Available online at http://scik.org
Commun. Math. Biol. Neurosci. 2015, 2015:30
ISSN: 2052-2541

THRESHOLD ANALYSIS OF A SWITCHED FISHERY MODEL WITH STAGE STRUCTURE AND FISHING MORATORIUM

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Abstract. Considering that fishing activities are forbidden during fishing moratorium and permitted after fishing moratorium, a fishery model with stage structure and fishing moratorium is formulated by a switched system which is composed of two subsystems, one does not have the term of harvest and the other one has the term of proportional harvest. After discussing the relations between two subsystems, the range of parameters considered in this paper is determined. The threshold condition for the stability of the switch system is given by a discrete system and Jury conditions. The region where the solutions tend to or are away from the common equilibrium is given by qualitative analyses. The positive periodic solution is also discussed. Finally, numerical simulations are given to verify the theoretical results and some discussions including the optimal problem are also given.

Keywords: fishery model; fishing moratorium; stage structure; switched system.

2010 AMS Subject Classification: 34C25, 34D05, 34H05, 92B05.

1. Introduction

Fishery resources refer to the economic plants and animals of a water area such as fish, shrimp, crab, shellfish, algae. They are the natural sources and foundation of fishery production.

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Received May 21, 2015

70% of them were consumed directly by people. According to the degree of development and utilization, the exploitation of fishery resources can be divided into the following four cases roughly: (1) Resource exhaustion. In quite a long time, the resources are difficult to return to normal level. (2) Excessive exploitation. The resources have a recession, but as long as the suitable protection measures are taken, the resources are able to restore. (3) Sustainable development. The natural ability of regeneration can adapt to sustaining the optimal production and satisfying the need of people. (4) Potential exploitation. The utilization of the resource is potential.

In order to prevent the fishery resources from the excessive exploitation and to avoid the resource exhaustion, the protection measures are usually taken, such as setting spatial and temporal closures of fishing, restricting the size of mesh, controlling the minimum body length of the catch, restricting the fishing effect.

Fishing moratorium is an important protection measure of fishery resources. It refers to the period during which the fishing activities are prohibited or restricted by the governmental regulations. It aims to protect the normal growth and reproduction of aquatic organisms and fish stocks to ensure continuous recovery and sustainable development. For example, the Yangtze river of China has three months fishing moratorium, from April 1 to June 30 for the water area of the lower reaches, and from February 1 to April 30 for the upper reaches.

Although all levels of government pay more attention to the fishing moratorium, the fishing ban brings some subsequent problems such as the employment and income of the fishers which must be considered during the fishing moratorium. Ref. [1] pointed out that adequate understanding of the economic conditions and considerations potentially affecting the fishermen is necessary to model their behaviours and address their concerns. Therefore, it is necessary to study the evolutionary process of fishery resources under fishing ban. This paper will consider the dynamical properties of a kind of single species fishery model with fishing moratorium.

Some references have investigated the fishery management by using mathematical models. For examples, Ref. [2] discussed the bioeconomic exploitation of a single species fishery by using a reasonable catch-rate function instead of the usual ‘catch-per-unit-effort hypothesis’. It is assumed that an external agency regulates the fishery by imposing a suitable tax per unit biomass of landed fish. The optimal harvesting policy was also studied from the view point of control theory in Ref. [2].

A ratio-dependent prey-predator model with the selective harvesting and the tax per unit biomass of prey species were considered in Refs. [3,4]. Ref. [5] studied a prey-predator system in a two-patch environment: one accessible to both prey and predators (patch 1) and the other one being a refuge for the prey (patch 2). The prey refuge (patch 2) constitutes a reserve zone of prey and fishing is not permitted, while the unreserved zone area is an open-access fishery zone. The possibilities of the existence of bionomic equilibrium were examined and an optimal harvesting policy was given by using Pontryagin's maximum principle in Ref. [5].

Since many fishery resources (e.g. some kinds of fish) have the immature and the mature, and the mature are the object of exploitation and utilization, thus this paper considers a kind of single species model with stage structure.

For the biological species model with stage structure, Ref. [6] investigated a predator-prey system with stage structure for the predator by using the theory of competitive systems and compound matrices. Ref. [7] analyzed the exploitation of the stage-structured single autonomous population model. Explicit expressions are obtained for the optimal harvesting effort, the maximum sustainable yield and the corresponding population level. Ref. [8] proposed an exploited single species model with stage structure for the dynamics in a fish population for which births occur in a single pulse once per time period, and showed that the timing of harvesting has a strong impact on the persistence of the fish population, on the volume of mature fish stock and on the maximum annual-sustainable yield. Moreover, the results given in Ref. [8] implied that the population can sustain much higher harvest rates if the mature fish is removed as early in the season (after the birth pulse) as possible. Of course, there are still many references to study the management measures of fishery resources from different aspects(e.g. Ref. [9–13]), here does not review and restate the relative results.

The continuous harvest was considered in the models of these references mentioned above. Since fishing activities are forbidden during fishing moratorium and permitted after fishing moratorium, then the evolutionary process of the related species can be described by a switched system which is a combination of two ordinary differential equations and a discrete switching event(e.g., fishing ban). The switched systems are applied to the fields of transmission system, traffic control system, craft control system and so on. There are many references(e.g. [14–21]) to study the switched system and its stability. But a few papers consider the biological system with fishing moratorium.

This paper will consider and investigate a kind of switched fishery model with fishing moratorium and stage structure. The rest of this paper is organized as follows. In Section 2, we will introduce and formulate a switched system to describe the evolutionary process of a single species with stage structure and fishing moratorium. The range of the parameters considered in this paper is determined by analyzing the dynamical properties of two subsystems. In Section 3, a discrete system is obtained by solving the equation in the form of matrix. In Section 4, the stability is discussed by Jury conditions and the threshold is given. In Section 5, the region where the solutions tend to or are away from the common equilibrium will be given by the qualitative analyses, and the positive periodic solution is discussed. Numerical simulations and some discussions including the optimal problem are given in Section 6.

2. Model formulation

Suppose that the fishery resource we consider is a kind of fish species and its evolutionary process can be described by the following model with stage structure when there is no exploitation.

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = by - d_1 x - cx, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = cx - d_2 y, \end{cases} \tag{2.1}$$

where $x = x(t)$ and $y = y(t)$ denote the populations of immature fish and mature fish, respectively. The birth rate of the immature fish is b, the death rate of the immature and the mature are d_1,d_2, respectively. The rate of the immature transferring into the mature is c. b, c, d_1, d_2 are positive constants.

If the exploitation of continuous harvest to the mature fish is considered and suppose that the harvest satisfies a proportional function, then system (2.1) becomes

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = by - d_1x - cx = by - ax, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = cx - d_2y - Ey = cx - dy, \end{cases} \tag{2.2}$$

where $a = d_1 + c$, $d = d_2 + E$ and $E > 0$ is the rate of harvest.

Since the fishing activity of harvest is forbidden during the fishing moratorium and permitted after fishing moratorium, then the mathematical model can be written as the following switched system:

$$\begin{cases} \dfrac{\mathrm{d}X}{\mathrm{d}t} = \begin{pmatrix} -a & b \\ c & -d_2 \end{pmatrix} X, t \in (n\tau, (n+l)\tau], \\ \dfrac{\mathrm{d}X}{\mathrm{d}t} = \begin{pmatrix} -a & b \\ c & -d \end{pmatrix} X, t \in ((n+l)\tau, (n+1)\tau], \end{cases} \tag{2.3}$$

where $X = X(t) = (x(t), y(t))^{\mathrm{T}}$, the initial value is $X(0) = (x_0, y_0)^{\mathrm{T}}$, τ is the period of management or the breeding cycle of the fish species, $n = 0, 1, 2, \cdots$; $l\tau$ is the length of fishing moratorium, $(1-l)\tau$ is the time of catching fish, $0 < l < 1$. For the convenience of simplicity, let

$$A_1 = \begin{pmatrix} -a & b \\ c & -d_2 \end{pmatrix}, A_2 = \begin{pmatrix} -a & b \\ c & -d \end{pmatrix}.$$

System (2.3) can also be written as the following switched system:

$$\dot{X} = A_{\sigma(i)}X, \tag{2.4}$$

where

$$\sigma(i) = \begin{cases} 1, & t \in (n\tau, (n+l)\tau], \\ 2, & t \in ((n+l)\tau, (n+1)\tau]. \end{cases}$$

The first and second subsystems of system (2.3) can be called as subsystems A_1 and A_2, respectively. Clearly, systems (2.1) and (2.2) have a common equilibrium $O(0,0)$. It is easily calculated that the eigenvalues of systems (2.1) and (2.2) are, respectively,

$$\begin{aligned} \lambda_1|_{(2.1)} &= \frac{1}{2}(-(a+d_2) + \sqrt{(a+d_2)^2 - 4(ad_2 - bc)}), \\ \lambda_2|_{(2.1)} &= \frac{1}{2}(-(a+d_2) - \sqrt{(a+d_2)^2 - 4(ad_2 - bc)}), \end{aligned} \tag{2.5}$$

and

$$\begin{aligned}\lambda_3|_{(2.2)} &= \frac{1}{2}(-(a+d_2+E)+\sqrt{(a+(d_2+E))^2-4(a(d_2+E)-bc)}),\\ \lambda_4|_{(2.2)} &= \frac{1}{2}(-(a+d_2+E)-\sqrt{(a+(d_2+E))^2-4(a(d_2+E)-bc)}).\end{aligned} \tag{2.6}$$

Comparing with system (2.2), system (2.1) has no the term of exploitation, that is, $E = 0$. For system (2.1), if $d_2 > b$, it is clear that $ad_2 - bc > 0$ (since $a = d_1 + c$) and then $0 < (a+d_2)^2 - 4(ad_2 - bc) < (a+d_2)^2$, further it follows that $\lambda_1 < 0$ and $\lambda_2 < 0$. Thus, the equilibrium $O(0,0)$ of system (2.1) is a stable node for $d_2 > b$. In this case, the populations of both the immature fish and the mature fish tend to 0. If $d_2 < b$ and $ad_2 - bc > 0$, that is, $b > d_2 > \frac{c}{a}b$, then it is easily proved that the equilibrium $O(0,0)$ is also stable. Those show that the fish species itself is in danger if one of the following cases holds:

1) $d_2 > b$,

2) $\frac{c}{a}b < d_2 < b$.

For two cases above, the fishing activity accelerates the extinction of the fish species, people only protect the resource and should not be exploited it. So we do not consider the above two cases.

If $d_2 < \frac{c}{a}b$ (or $ad_2 - bc < 0$), then $\lambda_1 < 0, \lambda_2 > 0$ and the equilibrium $O(0,0)$ of system (2.1) is a saddle point. In this case, the population of the species is increasing and there exists the possibility of restoring the species. So there are two cases:

1) There is the exploitation activity of fishing, and $d_2 + E < \frac{c}{a}b$, then the equilibrium $O(0,0)$ of system (2.2) is unstable although $E > 0$. This implies that the population of the fish species is increasing, so fishery ban will not exist and we also do not consider this case.

2) The population of the fish species is increasing if there is no exploitation, but it is decreasing if there exists the exploitation of continuous harvest. That is, the equilibrium $O(0,0)$ is unstable if $E = 0$, but $O(0,0)$ is stable if $E > 0$, $d_2 < \frac{c}{a}b$ and $d_2 + E > \frac{c}{a}b$.

For the excessive exploitation of fish species, when the activity of exploitation exists, the amount of the fish decreases and tends to zero if no control is taken. If the protection measures of fishing ban are taken, then it is possible to restore the fish species. According to the discussions mentioned above, the followings always suppose that

$$d_2 < \frac{c}{a}b < d_2 + E. \tag{2.7}$$

The evolutionary process of the fish species under fishing moratorium can be illustrated in Fig.2.1.

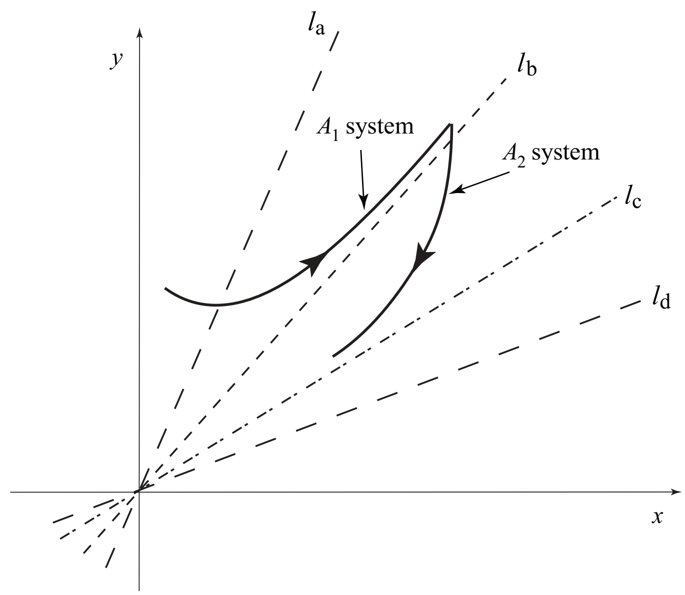

Fig.2.1. Illustration of the evolutionary process of the fish under system (2.3) and condition (2.7)
l_a: the isoclinic line $\frac{dy}{dt} = 0$ of subsystem A_1; l_b: the asymptotic line of subsystem A_1;
l_c: the asymptotic line of subsystem A_2; l_d: the isoclinic line $\frac{dx}{dt} = 0$ of subsystem A_2.

3. Discrete system

Clearly, two subsystems of system (2.3) have the same equilibrium $O(0,0)$. Since the equilibrium $O(0,0)$ of subsystem A_1 is an unstable saddle and the equilibrium $O(0,0)$ of subsystem A_2 is a stable node, then the solutions of system (2.3) either tend to $O(0,0)$, or keep away from $O(0,0)$.

For the matrixes A_1 and A_2, there exist invertible matrixes P and Q where

$$P = \begin{pmatrix} -c & -a-\lambda_1 \\ -c & -a-\lambda_2 \end{pmatrix}, \; Q = \begin{pmatrix} -c & -a-\lambda_3 \\ -c & -a-\lambda_4 \end{pmatrix},$$

and their corresponding inverse matrixes

$$P^{-1}=\begin{pmatrix} -\frac{1}{c}(1-\frac{a+\lambda_1}{\lambda_1-\lambda_2}) & -\frac{1}{c}\frac{a+\lambda_1}{\lambda_1-\lambda_2} \\ -\frac{1}{\lambda_1-\lambda_2} & \frac{1}{\lambda_1-\lambda_2} \end{pmatrix},\ Q^{-1}=\begin{pmatrix} -\frac{1}{c}(1-\frac{a+\lambda_3}{\lambda_3-\lambda_4}) & -\frac{1}{c}\frac{a+\lambda_3}{\lambda_3-\lambda_4} \\ -\frac{1}{\lambda_3-\lambda_4} & \frac{1}{\lambda_3-\lambda_4} \end{pmatrix},$$

such that

$$PA_1P^{-1}=\begin{pmatrix} \lambda_1 & \\ & \lambda_2 \end{pmatrix}=A,\ QA_2Q^{-1}=\begin{pmatrix} \lambda_3 & \\ & \lambda_4 \end{pmatrix}=B,$$

where $\lambda_1,\lambda_2,\lambda_3$ and λ_4 are the eigenvalues of systems (2.1) and (2.2), given in (2.5) and (2.6), respectively. It is easily to know that $\lambda_1>0,\lambda_2<0,\lambda_3<0$ and $\lambda_4<0$ for $d_2<\frac{c}{a}b<d=d_2+E$. From the subsystem A_1 of system (2.3), we can obtained that

$$X(t)=\mathrm{e}^{A_1(t-n\tau)}X(n\tau),t\in(n\tau,(n+l)\tau],n=0,1,2,\cdots, \tag{3.1}$$

where $X(n\tau)$ is the initial value of every period. Let $X_{n\tau}=X(n\tau)$. For $t=(n+l)\tau$, we have

$$X_{(n+l)\tau}=\mathrm{e}^{A_1l\tau}X_{n\tau}. \tag{3.2}$$

Similarly, we can obtain that

$$X(t)=\mathrm{e}^{A_2(t-(n+l)\tau)}X((n+l)\tau),t\in((n+l)\tau,(n+1)\tau],n=0,1,2,\cdots, \tag{3.3}$$

and for $t=(n+1)\tau$, it follows that

$$X_{(n+1)\tau}=\mathrm{e}^{A_2(1-l)\tau}X_{(n+l)\tau}=\mathrm{e}^{A_2(1-l)\tau}\mathrm{e}^{A_1l\tau}X_{n\tau}. \tag{3.4}$$

We can easily obtain that

$$\mathrm{e}^{A_2(1-l)\tau}\mathrm{e}^{A_1l\tau}=\mathrm{e}^{A_2(1-l)\tau+A_1l\tau}=\mathrm{e}^{A_2\tau+(A_1-A_2)l\tau}=\mathrm{e}^{A_2\tau}\mathrm{e}^{(A_1-A_2)l\tau}. \tag{3.5}$$

where

$$A_1-A_2=\begin{pmatrix} -a & b \\ c & -d_2 \end{pmatrix}-\begin{pmatrix} -a & b \\ c & -d_2-E \end{pmatrix}=\begin{pmatrix} 0 & 0 \\ 0 & E \end{pmatrix}.$$

Since $\mathrm{e}^{QA_2\tau Q^{-1}}=\mathrm{e}^{\mathrm{diag}(\lambda_3\tau,\lambda_4\tau)}=Q\mathrm{e}^{A_2\tau}Q^{-1}$, it follows that

$$\mathrm{e}^{A_2\tau}=Q^{-1}\begin{pmatrix} \mathrm{e}^{\lambda_3\tau} & \\ & \mathrm{e}^{\lambda_4\tau} \end{pmatrix}Q. \tag{3.6}$$

Let $g_1 = \frac{a+\lambda_3}{\lambda_3-\lambda_4}, g_2 = \frac{1}{\lambda_3-\lambda_4}$, then $1-g_1 = -\frac{a+\lambda_4}{\lambda_3-\lambda_4}$, and further Q^{-1} can be written as

$$Q^{-1} = \begin{pmatrix} -\frac{1}{c}(1-g_1) & -\frac{1}{c}g_1 \\ -g_2 & g_2 \end{pmatrix}.$$

Since $g_1 = g_2(a+\lambda_3), 1-g_1 = -g_2(a+\lambda_4)$, then it follows that

$$\begin{aligned} Q^{-1}\begin{pmatrix} \mathrm{e}^{\lambda_3\tau} & \\ & \mathrm{e}^{\lambda_4\tau} \end{pmatrix} Q &= \begin{pmatrix} -\frac{1}{c}(1-g_1) & -\frac{1}{c}g_1 \\ -g_2 & g_2 \end{pmatrix}\begin{pmatrix} \mathrm{e}^{\lambda_3\tau} & \\ & \mathrm{e}^{\lambda_4\tau} \end{pmatrix}\begin{pmatrix} -c & -(a+\lambda_3) \\ -c & -(a+\lambda_4) \end{pmatrix} \\ &= \begin{pmatrix} (1-g_1)\mathrm{e}^{\lambda_3\tau}+g_1\mathrm{e}^{\lambda_4\tau} & \frac{a+\lambda_3}{c}(1-g_1)\mathrm{e}^{\lambda_3\tau}+\frac{a+\lambda_4}{c}g_1\mathrm{e}^{\lambda_4\tau} \\ cg_2\mathrm{e}^{\lambda_3\tau}-cg_2\mathrm{e}^{\lambda_4\tau} & g_1\mathrm{e}^{\lambda_3\tau}+(1-g_1)\mathrm{e}^{\lambda_4\tau} \end{pmatrix} \\ &= \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}, \end{aligned} \tag{3.7}$$

where

$$\begin{aligned} b_{11} &= (1-g_1)\mathrm{e}^{\lambda_3\tau}+g_1\mathrm{e}^{\lambda_4\tau}, \\ b_{12} &= \frac{a+\lambda_3}{c}(1-g_1)\mathrm{e}^{\lambda_3\tau}+\frac{a+\lambda_4}{c}g_1\mathrm{e}^{\lambda_4\tau}, \\ b_{21} &= cg_2\mathrm{e}^{\lambda_3\tau}-cg_2\mathrm{e}^{\lambda_4\tau}, \\ b_{22} &= g_1\mathrm{e}^{\lambda_3\tau}+(1-g_1)\mathrm{e}^{\lambda_4\tau}. \end{aligned}$$

It can be verified that $b_{11} > 0, b_{12} > 0, b_{21} > 0$ and $b_{22} > 0$. From (3.5),(3.6) and (3.7), we can obtain that

$$\mathrm{e}^{A_2(1-l)\tau}\mathrm{e}^{A_1 l\tau} = Q^{-1}\begin{pmatrix} \mathrm{e}^{\lambda_3\tau} & \\ & \mathrm{e}^{\lambda_4\tau} \end{pmatrix} Q \begin{pmatrix} 1 & \\ & \mathrm{e}^{El\tau} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12}\mathrm{e}^{El\tau} \\ b_{21} & b_{22}\mathrm{e}^{El\tau} \end{pmatrix}. \tag{3.8}$$

Further, Eq.(3.4) can be rewritten as the following form.

$$\begin{pmatrix} x_{(n+1)\tau} \\ y_{(n+1)\tau} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12}\mathrm{e}^{El\tau} \\ b_{21} & b_{22}\mathrm{e}^{El\tau} \end{pmatrix}\begin{pmatrix} x_{n\tau} \\ y_{n\tau} \end{pmatrix}. \tag{3.9}$$

4. The stability of system (2.3)

Since the stability of system (2.3) can be determined by discussing the stability of the discrete system (3.9). Therefore, in the following, we will consider the stability of system (3.9) by analyzing Jury conditions. Let

$$L_1^* = \frac{1}{E\tau}\ln(\frac{1-b_{11}}{b_{22}+b_{12}b_{21}-b_{11}b_{22}}).$$

By calculating, we can obtain that

$$\begin{aligned}
1-b_{11} &= 1-((1-g_1)e^{\lambda_3\tau}+g_1e^{\lambda_4\tau}) = 1-e^{\lambda_3\tau}+g_1(e^{\lambda_3\tau}-e^{\lambda_4\tau}),\\
b_{12}b_{21} &= (\frac{a+\lambda_3}{c}(1-g_1)e^{\lambda_3\tau}+\frac{a+\lambda_4}{c}g_1e^{\lambda_4\tau})(cg_2e^{\lambda_3\tau}-cg_2e^{\lambda_4\tau})\\
&= g_2((a+\lambda_3)(1-g_1)e^{\lambda_3\tau}+(a+\lambda_4)g_1e^{\lambda_4\tau})(e^{\lambda_3\tau}-e^{\lambda_4\tau})\\
&= (g_1(1-g_1)e^{\lambda_3\tau}-(1-g_1)g_1e^{\lambda_4\tau})(e^{\lambda_3\tau}-e^{\lambda_4\tau})\\
&= g_1(1-g_1)(e^{\lambda_3\tau}-e^{\lambda_4\tau})^2,\\
b_{11}b_{22} &= [(1-g_1)e^{\lambda_3\tau}+g_1e^{\lambda_4\tau}][g_1e^{\lambda_3\tau}+(1-g_1)e^{\lambda_4\tau}],\\
&= g_1(1-g_1)e^{2\lambda_3\tau}+(g_1^2+(1-g_1)^2)e^{(\lambda_3+\lambda_4)\tau}+g_1(1-g_1)e^{2\lambda_4\tau},
\end{aligned}$$

and $b_{12}b_{21}-b_{11}b_{22} = -e^{(\lambda_3+\lambda_4)\tau}$. Further,

$$L_1^* = \frac{1}{E\tau}\ln(\frac{1-b_{11}}{b_{22}+b_{21}b_{12}-b_{11}b_{22}}) = \frac{1}{E\tau}\ln(\frac{1-e^{\lambda_3\tau}+g_1(e^{\lambda_3\tau}-e^{\lambda_4\tau})}{g_1e^{\lambda_3\tau}+(1-g_1)e^{\lambda_4\tau}-e^{(\lambda_3+\lambda_4)\tau}}).$$

Theorem 4.1. *The discrete system* (3.9) *is stable if* $l < L_1^*$ *and unstable if* $l > L_1^*$.

Proof. Let

$$C = \begin{pmatrix} b_{11} & b_{12}e^{El\tau} \\ b_{21} & b_{22}e^{El\tau} \end{pmatrix}.$$

Since $b_{11} > 0, b_{22} > 0$, then

$$\mathrm{tr}C = b_{11}+b_{22}e^{El\tau} > 0. \tag{4.1}$$

Since $b_{12}b_{21}-b_{11}b_{22} = -e^{(\lambda_4+\lambda_3)\tau}$, then

$$\det C = \begin{vmatrix} b_{11} & b_{12}e^{El\tau} \\ b_{21} & b_{22}e^{El\tau} \end{vmatrix} = (b_{11}b_{22}-b_{21}b_{12})e^{El\tau} = e^{(\lambda_4+\lambda_3)\tau}e^{El\tau} > 0. \tag{4.2}$$

The moduluses of two eigenvalues of system (3.9) are less than one if and only if all of the following Jury conditions are satisfied:

$$1-\mathrm{tr}C+\det C > 0,$$

$$1+\mathrm{tr}C+\det C>0,$$
$$1-\det C>0.$$

From (4.1) and (4.2), it is obvious that $1+\mathrm{tr}C+\det C>0$ always holds. Since $0<l<1$ and $\lambda_3+\lambda_4=-(a+d)=-(a+d_2+E)$, then $(\lambda_4+\lambda_3)\tau+El\tau=-(a+d_2+E)\tau+El\tau<0$. Therefore, we know from (4.2) that $\det C<1$ and $1-\det C=1-\mathrm{e}^{(\lambda_4+\lambda_3)\tau}\mathrm{e}^{El\tau}>0$ always hold.

In the following, we consider the sign of $1-\mathrm{tr}C+\det C$. Since

$$\begin{aligned}1-\mathrm{tr}C+\det C&=1-(b_{11}+b_{22}\mathrm{e}^{El\tau})+(b_{11}b_{22}-b_{21}b_{12})\mathrm{e}^{El\tau}\\&=1-b_{11}-(b_{22}+b_{12}b_{21}-b_{11}b_{22})\mathrm{e}^{El\tau},\end{aligned}$$

and

$$\begin{aligned}b_{22}+b_{12}b_{21}-b_{11}b_{22}=&g_1\mathrm{e}^{\lambda_3\tau}+(1-g_1)\mathrm{e}^{\lambda_4\tau}-\mathrm{e}^{(\lambda_3+\lambda_4)\tau}\\=&g_1(\mathrm{e}^{\lambda_3\tau}-\mathrm{e}^{\lambda_4\tau})+\mathrm{e}^{\lambda_4\tau}(1-\mathrm{e}^{\lambda_3\tau})>0,\end{aligned}$$

then $1-\mathrm{tr}C+\det C>0$ if $l<L_1^*$, further system (3.9) is stable $l<L_1^*$. Obviously, if $l>L_1^*$, then $1-\mathrm{tr}C+\det C<0$ and the modulus of at least one of the eigenvalues of system (3.9) is larger than one, further system (3.9) is unstable. This completes the proof.

Remark 4.1. Since $b_{22}+b_{12}b_{21}-b_{11}b_{22}>0$ and

$$\begin{aligned}&1-b_{11}-(b_{22}+b_{12}b_{21}-b_{11}b_{22})\\&\quad=1-\mathrm{e}^{\lambda_3\tau}+g_1(\mathrm{e}^{\lambda_3\tau}-\mathrm{e}^{\lambda_4\tau})-[g_1\mathrm{e}^{\lambda_3\tau}+(1-g_1)\mathrm{e}^{\lambda_4\tau}-\mathrm{e}^{(\lambda_3+\lambda_4)\tau}]\\&\quad=1-\mathrm{e}^{\lambda_3\tau}-\mathrm{e}^{\lambda_4\tau}+\mathrm{e}^{(\lambda_3+\lambda_4)\tau}\\&\quad=1-\mathrm{e}^{\lambda_3\tau}-\mathrm{e}^{\lambda_4\tau}(1-\mathrm{e}^{\lambda_3\tau})=(1-\mathrm{e}^{\lambda_3\tau})(1-\mathrm{e}^{\lambda_4\tau})>0,\end{aligned}$$

then

$$\frac{1-b_{11}}{b_{22}+b_{12}b_{21}-b_{11}b_{22}}>1,$$

which implies that there must exist some l such that

$$\mathrm{e}^{El\tau}<\frac{1-b_{11}}{b_{22}+b_{12}b_{21}-b_{11}b_{22}}$$

and further $1-\mathrm{tr}C+\det C>0$.

5. Qualitative analysis and positive periodic solution

Let

$$k_b = \frac{1}{2b}((a-d_2)+\sqrt{(a-d_2)^2+4bc}),\;\; k_c = \frac{1}{2b}((a-d)+\sqrt{(a-d)^2+4bc}).$$

It is easily verified that the line $l_b : y = k_b x$ is the asymptotic line of system (2.1). The asymptotic line l_b divides the first quadrant into two regions: $G_1^1 = \{(x,y)|y > l_b x, x > 0, y > 0\}$ and $G_1^2 = \{(x,y)|y < l_b x, x > 0, y > 0\}$(See Fig.2.1). Similarly, the line $l_c : y = k_c x$ is the asymptotic line of system (2.2) and also divides the first quadrant into two regions: $G_2^1 = \{(x,y)|y > l_c x, x > 0, y > 0\}$ and $G_2^2\{(x,y)|y < l_c x, x > 0, y > 0\}$.

The solutions of system (2.1) starting from G_1^1 will run along with the asymptotic line l_b, under the switching effect at the moment $(n+l)\tau$, the solutions will run along with the asymptotic line l_c of system (2.2). Since $k_c < k_b$, then the solutions will enter the region $G_1^2 \bigcap G_2^1$ after several switching and dwell there. Similarly, the solution starting from G_1^2 will also dwell in the region $G_1^2 \bigcap G_2^1$ after several switching. Therefore, after several switching, all the solutions of system (3.4) will enter the region $G_1^2 \bigcap G_2^1$, they either tend to or are away from the common equilibrium $O(0,0)$.

5.1. Asymptotic line of system (3.9)

In the following, we will consider the relations between (x_n, y_n) and (x_{n+1}, y_{n+1}). From system (3.9), it follows that

$$\begin{pmatrix} \Delta x_{(n+1)\tau} \\ \Delta y_{(n+1)\tau} \end{pmatrix} = \begin{pmatrix} x_{(n+1)\tau} - x_{n\tau} \\ y_{(n+1)\tau} - y_{n\tau} \end{pmatrix} = \begin{pmatrix} b_{11}-1 & b_{12}\mathrm{e}^{El\tau} \\ b_{21} & b_{22}\mathrm{e}^{El\tau}-1 \end{pmatrix} \begin{pmatrix} x_{n\tau} \\ y_{n\tau} \end{pmatrix},$$

we consider the asymptotic line of the discrete system (3.9). Denote the slope of the asymptotic line by $\bar{k}$, the slope $\bar{k}$ can be obtained by calculating

$$\frac{\Delta y_{(n+1)\tau}}{\Delta x_{(n+1)\tau}} = \frac{y_{(n+1)\tau} - y_{n\tau}}{x_{(n+1)\tau} - x_{n\tau}} = \frac{b_{21}x_{n\tau} + (b_{22}\mathrm{e}^{El\tau}-1)y_{n\tau}}{(b_{11}-1)x_{n\tau} + b_{12}\mathrm{e}^{El\tau}y_{n\tau}}.$$

When $n \to +\infty$, it follows that

$$\bar{k} = \frac{b_{21} + (b_{22}\mathrm{e}^{El\tau}-1)\bar{k}}{(b_{11}-1) + b_{12}\mathrm{e}^{El\tau}\bar{k}},$$

and

$$b_{12}\mathrm{e}^{El\tau}k^2+(b_{11}-b_{22}\mathrm{e}^{El\tau})k-b_{21}=0.$$

Further, we can obtain that

$$\begin{aligned}\bar{k}_1&=\frac{1}{2b_{12}\mathrm{e}^{El\tau}}\left(-(b_{11}-b_{22}\mathrm{e}^{El\tau})+\sqrt{(b_{11}-b_{22}\mathrm{e}^{El\tau})^2+4b_{12}\mathrm{e}^{El\tau}b_{21}}\right),\\ \bar{k}_2&=\frac{1}{2b_{12}\mathrm{e}^{El\tau}}\left(-(b_{11}-b_{22}\mathrm{e}^{El\tau})-\sqrt{(b_{11}-b_{22}\mathrm{e}^{El\tau})^2+4b_{12}\mathrm{e}^{El\tau}b_{21}}\right).\end{aligned}$$

It is easily to verify that $\bar{k}_1>0$ and $\bar{k}_2<0$. In the first quadrant, the points (x_n,y_n) will tend to or is away from (0,0) along with the line $y=\bar{k}_1x$ for $n\to\infty$. From

$$\begin{aligned}\bar{k}_1&=\frac{1}{2b_{12}\mathrm{e}^{El\tau}}\left(-(b_{11}-b_{22}\mathrm{e}^{El\tau})+\sqrt{(b_{11}-b_{22}\mathrm{e}^{El\tau})^2+4b_{12}\mathrm{e}^{El\tau}b_{21}}\right)\\ &=\frac{1}{2b_{12}}\left(-(b_{11}\mathrm{e}^{-El\tau}-b_{22})+\sqrt{(b_{11}\mathrm{e}^{-El\tau}-b_{22})^2+4b_{12}b_{21}}\right),\end{aligned}\tag{5.1}$$

we know that $\bar{k}_1$ is increasing as l increases.

5.2. The positive periodic solution

Theorem 5.1. *If system* (2.3) *has a positive periodic solution, then* $l=L_1^*$.

Proof. According to the theory of system of linear equations, we know that if Eq.(3.9) exists a positive fixed point, then the following equation must hold,

$$\left|\begin{pmatrix}1&0\\0&1\end{pmatrix}-\begin{pmatrix}b_{11}&b_{12}\mathrm{e}^{El\tau}\\b_{21}&b_{22}\mathrm{e}^{El\tau}\end{pmatrix}\right|=0.\tag{5.2}$$

By calculating, we have that

$$\left|\begin{pmatrix}1&0\\0&1\end{pmatrix}-\begin{pmatrix}b_{11}&b_{12}\mathrm{e}^{El\tau}\\b_{21}&b_{22}\mathrm{e}^{El\tau}\end{pmatrix}\right|=1-b_{11}-(b_{22}+b_{21}b_{12}-b_{11}b_{22})\,\mathrm{e}^{El\tau}.\tag{5.3}$$

Therefore, Eq.(5.2) holds if

$$\mathrm{e}^{El\tau}=\frac{1-b_{11}}{b_{22}+b_{12}b_{21}-b_{11}b_{22}}.\tag{5.4}$$

Further, if Eq.(3.9) exists a positive periodic solution, then $l = L_1^*$ must hold. This completes the proof.

Remark 5.1. From the qualitative analyses above, we know that all the solutions of system (3.4) will enter the region $G_1^2 \bigcap G_2^1$, they either tend to or are away from the equilibrium $O(0,0)$ after several switching. Therefore, it is possible that the solution starting from the point in the region $G_1^2 \bigcap G_2^1$ will be period if $l = L_1^*$.

6. Conclusions and discussions

Theorem 4.1 shows that system (2.3) is stable if $l < L_1^*$ and unstable if $l > L_1^*$. Take $b = 5, a = 0.9, c = 0.1, d_2 = 0.1, E = 0.5$, then $d = d_2 + E = 0.6$ and inequality (2.7) is satisfied. Further, we can calculate and obtain that $L_1^* \simeq 0.0863$. If we take $l = 0.07 < L_1^*$, then the numerical simulation can be seen in Fig.6.1 where the initial value is (7,8). Fig.6.1 gives the time series and phase portrait in 100 periods, and shows that the system is stable and the solution tends to $O(0,0)$. Fig.6.2 gives the time series and phase portrait in 50 periods for $l = 0.10 > L_1^*$ and shows that the system is unstable. From the phase portraits of Fig.6.1 and Fig.6.2, we can see that the solutions run in the region $G_1^2 \bigcap G_2^1$ where $G_1^2 = \{(x,y)|y < 0.2425x, x > 0, y > 0\}$ and $G_2^1 = \{(x,y)|y > 0.17457x, x > 0, y > 0\}$. The point (x_n, y_n) tends to or is away from $O(0,0)$ along with the asymptotic line $y = \bar{k}_1 x = 0.17464x$. These results above show that if the fishing moratorium $l\tau$ is less than the threshold $L_1^*\tau$, then the fishing ban is useless, the fish species does not adequately restore and the fish resource still tends to be exhausted. If the fishing moratorium $l\tau$ is larger than the threshold $L_1^*\tau$, then the population of the fish will be increasing although the activity of exploitation exists. Besides, from Eq.(5.1), we know that $\bar{k}_1$ is increasing as l increases, which implies that the amount of the fish species at the end of every period is increasing as the length of fishing moratorium increases.

From the view of mathematics, the existence of positive periodic solution is an important topic. Theorem 5.1 shows that l must be equal to L_1^* if system (2.3) exists a positive periodic solution. According to the analysis of the vector fields, it is possible that the periodic solution must exist in the region $G_1^2 \bigcap G_2^1$ if it exists. Fig.6.3 gives the times series and phase portraits of the solutions starting from the initial points (1,1), (5,0.873) and (8,1.3968), respectively, and

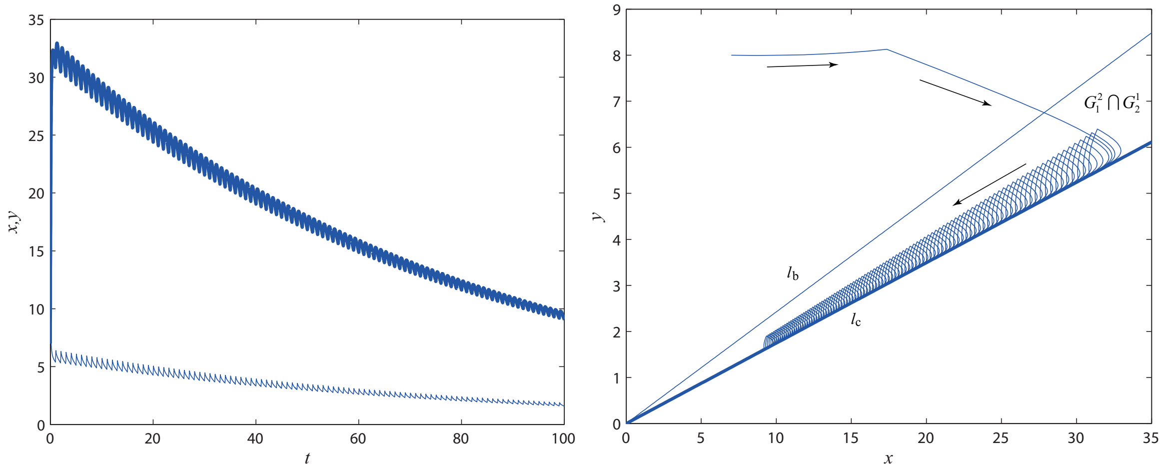

Fig.6.1. Time series and phase portrait of system (2.3) for $l = 0.07$

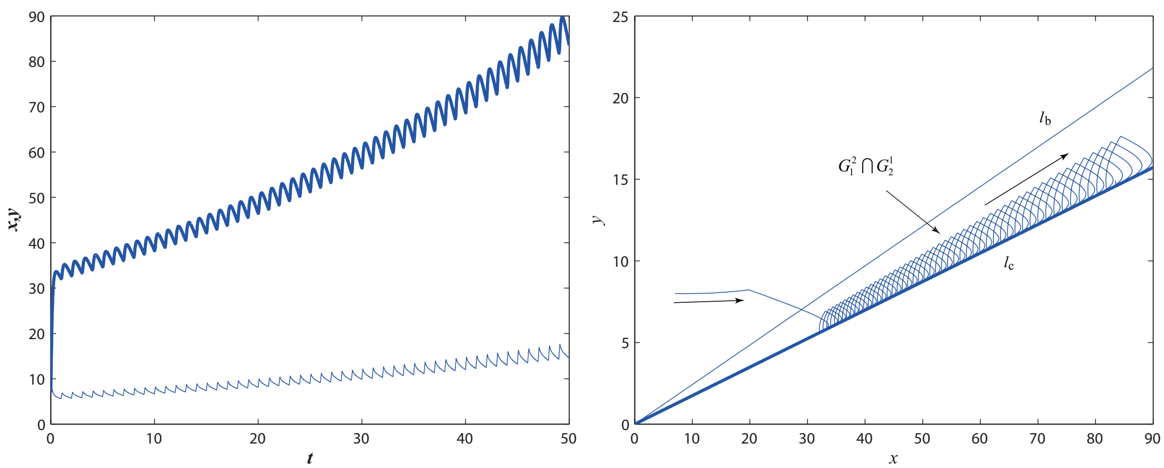

Fig.6.2. Time series and phase portrait of system (2.3) for $l = 0.10$

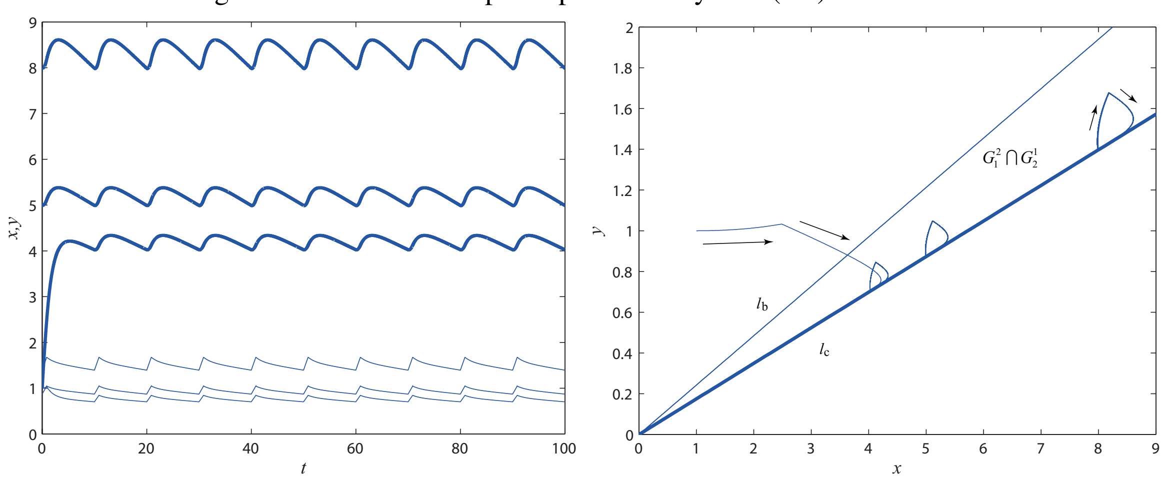

Fig.6.3. Time series and phase portrait of system (2.3) for $l = 0.87$

shows that the solutions starting from the points (5,0.873) and (8,1.3968) in the region $G_1^2 \bigcap G_2^1$ are periodic for $l = L_1^* \simeq 0.0863$. In the phase portrait of Fig.6.3, we can see that the trajectory starting from the point (1,1) enters the region $G_1^2 \bigcap G_2^1$ after one switching effect.

In practice, people always hope that they can get the maximum benefit by the minimum cost. In other words, the optimal problem is usually considered in the management strategy. It is well known that system (2.3) has different objective functions for different optimal problems. For example, we hope that, at the end of a period, the population of the fish $J_1 = y(\tau)$ is maximum, and the length of fishing moratorium $l\tau$ is minimum. In other words, $-l\tau$ is maximum, then the objective function can be written as $J = J_1 - l\tau = y(\tau) - l\tau$. Since $y(\tau) = b_{21}x_0 + b_{22}\mathrm{e}^{El\tau}y_0$, then

$$J = J_1 - l\tau = b_{21}x_0 + b_{22}\mathrm{e}^{El\tau}y_0 - l\tau. \tag{6.1}$$

Furthermore, from

$$\frac{\mathrm{d}J}{\mathrm{d}l} = E\tau b_{22}\mathrm{e}^{El\tau}y_0 - \tau = 0, \tag{6.2}$$

that is $Eb_{22}\mathrm{e}^{El\tau}y_0 - 1 = 0$ and $\mathrm{e}^{El\tau} = \frac{1}{Eb_{22}y_0}$, we know that $l = \frac{1}{E\tau}\ln\frac{1}{Eb_{22}y_0} := L_2^*$. It shows that if the solution of optimal problem (6.1) exists, then $Eb_{22}y_0 < 1$ and $l = L_2^*$, but it depends on the initial value y_0.

Of course, there are still other optimal problems. For example, the objective function is $J = \| X(\tau) \| - l\tau$, which implies that the modulus of $X(\tau)$ is maximum and the time length of fishing moratorium $l\tau$ is minimum. But there are some troubles to discuss the existence and the necessary conditions of the optimal solution.

Conflict of Interests

The authors declare that there is no conflict of interests.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (11371306, 11171284) and the Program for Science and Technology Innovation Talents in Universities of Henan Province (14HASTIT046).

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