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BOUNDARY CONTROLLABILITY OF NONLINEAR SIZE-STRUCTURED POPULATION DYNAMICS

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Abstract. This work is concerned with the existence of a unique solution and the controllability for a size-structured population model, which incorporates density-dependent immigration and boundary control. By means of semigroup theory of operators and fixed point reasoning, we show that the system is well-posed and exactly controllable.

Keywords: Population model; Size-structure; Controllability; Semigroup theory; Fixed point.

2010 AMS Subject Classification: 45K05, 92B05, 92D25.

1. Introduction

The controllability of a dynamic system is one of the most important internal features, which displays the function of structures and parameters in the system. If a system is not controllable in some sense, it may be of great risk. Unfortunately, although controllability is one of the most significant problems in system control, it is one of the most difficult problems as well. So is the case in biological population systems.

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Mathematical models of populations including age, size, spatial location, life stage or other structuring of individuals have an extensive history[1]-[3]. Compared with unstructured models, they are more realistic ecologically, and more challenging mathematically. As for the size-structured models, Sinko and Streifer formulated the first one in 1967[4]. Since then there have been a number of investigations on the topic, but most of the efforts has been focused on the behavioral analysis, such as persistence, extinction or other transient or long-run evolution. The authors in [5] established the well-posedness for their model and developed an approximation scheme for the solutions. Kato investigated the local existence, positivity and continuous dependence for solutions of a population models by means of characteristic curves methods, see[6]-[7]. In [8], the same author used Schauder's fixed point theorem to obtain the existence and uniqueness of local solutions, and the continuous dependence on initial data for a size-dependent population model with nonlinear growth rate. The stability and some optimal control problems for size-structured population dynamics have been studied in[9]-[12]. The authors in [13] proved an abstract result for a kind of semilinear problem and applied it to the controllability of a size-structured population model. However, there is only few work about the controllability for size-structured population models. As a special case, many results on the controllability of age-structured systems have been established, see[14]-[18] and the references therein. We are motivated in this paper to examine the problem for a type of size-dependent model, which incorporates the density-dependent immigration and boundary control policies.

The remainder of the present paper is organized as follows. In Section 2 we propose the model, provide a definition and list some hypotheses. The main results and its proofs are formulated in Section 3. Then final Section 4 consists of conclusions and some remarks.

2. **Preliminaries**

This work is concerned with the exact controllability in finite time for the following size-structured single species model:

$$
\begin{cases}
\dfrac{\partial p(s,t)}{\partial t}+\dfrac{\partial(g(s)p(s,t))}{\partial s}=-\mu(s,t)p(s,t)+f(p(s,t),t),(t,s)\in Q_T,\\
g(0)p(0,t)=\int_0^m\beta(s,t)p(s,t)ds+b(t)u(t),\quad t\in[0,T],\\
p(s,0)=p_0(s),\quad s\in[0,m],
\end{cases}
\tag{2.1}
$$

in which the unknown function $p(s,t)$ represents the density of population with respect to size s at time t, $Q_T=(0,m)\times(0,T)$. $m>0$ is the finite maximum size of any individual in the population. The size-specific functions $\mu(s,t)$ and $\beta(s,t)$ denote, respectively, the mortality and fertility. The term $f(p(s,t),t)$ models the migration progress, and $g(s)$ is the growth rate of size over time t. Let $X:=L^1(0,T),\partial X:=(0,T)$, b is a control operator which is bounded from a Banach space U to ∂X; $u(t)$ is the boundary control variable, and $p_0(s)$ is the initial size distribution of our target population. Without loss of generality, we suppose that the size of newborns is zero.

We state the basic assumptions as follows:

(A1) $\beta\in L^\infty(Q_{\mathrm{T}}),\beta(s,t)\geq 0$ a.e. $(s,t)\in Q_{\mathrm{T}}$;

(A2) $g\in C^1[0,m]$ and $0<g_*\leq g(s)\leq g^*,\mu(s,t)+g'(s)\geq 0$ a.e.$s\in[0,m]$, $g_*,\ g^*$ are constant and $\Gamma(s)=\int_0^s\frac{1}{g(v)}dv$;

(A3) $\mu(\cdot,t)\in L^1_{\mathrm{loc}}(0,m),\mu(s,t)\geq\nu>0$ a.e.$(s,t)\in Q_{\mathrm{T}}$, $\int_0^m\mu(\Gamma^{-1}(\Gamma(m)-s),t-s)ds=+\infty$;

(A4) $0\leq p_0(s)\leq p_0^*$ a.e. $s\in(0,m)$, p_0^* is a constant;

(A5) $f\in L^1(Q_{\mathrm{T}}),f(p(s,t),t)\geq 0$ a.e. $(s,t)\in Q_{\mathrm{T}}$; and

$$||f(\phi,t)-f(\psi,t)||\leq L||\phi-\psi||$$

where L is a constant;

(A6) The functions μ,β,f and g,p_0 are extended by zero in the outside of their domains.

In convenience of dealing with the control problem for systems (2.1), we first consider the case $f(p(s,t),t)=0$, then treat the general problem.

When $f(p(s,t),t)=0$, we fit the system (2.1) into the setting of abstract Cauchy problem. Take $D:=W^{1,1}(0.T)$, clearly it is dense and can be continuously embedded into X [19]. Let $x(t)=p(\cdot,t)$, then the size structured population model (2.1) can be changed into the following form:

$$\begin{cases} \dfrac{dx(t)}{dt} = A_{\max}(t)x(t), \quad t \in [0,T], \\ L(t)x(t) = h(t,x(t)) + b(t)u(t), \quad t \in [0,T], \\ x(0) = p_0, \end{cases} \tag{2.2}$$

the operators above are defined as follows:

$$A_{\max}(t)\varphi := -g'(\cdot)\varphi - g(\cdot)\frac{\partial}{\partial s}\varphi - \mu(\cdot,t)\varphi$$

and

$$L(t)\varphi := g(0)\varphi(0), \qquad h(t,x(t)) = \int_0^m \beta(s,t)x(t)ds$$

for $\varphi \in L^1(0,T)$.

For solutions to problem (2.2), we adopt the following definition.

Definition 1. [22] *The continuous function $x : [0,T] \longmapsto X$ given by variation of constants formula*

$$x(t) = U(0,t)x(0) + \lim_{\lambda \to +\infty} \int_0^t U(\sigma,t)\lambda L_{\lambda,\sigma}[h(\sigma,x(\sigma)) + b(\sigma)u(\sigma)]d\sigma, \qquad t \in [0,T] \tag{2.3}$$

is called the mild solution of the inhomogeneous boundary Cauchy problem (2.2).

Remark 1. *Under hypotheses (A1)-(A6), it will be shown that there is an evolution family $U(s,t)$ generated by the solution operators of system (2.2), such that $U(s,t)x \in D(A_{\max}(t))$ and $\frac{d}{dt}U(s,t)x = A_{\max}(t)U(s,t)x$ for all $x \in D(A_{\max}(s))$ and $t \geq s \geq 0$. And we also have the following estimate:*

$$||U(s,t)|| \leq Me^{w(t-s)},$$

where M and w are the stability constants.

Remark 2. $L_{\lambda,t} := (L(t)|_{\ker(\lambda - A_{\max}(t))})^{-1} : \partial X \to \ker(\lambda - A_{\max}(t))$.

3. Main Results

In this section, we show the well-posedness and the exact controllability for size-structured population model (2.1). Firstly, we consider the situation without migration process. Then we treat the general case.

3.1. **Existence of mild solutions.** The following result provides the existence of mild solutions for the problem (2.2).

Proposition 1. *Under the assumptions (A1)-(A6), the problem (2.2) has a unique mild solution* $x \in C([0,T],X)$.

Proof: According to Hille-Yosida's theorem, we need only to prove that $A_{\max}$ is closed and $\rho(A_{\max}) \supset (\omega,\infty)$, $||R(\lambda, A_{\max}(t))|| \leq \dfrac{M}{\lambda-\omega}$.

Taking $\varphi \in D$ and applying the closed graph theorem, we obtain that

$$
\begin{aligned}
||\varphi||_D &= \int_0^m |\varphi(a)|da + \int_0^m |\varphi'(a)|da \\
&\leq \int_0^m |\varphi(a)|da + \int_0^m |\varphi'(a) + \frac{g'(a)+\mu(a,t)}{g(a)}\varphi(a)|da \\
&\quad + \int_0^m |\frac{g'(a)+\mu(a,t)}{g(a)}\varphi(a)|da \\
&= \int_0^m |\varphi(a)|da + \int_0^m |\frac{A_{\max}(t)}{g(a)}\varphi(a)|da + \int_0^m |\frac{g'(a)+\mu(a,t)}{g(a)}\varphi(a)|da \\
&\leq ||\varphi||_X + \frac{1}{g_*}||A_{\max}\varphi||_X + \frac{1}{g_*}\sup_{t\in[0,T]}||g'(a)+\mu(a,t)||_\infty ||\varphi||_X \\
&\leq \max\{1, \frac{1}{g_*}\sup_{t\in[0,T]}||g'(a)+\mu(a,t)||_\infty, \frac{1}{g_*}\}(||\varphi||_X + ||A_{\max}\varphi||_X) \\
&= C_1(||\varphi||_X + ||A_{\max}\varphi||_X),
\end{aligned}
$$

where $C_1 = \max\{1, \frac{1}{g_*}\sup_{t\in[0,T]}||g'(a)+\mu(a,t)||_\infty, \frac{1}{g_*}\}$, and $||\varphi||_X = \int_0^m |\varphi(a)|da$.

On the other hand,

$$
\begin{aligned}
&||\varphi||_X+||A_{\max}\varphi||_X \\
&=\int_0^m|\varphi(a)|da+\int_0^m|-g(a)\varphi'(a)-(g'(a)+\mu(a,t))\varphi(a)|da \\
&\leq \int_0^m|\varphi(a)|da+\int_0^m|g(a)\varphi'(a)|da+\int_0^m|(g'(a)+\mu(a,t))\varphi(a)|da \\
&\leq \int_0^m|\varphi(a)|da+g^*\int_0^m|\varphi'(a)|da+\sup_{t\in[0,T]}||g'(a)+\mu(a,t)||_\infty\int_0^m|\varphi(a)|da \\
&\leq \max\{1+\sup_{t\in[0,T]}||g'(a)+\mu(a,t)||_\infty,g^*\}\left(\int_0^m|\varphi'(a)|da+\int_0^m|\varphi(a)|da\right) \\
&=C_2||\varphi||_D,
\end{aligned}
$$

where $C_2=\max\{1+\sup\limits_{t\in[0,T]}||g'(a)+\mu(a,t)||_\infty,g^*\}$.

Next, we will show that the resolvent operator associated to $A(t):=A_{\max}|_{\ker L(t)}$ meets the conditions of Hille-Yosida's theorem.

Let $\lambda>0$, the resolvent operator of $A(t)$ is given as follows:

$$R(\lambda.A(t))\varphi=\frac{1}{g(\cdot)}\int_0^{\cdot}e^{-\int_\tau^{\cdot}\frac{\lambda+\mu(\sigma,t)}{g(\sigma)}d\sigma}\varphi(\tau)d\tau$$

and

$$
\begin{aligned}
||R(\lambda.A(t))\varphi||_X &= \int_0^m|R(\lambda.A(t))\varphi(\xi)|d\xi \\
&= \int_0^m\frac{1}{|g(\xi)|}\left|\int_0^\xi e^{-\int_\tau^\xi\frac{\lambda+\mu(\sigma,t)}{g(\sigma)}d\sigma}\varphi(\tau)d\tau\right|d\xi \\
&\leq \int_0^m\frac{|\varphi(\tau)|}{g(\xi)}\int_\tau^m e^{-\int_\tau^\xi\frac{\lambda+\mu(\sigma,t)}{g(\sigma)}d\sigma}d\xi d\tau. \qquad (3.1)
\end{aligned}
$$

From the hypothesis (A3), we conclude that $-(\lambda+\mu(\sigma,t))\leq-(\lambda+v)$, i.e. the above inequality (3.1) can be changed into following form:

$$
\begin{aligned}
||R(\lambda.A(t))\varphi||_X &\leq \int_0^m\frac{|\varphi(\tau)|}{g(\xi)}\int_\tau^m e^{-(\lambda+v)\int_\tau^\xi\frac{1}{g(\sigma)}d\sigma}d\xi d\tau \\
&= \int_0^m\frac{|\varphi(\tau)|}{(\lambda+v)}\left(1-e^{-(\lambda+v)\int_\tau^m\frac{1}{g(\sigma)}d\sigma}\right)d\tau \\
&\leq \int_0^m\frac{|\varphi(\tau)|}{(\lambda+v)}d\tau \\
&= \frac{1}{(\lambda+v)}||\varphi||_X\leq\frac{1}{\lambda}||\varphi||_X.
\end{aligned}
$$

Therefore, the evolution family $U(s,t)$ exists and $A_{\max}$ is the infinitesimal generator of a C_0 semigroup of contractions on X.

Then by Hille-Yosida's theorem in [20], we claim that the problem (2.2) has a unique mild solution.

In fact, $A_{\max}\varphi$ is continuously differentiable for all $\varphi \in D$. If $L_{\lambda,\sigma}$ is well-defined, then the operator $L(t)$ is surjective and bounded, which means that the mild solution expression makes sense. In what follows, we will verify the point.

Setting, in (2.3), $\varphi(0) = -\int_0^m \frac{\partial \varphi(a)}{\partial a} da$ for all $\varphi \in W^{1,1}(0,T)$, we get that

$$||L(t)\varphi|| = |g(0)\varphi(0)| \leq g^*||\varphi||_D.$$

Then for $x \in [0,T]$, we take $\varphi(\cdot) = \frac{e^{-\cdot}}{g(\cdot)}x$, and conclude $L(t)\varphi = g(0)\varphi(0) = x$, which implies the surjectivity of $L(t)$. In addition, we also have

$$\begin{aligned} ||L(t)\varphi|| &= |g(0)\varphi(0)| = |\int_0^m \frac{\partial g(a)\varphi(a)}{\partial a} da| \\ &= |\int_0^m (-\lambda - \mu(a,t))\varphi(a) da| \\ &\geq (\lambda + v)||\varphi||_X > \lambda ||\varphi||_X. \end{aligned}$$

So, $L_{\lambda,\sigma}$ makes sense and satisfies $\lambda ||L_{\lambda,\sigma}|| \leq 1$.

When the migration progress $f(p(s,t),t) \neq 0$, we consider the existence of solutions for the following inhomogeneous Cauchy problem:

$$\begin{cases} \frac{dx(t)}{dt} = A_{\max}(t)x(t) + f(x(t),t), \quad t \in [0,T], \\ L(t)x(t) = h(t,x(t)) + b(t)u(t), \quad t \in [0,T], \\ x(0) = p_0. \end{cases} \tag{3.2}$$

From **Proposition 1**, it readily follows that

Theorem 1. *Under the assumptions (A1)-(A6), the size-structured population model (3.2) has a unique mild solution* $x \in C([0,T],X)$ *and*

$$\begin{aligned} x(t) &= U(0,t)x(0) + \lim_{\lambda \to +\infty} \int_0^t U(\sigma,t)\lambda L_{\lambda,\sigma}[h(\sigma,x(\sigma)) + b(\sigma)u(\sigma)]d\sigma \\ &\quad + \int_0^t U(\sigma,t) f(x(\sigma),\sigma) d\sigma, \; t \in [0,T]. \end{aligned} \tag{3.3}$$

3.2. **Boundary Controllability.** We now consider the controllability of the population model (2.1) (i.e. system (3.2)) by the boundary control variable. Here we assume that the control operators are such that $b(\cdot) \in L^2(0,T)$.

Definition 2. [21, 22] *The system (3.2) is said to be exactly controllable on $[0,T]$, for some $T>0$, if for every initial value x_0, $v \in X$ there is a control $u \in L^2(0,T;U)$ such that the solution $x(\cdot)$ of (3.2) satisfies $x(T)=v$.*

We define the operator W from $L^2([0,T],U)$ into X by

$$Wu := \lim_{\lambda\to+\infty}\int_0^T U(\sigma,T)\lambda L_{\lambda,\sigma}b(\sigma)u(\sigma)d\sigma, \tag{3.4}$$

then we can readily check that the operator W is well-defined. Using results in [23, 24], we induce that the inverse operator W^{-1} exists and is bounded.

With the help of the inverse operator W^{-1}, the following controllability result can be proved.

Proposition 2. *If the hypotheses (A1)-(A6) are satisfied, then the system (3.2) is exactly controllable on [0,T] provided $T(L+\|\beta\|_{L^\infty})(1+\|b(\cdot)\|_{L^2}\|W^{-1}\|)<1$.*

Proof: If the system (3.2) is exactly controllable on [0,T], then there is a u^* such that $x(T)=v$, i.e.

$$\begin{aligned} U(0,T)x(0)+\lim_{\lambda\to+\infty}\int_0^T U(\sigma,T)\lambda L_{\lambda,\sigma}[h(\sigma,x(\sigma)) \\ +b(\sigma)u^*(\sigma)]d\sigma+\int_0^T U(\sigma,T)f(x(\sigma),\sigma)d\sigma = v. \end{aligned}$$

Define a control as follows:

$$\begin{aligned} u^* = W^{-1}\Big[v-U(0,T)x(0)-\lim_{\lambda\to+\infty}\int_0^T U(\sigma,T)\lambda L_{\lambda,\sigma}h(\sigma,x(\sigma))d\sigma \\ -\int_0^T U(\sigma,T)f(x(\sigma),\sigma)d\sigma\Big]. \end{aligned}$$

To establish the controllability, we need only to show that the operator $\Phi : C([0,T],X)\to C([0,T],X)$ defined in the following (3.5) has a fixed point, which is the solution of the system (3.2) and satisfies $x(T)=v$:

$$(\Phi x)(t) \;=\; U(0,t)x(0)+\lim_{\lambda\to+\infty}\int_0^t U(\sigma,t)\lambda L_{\lambda,\sigma}[h(\sigma,x(\sigma))+b(\sigma)u^*(\sigma)]d\sigma$$

$$+\int_0^t U(\sigma,t)f(x(\sigma),\sigma)d\sigma,\ t\in[0,T]. \tag{3.5}$$

Let $x_i \in C([0,T],X)$ and u_i^* be the control associated to $x_i, i=1,2$. Then we see that

$$\begin{aligned}(\Phi x_1)(t)-(\Phi x_2)(t) &= \lim_{\lambda\to+\infty}\int_0^t U(\sigma,t)\lambda L_{\lambda,\sigma}\big[h(\sigma,x_1(\sigma))-h(\sigma,x_2(\sigma))+\\ &\quad b(\sigma)u_1^*(\sigma)-b(\sigma)u_2^*(\sigma)\big]d\sigma+\int_0^t U(\sigma,t)[f(x_1(\sigma),\sigma)\\ &\quad -f(x_2(\sigma),\sigma)]d\sigma.\end{aligned}$$

Consequently,

$$\begin{aligned}&\|(\Phi x_1)(t)-(\Phi x_2)(t)\| \le \lim_{\lambda\to+\infty}\left\|\textstyle\int_0^t U(\sigma,t)\lambda L_{\lambda,\sigma}\big[h(\sigma,x_1(\sigma))-h(\sigma,x_2(\sigma))\big]d\sigma\right\|\\ &+\lim_{\lambda\to+\infty}\left\|\textstyle\int_0^t U(\sigma,t)\lambda L_{\lambda,\sigma}b(\sigma)\big[u_1^*(\sigma)-u_2^*(\sigma)\big]d\sigma\right\|+\\ &\left\|\textstyle\int_0^t U(\sigma,t)[f(x_1(\sigma),\sigma)-f(x_2(\sigma),\sigma)]d\sigma\right\|\\ &\le \lim_{\lambda\to+\infty}\left\|\textstyle\int_0^t U(\sigma,t)\lambda L_{\lambda,\sigma}\big[h(\sigma,x_1(\sigma))-h(\sigma,x_2(\sigma))\big]d\sigma\right\|+\lim_{\lambda\to+\infty}\Bigg\|\textstyle\int_0^t U(\sigma,t)\\ &\lambda L_{\lambda,\sigma}b(\sigma)W^{-1}\left[\lim_{\lambda\to+\infty}\textstyle\int_0^T U(\tau,T)\lambda L_{\lambda,\tau}\big[h(\tau,x_1(\tau))-h(\tau,x_2(\tau))\big]d\tau\right]d\sigma\Bigg\|+\\ &\lim_{\lambda\to+\infty}\left\|\textstyle\int_0^t U(\sigma,t)\lambda L_{\lambda,\sigma}b(\sigma)W^{-1}\left[\textstyle\int_0^T U(\tau,T)[f(x_1(\tau),\tau)-f(x_2(\tau),\tau)]d\tau\right]d\sigma\right\|\\ &+\left\|\textstyle\int_0^t U(\sigma,t)[f(x_1(\sigma),\sigma)-f(x_2(\sigma),\sigma)]d\sigma\right\|.\end{aligned}$$

By hypotheses (A1)-(A6), we infer that $\|U(\sigma,t)\|\le 1, \|\lambda L_{\lambda,\sigma}\|\le\gamma\le 1$, and $\|h(\sigma,x_1(\sigma))-h(\sigma,x_2(\sigma))\|\le\|\beta\|_{L^\infty}\|x_1-x_2\|$. Then Hölder's inequality leads us to the following result:

$$\begin{aligned}&\|(\Phi x_1)(t)-(\Phi x_2)(t)\|\\ \le\ & \int_0^t\left\|\big[h(\sigma,x_1(\sigma))-h(\sigma,x_2(\sigma))\big]\right\|d\sigma+\|b(\cdot)\|_{L^2}\|W^{-1}\|\\ & \int_0^T\Big[\|h(\sigma,x_1(\sigma))-h(\sigma,x_2(\sigma))\|+\|f(x_1(\sigma),\sigma)\\ & -f(x_2(\sigma),\sigma)\|\Big]d\sigma+\int_0^t\left\|[f(x_1(\sigma),\sigma)-f(x_2(\sigma),\sigma)]\right\|d\sigma\\ \le\ & T\|\beta\|_{L^\infty}\|x_1-x_2\|+\|b(\cdot)\|_{L^2}\|W^{-1}\|\big[T\|\beta\|_{L^\infty}\\ & +TL\big]\|x_1-x_2\|+LT\|x_1-x_2\|\\ =\ & T(L+\|\beta\|_{L^\infty})(1+\|b(\cdot)\|_{L^2}\|W^{-1}\|)\|x_1-x_2\|.\end{aligned}$$

Since $T(L+\|\beta\|_{L^\infty})(1+\|b(\cdot)\|_{L^2}\|W^{-1}\|)<1$, the operator Φ is a contraction on $C([0,T],X)$. The existence of a unique fixed point for Φ ends the proof.

Combining the above result with **Theorem 1**, we obtain the controllability of the system (2.1) immediately.

Theorem 2. *If the hypotheses (A1)-(A6) are satisfied, then the system (2.1) is exactly controllable on [0,T] provided* $T(L+\|\beta\|_{L^\infty})(1+\|b(\cdot)\|_{L^2}\|W^{-1}\|)<1$.

4. Concluding Remarks

After a consideration of density-dependent immigration and boundary control, we have formulated a size-structured population dynamical system to model the evolution of the species. Semigroup theory of operators enables us to establish the existence and uniqueness of the state solutions, which are based up on some reasonable conditions. As one of the main research results in present paper, controllability implies that we can adjust the state of the population according to some practical needs, by a suitable choice of boundary control. In other words, we just need to choose appropriate function β and L, i.e. put in or taking out some newborn and immigration individuals to drive the population to a given distribution. This theoretical result is consistent with the practice. In addition, this kind of result should be useful for species stabilization, optimal management of renewable resources and other involved problems.

Conflict of Interests

The authors declare that there is no conflict of interests.

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