Available online at http://scik.org
Commun. Math. Biol. Neurosci. 2016, 2016:13
ISSN: 2052-2541

DYNAMIC MODEL OF INFLUENZA WITH AGE-STRUCTURED AND MEDIA COVERAGE

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Abstract: In this paper, we describe an SIR model with age-structured and media coverage for the spread of influenza, which transmits within a population in a short time, and the population is divided into two parts including the adolescent and the adult, respectively. The numerical simulations show that media coverage plays an important role in controlling the prevalence of influenza, moreover, the duration of media coverage should not last long when influenza epidemic appears, otherwise it may cause negative effects. And we also should consider the heterogeneity of the population when some interventions are proposed, if not, we may misestimate the size of the disease.

Keywords: Influenza; Age-structured; Media coverage; Numerical simulations; Heterogeneity of population.

2010 AMS Subject Classification: 34D20; 34K18.

1. Introduction

Influenza presents a significant morbidity and mortality burden in the world, a typical seasonal influenza epidemic kills up to 49,000 people per year in the USA [1], and from a quarter to half a million worldwide [2]. Influenza viruses have coexisted with humans for centuries and have historically been a cause of excessive morbidity and mortality [3]. Annually, the virus

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Received Jan 23, 2016

affects 25 to 50 million people, with an estimated 20 to 40 thousand influenza-related deaths, in the United States [3]. Influenza (also known as flu) is a respiratory disease caused by certain RNA viruses of the Orthomyxoviridae family [4]. Due to the influenza virus is RNA viruses, so it is prone to mistake every time a copy of its genome. Since these mutations usually not easy to be identified by the immune system, this will lead to influenza pandemic. It is well known that taking vaccination is an effective method to reduce the size of influenza and the economic loss. But, we often neglect the fact that large supplies of vaccine are needed in the early of influenza breakout. However, we can not make sure enough supply of the vaccine. In other words, if nations manage to secure large supplies, it still need to take several months to deliver, and the challenge posed by vaccine production restrictions and logistic distribution limitations will raise several questions, such as whether or not there is another way to alleviate the outbreak? In the early of influenza, what's the other control measure can replace the role of vaccine and lead to the least cost?

In 2003 years, Cui et al. considered the impact of media coverage in controlling infectious disease and illustrated media coverage is helpful for controlling the spread of diseases [5], which brings us some new ideas. Some functions describing media coverage are given in paper [6, 7, 8], such as $e^{-M(t)}$, $a-bf(I)$, $\frac{kI}{1+\alpha I^2}$, where $M(t)=max\{0, aI(t)+b\frac{dI(t)}{dt}\}$. In this paper, we use the function $\beta_1-\beta_2\frac{I}{m+I}$ to describe media coverage [6]. In 2012, Feng et al. refined the formula published by Jacquez et al. [9] to account for these newly-observed patterns and estimated age-specific fractions of contacts with each preferred group [10].

Inspired by paper [9, 10], we incorporate media coverage and age-structured into our model. In view of the paper [11], we divide the population into two parts, representing the adolescent and the adult, respectively. We regard the people younger than age 19 as adolescent, who are almost school students and lack of self-protection awareness, and this part of population has a higher degree of aggregation, the rest are regarded as adult.

2. Model

In this paper, we assume that the infected have immunity to make themselves away from the influenza pandemic in a short time, and we neglect the natural birth or death in our model as follows:

$$\begin{cases} \frac{dS_1}{dt} = -\lambda_1(t)S_1 \\ \frac{dS_2}{dt} = -\lambda_2(t)S_2 \\ \frac{dI_1}{dt} = \lambda_1(t)S_1 - rI_1 \\ \frac{dI_2}{dt} = \lambda_2(t)S_2 - rI_2 \\ \frac{dR_1}{dt} = rI_1 \\ \frac{dR_2}{dt} = rI_2 \end{cases}$$

Where

$$\lambda_1(t) = (\beta_1 - \beta_2 \frac{I_1 + I_2}{m + I_1 + I_2})(\frac{C_{11}I_1}{N_1} + \frac{C_{12}I_2}{N_2}).$$

$$\lambda_2(t) = (\beta_1 - \beta_2 \frac{I_1 + I_2}{m + I_1 + I_2})(\frac{C_{21}I_1}{N_1} + \frac{C_{22}I_2}{N_2}).$$

$C_{11}, C_{12}, C_{21}, C_{22}$ represent the elements in the contact matric, respectively. C_{11} means the contact of adolescent to adolescent, C_{12} means the contact of adolescent to adult, C_{21} means the contact of adult to adolescent, C_{22} means the contact of adult to adult. Specifically,

$$C_{ij} = \varepsilon_i \delta_{ij} + (1 - \varepsilon_i) \frac{(1 - \varepsilon_j) a_j N_j}{\sum_{k=1}^{2} (1 - \varepsilon_k) a_k N_k},$$

where δ_{ij} is the Kronecker delta ($\delta_{ij} = 1$ if $i = j$, and $\delta_{ij} = 0$ if $i \neq j$). If a proportion ε_i of i-group contacts is reserved for others in group i, called preference, and the complement $(1 - \varepsilon_i)$ is distributed among all groups, including i. N_i are group sizes and a_i are the average per capita contact rates of groups $(i = 1, 2)$ called activities [10]. In paper [12], we notice that mixing models should meet three criteria, the last criterion is $a_i N_i C_{ij} = a_j N_j C_{ji}$. If we add the impact of media into a_i we can not ensure the last criterion, hence we incorporate the impact of media into transmission rate, and then we choose function $\beta_1 - \beta_2 \frac{I_1 + I_2}{m + I_1 + I_2}$ to measure the role of the media. $\beta_1 - \beta_2 \frac{I_1 + I_2}{m + I_1 + I_2}$ is a continuous bounded function which takes into account disease saturation or psychological effects. m is the half-saturation constant, reflects the impact of media coverage on the contact transmission. β_2 reflects reduce of the contact transmission rate when susceptible people heard the news. r means recovery rate, $\frac{1}{r}$ represents infectious period, generally, influenza disease period about 10 days. From the infectious system above, we note that the last two equations are independent of the system, and so we omit them in our discussion.

3. The basic reproduction number

R_0 denotes the basic reproduction number, which is a measure of the transmissibility of the infectious disease when the population is completely susceptible. Specifically, R_0 accounts for the average number of secondary cases generated by a primary case during his/her infectious period. The calculation of basic reproduction number as follows:

$$F = \begin{bmatrix} \beta_1 C_{11} & \beta_1 \frac{C_{12}N_1}{N_2} \\ \beta_1 \frac{C_{21}N_2}{N_1} & \beta_1 C_{22} \end{bmatrix}, V = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}.$$

The basic reproduction number is the dominant eigenvalue of the next-generation matrix FV^{-1} [12], so the basic reproduction number is given by

$$R_0 = \frac{1}{2}[A + B + \sqrt{(A-B)^2 + 4CD}],$$

where $A = \frac{\beta_1 C_{11}}{r}, B = \frac{\beta_1 C_{22}}{r}, C = \frac{\beta_1 C_{12}}{r}, D = \frac{\beta_1 C_{21}}{r}$. Typically, we have that when $R_0 > 1$ an outbreak takes place and while $R_0 < 1$ indicates that an outbreak can not be sustained. Through the expression of R_0, we find media coverage does not influence R_0, this result accords with paper [5, 6].

4. Numerical simulations when $R_0 < 1$

Table 1 The value of parameters used in simulation

Parameters	Value	Original
$\frac{1}{r}$	10(days)	[2]
N_1	2.9(millions)	[13]
N_2	7.1(millions)	[13]
a_1	15.33	[14]
a_2	12.61	[14]
β_1	Varied	Assume
β_2	Varied	Assume
m	0.5	Assume

It's worth noting that we derive the value of a_1, a_2 by using the average method.

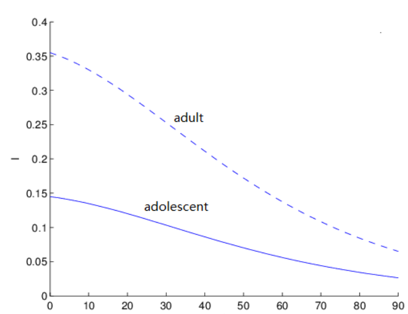

Figure 1: The change of infected adolescent and adult with time.

Here $\varepsilon_1 = 0.75, \varepsilon_2 = 0.25, \beta_2 = 0, R_0 = 0.7746, \beta_1 = 0.1$ [14]. From Figure 1, we find the infected people decreasing with time and finally the disease will tend to die out.

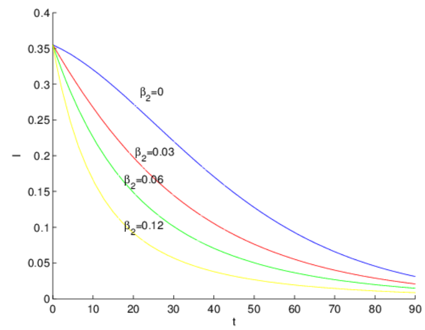

Figure 2: The impact of media coverage on all infected people.

In fact, media coverage mainly depends on the value of β_2, and so we assume β_2 =0, 0.03, 0.06, 0.12, respectively. We obtain that the number of all infected people decrease with the increase of media coverage. It may be the cause of people paying attention to the report, which may change their behavior during influenza pandemic. So we conclude that media coverage is helpful for controlling disease when $R_0 < 1$.

5. Numerical simulations when $R_0 > 1$

Since the system is so complex and the theoretical analysis is very difficult, we can do some numerical simulations to help us understanding the dynamic of influenza transmission better.

5.1 Proportionally-mixed

When $\varepsilon_1 = \varepsilon_2 = 0$, the mixing is proportional [16]. Here, we assume β_2=0, $\beta_1 = 0.18$, which deduces $R_0 = 1.5224$.

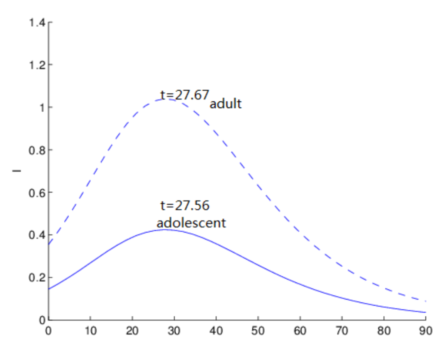

Figure 3: The change of infected adolescent and adult with time.

Figure 3 indicates that the time of arrival of the influenza peak, which of both adolescent and adult, is the same about 27 days. This phenomena will lead to vaccine supply limited and more people will be treated at hospital, so we must take some actions to reduce the loss of economic. Due to the limit of vaccine supply, we need to consider substitutable methods to solve this problem, thus, we study the impact of media coverage.

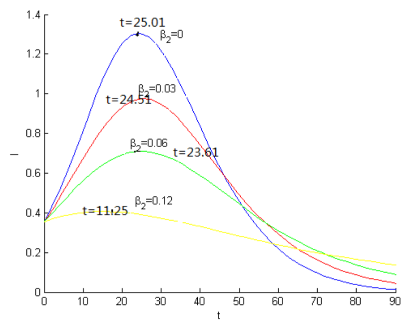

Figure 4: The impact of media coverage on all infected people.

Here we assume β_2=0, 0.03, 0.06, 0.12, R_0=1.5224, and the time of arrival of peak is shown in Figure 4. The same results as Figure 2, to some degree, the infected people decrease with the increase of media coverage. But we can also see the images intersecting at 50 days around, this may be the result from the long time report of media coverage, which may lead to a negative effect on people's psychological. People may think the media exaggerate the size of the outbreak of the disease so that they may neglect the report and relax of self-protection consciousness, which may result in the rebound of disease outbreaks. So we can make a conclusion that media coverage need not last too long, but in the peak of influenza, media coverage should be vastly reported in order to reduce the size of disease.

We also get that the media coverage can make the peak time advanced, which have both advantages and disadvantages. What the advantage of this phenomena is the final size of disease decreased and the loss of economic reduced, but the disadvantage is that we have no enough time to mass-produce vaccines before the bulk of the epidemic has passed, which is easy to lead to the next wave of influenza.

5.2 The preferential mixing

If $\varepsilon_1 \neq \varepsilon_2$ and both of them not equal to zero, we call the preferential mixing is heterogeneous. In this section we use $\beta_2 = 0, \varepsilon_1 = 0.75, \varepsilon_2 = 0.25$ [14] into the numerical simulation. In this case, $R_0 = 1.511$.

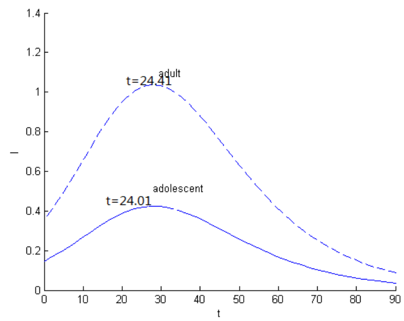

Figure 5: The change of all infected people with time.

The time of arrival of peak are shown in Figure 5. From Figure 5, we can conclude the average peak time of the epidemic occurred about 24 days, earlier than proportionally-mixed. There are some differences between Figure 3 and Figure 4. In order to compare the differences between those two Figures well, we can see Figure 6.

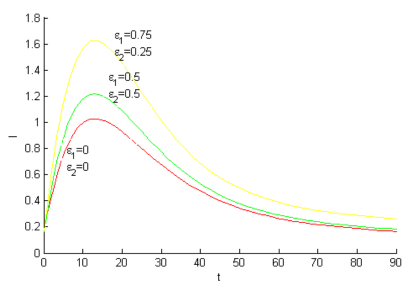

Figure 6: The change of infected adolescent and adult people with time and different ε.

In Figure 6, $\beta_2 = 0$, and the values of $\varepsilon_1, \varepsilon_2$ are shown. Through comparing different ε_i, it indicates that the size of preferential mixing is higher than that of proportionally-mixed, and the higher reserved contacts among adolescent, the larger final size of epidemic. It may be explained that most population in this part are almost school students. They contact closely and have poor immunity or no sense of protecting themselves from diseases, which cause outbreak of influenza among adolescent. As we all know that population is heterogeneous, so this simulation result is more reasonable.

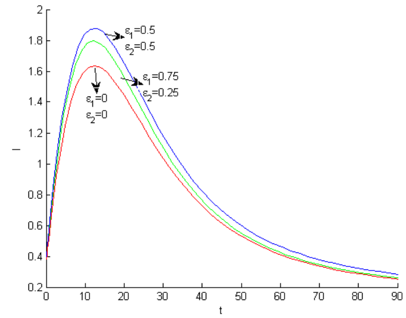

Figure 7: The change of infected adult with time and different ε_i.

In Figure 7, $\beta_2 = 0$ the values of $\varepsilon_1, \varepsilon_2$ are shown. Through comparing different of ε_i, it shows the size of preferential mixing is higher than that of proportionally-mixed, and the higher reserved contacts among adults, the larger final size of epidemic. So if we want to take some measures to control disease, we must consider the heterogonous of population, otherwise, we may misestimate the size of disease.

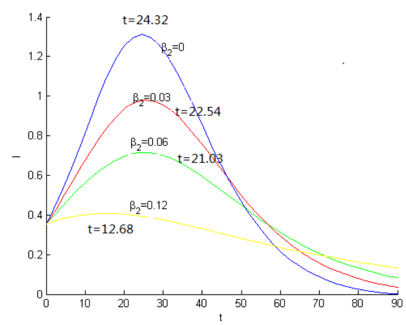

Figure 8: The impact of media coverage on all infected people.

The time of arrival of peak is shown in Figure 8. We assume β_2=0, 0.03, 0.06, 0.12. The same result can be found in Figure 4, we omit the explanation.

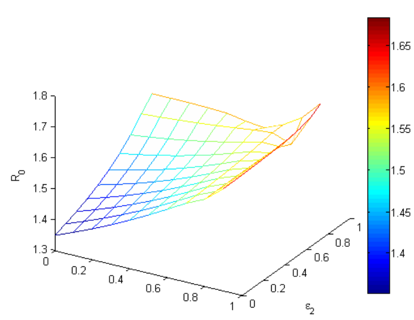

Figure 9: The relationship between R_0 and $\varepsilon_1, \varepsilon_2$.

By Figure 9, we can find that if we fixed ε_1, the value of R_0 becomes larger with the increase of ε_2, the same as ε_2. So we can conclude that the heterogonous of population can lead to a larger size of outbreak, and we must reduce the contact among peers in order to decrease the size of epidemic.

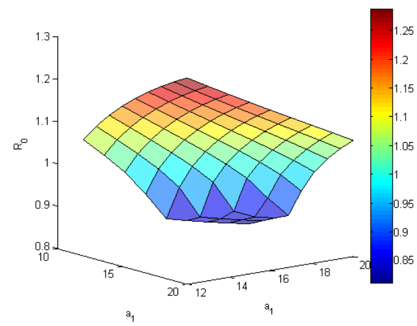

Figure 10: The relationship between R_0 and a_1, a_2.

Here, a_i are the average per capita contact rates of groups $i = 1,2$ called activities. Figure 10 implies that the more frequently of the activity, the bigger size of disease outbreaks. In the epidemic of influenza, we can reduce activities to control disease. This approach is applied to closure of school in order to to reduce activities among adolescent such as paper [13,16].

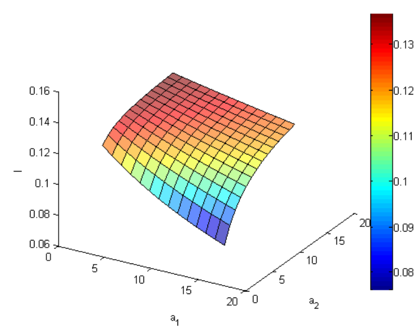

Figure 11: The relationship between infected people (adolescent and adult) with a_1, a_2.

Figure 11 shows the more frequently of the activity is, the more infected people emerge. So we believe if we can reduce the activities in the epidemic disease, we can reduce the final size of disease.

6. Discussion

Mathematical models are increasingly used to evaluate and inform infectious disease prevention and control policy. In this paper, we study the impact of media coverage on transmission dynamic of influenza. We find media coverage does not influence the basic reproduction number, but plays a significance role in controlling infectious diseases. One of the most important findings of our study is that the media coverage can influence the size of disease and the arrival time of peak. Another major insight gained from our study comes from the observation: during the influenza pandemic, media coverage should not last long time, because long time of reporting pandemic make the psychological of people boredom, which will bring negative effects. Media coverage can effectively reduce the average contact and activities among influenza pandemic, which can reduce the final size of disease and the loss of economic. We can not neglect the role of media coverage as a public health measures. Meanwhile, such results as the population of heterogeneity may substantially increase the basic reproduction number and the size of disease, especially when combined with preferential mixing, which accord with paper [16]. If we neglect the heterogeneous of population, we may misestimate the size of disease.

The paper also need to be improved, for example, the division of age-structured is too simple. But the actual condition is more complex. The media coverage specific impact of all ages are also needed to be study.

Conflict of Interests

The authors declare that there is no conflict of interests.

Acknowledgments

Supported by the National Natural Science Foundation of China (11371048), the Plan Project of Science and Technology of Beijing Municipal Education Committee (KM201610016018), and the Central Support Local Projects(21147515602).

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