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REVISITING LOGISTIC POPULATION MODEL FOR ASSESSING PERIODICALLY HARVESTED CLOSURES

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Abstract. Periodically harvested closures are frequently employed within the exploitation frameworks. Logistic model, with its variants, have been extensively investigated on evaluating different harvesting strategies. In this manuscript, we revisit Logistic model to evaluate the population dynamics and harvesting yields under periodically harvested frameworks. Sufficient conditions are obtained for global stability of positive periodic solutions. With the maximum annual-sustainable yield (MASY) as a management objective, the optimal harvest timing (or season) is determined for fixed harvesting effort and MASY is shown to depend solely on the duration of open harvesting season. Combining with the optimal harvesting effort, the higher annual-sustainable yield can be reached if the timing of harvesting is set as early as possible. Furthermore, the optimal harvesting policy, optimal harvesting effort, optimal harvest timing and their biological implications are discussed. Moreover, the paradoxical hydra effect observed in one-dimensional difference equation, where population size increases in response to additional mortality, is excluded for scalar continuous-time model. Our results offer a new perspective on setting the policy of harvesting closures.

Keywords: harvest timing; harvesting effort; logistic model; closed/open season; maximum annual-sustainable yield.

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1. Introduction

Fisheries and aquaculture serve as crucial resources of global development by providing nutritious food and high quality animal protein, and significantly contribute to the income and livelihood for hundreds of millions of people around the world by providing employment positions in ancillary activities. According to the United Nations Food and Agriculture Organization (FAO), global marine fisheries reached a peak of 86.4 million tonnes in 1996, and declined before stabilizing at about 80 million tones each year [1]. There is strong evidence that most fish stocks are fully exploited or overexploited, as reported that the percentage of fully exploited stocks and overexploited stocks respectively were 57% and 29.9% in 2009. As a consequence, marine stocks are in urgent need of effective fishery management.

In the fishery policies, time closures (also named closed seasons and seasonal closures) are extensively applied to control human activity. A closed season of a single species is the banning of fishing activity (in an area or an entire fishery) for a few weeks or months, usually to protect juveniles or spawners [2]. For instance, the spring time closures in Yangtze River. The fishery of common sardine and anchovy on the southern Chilean coast shows short periods with large captures, followed by relatively low ones at rest [2]. The pacific herring fishery [3] and village-base management program in Vanuatu [4, 5] pose good examples of long-term time closures. Management of harvesting and banning for a fixed period is proved to be effective on protecting juveniles, therefore, enhancing catched and sticks of fish products. Periodically harvested closures are also amendable to local implementation by co-management across the Indo-pacific region [6]. Despite their widespread popularity on harvesting strategies, the benefits of setting periodically harvested closed window have been largely untested from the population dynamical point of view. The success of periodically-harvested closure in fishery policies to get the maximum yields relies on setting the optimal timing and duration of harvest open season.

Mathematical biologists have promoted harvesting theory to predict the optimal harvesting strategies. The optimal harvesting policy for the Logistic population model with constant harvesting effort was firstly studied by Clark [7]

$$\dot{X} = rX\left(1-\frac{X}{K}\right) - qX,$$

where r and K represent the intrinsic growth rate and carrying capacity of fish population respectively while q denotes the harvesting effort. He showed that the harvesting effort q should be set at its optimal value $q_c^* = r/2$, to get the maximum sustainable yield (MSY) $Y_c^* = Kr/4$, and in this scenario, the population level is stabilizing at $X_c^* = K/2$. Following the pioneer work of Clark, great progresses have been made through extensive use of differential equations in predicting the consequences of harvest on population dynamics and designing optimal harvesting strategies. The classical results of Clark were extended to the periodic Logistic equations by Fan and Wang [8], where the optimal harvesting policy with the MSY as the management objective was examined, subject to constant and periodic effort harvesting. Impulsive harvesting policy was proposed based on the Logistic equation in [9]. The seasonal Logistic equation was also employed by Xu, Boyce and Daley [10] to investigate the harvest under five different exploitation rates (constant, linear, time-dependent, open/closed season and pulse). The effect of impulsive harvest on a general Kolomogorov-type periodic equation was investigated by Xiao, Cheng and Qin [11]. Stage structure of the population growth was included [12] with birth pulse and closed/open harvesting of mature population.

On the meantime, there is a parallel body of literature on modelling the impact of harvesting with discrete time by using difference equations. Difference equations can accommodate more detailed aspects of population dynamics, such as the explicit seperation of the ordering of events within a generation [13]. Tang, Cheke and Xiao proposed the optimal impulsive harvesting strategy for non-autonomous Beverton-Holt difference equations [14]. The developed results have also been applied to investigate the impulsive harvest on Logistic differential equation. Structured population difference model was proposed to investigate the density-dependence and timing of harvest on predict the long-term behaviour of population dynamics [13]. The impact of harvesting timing on population persistence was also investigated through a discrete model [15]. The population dynamics of one-dimensional difference models subject to harvesting were reviewed by Liz and Hilker reviewed [16]. It is interesting to predict the possibility of the hydra effect, where greater mortality increases population size [16, 17]. For the scenario of population subject to harvesting, this paradoxical phenomenon happens, where harvesting may increase the population size. However, for one-dimensional continuous model, regardless it being autonomous or nonautonomous, we are going to exclude the possibility of hydra effect in the discussion section.

For time-dependent growth models, timing of harvesting, as well as the order of harvesting in the life cycle, may pose a strong impact on the persistence of the population size and on the MSY [12, 13, 18, 19, 20, 21, 22, 23, 24]. Note that all of above works only focus on how the seasonal harvesting affects MSY which relates to the optimal harvesting effort. However, how do both the duration of closed season (or the starting point of the open season) and harvesting effort affect the persistence of population, and consequently affect the MSY? What is the optimal harvesting policy to achieve the maximum annual-sustainable yield (MASY) in the consideration of both the economical and biodiversity sustainable aspects? To address those questions theoretically, we will first adapt the classical Logistic model with closed/open seasonal harvesting which turns out to be a non-smooth Logistic model, and investigate the impact of harvest timing and harvesting effort on MASY and population persistence. Qualitative methods and optimal control methods are applied to analyze the population persistence and determine the optimal harvesting policy, in terms of the optimal harvesting effort and the optimal harvesting time-spectrum, and our results extend the classical results for the optimal resources management [7, 8, 10, 12].

Since we are focusing on the comparison of benefits between the static harvesting and periodically closure harvest, we use the differential equations, which give an explicit (thus more easily intepretable) solution to the effect of the harvesting strategies. We will evaluate the cost-effectiveness of the strategy of periodically harvested closures via the periodic Logistic equation

$$\frac{dx(t)}{dt} = \begin{cases} r(t)x(t)\left[1-\frac{x(t)}{k(t)}\right], & t \in [n, n+\alpha], \\ r(t)x(t)\left[1-\frac{x(t)}{k(t)}\right] - q(t)x(t), & t \in (n+\alpha, n+1]. \end{cases}$$

In this model, we assume the growth of population size follows the periodic Logistic equation with per-capita growth rate $r(t)$ and carrying-capacity $k(t)$, which are determined by biotic or abiotic factors. In particular, we assume the population growth is regulated by seasonal factors and therefore, $r(t)$ and $k(t)$ are periodic functions with period one year. We should also mention that other frequencies of environment fluctuations are possible and the period of these functions may be different from one. However, since our main goal is to compare benefits of static harvesting and periodically closure harvest, we may scale the period to be 1 year, and so as to the harvesting effort $q(t)$. Assume that every beginning part of the n-th year the period from n to $n+\alpha$ is the closed season, and the rest of each year is the open season ($n \in N$ with $N = \{0,1,2,\cdots\}$). Within each year, α is the starting point of harvest with $0 \leq \alpha < 1$. So the time-spectrums of closed season and open season are $[n, n+\alpha]$ and $(n+\alpha, n+1]$, respectively. In this model, we only describe the harvesting effort as fixed proportion harvesting, that is, individuals are removed at a constant proportion each year. Other types of harvesting efforts have been modelled, such as those dependent on the threshold population size (biomass at the limit) or the target oriented harvesting [10, 25], which may leave as our future work.

We start from the autonomous Logistic equation subject to periodically harvested closures, which illustrates to the theoretical analysis idea on evaluating the cost-effectiveness of harvesting strategies. Then, the results will be extended to periodic Logistic equation with periodically harvested closures.

2. **Harvest closures for species with constant growth rates**

When the population growth is not affected by other factors, such as the temperature, and the harvesting effort is constant during the open season, the population dynamics is given by the following non-smooth model

$$\frac{dX(t)}{dt} = \begin{cases} rX(t)(1-\frac{X(t)}{K}), & t \in [n, n+\alpha], \\ rX(t)(1-\frac{X(t)}{K}) - qX(t), & t \in (n+\alpha, n+1]. \end{cases} \tag{2.1}$$

In this section, we will focus on model (2.1) and address how the starting point α and harvesting effort q affect the optimal fishery resource management. Without loss of generality, the value K can be taken as the unit to measure fish stock. This is, if we let $x(t) = X(t)/K$, then the dynamic model (2.1) becomes

$$\frac{dx(t)}{dt} = \begin{cases} rx(t)(1-x(t)), & t \in [n, n+\alpha], \\ rx(t)(1-x(t)) - qx(t), & t \in (n+\alpha, n+1]. \end{cases} \tag{2.2}$$

It's easy to see that the solutions of model (2.2) remain nonnegative for any nonnegative initial conditions. For any $t \in [n, n+1]$ the solution of model (2.2) can be analytically obtained as follows

$$x(t) = \begin{cases} \frac{e^{r(t-n)}x(n)}{1+[e^{r(t-n)}-1]x(n)}, & t \in [n, n+\alpha], \\ \frac{e^{(r-q)(t-n-\alpha)}x(n+\alpha)}{1+\frac{r}{r-q}[e^{(r-q)(t-n-\alpha)}-1]x(n+\alpha)}, & t \in (n+\alpha, n+1], \end{cases}$$

with

$$x(n+\alpha) = \frac{e^{r\alpha}x(n)}{1+[e^{r\alpha}-1]x(n)}.$$

Therefore, we arrive a difference equation for the solutions at the end of year

$$x(n+1) = \frac{e^{r-q+q\alpha}x(n)}{1+[\frac{1}{r-q}(re^{r-q+q\alpha}-qe^{r\alpha})-1]x(n)}. \tag{2.3}$$

This equation takes the form of a discrete Beverton-Holt model

$$y(n+1) = \frac{ay(n)}{1+by(n)},$$

whose solutions are shown to either go to a globally stable positive equilibrium $M^* = (a-1)/b$ (when $a > 1$) or converge to zero ($a \leq 1$) [26]. With the help of this result, we can conclude the global dynamics of the autonomous Logistic model with closed/open harvesting seasons:

Theorem 2.1. *If $q > r$ and $\alpha \leq (q-r)/q$, then all solutions of model (2.2) converge to zero; If either (i) $q > r$ and $(q-r)/q < \alpha < 1$ or (ii) $q < r$, then model (2.2) admits a unique periodic solution $x^*(t)$ which is globally asymptotically stable, where*

$$x^*(t) = \begin{cases} \frac{e^{r(t-n)}x_0^*}{1+[e^{r(t-n)}-1]x_0^*}, & t \in [n\ n+\alpha], \\ \frac{e^{(r-q)(t-n-\alpha)}x^*(n+\alpha)}{1+\frac{r}{r-q}[e^{(r-q)(t-n-\alpha)}-1]x^*(n+\alpha)}, & t \in (n+\alpha\ n+1], \end{cases}$$

with

$$x_0^* = \frac{(q-r)(e^{r-q+q\alpha}-1)}{r-q+qe^{r\alpha}-re^{r-q+q\alpha}} = \frac{(q-r)(e^{r-q+q\alpha}-1)}{B_0}, \textit{and}$$

$$x^*(n+\alpha) = \frac{(q-r)(e^{r-q+q\alpha}-1)}{(q-r)e^{r-q+q\alpha}-qe^{(1-\alpha)(r-q)}+r} = \frac{(q-r)(e^{r-q+q\alpha}-1)}{B_\alpha}.$$

Proof. When (i) $q > r$ and $(q-r)/q < \alpha < 1$ or (ii) $q < r$, we have $e^{r-q+q\alpha} > 1$, and therefore, the difference equation (2.3) has a unique positive equilibrium x_0^*, which is globally stable. If we denote the solution through x_0^* as $x^*(t)$, then x^* is a periodic solution of (2.2). Next, we show that this positive periodic solution is global attractive. Let

$$\Phi(t)x(0) := x(t),\ x \geq 0$$

with $x(t)$ being the solution of (2.2) through the initial value $x(0)$, then $\Phi(t)$ is a periodic semiflow with period 1. Moreover, for any $\delta > 0$, there exists N such that whenever $n > N$

$$|\Phi(n)x(0) - \Phi(n)x_0^*(0)| = |\Phi(n)x(0) - x_0^*(0)| < \delta.$$

Based on the continuity of solutions in initial conditions, for any $\varepsilon > 0$, there exists $\delta > 0$ such that

$$|\Phi(t)x(0) - \Phi(t)x_0^*| < \varepsilon,\ \forall t \in [0,1]$$

provided $|x(0) - x_0^*(0)| < \delta$. Therefore, for any $\varepsilon > 0$, there exists N such that whenever $t > N$ (we may rewrite $t = N_1 + t_1$ with $N_1 > N$ and $t_1 \in (0,1)$) such that

$$\begin{aligned} |x(t) - x^*(t)| &= |\Phi(t)x(0) - \Phi(t)x_0^*(0)| \\ &= |\Phi(t_1)\Phi(N_1)x(0) - \Phi(t_1)\Phi(N_1)x_0^*(0)| \\ &= |\Phi(t_1)(\Phi(N_1)x(0)) - \Phi(t_1)x_0^*(0)| < \varepsilon. \end{aligned}$$

Therefore, the positive periodic solution is globally asymptotically stable.

The global attractivity of zero when $q > r$ and $\alpha \leq (q-r)/q$ can be proved by the similar arguments and we omit the details here. □

This theorem shows that it is important to design harvesting policy carefully in order to avoid extinction of the species: either allowing harvesting the whole year around with small harvesting effort $(q < r)$ or banning great effort harvesting for longer time ($q > r$ and $\alpha \leq (q-r)/q$).

Remark 2.1. *The case where the population persists was also illustrated in* [8] *by using a different approach, especially for the arguments of global stability of the positive periodic solution. Actually the global stability results can be proved by using the theory of monotone dynamical* *system with applications to the Poincare map, as the proof in* [27] *with a slight change. However, we reiterate it here by a different approach, which gives us an explicit expression for the periodic solution when the population persists.*

Remark 2.2. *Model (2.2) can be regarded as a switched model. In the literature* [28], *a switching law that incorporates an average dwell time approach was obtained to guarantee the exponential stability. According to Theorem 2 in* [28] *for $q > r$, we may obtain the following result: when*

$$\frac{1-\alpha}{\alpha} \geq \frac{r+\lambda^*}{q-r-\lambda^*},$$

i.e. $\alpha \leq (q-r+\lambda^*)/q$ *with any positive constant λ^* satisfying* $0 < \lambda^* < q-r$, *then the solutions of model (2.2) would be globally stable at zero. Obviously, Theorem 2.1 gives a sharper condition for α.*

In the harvest management, sustainable development should be given priority over other considerations, that is, people should pay attention to the harvesting effort and the length of open season to avoid over harvesting to accommodate their sustainable commercial benefits. To make sure harvesting can not drive the species to extinction, throughout this section, we assume

$$\text{either (i) } q > r \text{ and } \frac{q-r}{q} < \alpha < 1 \text{ or (ii) } q < r \text{ and } 0 \leq \alpha < 1.$$

This section devotes to the optimal seasonal harvesting policy for maximum stock level of the fish at the end of fishing season and MASY.

Firstly, since the population size will eventually oscillate at a periodic state (Theorem 2.1), in the end of the fishing season, the stock level can be expressed as

$$x(n) = x_0^* = \frac{(q-r)(e^{r-q+q\alpha}-1)}{r-q+qe^{r\alpha}-re^{r-q+q\alpha}}. \tag{2.4}$$

Taking the derivation of function $x(n)$ with respect to α, one obtains

$$
\begin{aligned}
\frac{dx(n)}{d\alpha} &= \frac{q(q-r)\left[e^{r-q+q\alpha}B_0 + r(e^{r-q+q\alpha}-1)(e^{r-q+q\alpha}-e^{r\alpha})\right]}{B_0^2}, \\
&= \frac{q(q-r)\left[(q-r)(e^{r-q+q\alpha}-1)(e^{r\alpha}-1)+B_0\right]}{B_0^2}, \qquad (2.5)\\
&= \frac{q(q-r)\left[(e^{r\alpha}-1)x_0^*+1\right]}{B_0}.
\end{aligned}
$$

in which $B_0 = r - q + qe^{r\alpha} - re^{r-q+q\alpha}$. It is easy to verify that $\frac{dx(n)}{d\alpha} > 0$, either for condition (i) or (ii). Thus unlike impulsive harvesting policies [29], there is no optimal harvesting timing for closed/open harvesting policy. In addition, the latter the harvesting takes, the greater fish stock will be left.

Secondly, MASY can be expressed as

$$
\begin{aligned}
Y(\alpha,q) &= q\int_\alpha^1 x^*(t)dt = q\int_\alpha^1 \frac{e^{(r-q)(t-\alpha)}x^*(\alpha)}{1+\frac{r}{r-q}[e^{(r-q)(t-\alpha)}-1]x^*(\alpha)}dt \\
&= q\int_\alpha^1 \frac{(q-r)e^{(r-q)(t-\alpha)}(e^{(1-\alpha)(r-q)+r\alpha}-1)}{q(e^{r\alpha}-1)e^{(1-\alpha)(r-q)}-r(e^{(1-\alpha)(r-q)+r\alpha}-1)e^{(r-q)(t-\alpha)}}dt \qquad (2.6)\\
&= \frac{q}{r}\ln\frac{q(e^{r\alpha}-1)e^{(1-\alpha)(r-q)}-r(e^{(1-\alpha)(r-q)+r\alpha}-1)e^{(r-q)(1-\alpha)}}{q(e^{r\alpha}-1)e^{(1-\alpha)(r-q)}-r(e^{(1-\alpha)(r-q)+r\alpha}-1)}, \\
&= \frac{q}{r}\ln\frac{e^{(r-q)(1-\alpha)}B_0}{B_\alpha}.
\end{aligned}
$$

For the MASY, denote $Y^q(\alpha) := Y(\alpha,q)$ as a function of α with fixed harvesting effort q and we have the following result.

Theorem 2.2. *For any given harvesting effort $q > r/2$, there exists a unique optimal harvesting time $\alpha^* \in [0,1)$ such that the annual-sustainable yield $Y^q(\alpha)$ reaches its maximum value. If $q \le r/2$, $Y^q(\alpha)$ is a decreasing function with respect to α.*

Proof. Here we prove the case when $0 < \frac{q-r}{q} < \alpha < 1$. The result for $q < r$ can be obtained by similar arguments. We need to show $\widetilde{Y}^q(\alpha)$ reaches its maximum value at α^* with

$$\widetilde{Y}^q(\alpha) = \frac{e^{(r-q)(1-\alpha)}B_0}{B_\alpha}.$$

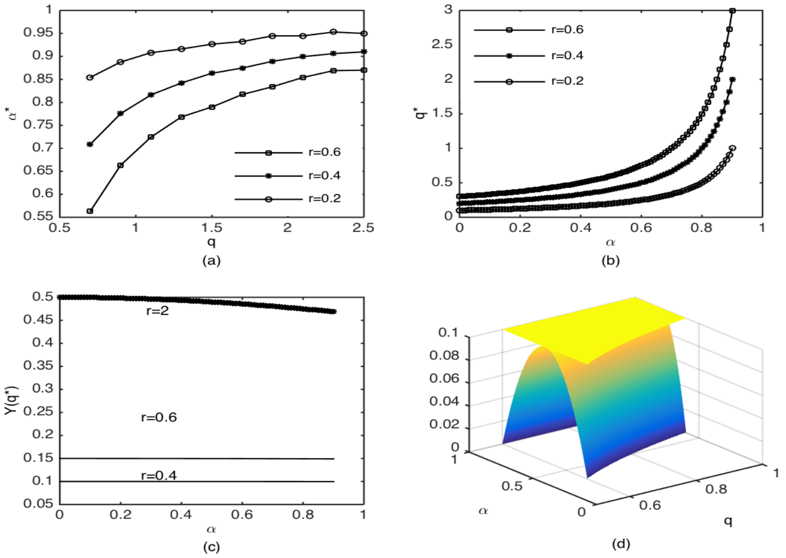

FIGURE 1. Illustrations of the relationships among harvest timing, harvesting effort and MASY for model (2.2). (a) The effects of the intrinsic growth rates and the harvesting effort on the optimal harvest timing α^*; (b) The effects of the intrinsic growth rates and the harvesting timing on the optimal harvesting effort q^*; (c) With intrinsic growth rate $r = 0.4$ and the optimal harvesting effort given in Figure 1(b), the plot of MASY $Y(q^*)$ with respect to α from $0 \le \alpha < 0.9$; (d) For $r = 0.4$ and $0.6 \le q \le 1$ with α satisfying $(q-r)/q < \alpha \le 1$, the plot of annual-sustainable yield $Y(\alpha, q)$ against α and q.

Taking the first derivative of $\widetilde{Y}^q(\alpha)$ with respect to α, one yields

$$\widetilde{Y}_1^q(\alpha): \;=\; \frac{d\widetilde{Y}^q(\alpha)}{d\alpha} = \frac{re^{(r-q)(1-\alpha)}\hat{Y}^q(\alpha)}{B_\alpha^2}.$$

where

$$\hat{Y}^q(\alpha) = (q-r)(1-e^{r-q+q\alpha})B_0 + e^{r\alpha}(1-e^{(r-q)(1-\alpha)})B_\alpha.$$

Since $\widetilde{Y}_1^q((q-r)/q) > 0$ follows from $\hat{Y}^q((q-r)/q) = qe^{r(q-r)/q}(e^{r(q-r)/q}-1)^2 > 0$ and $\widetilde{Y}_1^q(1) < 0$ follows from $\hat{Y}^q(1) = -(q-r)^2(1-e^r)^2 < 0$, therefore, the function $\widetilde{Y}_1^q(\alpha)$ has at least one root on $\alpha \in (\frac{q-r}{q}, 1)$. Furthermore, $\frac{d\hat{Y}^q(\alpha)}{d\alpha} < 0$ for any $0 \le \alpha < 1$. Thus, the function $Y^q(\alpha)$ concaves downward and there is a unique $\alpha^* \in (\frac{q-r}{q}, 1)$ such that the function $Y^q(\alpha)$ reaches its maximum value. $\square$

It is difficult to find a unique q^* for fixed α which maximizes the annual-sustainable yield theoretically, so we turn to investigate the existence of q^* numerically in the following. For the parameter set as those shown in Figure 1, we can see that the optimal harvest timing α^* increases as the harvesting effort q increases, which indicates that in order to avoid overfishing with larger magnitude of captures, a shorter duration of open seasons should be set. On the meanwhile, Figure 1(b) shows that the optimal harvesting effort q^* increases when the harvest timing α becomes later. Further, it follows from Figure 1(a) and (b) that for a given fixed harvesting effort q (or harvest timing α), the optimal harvest timing α^* (or the optimal harvesting effort q^*) is a monotonically increasing function with respect to the intrinsic growth rate r. Those imply that the intrinsic growth rate is the key factor that influence the economic benefits. Long-term closures and low intensity of capture can ensure the bioeconomic sustainability for species with low intrinsic growth rate.

To address the effects of harvest timing α and harvesting effort q synthetically on the $Y(\alpha,q)$, we fixed r in Figures 1(c) and (d). It is seen that MASY $Y(q^*)$ is $r/4$ when $\alpha=0$, and $Y(q^*)$ is decreasing when harvest timing is released later. However, when the duration of harvesting time becomes shorter (i.e. $1-\alpha$ becomes smaller), we can increase the harvesting effort q to obtain the almost same MASY when the species population only has a relatively small intrinsic growth rate, as shown in Figure 1(c) for $r=0.4$ and $r=0.6$. Figure 1(d) not only validates the existence of optimal harvest timing for given harvesting effort q, but also shows that the approximation of maximum $Y(\alpha,q)$ is $r/4$. Based on those numerical results, we could address the following remark about $Y(\alpha,q)$ theoretically.

Figures 1(b), (d) indicate that $Y(\alpha,q)$ reaches its maximal value $r/4$ at $(\alpha,q)=(0,r/2)$, which motivates us to propose the following theoretical result.

Theorem 2.3. *If $(\alpha,q)=(0,r/2)$, then $Y(\alpha,q)$ defined by Eq. (2.6) reaches its maximum value with $Y_{\max}(0,r/2)=r/4$, and the population level is $x^*(t)=1/2$.*

Proof. If $\alpha=0$ and $q=r/2$, then it is easy to verify that the first derivatives of $Y(\alpha,q)$ with respect to α and q are $Y_\alpha(0,r/2)=Y_q(0,r/2)=0$. And the second-order partial derivatives of

$Y(\alpha,q)$ at $(0,r/2)$ can be calculated as follows:

$$Y_{qq} = -\frac{2}{r} < 0, \quad Y_{q\alpha} = 1, \quad Y_{\alpha\alpha} = \frac{r^2(e^{\frac{r}{2}}+1)}{8(e^{\frac{r}{2}}-1)}, \quad Y_{q\alpha}^2 - Y_{qq}Y_{\alpha\alpha} = 1 - \frac{r(e^{\frac{r}{2}}+1)}{4(e^{\frac{r}{2}}-1)}.$$

Let $g(r) = 4(e^{\frac{r}{2}}-1) - r(e^{\frac{r}{2}}+1)$, then the first and second order derivatives of the function $g(r)$ with respect to r can be calculated as

$$g'(r) = e^{\frac{r}{2}}(1-\frac{r}{2}) - 1, \quad g''(r) = -\frac{r}{4}e^{\frac{r}{2}}.$$

Further, for any given $r > 0$ we have $g''(r) < 0$. Thus, $g'(r) < g'(0) = 0$. This shows that for any given $r > 0$ we have $g(r) < g(0) = 0$. Therefore, if $r > 0$, then $Y_{q\alpha}^2 - Y_{qq}Y_{\alpha\alpha} < 0$, and this indicates that $Y(\alpha,q)$ reaches its maximal value at $(\alpha,q) = (0,r/2)$. In addition, $Y_{\max}(0,r/2) = r/4$ and the optimal population level is $x^*(t) = 1/2$. $\square$

Thus, neither for the maximum stock level nor MASY there is no advantage in setting closure season for species with constant growth rate. In the next section, the scenario where population growth is affected by seasonal environment changes was studied.

3. **Harvest closures for species with seasonal changing growth rates**

Many biological populations experience some kind of environmental seasonality, which often results from periodic fluctuations in the availability of resources (i.e. food, water and nutrients), territories and cover [10]. In this section, we assume that the population follows Logistic growth with periodic intrinsic growth rate and the carrying capacity. With seasonally varying harvesting, the mathematical model can be written as

$$\frac{dx(t)}{dt} = \begin{cases} r(t)x(t)\left[1-\frac{x(t)}{k(t)}\right], & t \in [n, n+\alpha], \\ r(t)x(t)\left[1-\frac{x(t)}{k(t)}\right] - q(t)x(t), & t \in (n+\alpha, n+1], \end{cases} \tag{3.1}$$

where $r(t)$, $k(t)$ and $q(t)$ are positive and continuous period-1 functions and $0 \le \alpha < 1$.

By taking advantage of the explicit solutions for Bernoulli type equations, the solution of (3.1) in the interval $t \in [n, n+1]$ through $x(n)(x(n) > 0)$ in $[n, n+1]$ can be explicitly written as

$$x(t) = \begin{cases} \left[\frac{1}{x(n)} e^{-\int_n^t r(s)ds} + \int_n^t \frac{r(s)}{k(s)} e^{-\int_s^t r(\tau)d\tau} ds\right]^{-1}, & t \in [n, n+\alpha] \\ \left[\frac{1}{x(n+\alpha)} e^{-\int_{n+\alpha}^t (r(s)-q(s))ds} + \int_{n+\alpha}^t \frac{r(s)}{k(s)} e^{-\int_s^t (r(\tau)-q(\tau))d\tau} ds\right]^{-1}, & t \in (n+\alpha, n+1] \end{cases}$$

Therefore, we can construct a difference equation for $x(n)$, $n = 0, 1, \ldots$, as

$$x(n+1) = \frac{Ax(n)}{1+Bx(n)}$$

with

$$A = e^{\int_0^1 r(\tau)d\tau - \int_\alpha^1 q(\tau)d\tau} \text{ and } B = \int_0^\alpha \frac{r(s)}{k(s)} e^{\int_0^s r(\tau)d\tau} ds + A\int_\alpha^1 \frac{r(s)}{k(s)} e^{\int_s^1 q(\tau)-r(\tau)d\tau} ds.$$

Analogous to the proof of Theorem 2.1, we can get the following result.

Theorem 3.1. *If* $\int_0^1 r(\tau)d\tau - \int_\alpha^1 q(\tau)d\tau \le 0$, *all the solutions of model (3.1) converge to zero; If* $\int_0^1 r(\tau)d\tau - \int_\alpha^1 q(\tau)d\tau > 0$, *model (3.1) has a unique periodic solution* $x^*(t)$ *which is globally asymptotically stable. Moreover,* $x^T(t)$ *can be represented as*

$$x^*(t) = \begin{cases} \left[\frac{1}{x_0^*} e^{-\int_n^t r(\tau)d\tau} + \int_n^t \frac{r(s)}{k(s)} e^{-\int_s^t r(\tau)d\tau} ds\right]^{-1}, & t \in [n, n+\alpha], \\ \left[\frac{1}{x^*(n+\alpha)} e^{\int_{n+\alpha}^t q(\tau)-r(\tau)d\tau} + \int_{n+\alpha}^t \frac{r(s)}{k(s)} e^{\int_s^t q(\tau)-r(\tau)d\tau} ds\right]^{-1}, & t \in (n+\alpha, n+1]. \end{cases}$$

with

$$x^* = \tfrac{A-1}{B},\ x^*(n+\alpha) = \tfrac{(A-1)e^{\int_\alpha^1 q(\tau)-r(\tau)d\tau}}{Z(\alpha,\alpha)} \text{ and}$$
$$Z(\alpha,t) = \int_0^\alpha \tfrac{r(s)}{k(s)} e^{\int_0^s r(\tau)d\tau} ds + A\int_\alpha^t \tfrac{r(s)}{k(s)} e^{\int_s^1 q(\tau)-r(\tau)d\tau} ds + \int_t^1 \tfrac{r(s)}{k(s)} e^{\int_s^1 q(\tau)-r(\tau)d\tau} ds.$$

3.1. **Sustainable harvesting.** This section devotes to the optimal harvesting policy in the consideration of the MASY. Throughout the section, we assume $\int_0^1 r(\tau)d\tau - \int_\alpha^1 q(\tau)d\tau > 0$ to ensure the population persists. Then the size of the species may stabilize at a positive periodic state $x^*(t)$. With fixed $q(t)$ the annual- sustainable yield becomes

$$Y^{q(t)}(\alpha) \quad = \int_\alpha^1 q(t)x^*(t)dt = (A-1)\int_\alpha^1 \tfrac{q(t)e^{\int_t^1 q(\tau)-r(\tau)d\tau}}{Z(\alpha,t)} dt.$$

Taking the derivative of $Z(\alpha,t)$ with respect to t, one yields

$$\frac{dZ(\alpha,t)}{dt} = (A-1)\frac{r(t)}{k(t)} e^{\int_t^1 q(\tau)-r(\tau)d\tau}. \tag{3.2}$$

Rearranging (3.2) we obtain

$$(A-1)e^{\int_t^1 q(\tau)-r(\tau)d\tau} = \frac{\frac{dZ(\alpha,t)}{dt}k(t)}{r(t)}.$$

Hence Eq. (3.1) becomes

$$Y^{q(t)}(\alpha) = \int_\alpha^1 \varphi(t) f(\alpha,t)dt,$$

where

$$\varphi(t) = \frac{q(t)k(t)}{r(t)}, \text{ and } f(\alpha,t) = \frac{\frac{dZ(\alpha,t)}{dt}}{Z(\alpha,t)} = \frac{\frac{r(t)}{k(t)}(A-1)e^{\int_t^1 q(\tau)-r(\tau)d\tau}}{Z(\alpha,t)}.$$

Thus, we have the following result.

Theorem 3.2. *If* $0 < \int_0^1 r(\tau)d\tau - \int_\alpha^1 q(\tau)d\tau < \ln(\frac{2q(\alpha)}{r(\alpha)})$, *then there exists a unique harvest timing* α^* *such that the annual-sustainable yield* $Y^{q(t)}(\alpha)$ *reaches its maximum value.*

Proof. The first derivative of function $Y^{q(t)}(\alpha)$ with respect to α can be expressed as

$$Y_1^{q(t)}(\alpha) := \frac{dY^{q(t)}(\alpha)}{d\alpha} = -\varphi(\alpha)f(\alpha,\alpha) + \int_\alpha^1 \varphi(t)\frac{df(\alpha,t)}{d\alpha}dt.$$

To show the existence and uniqueness of α^* that maximizes ASY, we first discuss the existence of the solution for equation $Y_1^{q(t)}(\alpha) = 0$ with respect to α, i.e.

$$\varphi(\alpha)f(\alpha,\alpha) = \int_\alpha^1 \varphi(t)\frac{df(\alpha,t)}{d\alpha}dt. \tag{3.3}$$

Rearranging (3.3), one yields

$$G(\alpha) = \frac{(A-1)e^{\int_\alpha^1 q(\tau)-r(\tau)d\tau}}{AZ^2(\alpha,\alpha)} - \int_\alpha^1 \frac{q(t)e^{\int_t^1 q(\tau)-r(\tau)d\tau}}{Z^2(\alpha,t)}dt = 0.$$

It's easy to verify $G(\alpha) < 0$ when $\int_0^1 r(t)dt = \int_\alpha^1 q(t)dt$, and $G(\alpha) > 0$ when $\alpha = 1$. The first derivative of $G(\alpha)$ can be expressed as

$$\begin{aligned} G'(\alpha) = & \left[2q(\alpha) + (A-1)r(\alpha) - 2(A-1)^2\frac{r(\alpha)e^{\int_\alpha^1 q(\tau)-r(\tau)d\tau}}{k(\alpha)Z(\alpha,\alpha)}\right]\frac{e^{\int_\alpha^1 q(\tau)-r(\tau)d\tau}}{AZ^2(\alpha,\alpha)} \\ & + 2q(\alpha)A\int_\alpha^1 \frac{q(t)e^{\int_t^1 q(\tau)-r(\tau)d\tau}}{Z^3(\alpha,t)}\int_\alpha^t \frac{r(s)}{k(s)}e^{\int_s^1 q(\tau)-r(\tau)d\tau}dsdt \\ > & \left[2q(\alpha) + (A-1)r(\alpha)[1 - \frac{2(A-1)e^{\int_\alpha^1 q(\tau)-r(\tau)d\tau}}{k(\alpha)Z(\alpha,\alpha)}]\right]\frac{e^{\int_\alpha^1 q(\tau)-r(\tau)d\tau}}{AZ^2(\alpha,\alpha)}. \end{aligned}$$

Substituting $x^*(n+\alpha)$ into above inequality, one obtains

$$\begin{aligned} G'(\alpha) &> \left[2q(\alpha)+(A-1)r(\alpha)[1-\frac{2x^*(n+\alpha)}{k(\alpha)}]\right]\frac{e^{\int_\alpha^1 q(\tau)-r(\tau)d\tau}}{AZ^2(\alpha,\alpha)} \\ &> [2q(\alpha)-(A-1)r(\alpha)]\frac{e^{\int_\alpha^1 q(\tau)-r(\tau)d\tau}}{AZ^2(\alpha,\alpha)} \\ &> [2q(\alpha)-Ar(\alpha)]\frac{e^{\int_\alpha^1 q(\tau)-r(\tau)d\tau}}{AZ^2(\alpha,\alpha)}. \end{aligned}$$

It follows from $\int_0^1 r(\tau)d\tau-\int_\alpha^1 q(\tau)d\tau<\ln(\frac{2q(\alpha)}{r(\alpha)})$ that $2q(\alpha)-Ar(\alpha)>0$, i.e. $G'(\alpha)>0$. Thus, there exists a unique α^* such that ASY reaches its maximum value. □

For model (3.1) with fixed harvest timing α, the following result shows the optimal harvesting effort which maximizes the ASY $Y^\alpha(q(t))$ defined by Eq. (3.1). []

Theorem 3.3. *If $r(t)\geq 2k(t)/k'(t)$ for $t\in(\alpha,1]$, then the $Y^\alpha(q(t))$ reaches its maximum value with $q^*(t)=\frac{1}{2}r(t)-k'(t)/k(t)$, and the maximum value is*

$$Y^\alpha(q^*(t))=\frac{1}{4}\int_\alpha^1 r(t)k(t)dt-\frac{1}{2}[k(1)-k(\alpha)].$$

The corresponding population level is

$$x^*(t)=\begin{cases}\frac{k(1)e^{\int_0^t r(\tau)d\tau}}{2+k(1)\int_0^t\frac{r(s)}{k(s)}e^{\int_0^s r(\tau)d\tau}ds}, & t\in[0,\alpha],\\ \frac{1}{2}k(t), & t\in(\alpha,1].\end{cases}$$

Proof. Let

$$y(t)=e^{\int_\alpha^t q(\tau)d\tau}\text{ and }R=e^{\int_0^1 r(\tau)d\tau},$$

then

$$q(t)=(\ln(y(t)))'.$$

In this case, the ASY $Y^\alpha(q(t))$ can be written as

$$\begin{aligned} Y^\alpha(y(t)) &= \int_\alpha^1\frac{(\frac{R}{y(1)}-1)\frac{y(1)e^{-\int_t^1 r(\tau)d\tau}}{y(t)}(\ln(y(t)))'}{Z(t)}dt \\ &= -\int_\alpha^1\frac{(R-y(1))e^{-\int_t^1 r(\tau)d\tau}(\frac{1}{y(t)})'}{Z(t)}dt, \end{aligned}$$

where

$$Z(t)=\int_0^{\alpha}\frac{r(s)}{k(s)}e^{\int_0^s r(\tau)d\tau}ds+R\int_{\alpha}^{t}\frac{r(s)e^{-\int_s^1 r(\tau)d\tau}}{k(s)y(s)}ds+y(1)\int_t^1\frac{r(s)e^{-\int_s^1 r(\tau)d\tau}}{k(s)y(s)}ds.$$

The first derivative of $Z(t)$ with respect to t can be expressed as

$$Z'(t)=(R-y(1))\frac{r(t)e^{-\int_t^1 r(\tau)d\tau}}{k(t)y(t)}. \tag{3.4}$$

Then we obtain

$$(R-y(1))\frac{e^{-\int_t^1 r(\tau)d\tau}}{y(t)}=\frac{Z'(t)k(t)}{r(t)},$$

and

$$(R-y(1))e^{-\int_t^1 r(\tau)d\tau}\left[\frac{1}{y(t)}\right]'=\left[\frac{Z'(t)k(t)}{r(t)}\right]'-Z'(t)k(t).$$

Therefore, $Y^{\alpha}(q(t))$ becomes

$$\begin{aligned} Y^{\alpha}(Z(t)) &= -\int_{\alpha}^{1}\left[\frac{(\frac{Z'(t)k(t)}{r(t)})'}{Z(t)}-\frac{Z'(t)k(t)}{Z(t)}\right]dt \\ &= -\frac{k(t)}{r(t)}\left[\ln(Z(t))\right]'\,|_{\alpha}^{1}-\int_{\alpha}^{1}\frac{k(t)}{r(t)}\left[(\ln(Z(t)))'\right]^2-k(t)\left[\ln(Z(t))\right]'dt. \end{aligned} \tag{3.5}$$

Denote

$$W(t)=[\ln(Z(t))]',$$

then Eq. (3.5) becomes

$$\begin{aligned} Y^{\alpha}((W(t))) &= -\frac{k(t)}{r(t)}W(t)\,|_{\alpha}^{1}-\int_{\alpha}^{1}\frac{k(t)}{r(t)}W^2(t)-k(t)W(t)dt \\ &= -\int_{\alpha}^{1}\left[\frac{k(t)}{r(t)}W(t)\right]'+\frac{k(t)}{r(t)}W^2(t)-k(t)W(t)dt \\ &= -\int_{\alpha}^{1}F(t,W,W')dt. \end{aligned} \tag{3.6}$$

To obtain the optimal harvesting effort for model (3.1), we follow three steps: (1) find $W^*(t)$ which maximizes $Y^{\alpha}(W(t))$ in Eq. (3.6); (2) determine $q^*(t)$ corresponding to $W^*(t)$; (3) prove $q^*(t)$ satisfies $\int_0^1 r(\tau)d\tau-\int_{\alpha}^1 q(\tau)d\tau>0$. According to the optimal control theory, a $W^*(t)$ that maximizes $Y^{\alpha}(W(t))$ in Eq. (3.6) must satisfy the *Euler-Lagrange Equation* [30]:

$$\frac{\partial F}{\partial W}-\frac{d}{dt}\left(\frac{\partial F}{\partial W'}\right)=0. \tag{3.7}$$

Substituting F in Eq. (3.6) into Eq. (3.7), we obtain

$$W^*(t) = \frac{1}{2}r(t), \quad t \in (\alpha, 1]$$

which is unique. Substituting $W^*(t)$ into $W(t) = [\ln(Z(t))]'$, we get

$$Z^*(t) = Z(\alpha)e^{\frac{1}{2}\int_\alpha^t r(s)ds}, \quad t \in (\alpha, 1]. \tag{3.8}$$

Using Eq. (3.4) and Eq. (3.8), we obtain

$$y^*(t) = \frac{2(R - y^*(1))e^{-\int_t^1 r(s)ds - \frac{1}{2}\int_\alpha^t r(s)ds}}{k(t)Z(\alpha)}, \quad t \in (\alpha, 1].$$

Then the optimal harvesting effort, determined by $W^*(t)$, is

$$q^*(t) = [\ln(y^*(t))]' = \frac{1}{2}r(t) - \frac{k'(t)}{k(t)}, \quad t \in (\alpha, 1]. \tag{3.9}$$

If $r(t) \geq \frac{2k'(t)}{k(t)}$ holds true for any $t \in (\alpha, 1]$, then $q^*(t) > 0$. By Eq. (3.9) and the periodicity of $k(t)$, we have

$$\begin{aligned}
& \textstyle\int_0^1 r(\tau)d\tau - \int_\alpha^1 q^*(\tau)d\tau \\
> \; & \textstyle\int_0^1 r(\tau) - q^*(\tau)d\tau \\
= \; & \textstyle\frac{1}{2}\int_0^1 r(t)dt + \int_0^1 [\ln(k(t))]'dt \\
= \; & \textstyle\frac{1}{2}\int_0^1 r(t)dt > 0.
\end{aligned}$$

Therefore, $q^*(t)$ is the unique optimal harvesting effort of model (3.1) which maximizes annual-sustainable yield $Y^\alpha(q^*(t))$ at

$$Y^\alpha(q^*(t)) = \frac{1}{4}\int_\alpha^1 r(t)k(t)dt - \frac{1}{2}[k(1) - k(\alpha)].$$

To calculate the optimal population level $x^*(t)$, substituting Eq. (3.8) and Eq. (3.4) into $W(t) = [\ln(Z(t))]'$, we have

$$W^*(t) = \frac{(Z^*(t))'}{Z^*(t)} = \frac{(R - y^*(1))\frac{r(t)e^{-\int_t^1 r(\tau)d\tau}}{k(t)y^*(t)}}{Z(\alpha)e^{\frac{1}{2}\int_\alpha^t r(s)ds}} = \frac{1}{2}r(t).$$

Therefore, we get the following periodic population size subject to harvesting:

$$x^*(t) = \begin{cases} \frac{(A-1)e^{\int_0^t r(\tau)d\tau}}{Z(1)+(A-1)\int_0^t \frac{r(s)}{k(s)}e^{\int_0^s r(\tau)d\tau}ds} = \frac{k(1)e^{\int_0^t r(\tau)d\tau}}{2+k(1)\int_0^t \frac{r(s)}{k(s)}e^{\int_0^s r(\tau)d\tau}ds}, & t \in [0, \alpha], \\ \frac{(A-1)e^{\int_t^1 q^*(\tau)-r(\tau)d\tau}}{Z^*(t)} = \frac{(A-1)e^{\int_t^1 q^*(\tau)-r(\tau)d\tau}}{Z(\alpha)e^{\frac{1}{2}\int_\alpha^t r(s)ds}} = \frac{1}{2}k(t), & t \in (\alpha, 1]. \end{cases}$$

□

Remark 3.1. *In model (3.1), if $\alpha = 0$, then*

$$q^*(t) = \frac{1}{2}r(t) - \frac{k'(t)}{k(t)}, \quad x^*(t) = \frac{k(t)}{2}, \quad Y^0(q^*(t)) = \frac{1}{4}\int_0^1 r(t)k(t)dt,$$

which is proposed by Fan and Wang [8]. *Therefore, the results in this paper generalize the classical results for non-autonomous harvested model. In addition, the results also reveal that the optimal harvesting effort for model (3.1) is independent from harvest timing. Moreover, the longer harvesting time-spectrum the larger annual-sustainable yield becomes.*

In fact, the first derivative of $Y^\alpha(q^*(t))$ with respect to α is

$$\frac{dY^\alpha(q^*(t))}{d\alpha} = -\frac{1}{4}[r(\alpha)k(\alpha) - 2k'(\alpha)] \le 0.$$

Therefore the optimal harvesting policy is

$$q^*(t) = \frac{1}{2}r(t) - \frac{k'(t)}{k(t)}, \quad \alpha^* = 0,$$

and MASY is

$$Y_{\max}(\alpha, q(t)) = \frac{1}{4}\int_0^1 r(t)k(t)dt.$$

The optimal population level is $x^*(t) = \frac{k(t)}{2}$.

3.2. **The effect of harvest timing on average harvesting effort.**

When $q^*(t) = \frac{1}{2}r(t) - \frac{k'(t)}{k(t)}$, the average harvesting effort on $(\alpha, 1]$ can be described as

$$\begin{aligned}\overline{q}(\alpha) &= \frac{1}{1-\alpha}\int_\alpha^1 \left[\frac{1}{2}r(t) - \frac{k'(t)}{k(t)}\right] dt \\ &= \frac{1}{2(1-\alpha)}\int_\alpha^1 r(t)dt - \frac{1}{1-\alpha}\ln\frac{k(1)}{k(\alpha)}.\end{aligned}$$

It is easy to see that the first derivative of $\overline{q}(\alpha)$ with respect to α can be expressed as

$$\begin{aligned}\overline{q}'(\alpha) &= \tfrac{1}{(1-\alpha)^2}\int_\alpha^1 \left[\tfrac{1}{2}r(t) - \tfrac{k'(t)}{k(t)}\right] dt - \tfrac{1}{1-\alpha}\left[\tfrac{1}{2}r(\alpha) - \tfrac{k'(\alpha)}{k(\alpha)}\right] \\ &= \tfrac{1}{(1-\alpha)^2}\left[\int_\alpha^1 \tfrac{1}{2}r(t)dt - \ln\tfrac{k(1)}{k(\alpha)} - (1-\alpha)\left[\tfrac{1}{2}r(\alpha) - \tfrac{k'(\alpha)}{k(\alpha)}\right]\right] \\ &=: \tfrac{1}{(1-\alpha)^2}M(\alpha).\end{aligned}$$

To explore the effect of α on function $\overline{q}(\alpha)$, we take the forms for $r(t)$, $k(t)$ as

$$r(t) = r_0 + \mu\cos(2\pi t + \phi), \quad k(t) = k_0 + \eta\cos(2\pi t + \varphi),$$

such that $r(t)$ and $k(t)$ satisfy $C(t) := \frac{1}{2}r(t) - \frac{k'(t)}{k(t)} > 0$. Then

$$\begin{aligned} M(\alpha) = & \frac{\mu}{4\pi}[\sin(\phi) - \sin(2\pi\alpha + \phi)] - \ln \frac{k_0 + \eta\cos(\varphi)}{k_0 + \eta\cos(2\pi\alpha + \varphi)} \\ & - (1-\alpha)\left[\frac{1}{2}\mu\cos(2\pi\alpha + \phi) + \frac{2\pi\eta\sin(2\pi\alpha + \varphi)}{k_0 + \eta\cos(2\pi\alpha + \varphi)}\right]. \end{aligned}$$

If we take $r_0 = 1.5$, $\mu = 0.2$, $k_0 = 100$, $\eta = 10$, $\phi = \pi/6$, $\varphi = 3\pi/4$, then Figure 2(a) shows that $C(t) = \frac{1}{2}r(t) - \frac{k'(t)}{k(t)} > 0$ holds true for any given $t \in [0,1]$. Figure 2(b-c) show that there exists two harvesting timings $\alpha_1 < \alpha_2$ such that the average harvesting effort $\overline{q}(\alpha)$ reaches its minimum ($\alpha = \alpha_1$) and maximum ($\alpha = \alpha_2$) value on $\alpha \in [0,1]$, respectively. Thus either $\alpha < \alpha_1$ or $\alpha > \alpha_2$, the average harvesting effort is decreasing as the length of harvesting time decreases, and consequently the annual-sustainable yield decreases very fast. On the other hand, if $\alpha_1 \leq \alpha \leq \alpha_2$, the average harvesting effort increases as the length of harvesting duration decreases. For this case, the length of harvesting open season has relatively moderate effect on the annual-sustainable yield, as shown in Figure 2(d). In conclusion, although the optimal harvesting effort is independent from harvest timing, the latter has significant effect on the average optimal harvesting effort. However, due to the different relationships between harvest timing and average harvesting effort, the decreasing speed of the annual-sustainable yield is different as harvest timing increases.

4. **Discussion and biological conclusions**

Closed seasons are often employed and implemented to protect the renewable resources, and it had been shown that seasonal closures have significant effects on population persistence and sustainable yields. With the assumption that harvesting is banned at one period of each year and the rest is open season, a non-smooth Logistic model is formulated to investigate the harvesting problems in constant and periodic environments. We are focusing on the effect of the harvest timing on the stability properties, as well as the optimal harvesting policy with the MASY as the optimal objective.

In constant environment, both of the harvest timing and harvesting effort have the key effects on the MASY . When the harvesting intensity is small, namely harvesting effort not exceed

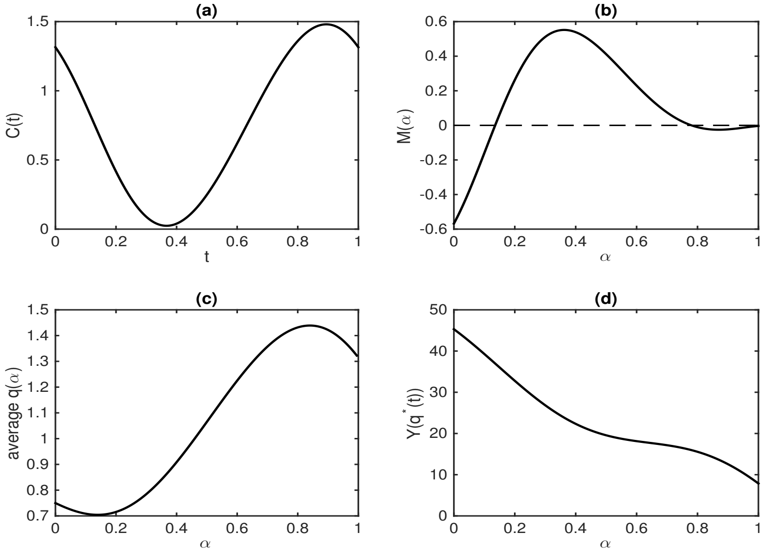

FIGURE 2. Illustrations of the effect of harvest timing on average harvesting effort for model (3.1) with fixed $r(t) = 1.5+0.5\cos(2\pi t+\pi/6)$ and $k(t) = 100+10\cos(2\pi t+3\pi/4)$. (a) The curve of function $C(t) == \frac{1}{2}r(t) - \frac{k'(t)}{k(t)}$ with $C(t) > 0$ for $t \in [0,1]$; (b-c) Existence of positive roots of the function $M(\alpha)$ with $\alpha_1 < \alpha_2$ such that the average harvesting $q(\alpha)$ reaches its minimum and maximum value; (d) The plot of annual-sustainable yield $Y(q^*(t))$ against α.

intrinsic growth rate, then the population persists regardless of the length of the harvesting open season. On the contrary, if the harvesting effort is greater than intrinsic growth rate, suitable length of closed season is allowed to ensure population persistence. Consequently, when the harvesting effort is small, then the longer the open season is the larger MASY can obtain. When the harvesting effort is greater than half of intrinsic growth rate, then there exists a unique optimal harvest timing which can maximize the annual-sustainable yield. Our results also indicate that corresponding to the optimal harvesting effort the higher MASY can be achieved if harvesting occurs as early as possible. This is consistent with the results obtained by Tang and Chen [12] and Xu, Boyce and Daley [10]. Based on the results obtained from Clark [7], the optimal harvesting policy is keeping the whole year as open season and harvesting effort at half of the intrinsic growth rate. Therefore, intensive harvesting combined with short-term open season takes no advantage over the moderate harvesting with long-term open season with respect to economic benefits.

When the population growth is affected by seasonal (or fluctutant) environments, the results also show that the length of harvesting time-spectrum and harvesting effort synthetically play an important role on the population persistence and annual yields. Once again, our main results extend the results obtained by Fan and Wang (1998) for periodic Logistic model with periodic harvest. Similar with the autonomous model, in the non-autonomous model with respect to the optimal harvesting effort MASY always decreases as the length of open harvesting season decreases. The different aspects between autonomous models and non-autonomous model lie in the relationship between the optimal harvesting effort and harvest timing. In autonomous model, the optimal harvesting effort has direct relationship with harvest timing, a shorter open season corresponds to a greater optimal harvesting effort. However, the optimal harvesting effort in non-autonomous model is independent on harvest timing. The harvest timing affects the average value of harvesting effort.

In the present work, we only focus on the harvesting effort in model (2.2) which is independent of the harvesting time parameter α. Obviously, this is not realistic in practice. Our results show that the optimal harvesting effort is increasing as the harvest timing increases. In fact, in order to obtain maximum yields once the closed season policy is implemented, the fish-men will increase the harvesting effort. Therefore, it is more reasonable to assume that the harvesting effort is an increasing function of the duration of the harvest closing season (α):

$$q = g(\alpha),$$

where $g(\alpha)$ is a increasing function with respect to α. Thus the model (2.2) can be expressed as following:

$$\frac{dx(t)}{dt} = \begin{cases} rx(t)(1-x(t)), & t \in [n, n+\alpha], \\ rx(t)(1-x(t)) - g(\alpha)x(t), & t \in (n+\alpha, n+1]. \end{cases}$$

Córdova-Lepe et al. [2] firstly considered the functional relationship between the harvest timing and harvesting effort and analysed the harvesting effort threshold of sustainability and the extinction of the fishery resources. We may leave this for further investigation.

It is always predicted the existence of seemingly paradoxical hydra effect, where population size increases in response to an increase in its per-capita mortality rate, in some one-dimensional difference equations [16, 17]. However, it is not possible for continuous-time growth model with growth function $x' = f(t,x)$. Considering additional mortality rate g, the system becomes $y' = f(t,y) - g(t,y)$. It then follows from the comparison argument (similar to the proof of proposition 3.1.1 in [31]), that for any $x(0) > y(0)$, if solutions for respective systems $x(t)$ and $y(t)$ exists for all $t \in [0,\infty)$, then $x(t) \geq y(t)$. Therefore, hydra effect is impossible for one-dimensional continuous time system. In the present paper, the unstructued population growth excludes the hydra effect. However, the set of harvest closures is aimed to protect juvenile population and therefore, a stage-structured population should be more suitable, which is not considered in the present manuscript, we also leave this interesting topic as our future work.

In this manuscript, the cost-effectiveness of the proportional harvest with closed/open harvest seasons, which is easy to set as a policy, is evaluated. In reality, other types of harvesting efforts have been employed, such as those dependent on the threshold population size (biomass at the limit), under which harvesting is banned, and harvesting is allowed when the threshold is surpassed, or the target oriented harvesting [25]. We hope the theoretical results developed here can be adapted to other suitable harvesting efforts in various scenarios, which may leave as our future work.

Conflict of Interests

The authors declare that there is no conflict of interests.

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