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EFFECT OF INSECTICIDE SPRAYING ON *JATROPHA CURCAS* PLANT TO CONTROL MOSAIC VIRUS: A MATHEMATICAL STUDY

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Abstract. *Jatropha curcas L.* is one of the wonder plant with a variety of applications and economic potentiality. Biodiesel, an alternative fuel from non-edible oil of *Jatropha curcas* plants, is significantly considered as an alternative fuel to diesel oil as it has the anticipated physicochemical and environment friendly characteristics compared to diesel. To get oil from *Jatropha curcas* plant quantitatively and qualitatively, proper plantation and protection of this plant, mainly from virus carrying vector (white-fly), is essential. In this research article, an epidemic model is formulated for *Jatropha curcas* plant, which describes the vector-borne diseases with the aim to control the effect of vectors i.e., white-fly on the spread of mosaic disease. The reliability of the mathematical model is established by sensitivity analysis. Here, spraying of an insecticide (Insecticidal Soap) is considered to prevent vector white-fly in two avenues viz. continuous and impulsive strategy. The objective is to find the suitable and effective method that can serve as an integrating measure to identify and design appropriate plant disease control strategies. Numerical simulations are employed to support the analytical results.

Keywords: Jatropha curcas; Mosaic virus; White-fly; Insecticidal soap; Impulsive control approach; Continuous control approach.

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1. Introduction

Jatropha (*Jatropha curcas L.*), known as physic nut, is a drought resistant perennial plant, which is popularly cultivated in the tropics as a living fence. The tree is of significant economic importance for its numerous industrial and medicinal uses. The oil extracted from Jatropha seeds is being used as bio-fuel for diesel engines. Thus Jatropha has a great potential to contribute to the renewable energy source [1, 2, 3]. In India, the area under the cultivation of Jatropha is increasing in recent years with the ever increasing demand for fossil fuels that are exhausting at a rapid rate. Jatropha suffers from several fungal and bacterial diseases and more recently by the Jatropha Mosaic India Virus (JMIV), which causes Jatropha Mosaic Disease (JMD) [4, 5, 6].

Mosaic virus is one type of plant virus that causes the leaves of plants with a spotted and speckled look. They move frequently in the environment. The mosaic virus spreading mainly depends on the vector white-fly. Virus frequency is raised whenever the plants are growing robustly [7]. Therefore, the spread of the virus is highly dependent basically on the plant density. A single white-fly is adequate to infect the host plants but transmission of the disease is scattered when numerous infected white-flies feed on the host plants through massive flux of saliva [8]. Thus, the host plants (*Jatropha curcas*) are facing a lot of difficulties during such feeding. Feeding of white-flies comprises of leaf damage and sap drainage. White-flies are tremendously productive, if once they get conventional on any part of the plants around the home or garden, they will voluntarily roam and try to attack any other immediate vegetation [9, 10]. Normally white-fly needs three hours feeding time to procure the virus and a latent phase of eight hours. It requires ten minutes time to contaminate the young leaves. Symptoms seem to be appeared after a latent period of three to five weeks [7].

There are several constituents that can be sprayed over white-flies to control its population. The Insecticidal Soap belongs to that group of organic insecticide. It is too safe and harmless that one can easily eat the fruits or vegetables on which the soap has been sprayed. It performs in many avenues to control or block the spreading of the white-fly population. It breaks the gatherings from the laying more amounts of eggs. It also prevents adults from flying that helps for not migrating to the neighbouring plants [5, 11].

Plant virus causes serious losses in yield and quality of cultivated plants and therefore, plant diseases are an important constraint to crop production. Epidemiological information can be used to build up an effective integrated disease management strategies for various situations. Huge population or intense feeding by white-flies can harm plants causing Pigment dysfunction in leaves and death to the host plants [12].

Mathematical models of plant-virus epidemics are developed to provide detailed explanation on how to describe, analyze, and predict epidemics of plant disease for the ultimate purposes of developing and testing control strategies and tactics for crop protection [13, 14]. In recent times, mathematical modeling and analysis of vector-borne plant diseases have attracted the interest of many researchers. For instance, Bosch and Jeger have researched plant virus characteristics and population dynamics and they have analyzed the dynamics of virus plant diseases [15]. Grill has discussed the influence of the timings of insects mediators feeding on plant virus' infection rate [16]. Jeger et al. have presented some control strategies and they pointed out that biological control method has become an important part of the integrated pest management [17].

Control strategy to prevent diseases of plant has been considered by many researchers through mathematical modeling. The selection and application of a wide range of control strategies are adopted that minimize losses and maximize yield. Gilligan et al. considered the effect of biological control on soil-borne plant pathogens. An antagonist is included in their model to control plant diseases. They obtained invasion criteria for all species [18]. A cultural control approach including replanting and removing of diseased plants is a widely accepted treatment policy for plant epidemics. It involves periodic inspections that is followed by removal of the detected infected plants [17, 19].

In this article, a mathematical model is formulated to analyze the dynamics of the epidemics of *Jatropha curcas* plant. The insecticide, Insecticidal Soap is sprayed on the host plants in two different modes. We investigate both the continuous spraying and the impulsive control approach. A comparative analysis between two different approaches has been analyzed to find the most effective method to control the vector white-flies. Analytical outcomes are supported numerically through Matlab.

2. The mathematical model

The following assumption are made to formulate the mathematical model. The model, we analyze in this paper, has four population densities, viz.

(i) The healthy Jatropha plant, $x(t)$,

(ii) The latent plant, $l(t)$,

(iii) The infected plant, $y(t)$,

(iv) The healthy vector (white-fly), $u(t)$ and.

(v)The infected vector, v(t).

The mosaic virus is carried through the vector white-flies. After being penetrated of mosaic virus, the vectors are infected. When this infected vectors come in touch with Jatropha plants, the virus enters into the plants. As a result, the plants become infected after the latent period. So the disease is mainly transmitted through the infected white-flies. Thus we are not taking into consideration the virus population into our mathematical model. When we spray the insecticide in to the environment, it affects both non-infected and infected population simultaneously because we are unable to distinguish between two kinds of vector population.

Logistic fashion in the growth equation of healthy plants is taken as the plants can not grow unboundedly. Here, r is the maximum plantation rate and k is the maximum plant density i.e., the carrying capacity of the plants. Infected vectors interacts with healthy plants at a rate k_1. The rate of recovery of latent class of Jatropha plant is denoted by δ. Recovery may occur with very low vector population in specific biological situation. Various environmental factors like climatic conditions, water and nutrient availability, biotic and abiotic stresses etc. can play a significant role in plant recovery [20, 21, 22]. The susceptible plants can be infected not only by the infected vectors but also by the infected plants. A susceptible vector can be infected only by an infected plant host and after it is infected, it will hold the virus for rest of its life. It is considered that there is no vertical infection. The recruitment rate of insect vectors is a positive constant and all of the new born vectors are susceptible. The rate of transfer of latent plants to the infected state is a. Cutting of infected plants is considered at a rate g. The rate of normal plant loss is treated as β.

Based on the above assumptions, the following rate equations are obtained:

$$\begin{aligned}\frac{dx}{dt} &= rx[1-\frac{x+l+y}{k}]-k_1xv+\delta l,\\ \frac{dl}{dt} &= k_1xv-al-\delta l,\\ \frac{dy}{dt} &= al-gy-\beta y.\end{aligned} \tag{2.1}$$

Logistic type growth is assumed in the non-infected vectors white-fly with b as the maximum vector birth rate and m as the maximum vector abundance per plant as the population do not grow unboundedly. The interaction between infected plants and non-infected vectors is also considered. The non-infected vector population is reduced with k_2 as the interaction rate between infected plant and non-infected vector. Finally, c is considered as the vector mortality rate for both non-infected and infected vectors.

Based on the above assumptions, the complete mathematical model is obtained as below:

$$\begin{aligned}\frac{dx}{dt} &= rx[1-\frac{x+l+y}{k}]-k_1xv+\delta l,\\ \frac{dl}{dt} &= k_1xv-al-\delta l,\\ \frac{dy}{dt} &= al-gy-\beta y,\\ \frac{du}{dt} &= b(u+v)[1-\frac{u+v}{m(x+l+y)}]-k_2yu-cu,\\ \frac{dv}{dt} &= k_2yu-cv,\end{aligned} \tag{2.2}$$

with initial condition:

$$x(0)>0, l(0)>0, y(0)>0, u(0)>0, v(0)>0. \tag{2.3}$$

3. Boundedness of the System

Lemma 3.1 Let us assume that $W_1(x(t),l(t)) = x(t)+l(t)$. Then for all $t>0$, $W_1 \leq M_1$, where $M_1 = \frac{k(a+r)^2}{4r} + W_1(x(0),l(0))e^{-at}$ is a positive constant. Thus $x(t)$ and $l(t)$ are bounded. Hence y(t) is also bounded for all $t>0$.

Proof: Let, $(x(t),l(t),y(t),u(t),v(t))$ be any solution of the system (2.2). Let, $W_1(x(t),l(t)) = x(t)+l(t)$. Then, from first and second equation of system (2.2), we have

$$\begin{aligned} \frac{dW_1}{dt} &= rx(1-\tfrac{x+l+y}{k}) - k_1xv + \delta l + k_1xv - al + \delta l, \\ &= rx(1-\tfrac{x+l+y}{k}) - al, \\ &= rx(1-\tfrac{W_1+y}{k}) - al, \\ &= rx(1+\tfrac{a}{r}-\tfrac{W_1+y}{k}) - a(x+l). \end{aligned} \tag{3.1}$$

Therefore,

$$\begin{aligned} \frac{dW_1}{dt} + aW_1 &= rx(1+\tfrac{a}{r}-\tfrac{W_1+y}{k}), \\ &\le rW_1(1+\tfrac{a}{r}-\tfrac{W_1}{k}). \end{aligned} \tag{3.2}$$

By comparison theorem, $W_1 \le \frac{k(a+r)^2}{4r} + W_1(x(0),l(0))e^{-at}$.

Now, l(t) is bounded, then there exists a positive constant M_1 such that $l \le M_1$.

Now,

$$\begin{aligned} & \frac{dy}{dt} = al - gy - \beta y, \\ \Rightarrow \quad & \frac{dy}{dt} \le aM_1 - (g+\beta)y, \\ \Rightarrow \quad & \frac{dy}{dt} + (g+\beta)y \le aM_1, \end{aligned}$$

By comparison theorem,

$$y \le \frac{aM_1}{g+\beta} + y(0)e^{-(g+\beta)t}. \tag{3.3}$$

Therefore, y(t) is bounded.

Lemma 3.2

Let $W_2(u(t),v(t)) = u(t)+v(t)$. Then for all $t>0$, $W_1 \le M_2$, where $M_2 = \frac{bmM}{4} + W_2(u(0),v(0))e^{-ct}$, which is a positive constant. Hence $u(t)$ and $v(t)$ are bounded for all $t>0$.

Proof: Let, $(x(t),l(t),y(t),u(t),v(t))$ be any solution of the system (2.2). Since $x(t),l(t),y(t)$ are bounded, so $W_3(x(t),l(t),y(t)) = x(t)+l(t)+y(t)$ is also bounded. Thus there exists a positive constant M such that $W_3 \le M$. Now, $W_2(u(t),v(t)) = u(t)+v(t)$.

Again, from system (2.2) we have:

$$\frac{dW_2}{dt} = b(u+v)[1-\frac{u+v}{m(x+l+y)}] - k_2yu - cu + k_2yu - cv,$$
$$\Rightarrow \quad \frac{dW_2}{dt} = bW_2(1-\frac{W_2}{mW_3}) - cW_2,$$

Thus,

$$\frac{dW_2}{dt} + cW_2 = bW_2(1-\frac{W_2}{mW_3}),$$
$$\frac{dW_2}{dt} + cW_2 \leq bW_2(1-\frac{W_2}{mM}) = \frac{bW_2}{mM}(mM - W_2).$$

By comparison theorem we have,

$$W_2(u(t),v(t)) \quad \leq \quad \frac{bmM}{4} + W_2(u(0),v(0))e^{-ct}.$$

Thus, $W_2(u(t),v(t)) \to \frac{bmM}{4}$ as $t \to \infty$ implies that $W(t)$ is bounded. Thus, $u(t)$ and $v(t)$ are bounded.

Theorem 3.1

All solutions of the system (2) that start in R^5_+ are uniformly bounded.

Proof: The proof directly follows from **Lemma 3.1** and **Lemma 3.2**.

4. The System with Continuous Spraying

Now, we observe the effect of optimal control i.e., continuous spraying strategy on vector population. Spraying of insecticide can reduce both the uninfected and infected vector population. We consider γ as control parameter due to spraying i.e. strength of spraying. Introducing continuous spraying, the system (2.2) becomes:

$$\begin{aligned}
\frac{dx}{dt} &= rx[1-\frac{x+l+y}{k}] - k_1xv + \delta l,\\
\frac{dl}{dt} &= k_1xv - al - \delta l,\\
\frac{dy}{dt} &= al - gy - \beta y,\\
\frac{du}{dt} &= b(u+v)[1-\frac{u+v}{m(x+l+y)}] - k_2yu - cu - \gamma u,\\
\frac{dv}{dt} &= k_2yu - cv - \gamma v,
\end{aligned} \tag{4.1}$$

with initial values $x(0) > 0$, $l(0) > 0$, $y(0) > 0$, $u(0) > 0$ and $v(0) > 0$.

Dynamics of the System

The variational matrix for the equilibrium point $E^*(x^*, l^*, y^*, u^*, v^*)$ of the system (4.1) is given by,

$$J_{E^*} = \begin{bmatrix} r(1-\frac{2x^*+l^*+y^*}{k}) & -\frac{rx^*}{k}+\delta & -\frac{rx^*}{k} & 0 & -k_1x^* \\ k_1v^* & -(a+\delta) & 0 & 0 & k_1x^* \\ 0 & a & -(g+\beta) & 0 & 0 \\ B & B & B-k_2u^* & A & C \\ 0 & 0 & k_2u^* & k_2y^* & -(c+\gamma) \end{bmatrix},$$

where,

$$\begin{aligned} A &= b[1-\tfrac{2(u^*+v^*)}{m(x^*+l^*+y^*)}]-k_2y^*-(c+\gamma), \\ B &= \tfrac{(u^*+v^*)^2}{(x^*+l^*+y^*)^2}, \text{ and } C = A+k_2y^*+(c+\gamma). \end{aligned} \tag{4.2}$$

Now, the *Lyapunov function* is considered as follows:

$$\psi(x,l,y,u,v) = \tfrac{1}{2}(c_1x^2+c_2l^2+c_3y^2+c_4u^2+c_5v^2),$$

where, $c_i > 0$; $i = 1,2,3,4,5$, is to be chosen suitably. Obviously, ψ is positive definite. Derivative of ψ along the solution of the equation $\dot{X}(t) = J_{E^*}X(t)$, where $X(t) = (x(t), l(t), y(t), u(t), v(t))^T$ is as follows:

$$\begin{aligned} \dot{\psi} &= c_1x\dot{x}+c_2l\dot{l}+c_3y\dot{y}+c_4u\dot{u}+c_5v\dot{v}, \text{i.e.} \\ \dot{\psi} &= c_1r(1-\tfrac{2x^*+l^*+y^*}{k})x^2-c_2(a+\delta)l^2-c_3(g+\beta)y^2+c_4Au^2-c_5(c+\gamma)v^2 \\ &\quad +[c_2k_1v^*-c_1l(\tfrac{rx^*}{k}-\delta)]xl-\tfrac{c_1rx^*}{k}xy+c_4Bxu-c_1k_1x^*xv+c_3ayl+c_4Blu \\ &\quad +c_2k_1x^*lv+c_4(B-k_2u^*)uy+c_5k_2u^*vy+[c_4(A+k_2y^*+(c+\gamma))+c_5k_2y^*]uv. \end{aligned}$$

Thus, symmetric matrix corresponding to $\dot{\psi}$ is given as:

$$M = \frac{1}{2}\begin{bmatrix} m_{11} & c_2k_1v^*-c_1l(\frac{rx^*}{k}-\delta) & \frac{-c_1rx^*}{k} & c_4B & -c_1k_1x^* \\ m_{21} & -2c_2(a+\delta) & c_3a & c_4B & c_2k_1x^* \\ \frac{-c_1rx^*}{k} & c_3a & -2c_3(g+\beta) & c_4(B-k_2u^*) & c_5k_2u^* \\ c_4B & c_4B & c_4(B-k_2u^*) & 2c_4A & P \\ -c_1k_1x^* & c_2k_1x^* & c_5k_2u^* & P & -2c_5(c+\gamma) \end{bmatrix},$$

where,

$$m_{11} = 2c_1r(1-\frac{2x^*+l^*+y^*}{k}),\ m_{21}+c_2k_1v^*-c_1l(\frac{rx^*}{k}-\delta),$$
$$P = c_4(A+k_2y^*+(c+\gamma))+c_5k_2y^*.$$

The above matrix, M, is a symmetric matrix. Now, the system is stable, if the matrix is negative definite. The matrix is negative definite if determinants of all its principal minors are alternatively negative and positive, i.e., if

$$2c_1r(1-\frac{2x^*+l^*+y^*}{k}) < 0, \tag{4.3}$$
$$-4c_1c_2r(a+\delta)(1-\frac{2x^*+l^*+y^*}{k})-[c_2k_1v^*-c_1l(\frac{rx^*}{k}-\delta)]^2 > 0, \tag{4.4}$$
$$P_1 < 0, \tag{4.5}$$
$$2c_4AP_1-c_4(B-k_2u^*)A^3_{124}+c_4BA^3_{134}-c_4BA^3_{234} > 0, \tag{4.6}$$
$$\begin{aligned}&-2c_5(c+\gamma)P_2-[c_4(A+k_2y^*+(c+\gamma))+c_5k_2y^*]A^4_{1235}+c_5k_2u^*A^4_{1245}\\&-c_2k_1x^*A^4_{1345}-c_1k_1x^*A^4_{2345} < 0,\end{aligned} \tag{4.7}$$

where,

$$\begin{aligned}P_1 \;&=\; 2c_1r(1-\frac{2x^*+l^*+y^*}{k})[4c_2c_3(a+\delta)(g+\beta)-(c_3a)^2]-[c_2k_1v^*-c_1l(\frac{rx^*}{k}-\delta)]\\&\quad[-(c_2k_1v^*-c_1l(\frac{rx^*}{k}-\delta))2c_3(g+\beta)+c_3a\frac{c_1rx^*}{k}]-\frac{c_1rx^*}{k}[(c_2k_1v^*-c_1l(\frac{rx^*}{k}-\delta)c_3a\\&\quad-2c_1c_2\frac{rx^*}{k}(a+\delta)],\\P_2 \;&=\; 2c_4AP_1-c_4(B-k_2u^*)A^3_{124}+c_4BA^3_{134}-c_4BA^3_{234},\end{aligned}$$

A^3_{124} = Minor with respect to the element (4,3) of $matrix\ M_1$,

A^3_{124} = Minor with respect to the element (4,2) of $matrix\ M_1$,

A^3_{234} = Minor with respect to the element (4,1) of $matrix\ M_1$,

A^4_{1235} = Minor with respect to the element (5,4) of $matrix\ M$,

A^4_{1245} = Minor with respect to the element (5,3) of $matrix\ M$,

A^4_{1345} = Minor with respect to the element (5,2) of $matrix\ M$,

A^4_{2345} = Minor with respect to the element (5,1) of $matrix\ M$,

where, M_1 =

$$\frac{1}{2}\begin{bmatrix} 2c_1r(1-\frac{2x^*+l^*+y^*}{k}) & c_2k_1v^*-c_1l(\frac{rx^*}{k}-\delta) & \frac{-c_1rx^*}{k} & c_4B \\ c_2k_1v^*-c_1l(\frac{rx^*}{k}-\delta) & -2c_2(a+\delta) & c_3a & c_4B \\ \frac{-c_1rx^*}{k} & c_3a & -2c_3(g+\beta) & c_4(B-k_2u^*) \\ c_4B & c_4B & c_4(B-k_2u^*) & 2c_4A \end{bmatrix}.$$

Now, we choose c_1, c_2, c_3, c_4, c_5 such that

$$\begin{aligned} c_2k_1v^*-c_1l(\tfrac{rx^*}{k}-\delta) &= 0, \\ c_4(A+k_2y^*+(c+\gamma))+c_5k_2y^* &= 0. \end{aligned} \tag{4.8}$$

From (4.3) and (4.4), we get $k < 2x^*+l^*+y^*$.

Now we choose c_1 and c_2 from (10) as

$$\begin{aligned} c_1 &= k_1v^* = m_3 \text{ (say)}, \\ c_2 &= l(\tfrac{rx^*}{k}-\delta) = m_1 \text{ (say)} \end{aligned} \tag{4.9}$$

It is also clear from (4.3) and (4.5) that (4.6) holds if

$$4c_2(a+\delta)(g+\beta)-c_3a^2 > 0. \tag{4.10}$$

Putting the value of c_2 in (4.10), the following relation can be obtained,

$$0 < c_3 < \frac{4l(\frac{rx^*}{k}-\delta)(a+\delta)(g+\beta)}{a^2}. \tag{4.11}$$

Now, c_3 is chosen from the equation (4.11) as below,

$$c_3 = \frac{2l(\frac{rx^*}{k}-\delta)(a+\delta)(g+\beta)}{a^2} = m_2 \text{ (say)}, \tag{4.12}$$

and c_4 and c_5 are chosen from condition (4.12).

From the the above discussion, we get following conditions:

$$\begin{aligned} &(1) \quad k < 2x^*+l^*+y^*, \\ &(2) \quad 2rm_2(1-\tfrac{2x^*+l^*+y^*}{k})[4m_1(a+\delta)(g+\beta)-m_2a^2]+m_1m_3(\tfrac{rx^*}{k})^2(a+\delta) < 0, \\ &(3) \quad 2c_4AP_1-c_4(B-k_2u^*)A^3_{124}+c_4BA^3_{134}-c_4BA^3_{234} > 0, \\ &(4) \quad -2c_5(c+\gamma)P_2-[c_4(A+k_2y^*+(c+\gamma))+c_5k_2y^*]A^4_{1235} \\ &\qquad +c_5k_2u^*A^4_{1245}-c_2k_1x^*A^4_{1345}-c_1k_1x^*A^4_{2345} < 0. \end{aligned} \tag{4.13}$$

Theorem 4.1 If the conditions, given in (4.13), hold then the system is locally asymptotically stable around the interior equilibrium $E^*(x^*, l^*, y^*, u^*, v^*)$.

5. Impulsive Control Approach

The virus population is not taken into concern for our mathematical model as the mosaic virus is carried by the vector white-flies. Simultaneously, we spray the insecticide Insecticidal Soap on the host plant to reduce the white-fly population. We introduce the impulsive method for spraying to control the vector population (white-fly) to protect the natural resource *Jatropha curcas* plants. Thus, we have the following impulsive system:

$$\begin{aligned}
\tfrac{dx}{dt} &= rx[1-\tfrac{x+l+y}{k}] - k_1xv + \delta l,\ t \neq nT \\
\tfrac{dl}{dt} &= k_1xv - al - \delta l,\ t \neq nT \\
\tfrac{dy}{dt} &= al - gy - \beta y,\ t \neq nT \\
\tfrac{du}{dt} &= b(u+v)[1-\tfrac{u+v}{m(x+l+y)}] - k_2yu - cu,\ t \neq nT \\
\tfrac{dv}{dt} &= k_2yu - cv,\ t \neq nT,
\end{aligned} \tag{5.1}$$

where,

$$\begin{aligned}
\Delta x(t) &= 0,\ t = nT \\
\Delta l(t) &= 0,\ t = nT \\
\Delta y(t) &= 0,\ t = nT \\
\Delta u(t) &= -s,\ t = nT \\
\Delta v(t) &= -s,\ t = nT
\end{aligned} \tag{5.2}$$

and

$$\begin{aligned}
\Delta x(t) &= x(t^+) - x(t), \\
\Delta l(t) &= l(t^+) - l(t), \\
\Delta y(t) &= y(t^+) - y(t), \\
\Delta u(t) &= u(t^+) - u(t), \\
\Delta v(t) &= v(t^+) - v(t).
\end{aligned} \tag{5.3}$$

We assume w as the sum of non-infected and infected vectors (white-fly), i.e., $u+v=w$ and we also denote the sum of healthy, latent class and infected Jatropha Curcas plants as N, i.e., $x+l+y=N$. The one-dimensional impulsive differential equation takes the form:

$$\begin{aligned}&\tfrac{dw}{dt}=bw(1-\tfrac{w}{mN})-cw(t), t\neq nT,\\&\Delta w=-s,\ t=nT.\end{aligned}$$

The number of vector in the environment is reduced at a rate s at $t=nT$, $n\in N=0,1,2,...$, where T is the period of the impulsive control.

We consider the following sub-system given below:

$$\begin{aligned}&\tfrac{dw(t)}{dt}=-bw,\ t\neq nT,\\&w(t^+)=w(t)-s,\\&w(0^+)=w_0.\end{aligned}\tag{5.4}$$

Thus the following **Lemma** holds.

Lemma For a positive periodic solution $w^*(t)$ of system (5.4) and the solution $w(t)$ of system (5.4) with initial value $w_0=w(0^+)\geq 0$, $\mid w(t)-w^*(t)\mid\to 0$ as $t\to\infty$, where

$$\begin{aligned}&w^*(t)=\big(\tfrac{-s\,\exp(-b(t-nT))}{1-\exp(-bT)}\big),\ t\in(nT,(n+1)T],\\&w^*(0^+)=\big(\tfrac{-s}{1-\exp(-bT)}\big),\\&w(t)=\big(w(0^+)-\big(\tfrac{-s}{1-\exp(-bT)}\big)\big)\exp(-bT)+w^*(t).\end{aligned}$$

Theorem 5.1

The periodic solution $(x^*(t),0,0,w^*(t))$ is locally stable if $T<\frac{2q}{k}$, where $q=\int_0^T x^*(t)dt$.

Proof: The solution $(x^*(t),0,0,0,w^*(t))$ the system (5.1),

where $w^*(t)=\big(\frac{-s\,\exp(-b(t-nT))}{1-\exp(-bT)}\big)$, $t\in(nT,(n+1)T]$ with initial condition $w^*(0^+)$ as of above

Lemma. Variational matrix at $(x(t),0,0,0,w(t))$ is given by,

$$J(t) = \begin{pmatrix} r-\frac{2rx^*(t)}{k} & a_1 & 0 & 0 \\ 0 & -a-\delta & 0 & 0 \\ 0 & a_2 & -g-\beta & 0 \\ 0 & 0 & 0 & -b \end{pmatrix},$$

Where, a_1 and a_2 are some constants. The monodromy matrix $M(t)$ of variational matrix $J(t)$ is $M(T) = Ie^{\int_0^T X_1(t)dt}$, where I is the identity matrix and $M(T)$ is given by:

$$\begin{pmatrix} \exp[rT-\int_0^T \frac{2rx^*(t)}{k}dt] & 0 & 0 & 0 \\ 0 & \exp[-aT-\delta T] & 0 & 0 \\ 0 & 0 & \exp[-gT-\beta T] & 0 \\ 0 & 0 & 0 & \exp(-bT) \end{pmatrix},$$

If the absolute values of all eigenvalues are less than one, then the periodic solution $(x^*(t),0,0,w^*(t))$ is locally stable. If we denote the eigenvalues of J by $\lambda_1,\lambda_2,\lambda_3$ and λ_4, then $\lambda_1 = \exp(rT - \frac{2r}{k}\int_0^T x^*(t)dt)$, $\lambda_2 = \exp[-(a+\delta)T] < 1$, $\lambda_3 = \exp[-(g+\beta)T] < 1$ and $\lambda_4 = \exp(-bT) < 1$.

According to the Floquet theory [32] of impulsive differential equations, the periodic solution $(x^*(t),0,0,w^*(t))$ is locally stable if $|\lambda_1| < 1$ i.e., when $T < \frac{2q}{k}$, and $q = \int_0^T x^*(t)dt$. Hence the theorem.

6. Sensitivity Analysis of the Model Parameters

The reliability of a mathematical model is reflected in the numerical values of the sensitivity coefficients. A model must be sensitive to large (relative to typical experimental error) changes in parameter values. Otherwise, a wide range of values will produce substantially the same behavior. It will not be possible to verify the correct parameter values that have been used in the simulation. Thus, the structure of the model will be suspected. Small errors in the parameter values will produce large supplementary motions and the model will not be testable. Thus, the predictions of the model will not be reliable [24, 25, 26]. If a model has a species x and two parameters y and z, the time-dependent sensitivities of x with respect to each parameter value are the time-dependent derivatives: $\frac{\partial x}{\partial y}$ and $\frac{\partial x}{\partial z}$ respectively. The numerator is the sensitivity

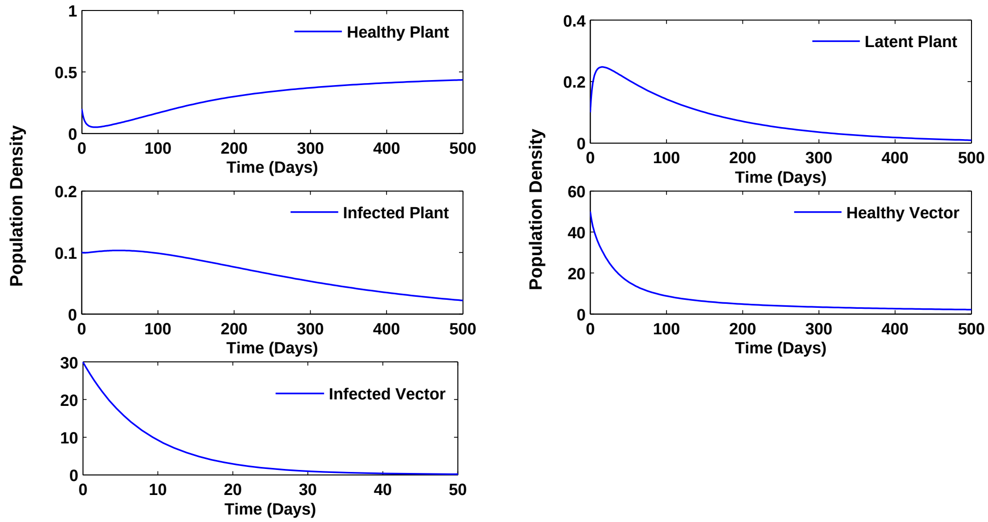

FIGURE 1. Population densities are plotted as a function of time using the values of the parameters as in Table. We consider $x(0) = 0.2\ m^{-2},\ l(0) = 0.1\ m^{-2}\ y(0) = 0.1\ m^{-2},\ u(0) = 150\ plant^{-1},\ v(0) = 50\ plant^{-1}$ as initial conditions.

Table: Values of the parameters used in the model equation [27, 28].

Parameters	Definition	Default Values Assigned
r	The maximum rate of plantation	0.05 Day^{-1}
k	The plant density	0.5 Metre^{-2}
k_1	The infection rate	0.001 Vector^{-1} Day^{-1}
δ	The rete of recovery	8.5410^{-4} Day^{-1}
a	The rate of transfer to the infected state	0.5 Day^{-1}
g	The rate of cutting	0.03 Day^{-1}
β	The rate of plant loss	0.003 Day^{-1}
b	The rate of maximum vector birth	0.8 Day^{-1}
k_2	Uninfected Vector and infected plant interaction rate	0.008 Plant^{-1} Day^{-1}
m	Maximum vector abundance	300 Plant^{-1}
c	The vector mortality rate	0.12 Day^{-1}

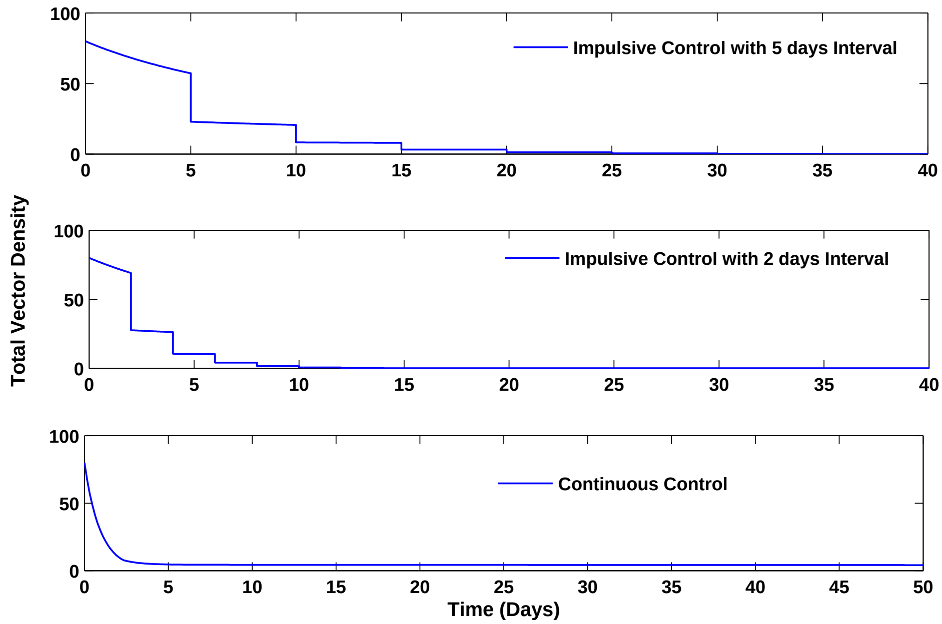

FIGURE 2. Comparative representation between impulsive control and continuous control.

output and the denominator is the sensitivity input to sensitivity analysis.

Sensitivity analysis is supported only by the ordinary differential equation (ODE) solvers. The software calculates local sensitivities by combining the original ODE system with the auxiliary differential equations for the sensitivities. The additional equations are derivatives of the original equations with respect to parameters. This method is sometimes called "Forward Sensitivity Analysis" or "Direct Sensitivity Analysis".

7. Numerical Simulation

In this section, we perform the different figures of the models based on analytical calculation. The Values of parameters in Table 1 are mainly taken from Holt et al. [28]. We use the fact that the viruses that attack the Cassava plants and the Jatropha plants have been found to be almost identical in nature. In fact in [29, 30, 31], it is observed that the Indian Cassava mosaic virus can cause the mosaic disease on the *Jatropha Curcas* plants. For this reason, we chose the

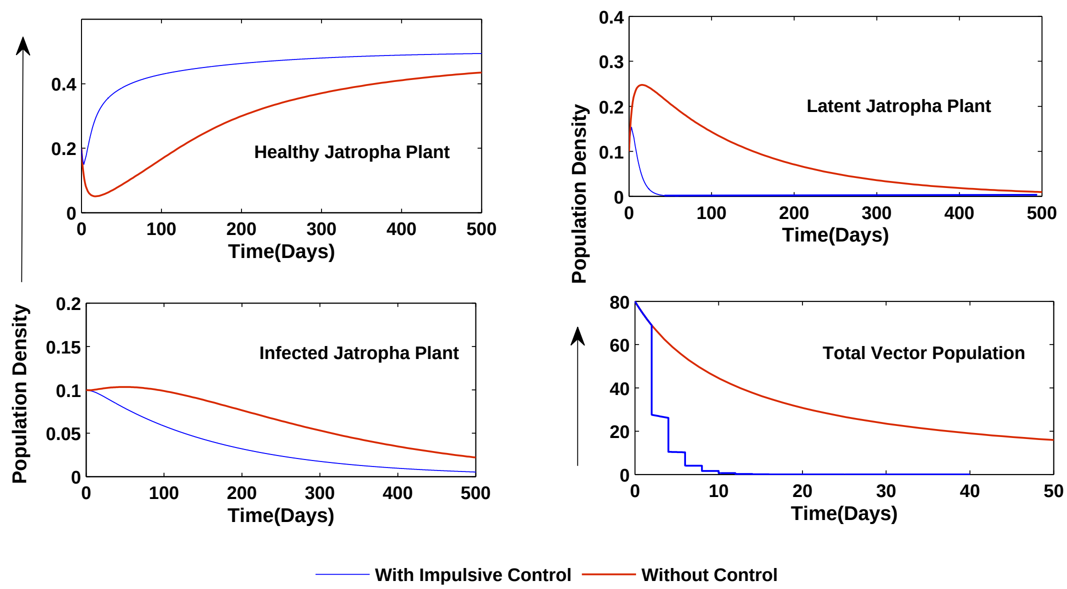

FIGURE 3. Comparative representation between impulsive control and without control.

parameters for the the virus and the vector from Table 1.

Furthermore, although the phylogenetic nature of Cassava plants and the Jatropha Plants are not same, so that their growth and death rates differ, the plantation processes are very similar. We have estimated their values from the literature by Holt et al. [28] for the numerical simulations, as no real data is available for Jatropha plant.

In Figure 1, the population densities are plotted as a function of time. The healthy plant population density is initially decreased for transfer to the latent class and then is gradually increased due to cause of replantation. The latently infected plant population is reduced because of transfer to the infected class. The population density of infected plants is initially increased due to transfer of plants from latently infected class and then it is reduced due to cutting of infected plants. The total vector population is reduced because of removal of infected plants.

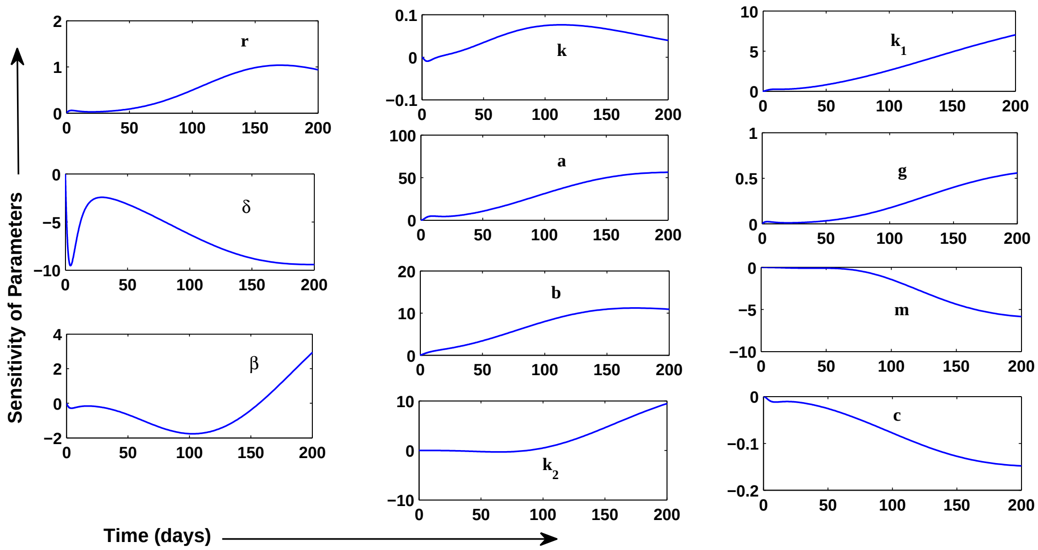

FIGURE 4. Sensitivity index (w.r.t. healthy plant) as a function of time for different parameters of system (2.2).

Figure 2 reveals the comparative analysis amongst impulsive control with 5 days interval, impulsive control with 2 days interval and continuous control through spraying. When impulsive control with 5 days interval through spraying of insecticide is applied, total vector population dies out within 20 days but when impulsive control with 2 days interval is used, total vector population dies out within 10 days. But continuous spraying unable to exterminate the vector population but it can lead to reduced population growth. Thus, it is seen that if the strength of insecticide spraying is increased, the system moves towards stability soon and complete eradication is done by impulsive control strategy.

In Figure 3, we have taken same strength for the spraying of insecticide (Insecticidal Soap). We have observed that impulsive control with 2 days interval has achieved the better results. Now in Figure 3, we display the comparison between the impulsive control with 2 days interval and without control approach.

We have done a local sensitivity analysis of the system (2.2) with respect to healthy plant population. In Figure 4, we display the sensitivity characteristics of all parameters introduced into our proposed ecological system. Here we observe that all parameters of the model are sensitive (some are positively and some are negatively sensitive). Thus we can conclude that all parameters are required to formulate our mathematical model. This actually validates our formulated mathematical model.

8. Discussion and Conclusion

In this research article, a comparative analysis between two types of controlling procedures to reduce the white-fly population is provided. One is insecticide spraying with an impulsive mode and another one is continuous control strategy. On that basis, mathematical models are developed for Jatropha plantation and control strategy are given on the system in two ways. The focus is to propose the most effective method to reduce the vector, white fly, population that affects the plants. Actually, cutting of infected plant can perform as the measure of control, which is introduced into our model system. Along with cutting, we have applied spraying on the *Jatropha curcas* plants and achieved more better results. It is established that impulsive spraying is more suitable strategy rather than continuous spraying. Spraying, in impulsive mode, is the best possible way to eradicate the vector population, which causes the movement of virus to the *Jatropha curcas* plants. Spraying of Insecticidal soap improves the system towards the stability more efficiently in case of impulsive control approach. So the insecticide spraying with impulsive control in two days interval ultimately leads towards the stable position in a disease-free situation of *Jatropha curcas* plants that finally contribute a good renewable source for biodiesel.

Conflict of Interests

The authors declare that there is no conflict of interests.

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REFERENCES

[1] P. K. Roy, S. Datta, S. Nandi, F. A. Basir, Effect of Mass Transfer Kinetics for Maximum Production of Biodiesel from Jatropha Curcas Oil: A Mathematical Approach, Fuel, 134 (2014), 39-44.

[2] Basir, F. A., S. Datta, P. K. Roy, Studies on biodiesel production from Jatropha Curcas oil using chemical and biochemical methodsCA mathematical approach, 158 (2015) 503C511.

[3] F. A. Basir, P. K. Roy, Production of Biodiesel using Enzymatic Transesterification of Jatropha Curcas Oil: A Mathematical Study, Mathematics In Engineering, Science And Aerospace, 5 (2014), 175-184.

[4] James E. Duffus, Whitefly Transmission of Plant Viruses, Current Topics in Vector Research, 4 (1987), 73-91.

[5] bugspray.com/article/whiteflies.html.

[6] Fauquet C., Fargette D., African Cassava Mosaic Virus: Etiology, Epidemiology and Control, Laboratoire de Phytovirologie, ORSTOM, Plant Disease, 74 (1990), 404 - 411.

[7] D. Fargette, M. Jeger, C. Fauquet and L. D. C. Fishpool, Analysis of Temporal Disease Progress of African Cassava Mosaic Virus, Phytopathology, 54 (1994), 91 - 98.

[8] H. D. Thurston, Tropical Plant Diseases, APS Press, (1998).

[9] Gao S.Q., Qu J., Chua N.H., Ye J., A new strain of Indian cassava mosaic virus causes a mosaic disease in the biodiesel crop *Jatropha curcas*. Arch. Virol. 155 (2010), 607-612.

[10] Narayana D.S.A., Shankarappa K.S., Govindappa M.R., Prameela H.A., Rao M.R.G. Rangaswamy K.T., Natural occurrence of Jatropha mosaic virus disease in India. Curr. Sci. 91 (2006), 584-586.

[11] J. M. Thresh, R. J. Cooter, Strategies for controlling cassava mosaic virus disease in Africa, Plant Pathology, Vol. 54 (2005) 587-614, 2005.

[12] R. A. C. Jones, Using epidemiological information to develop effective integrated virus disease management strategies. Virus Research, 100 (2004), 5-30.

[13] F. Brauer, P. Van den Driessche, Models for transmission of disease with immigration of infectives, mathematical Biosciences, 171 (2001), 143-154.

[14] Tang S. Y., Xiao Y. N., Cheke R. A., Dynamical analysis of plant disease models with cultural control strategies and economic thresholds, Mathematics and Computers in Simulation, 80 (2010), 894-921.

[15] M. J. Jeger, J. Holt, F. V. D. Bosch and L. V. Madden, Epidemiology of insect-transmitted plant viruses: modelling disease dynamics and control interventions, Physiological Entomology, 29 (2004), 291 - 304.

[16] M. P. Grilli, J. Holt, Vector feeding period variability in epidemiological models of persistent plant viruses, Ecol. Modelling, 126 (2004), 49-57.

[17] Suen C. M., M. J. Jeger, An analytical model of plant virus disease dynamics with roguing and replanting,Journal of Applied Ecology, 31 (1994), 413-427.

[18] Van den Bosch F., Jeger M. J., Gilligan C. A., Disease control and its selection for damaging plant virus strains in vegetatively propagated staple food crops; a theoretical assessment, Proceedings of the Royal Society B, 274 (2007), 11-18.

[19] S. Fishman, R. Marcus, H. Talpaz, Epidemiological and economic models for spread and control of citrus tristeza virus disease, Phytoparasitica, 11, (1983), 39-49.

[20] C. Morone, M. Boveri, S. Giosue, P. Gotta, V. Rossi, I. Scapin, C. Marzachi, Epidemiology of Flavescence dore in vineyards in northwestern Italy. Phytopathology, 97 (2007), 1422-1427.

[21] R. Musetti, R. Marabottini, M. Badiani, M. Martini, L. Sanit di Toppi, S. Borselli, M. Borgo, R. Osler, On the role of H2O2 in the recovery of grapevine (Vitis vinifera cv. Prosecco) from Flavescence dore disease. Functional Plant Biology, 34 (2007), 750-758.

[22] C. Bellomo, L. Carraro, P. Ermacora, F. Pavan,R. Osler, C. Frausin, G. Governatori, Recovery phenomena in grapevines affected by grapevine yellows in Friuli Venezia Giulia, Bulletin of Insectology, 60 (2007), 235-236.

[23] M. Zhao, Y. Wang, L. Chen, Dynamic Analysis of a Predator-Prey (Pest) Model with Disease in Prey and Involving an Impulsive Control Strategy, Journal of Applied Mathematics, 2012 (2012).

[24] P. K. Roy , A. N. Chatterjee, Effect of HAART on CTL Mediated Immune Cells: An Optimal Control Theoretic Approach, Electrical Engineering and Applied Computing, Springer, 90 (2011), 595-607.

[25] A. Saltelli, M. Scott, The role of sensitivity analysis in the corroboration of models and itslink to model structural and parametric uncertainty, Reliability Engineering and System Safety, 57 (1999), 1-4.

[26] A. Saltelli, S. Tarantola, K. S. Chan, A quantitative model-independent method for global sensitivity analysis of model output, Technometrics, 41 (1999), 39-56.

[27] M. Federico, B. Domenico, M. Cristina, Conceptual and mathematical modeling of insect-borne plant diseases: theory and application to flavescence doree in grapevine, arXiv preprint arXiv:1408.6616 (2014).

[28] Holt J., Jeger M. J., Thresh J. M., Otim-Nape G.W., An epidemiological model incorporating vector population dynamics applied to African cassava mosaic virus disease, Journal of Applied Ecology, 1997.

[29] Gao, S., Qu, J., Chua, N. H., Ye, J., A new strain of Indian cassava mosaic virus causes a mosaic disease in the biodiesel crop *Jatropha curcas*. Archives of virology,?155(4) (2010), 607-612.

[30] Kashina, B. D., Alegbejo, M. D., Banwo, O. O., Nielsen, S. L., Nicolaisen, M., Molecular identification of a new begomovirus associated with mosaic disease of *Jatropha curcas* L. in Nigeria.?Archives of virology,?158 (2013), 511-514.

[31] Ramkat, R., Calari, A., Maghuly, F., Laimer, M., Occurrence of African cassava mosaic virus (ACMV) and East African cassava mosaic virus-Uganda (EACMV-UG) in *Jatropha curcas*. BMC Proceedings, BioMed Central Ltd,5,(2011),93-103.

[32] Bainov, D., and Simeonov P., Impulsive differential equations: periodic solutions and applications, CRC Press, 66 (1993).