analyze the dynamics of this model in Section 3 and 4, including the existence of equilibria and stability. Finally, we present some numerical simulations and comments on our findings in Section 5.

2. Preliminaries

In this section, we will establish a new model with stage-structured mosquitoes to study the impact of resource and temperature on the malaria transmission dynamics.

The basic assumptions for our model are as follows:

- The total number of host population and adult female mosquito population are denoted by N_b and N_m respectively.
- The adult stage of the female mosquito population is divided into susceptible M_s and infectious M_i compartments. We assume that the pathogen can cause reservoir death and this population is divided into susceptible B_s , infectious B_i and recovered B_r compartments. The recovered host have immunity.
- Mosquitoes never recover from infection, that is, their infective period ends with their death due to their relatively short life.
- For the transmission of the pathogen, it is assumed that a susceptible host can receive the infection only by contacting with infective mosquitos, vice versa.

With the assumptions above and parameters listed in Table 1, the dynamics are given by the following equations:

$$(2.1) \quad \left\{ \begin{array}{l} \frac{dM_s}{dt} = r_m M_s(t - \tau) e^{-\delta\tau} + (1 - q) r_m M_i(t - \tau) e^{-\delta\tau} - b\beta_m \frac{M_s B_i}{N_b + A} \\ \quad - d_m M_s - \varepsilon_m M_s N_m, \\ \frac{dM_i}{dt} = q r_m M_i(t - \tau) e^{-\delta\tau} + b\beta_m \frac{M_s B_i}{N_b + A} - d_m M_i - \varepsilon_m M_i N_m, \\ \frac{dB_s}{dt} = \Lambda - d_b B_s - b\beta_b \frac{M_i B_s}{N_b + A}, \\ \frac{dB_i}{dt} = b\beta_b \frac{M_i B_s}{N_b + A} - (d_b + \nu + r) B_i, \\ \frac{dB_r}{dt} = r B_i - d_b B_r, \end{array} \right.$$

with initial data $M_s(\theta) = \phi_s(\theta) > 0$ and $M_i(\theta) = \phi_i(\theta) > 0$, where $\phi_s(\theta)$ and $\phi_i(\theta)$ are positive continuous functions on $[-\tau, 0]$ respectively, $0 \leq B_s(0) \leq \frac{\Lambda}{d_b}$, $0 \leq B_i(0) \leq \frac{\Lambda}{d_b}$, $0 \leq B_r(0) \leq \frac{\Lambda}{d_b}$, and also $N_m = M_s + M_i$ and $N_b = B_s + B_i + B_r$.

Parameters in the models	Mosquito	Host
Per capita birth rate	r_m	
Recruitment rate		Λ
Proportion of births that are infected	q	
Natural death rate for adult (pre-adult) stage	$d_m(\delta)$	d_b
Disease-induced death rate		v
Maturation delay	τ	
Recovery rate		r
Biting rate (the average number of bites per mosquito per unit time)	b	
Transmission probability (from vectors to birds)		β_b
Transmission probability (from birds to vectors)	β_m	
The number of other animals as blood meal resource	A	
Intraspecific competition rate	ϵ_m	

TABLE 1. Parameters involved in the model.

It is not difficult to prove the following theorem:

Theorem 2.1. *With an initial value condition in (2.1), there is a unique solution, and the solution remains positive and bounded for any finite time $t \geq 0$.*

Therefore, Model (2.1) is mathematically well-defined and biologically reasonable.

3. Equilibria

A straightforward calculation shows there two disease-free equilibria for model (2.1):

$$P_{01} = (0, 0, \Lambda/d_b, 0, 0)$$

and

$$P_{02} = (\widehat{N}_m, 0, \Lambda/d_b, 0, 0),$$

where

$$\widehat{N}_m = (r_m e^{-\delta\tau} - d_m) / \varepsilon_m.$$

In addition, P_{02} exists if and only if $\tau < \tau^*$, where

$$\tau^* = \frac{1}{\delta} \ln \frac{r_m}{d_m}.$$

By equating the derivatives on the left-hand side to zero and solving the resulting algebraic equations. The points of any equilibrium $\widehat{P} = (\widehat{M}_s, \widehat{M}_i, \widehat{B}_s, \widehat{B}_i, \widehat{B}_r)$ satisfy the following relations

$$(3.2) \quad \widehat{B}_s = \frac{\Lambda - (d_b + v + r)\widehat{B}_i}{d_b}$$

$$(3.3) \quad \widehat{N}_b = \frac{\Lambda - v\widehat{B}_i}{d_b}$$

$$(3.4) \quad \widehat{M}_s = \widehat{N}_m - \widehat{M}_i$$

$$(3.5) \quad \widehat{M}_i = \frac{b\beta_m d_b \widehat{N}_m \widehat{B}_i}{b\beta_m d_b \widehat{B}_i + r_m e^{-\delta\tau} (1 - q) (\Lambda - v\widehat{B}_i + Ad_b)}$$

Substituting (3.2)-(3.5) in the corresponding fourth equilibrium equation of (2.1), we obtain that $\widehat{B}_i = 0$ or \widehat{B}_i is a positive root of the quadratic polynomial

$$(3.6) \quad r(B_i) = a_2 B_i^2 + a_1 B_i + a_0,$$

where

$$a_2 = [b\beta_m d_b - r_m e^{-\delta\tau} (1 - q)v]v,$$

$$\begin{aligned} a_1 &= 2r_m e^{-\delta\tau} (1 - q) (\Lambda + Ad_b)v - b\beta_m d_b (\Lambda + Ad_b) - b^2 \beta_m \beta_b \widehat{N}_m d_b \\ &= -a_2 (\Lambda + Ad_b) / v + r_m (1 - q e^{-\delta\tau}) (\Lambda + Ad_b) [v - (d_b + v + r) \frac{\Lambda + Ad_b}{\Lambda} R_0], \end{aligned}$$

$$a_0 = r_m e^{-\delta\tau} (1 - q) (\Lambda + Ad_b)^2 (R_0(\tau) - 1)$$

and

$$(3.7) \quad R_0(\tau) = \frac{b^2 \beta_m \beta_b \widehat{N}_m d_b \Lambda}{r_m e^{-\delta\tau} (1 - q) (d_b + v + r) (\Lambda + Ad_b)^2}.$$

It is easy to see that $a_2 < 0$ if $a_1 > 0$ and $R_0(\tau) \geq 1$.

Remark 3.1. R_0 is the basic reproduction number which is a monotone decreasing function with respect to the maturation τ .

The solution $\widehat{B}_i = 0$ gives the disease-free equilibrium point P_{01} and P_{02} .

We are looking for nontrivial equilibrium solutions. From (3.2)-(3.5) it can be seen that $\widehat{B}_i \in I \triangleq (0, B_0)$, where

$$B_0 = \frac{\Lambda}{d_b + v + r}.$$

Evaluating $r(B_i)$ at the end points of the interval I gives

$$\begin{aligned} r(0) &= r_m e^{-\delta\tau} (1-q)(\Lambda + Ad_b)^2 (R_0(\tau) - 1) \\ r(B_0) &< 0. \end{aligned}$$

When $R_0(\tau) > 1$, then $r(0) > 0$, therefore there exists a unique root in the interval I , which implies the existence of a unique equilibrium point $P^*(\widehat{M}_s^*, \widehat{M}_i^*, \widehat{B}_s^*, \widehat{B}_i^*, \widehat{B}_r^*)$, where \widehat{i}_a satisfies: If $a_2 = 0$,

$$\widehat{B}_i^* = -\frac{a_0}{a_1}.$$

If $a_2 \neq 0$,

$$\widehat{B}_i^* = \frac{-a_1 - \sqrt{\Delta}}{2a_2},$$

where $\Delta \triangleq a_1^2 - 4a_2a_0$.

When $R_0(\tau) = 1$, the roots of (3.6) are 0 and $-\frac{a_1}{a_2}$. It is easy to see that there exists a unique root in the interval I if and only if $a_1 > 0$, which implies the existence of a unique equilibrium point P^* .

When $R_0(\tau) < 1$, $r(0) < 0$. The conditions to have at least one root in the mentioned interval are: $a_2 < 0$, $0 < -\frac{a_1}{2a_2} < B_0$ and $\Delta \geq 0$. If $\Delta > 0$, there exist two roots in the interval I , which implies the existence of two equilibria $P_1(\widehat{M}_{s1}, \widehat{M}_{i1}, \widehat{B}_{s1}, \widehat{B}_{i1}, \widehat{B}_{r1})$ and $P_2(\widehat{M}_{s2}, \widehat{M}_{i2}, \widehat{B}_{s2}, \widehat{B}_{i2}, \widehat{B}_{r2})$, where

$$\begin{aligned} \widehat{B}_{i1} &= \frac{-a_1 + \sqrt{\Delta}}{2a_2}, \\ \widehat{B}_{i2} &= \frac{-a_1 - \sqrt{\Delta}}{2a_2}. \end{aligned}$$

If $\Delta = 0$, there is a unique root in the interval I , which implies the existence of unique equilibrium.

Then, we can conclude the above results in the following theorem:

Theorem 3.2. *1. The boundary equilibrium, the disease free equilibrium P_{01} always exists and P_{02} exists when $\tau < \tau^*$.*

2. If $R_0(\tau) > 1$, there exists a unique positive equilibrium P^ .*

3. If $R_0(\tau) = 1$, then there is a positive equilibrium when $a_1 > 0$, otherwise there is no positive equilibrium.

4. If $R_0(\tau) < 1$, then (a) if $a_2 \geq 0$, there is no positive equilibrium; (b) if $a_2 < 0$, the system (2.1) has two positive equilibria P_1 and P_2 if and only if $\Delta > 0$ and $0 < \frac{-a_1}{2a_2} < B_0$; And these two equilibria coalesce if and only if $0 < \frac{-a_1}{2a_2} < B_0$ and $\Delta = 0$; otherwise there is no positive equilibrium.

4. Stability

For the equilibrium P_{01} , the eigenvalues of the Jacobian matrix are: $-d_b$ (multiplicity 2), $-(d_b + v + r)$ and the roots of

$$(4.8) \quad (\lambda - r_m e^{-(\delta+\lambda)\tau} + d_m)(\lambda - r_m q e^{-(\delta+\lambda)\tau} + d_m) = 0$$

When $\tau = 0$, one can easily get that one of the eigenvalues of the Jacobian matrix at P_{01} is: $r_m - d_m > 0$. Therefore, P_{01} is always unstable.

When $\tau > 0$, we can easily prove that P_{01} is unstable stable if $0 < \tau < \tau^*$ and is stable if $\tau > \tau^*$

Then, we have the following theorem:

Theorem 4.1. *The equilibrium P_{01} is always unstable if $0 \leq \tau < \tau^*$; P_{01} is stable if $\tau > \tau^*$.*

For the equilibrium P_{02} , the eigenvalues of the Jacobian matrix are: $-d_b$ (multiplicity 2) and the roots of

$$(4.9) \quad \lambda - r_m e^{-(\delta+\lambda)\tau} + 2r_m e^{-\delta\tau} - d_m = 0,$$

$$(4.10) \quad (\lambda + d_b + v + r)(\lambda - r_m q e^{-(\delta+\lambda)\tau} + r_m e^{-\delta\tau}) - r_m(1-q)e^{-\delta\tau}(d_b + v + r)R_0(\tau) = 0.$$

When $\tau = 0$, one can easily get that the eigenvalues of the Jacobian matrix at P_{02} are: $-(r_m - d_m) < 0$, $-d_b < 0$ (multiplicity 2), and the roots of

$$(4.11) \quad \lambda^2 + [d + v + r + r_m(1-q)]\lambda + r_m(1-q)(d + v + r)(1 - R_0(0)) = 0$$

It is easy to see that P_{02} is unstable if $R_0 > 1$, P_{02} is locally stable if $R_0 < 1$. Then, we have the following theorem:

Theorem 4.2. *When $\tau = 0$, the equilibrium P_{02} is locally stable (unstable) if $R_0(0) < 1$ ($R_0(0) > 1$).*

In the following, will discuss the situation when $0 < \tau < \tau^*$. Let

$$\tau^{**} \triangleq \frac{1}{\delta} \ln \frac{r_m(R_0(0) - 1) + d_m}{d_m R_0(0)},$$

$$\tau^{***} \triangleq \frac{1}{\delta} \ln \left(\frac{r_m}{d_m} - \frac{\varepsilon_m r_m (1+q)(d_b + v + r)(\Lambda + A d_b)^2}{d_m \Lambda \beta_m \beta_b b^2 d_b} \right).$$

Obviously, $\tau^{**} < \tau^*$ and

$$\tau = \tau^{**} \Leftrightarrow R_0(\tau) = 1,$$

$$\tau^{**} > 0 \Leftrightarrow R_0(0) > 1,$$

$$\tau^{**} = 0 \Leftrightarrow R_0(0) = 1,$$

and $\tau^{**} < 0$ or τ^{**} does not exist if and only if $R_0(0) < 1$. We can prove that:

Lemma 4.3. *No stability switch may occur when $0 < \tau \leq \tau^{***}$ or $\tau^{**} < \tau < \tau^*$.*

Proof. One can easily prove that 0 can be the root of (4.12) when $\tau^{**} < \tau < \tau^*$ or $0 < \tau \leq \tau^{***}$.

Assuming that there is a pure imaginary root $\lambda = i\omega$ for (4.10). Let $z = \omega^2$. Then, we have

$$(4.12) \quad z^2 + aZ + b = 0,$$

where

$$\begin{aligned} a &= r_m^2(1-q^2)e^{-2\delta\tau} + (d_b + v + r)^2 + \frac{2b^2\beta_m\beta_b\Lambda d_b(r_me^{-\delta\tau} - d_m)}{\epsilon_m(\Lambda + Ad_b)^2}, \\ b &= r_me^{-\delta\tau}(1-q)(d_b + v + r)(1 - R_0(\tau))[r_me^{-\delta\tau}(1+q)(d_b + v + r) \\ &\quad - \frac{b^2\beta_m\beta_b\Lambda d_b(r_me^{-\delta\tau} - d_m)}{\epsilon_m(\Lambda + Ad_b)^2}]. \end{aligned}$$

It is easy to see $a > 0$. If $\tau^{**} \leq \tau < \tau^*$, then $b \geq 0$, there is no positive root for (4.12) which implies that there is no pure imaginary root for (4.10). Therefore, no stability switch may occur when $0 < \tau \leq \tau^{***}$ or $\tau^{**} < \tau < \tau^*$.

□

Then, we have the following result.

Theorem 4.4. *Case 1: $R_0(0) < 1$ ($\tau^{**} < 0$ or τ^{**} not exist). According to Theorem 4.2 and Lemma 4.3, P_{02} is stable when $0 < \tau < \tau^*$.*

*Case 2: $R_0(0) > 1$ ($\tau^{**} > 0$).*

*(i) $\tau^{***} > 0$. When $0 < \tau < \tau^{***}$, according to Theorem 4.2 and Lemma 4.3, P_{02} is unstable.*

*When $\tau^{***} < \tau < \tau^{**}$, there may be Hopf bifurcation.*

*(ii) $\tau^{***} \leq 0$. The situation is similar to (i).*

5. Simulation and discussion

In this paper, a mosquito-borne disease model, which address the impacts of the aquatic stage of mosquitoes and the seasonal climate effects on mosquito-borne disease transmission, has been proposed and analyzed. Our model provides a baseline to understand the risk and plan

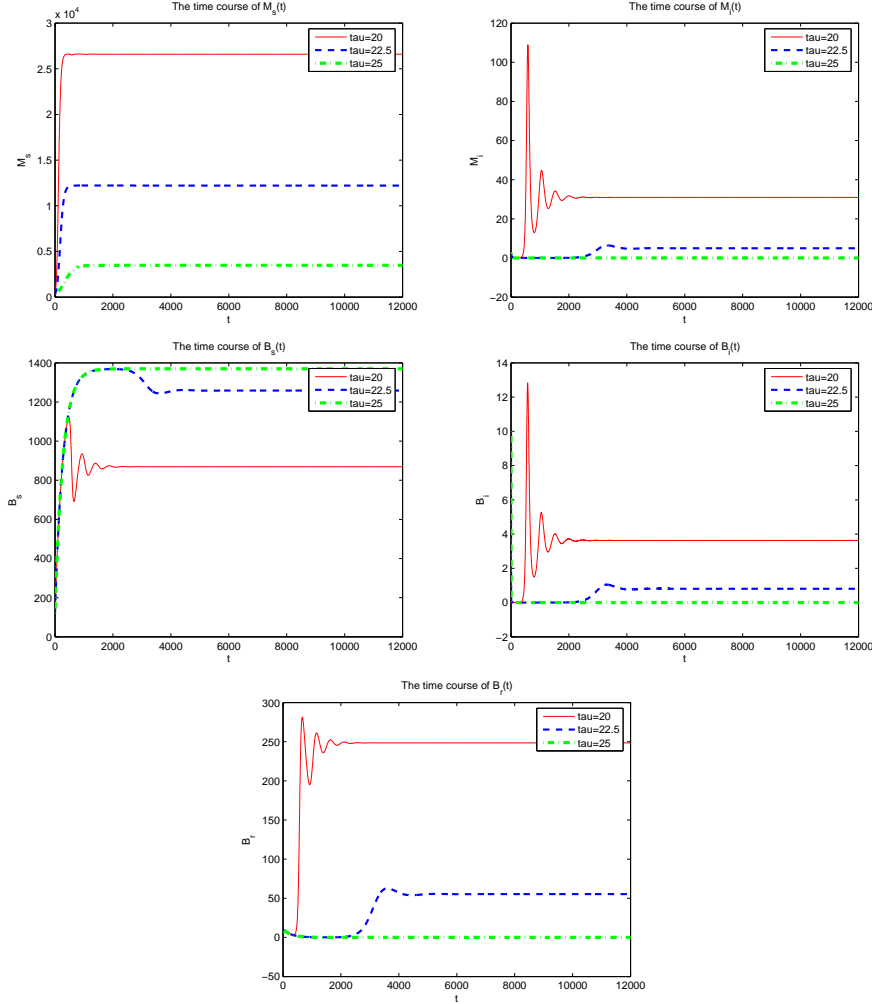


FIGURE 1. The time course of state variables. The unit of time t is day. $r_m = 10$; $q = 0.06$; $\delta = 0.2$; $A = 3000$; $b = 0.35$; $\beta_m = 0.65$; $d_m = 0.05$; $\varepsilon_m = 0.000005$; $\Lambda = 5$; $d_b = 3.65 \times 10^{-3}$; $\beta_b = 0.8$; $\nu = 0.25$; $r = 0.25$. Note that $\tau^* = 26.49158684$, $\tau^{**} = 22.84233291$ and $\tau^{***} = 22.10444322$.

for mosquito-borne disease control under the condition of global warming, since the maturation delay becomes shorter with the increasing of temperature.

The basic reproduction number which is a monotone decreasing function with respect to mosquito maturation delay has been figured out. This result implies that the increasing of temperature indeed exacerbates mosquito-borne disease transmission.

For many mosquito-borne disease models, the threshold condition, the basic reproduction number was calculated which is a crucial control threshold for disease eradication. However,

our study suggests that backward bifurcation can be an intrinsic property. There exists a sub-threshold condition for the outbreak of the virus due to the existence of backward bifurcation. The infection will not be cleared from the population provided even if the basic reproduction number is smaller than unity. Due to backward bifurcation of the model and the basic reproductive number itself is not enough to describe whether the disease will prevail or not and we should pay more attention to the initial sizes and density of the infected hosts and mosquitos.

It should be mentioned that the stability of endemic equilibria is still unknown even if $\tau = 0$, theoretically. In order to demonstrate the theoretical results and figure out the stability of endemic equilibria for some special parameter values, we carry out numerical simulations for the model (2.1). Fig. 1 implies that there exists a stable endemic equilibrium and the disease-free equilibrium P_{02} is unstable if $0 < \tau < \tau^{***}$ and $\tau^{**} < \tau < \tau^*$, no stability switch may occur and P_{02} is stable if the value of maturation delay increases such that $\tau^{**} < \tau < \tau^*$. On the other hand, Fig. 1 also shows that both of the number of infected mosquitoes and infected hosts converge to a higher positive value with the decreasing of maturation delay, which manifests the increasing of temperature indeed exacerbates the transmission of mosquito-borne disease numerically.

Conflict of Interests

The authors declare that there is no conflict of interests.

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