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EXACT NULL CONTROLLABILITY OF SEMI-LINEAR STOCHASTIC DIFFERENTIAL SYSTEMS WITH NONLOCAL CONDITION

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Abstract. The main purpose of this paper is to study the exact null controllability for semi-linear stochastic differential systems with nonlocal conditions. By using the Banach fixed point theorem, sufficient conditions for the exact null controllability were established. The included application to stochastic differential control system provides a motivation for abstract results.

Keywords: exact null controllability; semi-linear stochastic differential system; nonlocal condition.

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1. Introduction

In this paper, we study the exact null controllability of the following semi-linear stochastic differential systems with nonlocal conditions

$$\begin{cases} dx(t) = [Ax(t) + Bu(t) + f(t,x(t))]dt + g(t,x(t))dw(t),\ t \in J = [0,T], \\ x(0) + h(x) = x_0, \end{cases} \tag{1.1}$$

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where the state variable $x(\cdot)$ takes values in a Hilbert space H and the control function $u(\cdot)$ is given in a Banach space $L_2^{\mathfrak{F}_t}(J,U)$ of admissible control functions, U is also a Hilbert space. A is the infinitesimal generator of a strongly continuous semigroup $S(t)$ in H. B is a bounded linear operator from U into H. Define K be an another separable Hilbert space. Suppose that $w(t)$ is an K-valued Winer process associated with a finite trace nuclear covariance operator $Q \geq 0$. $f : J \times H \longrightarrow H, g : J \times H \longrightarrow L_2^0(K,H)$ and $h : C(J,H) \longrightarrow H$ are appropriate functions to be specified later.

The problem of exact null controllability was studied by several authors. Balachandran et al. [1] considered the local null controllability for nonlinear functional differential systems in Banach spaces. They obtained the results under the assumption that the semigroup $T(t), t > 0$, associated with the linear part of the functional equation is compact and the linear convolution operator $L_0^T u = \int_0^T T(t-s)Bu(s)ds$ has a bounded inverse operator $(L_0)^{-1}$ with values in $L_2(J,U)/ker(L_0^T)$. Dauer and Mahmudov [2] obtained sufficient conditions for exact null controllability of the semi-linear integrodifferential systems in Hilbert spaces. They pointed out that the bounded invertibility assumption is rather strong and can only be applied to finite dimensional systems. In fact, exact null controllability of this system does not guarantee the boundedness of $(L_0)^{-1}$, but it guarantees the boundedness of the operator $(L_0)^{-1}N_0^T$. In many cases, deterministic models often fluctuate due to noise, which is random or at least to be so. So, we have to move from deterministic problems to stochastic ones. Park and Balasubramaniam [3] studied the exact null controllability for abstract semi-linear functional integrodifferential stochastic evolution equations in Hilbert spaces. They removed the bounded invertibility condition replacing it by the exact null controllability of the associated linear system with additive term in the stochastic settings based on the observation [2]. They first established the boundedness result on $(L_0)^{-1}N_0^T$ as in [2]. Using this operator and Schauder fixed point theorem, Then the authors obtained the result of exact null controllability for stochastic evolution equations.

On the other hand, the importance of nonlocal conditions in different fields was discussed in [4,5] and the references therein. In the past several years, several authors have investigated the controllability for differential systems with nonlocal conditions, see[6,7,8]. Recently, Fu and Zhang [9] established a sufficient result with nonlocal conditions. They first established the boundedness result on $(L_0)^{-1}N_0^T$ as in [2] and then studied the exact null controllability using this result and theory of linear evolution operator.

Controllability is one of the fundamental concepts in mathematical control theory. Recently, several authors have investigated the exact null controllability for different classes of systems, see$[10-13]$. Up to now, there is few work on exact null controllability stochastic system with nonlocal conditions. The purpose of this article is to investigate the exact null controllability for the system (1.1). We first guarantees the boundedness of the operator $(L_0)^{-1}N_0^T$ which is defined in lemma 2.3. Then, with the help of Banach fixed point theorem, some sufficient conditions will be obtained.

This paper is organized as follows. In section 2, we give the preliminaries for the paper. In section 3, we investigate the exact null controllability of system (1.1). In section 4, we end this paper by the study of the exact null controllability for stochastic differential control system.

2. Preliminaries

Throughout this paper, H will be a Hilbert space with norm $\|\cdot\|$. Let $(\Omega,\mathfrak{F},P)$ be a complete probability space furnished with complete family of right continuous increasing sub-σ-algebras $\{\mathfrak{F}_t : t\geq 0\}$ satisfying $\mathfrak{F}_t\subset\mathfrak{F}$. The collection of all square integrable and $\mathfrak{F}_t$-adapted processes with values in U is denoted by $L_2^{\mathfrak{F}_t}(J,U)$. Let $\beta_n(t)(n=1,2,\cdots)$ be a sequence of real valued one dimensional standard Brownian motions mutually independent over $(\Omega,\mathfrak{F},P)$. We assume there exists a complete orthonormal basis $\{e_n\}$ in K and a bounded sequence of nonnegative real numbers λ_n such that $w(t)=\sum_{n=1}^{\infty}\sqrt{\lambda_n}\beta_n(t)e_n,\ t\geq 0$. Let $Q\in L(K,K)$ be an operator defined by $Qe_n=\lambda_n e_n, (n=1,2,3\ldots)$ with finite trace $trQ=\sum_{n=1}^{\infty}\lambda_n<\infty$. Then the above K-valued stochastic process $w(t)$ is called a Q-Wiener process. We assume that $\mathfrak{F}_t=\sigma(w(s):0\leq s\leq t)$ is the σ-algebra generated by w and $\mathfrak{F}_t=\mathfrak{F}$. Let $\Psi\in L_2^0(K,H)$ with the norm

$$\|\Psi\|_Q^2=tr(\Psi Q\Psi^*)=\sum_{n=1}^{\infty}\|\sqrt{\lambda_n}\Psi e_n\|^2,$$

where $L_2^0(K,H)$ is the space of all Q- Hilbert -Schmidt operators from K into H. If $\|\Psi\|_Q<\infty$, then Ψ is called a Q-Hilbert-Schmidt operator.

Let $L_2(\Omega,H)$ be the space of all $\mathfrak{F}_t$-measurable square integrable random variable with value in H. Let $C(J,L_2(\Omega,H))$ be the space of all continuous from J into $L_2(\Omega,H)$ satisfying the condition $\sup_{t\in J}\mathbb{E}\|x(t)\|^2<\infty$. Then define Y be the closed subspace of $C(J,L_2(\Omega,H))$ consisting of continuous H-valued processes $\{\xi(t):t\in J\}$ which are measurable and $\mathfrak{F}_t$-adapted. Let $\|\cdot\|_Y$ be a seminorm in Y defined by $\|\xi\|_Y=\sup_{t\in J}(\mathbb{E}\|\xi(t)\|^2)^{1/2}<\infty$. It is easy to verify that Y furnished with the norm topology as defined above is a Banach space.

To study the exact null controllability for (1.1), we consider the linear system

$$\begin{cases} z'(t)=Az(t)+Bu(t)+F(t),\ t\in J=[0,T], \\ z(0)=z_0, \end{cases} \tag{2.1}$$

associated with the system (1.1), where $F(t)=f(t)+g(t)dw(t)$.

Define the operator $L_0^T:L_2(J,U)\longrightarrow H$ and $N_0^T:H\times L_2(J,H)\longrightarrow H$, respectively, as

$$L_0^T u=\int_0^T S(T-s)Bu(s),\ u\in L_2(J,U),$$

$$N_0^T(z_0,F)=S(T)z_0+\int_0^T S(T-s)F(s)ds.$$

Definition 2.1. The system (2.1) is said to be exact null controllability if

$$ImL_0^T\supset ImN_0^T.$$

Remark 2.2. It is proved that, see [14], system (2.1) is exact null controllability if and only if there is a positive number γ such that

$$\|(L_0^T)^*z\|\geq\gamma\|(N_0^T)^*z\|$$

for all $z\in H$.

The following lemma is crucial to obtain our main result:

Lemma 2.3. *Suppose that the system (2.1) is exact null controllability on J. Then the linear operator $W:=(L_0)^{-1}(N_0^T):H\times L_2(J,H)\longrightarrow L_2(J,U)$ is bounded and the control*

$$u(t)=-(L_0)^{-1}(N_0^T(z,F))(t)=-W(z_0,f,g)(t)$$

$$=-(L_0)^{-1}[S(T)z_0+\int_0^T S(T-s)f(s)ds+\int_0^T S(T-s)g(s)dw(s)](t)$$

transfers the system (2.1) from z_0 *to* 0*, where* L_0 *is the restriction of* L_0^T *to the* $[kerL_0^T]^{\perp}$*,* $f \in L_2(J,H)$ *and* $g \in L_2(J, L_2^0(K,H))$.

Proof. The proof of this lemma is similar to that of Lemma 3 in [2] and we omit it here.

3. Main results

In this section, we consider the exact null controllability problem for (1.1). First, we define the mild solution and exact null controllability for it.

Definition 3.1. A continuous stochastic process $x(\cdot) \in C(J, L_2(\Omega, H))$ is a mild solution of (1.1) if the following conditions are satisfied:

(i) $x(t)$ is measurable and $\mathfrak{F}_t$-adapted for each $t \in J$,

(ii) $\int_0^T \|x(s)\|^2 ds < \infty$ for each $s \in J$,

(iii) for each $u \in L_2^{\mathfrak{F}}(J,U)$ the process $x(\cdot)$ satisfies the following integral equation

$$x(t) = S(t)[x_0 - h(x)] + \int_0^t S(t-s)[Bu(s) + f(s,x(s))]ds + \int_0^t S(t-s)g(s,x(s))dw(s),\ t \in J.$$

Definition 3.2. The system (1.1) is said to be exact null controllable on J if there is a stochastic control $u \in L_2^{\mathfrak{F}}(J,U)$ such that the solution $x(\cdot)$ satisfies $x(T) = 0$.

For the proof of the main result, we introduce the following assumptions.

(H_1) $A : D(A) \subset H \to H$ generates a strongly continuous semigroup $(S(t))_{t\geq 0}$ in H. There exists a constant $M \geq 1$, such that

$$\|S(t)\| \leq M, \ \text{for all } t \in J.$$

(H_2) The function $f : J \times H \longrightarrow H$ is locally Lipschitz continuous, for all $t \in J, x, x_1, x_2 \in H$, there exists a constant $L_1 > 0$, such that

$$\|f(t,x_2) - f(t,x_1)\|^2 \leq L_1 \|x_2 - x_1\|^2,$$

$$\|f(t,x)\|^2 \leq L_1(\|x\|^2 + 1).$$

(H_3) The function $g : J \times H \longrightarrow L_2^0(K,H)$ is locally Lipschitz continuous, for all $t \in J, x, x_2, x_1 \in H$, there exists a constant $L_2 > 0$, such that

$$\|g(t,x_2) - g(t,x_1)\|_Q^2 \leq L_2\|x_2 - x_1\|^2,$$

$$\|g(t,x)\|^2 \leq L_2(\|x\|^2 + 1).$$

(H_4) The function $h : C(J,H) \to H$ is continuous, for any $\eta, \eta_1, \eta_2 \in C(J,H)$ there exists a constant $L_3 > 0$, such that

$$\|h(\eta_2) - h(\eta_1)\|^2 \leq L_3\|\eta_2 - \eta_1\|^2,$$

$$\|h(\eta)\|^2 \leq L_3(\|\eta\|^2 + 1).$$

(H_5) The linear system (2.1) is exact null controllability on J in H.

Theorem 3.3. *Assume the conditions $(H_1) - (H_5)$ are satisfied. Then, the system (1.1) is exact null controllable on J provided that*

$$4M^2[L_3 + T^2\|B\|^2\|W\|^2(L_3 + L_2 + L_1) + T^2L_1 + TL_2] < 1.$$

Proof. For any $x \in Y$ we take the control u as

$$\begin{aligned} u(t) =& -W(x_0 - h(x), f, g)(t) \\ =& -(L_0)^{-1}\{S(T)[x_0 - h(x)] + \int_0^T S(T-s)f(s,x(s))ds \\ &+ \int_0^T S(T-s)g(s,x(s))dw(s)\}(t) \end{aligned}$$

Then, we can see that this control steers x_0 to 0. In fact, if $x(t,u)$ is a mild solution to (1.1) with u, then,

$$\begin{aligned}
x(T,u) =&S(T)[x_0-h(x)]+\int_0^T S(T-s)[Bu(s)+f(s,x(s)]ds\\
&+\int_0^T S(T-s)g(s,x(s))dw(s)\\
=&S(T)[x_0-h(x)]-\int_0^T S(T-s)[BW(x_0-h(x),f,g)(s)+f(s,x(s)]ds\\
&+\int_0^T S(T-s)g(s,x(s))dw(s)\\
=&0.
\end{aligned}$$

We define the operator Φ on Y as follows,

$$\begin{aligned}
(\Phi x)(t) =&S(t)[x_0-h(x)]-\int_0^t S(t-s)[BW(x_0-h(x),f,g)(s)+f(s,x(s)]ds\\
&+\int_0^t S(t-s)g(s,x(s))dw(s),
\end{aligned}$$

for $t\in(0,T]$.

It will be shown that the operator Φ has a fixed point on Y, which is a mild solution of (1.1). The proof will be given in several steps.

Step 1. $\Phi(Y)\subset Y$.

First, for any $x\in y$, the control $u=-W(x_0-h(x),f,g)$ is bounded on Y. Indeed,

$$\begin{aligned}
\mathbb{E}\|u\|^2 &= \mathbb{E}\|-W(x_0-h(x),f,g)\|^2\\
&\le \|W\|^2[\mathbb{E}\|x_0+h(x)\|^2+\mathbb{E}\|f\|^2+\mathbb{E}\|g\|^2]\\
&\le \|W\|^2[\mathbb{E}\|x_0\|^2+L_3(\|x\|_Y^2+1)+L_1(\|x\|_Y^2+1)+L_2(\|x\|_Y^2+1)]. \qquad (3.1)
\end{aligned}$$

Then,

$$
\begin{aligned}
\mathbb{E}\|(\Phi x)(t)\|^2 \leq & 4\mathbb{E}\|S(t)[x_0 - h(x)\|^2 \\
& + 4\mathbb{E}\|\int_0^t S(t-s)[BW(x_0-h(x),f,g)(s)]ds\|^2 \\
& + 4\mathbb{E}\|\int_0^t S(t-s)f(s,x(s))ds\|^2 \\
& + 4\mathbb{E}\|\int_0^t S(t-s)g(s,x(s))dw(s)\|^2 \\
\leq & 4M^2[\mathbb{E}\|x_0\|^2 + L_3(\|x\|_Y^2+1)] \\
& + 4M^2\|B\|^2T^2\mathbb{E}\|W(x_0-h(x),f,g)\|^2 \\
& + 4M^2\mathbb{E}(\int_0^t \|f(s,x(s))\|ds)^2 + 4M^2\mathbb{E}(\int_0^t \|g(s,x(s)\|_Q^2 ds) \\
\leq & 4M^2[\mathbb{E}\|x_0\|^2 + L_3(\|x\|_Y^2+1)] \\
& + 4M^2\|B\|^2T^2\mathbb{E}\|W(x_0-h(x),f,g)\|^2 \\
& + 4M^2T^2L_1(\|x\|_Y^2+1) + 4M^2TL_2(\|x\|_Y^2+1),
\end{aligned}
$$

from (3.1) we can imply that $\|\Phi x\|_Y^2 < \infty$. So $\Phi(Y) \subset Y$.

Step 2. $(\Phi x)(t)$ is continuous on J for any $x \in Y$.

Let $0 < t \leq T$ and $\varepsilon > 0$ be sufficiently small, then,

$\mathbb{E}\|(\Phi x)(t+\varepsilon) - (\Phi x)(t)\|^2$

$$
\begin{aligned}
\leq & 4\mathbb{E}\|[S(t+\varepsilon)-S(t)][x_0-h(x)]\|^2 \\
& + 4\mathbb{E}\|\int_0^{t+\varepsilon} S(t+\varepsilon-s)[BW(x_0-h(x),f,g)(s)]ds \\
& - \int_0^t S(t-s)[BW(x_0-h(x),f,g)(s)]ds\|^2 \\
& + 4\mathbb{E}\|\int_0^{t+\varepsilon} S(t+\varepsilon-s)f(s,x(s))ds - \int_0^t S(t-s)f(s,x(s))ds\|^2 \\
& + 4\mathbb{E}\|\int_0^{t+\varepsilon} S(t+\varepsilon-s)g(s,x(s))dw(s) - \int_0^t S(t-s)g(s,x(s))dw(s)\|^2 \\
= & I_1 + I_2 + I_3 + I_4.
\end{aligned}
$$

From (H_1), we have,

$$\begin{aligned} I_2 \leq & 8\mathbb{E}\|\int_0^t [S(t+\varepsilon-s)-S(t-s)][BW(x_0-h(x),f,g)(s)]ds\|^2 \\ & +8\mathbb{E}\|\int_t^{t+\varepsilon} S(t+\varepsilon-s)[BW(x_0-h(x),f,g)(s)]ds\|^2 \\ \leq & 8\|B\|^2\mathbb{E}\|W(x_0-h(x),f,g)\|^2\mathbb{E}\|\int_0^t S(t-s)[S(\varepsilon)-I]ds\|^2 \\ & +8M^2\|\varepsilon\|^2\|B\|^2\mathbb{E}\|W(x_0-h(x),f,g)\|^2. \end{aligned}$$

From (H_1) and (H_2), we have,

$$\begin{aligned} I_3 \leq & 8\mathbb{E}\|\int_0^t [S(t+\varepsilon-s)-S(t-s)]f(s,x(s))ds\|^2 \\ & +8\mathbb{E}\|\int_t^{t+\varepsilon} S(t+\varepsilon-s)f(s,x(s))ds\|^2 \\ \leq & 8\mathbb{E}\|\int_0^t S(t-s)[S(\varepsilon)-I]f(s,x(s))ds\|^2 \\ & +8M^2\|\varepsilon\|^2L_1(\|x\|_Y^2+1). \end{aligned}$$

Similarly, from (H_1) and (H_3), we can also obtain that,

$$\begin{aligned} I_4 \leq & 8\mathbb{E}\|\int_0^t [S(t+\varepsilon-s)-S(t-s)]g(s,x(s))dw(s)\|^2 \\ & +8\mathbb{E}\|\int_t^{t+\varepsilon} S(t+\varepsilon-s)g(s,x(s))dw(s)\|^2 \\ \leq & 8\mathbb{E}(\int_0^t \|S(t-s)[S(\varepsilon)-I]\|^2\|g(s,x(s))\|_Q^2 ds) \\ & +8M^2\|\varepsilon\|^2L_2(\|x\|_Y^2+1). \end{aligned}$$

Clearly, $\varepsilon \to 0$ and $S(t)$ is strongly continuous. So $I_1 \to 0, I_2 \to 0, I_3 \to 0$ and $I_4 \to 0$. Hence, $(\Phi x)(t)$ is continuous on J.

Step 3. Φ is a contraction in Y.

Let $x_1, x_2 \in Y$, for any $t \in (0,T]$ be fixed, then,

$$
\begin{aligned}
&\mathbb{E}\|(\Phi x_2)(t)-(\Phi x_1)(t)\|^2 \\
&\qquad \le 4\mathbb{E}\|S(t)[h(x_2)-h(x_1)]\|^2 \\
&\qquad\quad +4\mathbb{E}\|\int_0^t S(t-s)B[W(x_0-h(x_2),f,g)-W(x_0-h(x_1),f,g)(s)ds\|^2 \\
&\qquad\quad +4\mathbb{E}\|\int_0^t S(t-s)[f(s,x_2(s))-f(s,x_1(s))]ds\|^2 \\
&\qquad\quad +4\mathbb{E}(\int_0^t \|S(t-s)\|^2\|g(s,x_2(s))-g(s,x_1(s))\|_Q^2 ds) \\
&\qquad \le 4M^2L_3\|x_2-x_1\|_Y^2+4M^2T^2\|B\|^2\|W\|^2(L_1+L_2+L_3)\|x_2-x_1\|_Y^2 \\
&\qquad\quad +4M^2T^2L_1\|x_2-x_1\|_Y^2+4M^2TL_2\|x_2-x_1\|_Y^2 \\
&\qquad =4M^2[L_3+T^2\|B\|^2\|W\|^2(L_3+L_2+L_1)+T^2L_1+TL_2]\|x_2-x_1\|_Y^2.
\end{aligned}
$$

Thus,

$$
\|(\Phi x_2)-(\Phi x_1)\|_Y^2 \le 4M^2[L_3+T^2\|B\|^2\|W\|^2(L_3+L_2+L_1)+T^2L_1+TL_2]\|x_2-x_1\|_Y^2.
$$

Hence, Φ is a contraction in Y. From the Banach fixed point theorem, Φ has a unique fixed point. So system (1.1) is exact null controllability on J. This completes the proof.

4. Application

To illustrate our abstract results, we consider the following stochastic differential control system:

$$
\begin{cases}
dx(t,\theta)=[\frac{\partial^2}{\partial\theta^2}x(t,\theta)+u(t,\theta)+F(t,x(t,\theta))]dt+G(t,x(t,\theta))dw(t), \\
\qquad t\in J=[0,T],\ 0<\theta<1, \\
x(t,0)=x(t,1)=0, \quad t\in J, \\
x(0,\theta)+\sum_{i=1}^p c_i x(t_i,\theta)=x_0, \quad \theta\in[0,1],
\end{cases}
\tag{4.1}
$$

where $t_i(i=1,2\cdots p)\in(0,T)$, $w(t)$ is a Wiener process.

Let $H=L_2[0,1]$ and $A:H\to H$ be operator defined by

$$
Az=\frac{\partial^2}{\partial\theta^2}z,
$$

with domain

$$D(A)=\{z\in H: z, \frac{\partial}{\partial\theta}z \text{ are absolutely continuous}, \frac{\partial^2}{\partial\theta^2}z\in H, z(0)=z(1)=0\}.$$

It is well known that A is self-adjoint and A has the eigenvalues $\lambda_n=-n^2\pi^2$, $n\in\mathbb{N}$ and the corresponding eigenvectors $e_n(\theta)=\sqrt{2}\sin(n\pi\theta)$. Further, it is known that A generates a compact C_0-semigroup $(S(t))_{t\geq 0}$ given by

$$\begin{aligned}S(t)z &= \sum_{n=1}^{\infty}\langle z,e_n\rangle e_n\\ &= \sum_{n=1}^{\infty}2e^{-n^2\pi^2 t}\sin(n\pi\theta)\int_0^1 \sin(n\pi\alpha)z(\alpha)d\alpha, \quad z\in H.\end{aligned}$$

So, the condition (H_1) is satisfied. To write system (4.1) in the form (1.1), define $f: J\times H\to H, g: J\times H\to L_2^0(K,H)$ as follows:

$$f(t,x)=F(t,x(t,\theta)),$$

$$g(t,x)=G(t,x(t,\theta)),$$

$$h(x(t,\theta))=\sum_{i=1}^{p}c_i x(t_i,\theta).$$

Then, $h(\cdot)$ clearly satisfies condition (H_4) . Suppose $F: J\times R\to R, G: J\times R\to R$ are continuous and global Lipschitz continuous in the second variable. It is easy to show that f,g satisfies $(H_2),(H_3)$.

Now, Let $u\in L_2^{\mathfrak{F}}(J,U)$, $B=I$, So $B^*=I$. Thus, system (4.1) can be written in the form of (1.1). Let us consider the following linear system with additive term F, G:

$$\begin{cases} dz(t,\theta)=[\frac{\partial^2}{\partial\theta^2}z(t,\theta)+u(t,\theta)+F(t,x(t,\theta))]dt+G(t,x(t,\theta))dw(t),\\ \qquad t\in J=[0,T],\ 0<\theta<1,\\ z(t,0)=z(t,1)=0, \quad t\in J,\\ z(0,\theta)=z_0, \quad \theta\in[0,1].\end{cases} \tag{4.2}$$

Due to Remark 2.2, the exact null controllability of system (4.2) is equivalent to that there is a $\gamma>0$, such that

$$\int_0^T \|B*S*(T-s)z\|^2ds\geq\gamma(\|S^*(T)z\|^2+\int_0^T\|S^*(T-s)z\|^2ds),$$

or equivalently

$$\int_0^T \|S(T-s)z\|^2 ds \geq \gamma(\|S(T)z\|^2 + \int_0^T \|S(T-s)z\|^2 ds). \tag{4.3}$$

It can be showed that (4.3) hold for some $\gamma > 0$. In fact, considering

$$\int_0^T e^{-2n^2\pi^2(T-s)} ds = Te{-}2n^2\pi^2(T-\varepsilon), \quad \varepsilon \in J,$$

$$\geq Te^{-2n^2\pi^2 T}.$$

We see that

$$\sum_{n=1}^{\infty} \int_0^T 2e^{-2n^2\pi^2(T-s)} ds (\int_0^1 \sin(n\pi\alpha)z(\alpha)d\alpha)^2 \geq \sum_{n=1}^{\infty} 2Te^{-2n^2\pi^2 T}(\int_0^1 \sin(n\pi\alpha)z(\alpha)d\alpha)^2,$$

that means,

$$\int_0^T \|S(T-s)z\|^2 ds \geq T\|S(T)z\|^2.$$

Hence,

$$\int_0^T \|S(T-s)z\|^2 ds \geq \frac{T}{T+1}(\|S(T)z\|^2 + \int_0^T \|S(T-s)z\|^2 ds).$$

So, (4.3) holds with $\gamma = \frac{T}{T+1}$. Thus, the linear system (4.2) is exactly null controllable on J . So the condition (H_5) is satisfied. Using the construction presented in [2], we can assume that the bounded linear operator W exists. Thus, by choosing the constants $c_i, i = 1, 2, \cdots, p, M, L_1, L_2$ such that

$$4M^2[\sum_{i=1}^{p} |c_i| + \|W\|^2 T^2 (\sum_{i=1}^{p} \|c_i\| + L_1 + L_2) + T^2 L_1 + TL_2] < 1.$$

Therefore, all the conditions stated in Theorem 3.3 are satisfied. Hence, the system (4.1) is exactly null controllable on J.

5. Conclusions

We have considered the exact null controllability of semi-linear stochastic differential systems with nonlocal conditions. By using Banach fixed point theorem, sufficient conditions have been given. Moreover, an application is provided to illustrate the obtained theoretical results.

We point out here that, our results are applicable to investigate the exact null controllability of stochastic size-structured population model. In the future research, the exact null controllability of stochastic size-structured population model with nonlocal condition may be considered. In addition, it is interesting to investigate the case with both delays and impulsive effects.

Conflict of Interests

The authors declare that there is no conflict of interests.

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