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POSITIVE PERIODIC SOLUTION OF A DISCRETE COMMENSAL SYMBIOSIS MODEL WITH HOLLING II FUNCTIONAL RESPONSE

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Abstract. Sufficient conditions are obtained for the existence of positive periodic solution of the following discrete commensal symbiosis model with Holling II functional response

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\left\{a_1(k)-b_1(k)x_1(k)+\frac{c_1(k)x_2(k)}{e_1(k)+f_1(k)x_2(k)}\right\}, \\ x_2(k+1) &= x_2(k)\exp\left\{a_2(k)-b_2(k)x_2(k)\right\}, \end{aligned}$$

where $\{b_i(k)\}, i=1,2, \{c_1(k)\}\{e_1(k)\}, \{f_1(k)\}$ are all positive ω-periodic sequences, ω is a fixed positive integer, $\{a_i(k)\}$ are ω-periodic sequences, which satisfies $\overline{a}_i = \frac{1}{\omega}\sum\limits_{k=0}^{\omega-1} a_i(k) > 0, i=1,2$. The results obtained in this paper generalized the main results of Xiangdong Xie, Zhansshuai Miao, Yalong Xue (Commun. Math. Biol. Neurosci. 2015 (2015), Article ID 2).

Keywords: commensal symbiosis model; positive periodic solution; functional response.

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1. Introduction

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In the past decade, numerous works on the mutualism model has been published([1-14]) and many excellent works concerned with the persistence, existence of positive periodic solution, and stability of the system were obtained. However, only recently did scholars paid attention to the commensal symbiosis model([17-23]), a model which describe a relationship which is only favorable to the one side and have no influence to the other side.

Sun and Wei[15] first time proposed a intraspecific commensal model:

$$\begin{aligned} \frac{dx}{dt} &= r_1 x\Big(\frac{k_1 - x + ay}{k_1}\Big), \\ \frac{dy}{dt} &= r_2 y\Big(\frac{k_2 - y}{k_2}\Big). \end{aligned} \tag{1.1}$$

They investigated the local stability of all equilibrium points.

Stimulated by the works of Sun and Wei[15] and Fan and Wang[16], Xie et al. [17] proposed the following discrete commensal symbiosis model

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\big\{a_1(k) - b_1(k)x_1(k) + c_1(k)x_2(k)\big\}, \\ x_2(k+1) &= x_2(k)\exp\big\{a_2(k) - b_2(k)x_2(k)\big\}, \end{aligned} \tag{1.2}$$

where $\{b_i(k)\}, i = 1,2, \{c_1(k)\}$ are all positive ω-periodic sequences, ω is a fixed positive integer, $\{a_i(k)\}$ are ω-periodic sequences, which satisfies $\overline{a}_i = \frac{1}{\omega}\sum\limits_{k=0}^{\omega-1} a_i(k) > 0, i = 1,2$. By applying the coincidence degree theory, they showed that the system (1.2) admits at least one positive ω-periodic solution.

In system (1.1) and (1.2), the authors made the assumption that the influence of the second species to the first one is linearize. Generally speaking, a suitable relationship between two species should be a nonlinear one. Already, in predator-prey system, Holling type functional response has been widely used to describe the relationship between two species ([24-29]). Now, by adapting the Holling II functional response to system (1.2), we propose the following two species discrete commensal symbiosis model

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\Big\{a_1(k) - b_1(k)x_1(k) + \frac{c_1(k)x_2(k)}{e_1(k) + f_1(k)x_2(k)}\Big\}, \\ x_2(k+1) &= x_2(k)\exp\big\{a_2(k) - b_2(k)x_2(k)\big\}, \end{aligned} \tag{1.3}$$

where $\{b_i(k)\}, i=1,2, \{c_1(k)\}\{e_1(k)\}, \{f_1(k)\}$ are all positive ω-periodic sequences, ω is a fixed positive integer, $\{a_i(k)\}$ are ω-periodic sequences, which satisfies $\overline{a}_i = \frac{1}{\omega}\sum\limits_{k=0}^{\omega-1} a_i(k) > 0, i=1,2$. Here we assume that the coefficients of the system (1.3) are all periodic sequences, which have a common integer period. Such an assumption seems reasonable in view of seasonal factors, e.g., mating habits, availability of food, weather conditions, harvesting, and hunting, etc. The aim of this paper is to obtain a set of sufficient conditions which ensure the existence of positive periodic solution of system (1.3). To the best of our knowledge, this is the first time that such kind of commensal symbiosis model is proposed and studied.

2. Main results

In the proof of our existence theorem below, we will use the continuation theorem of Gaines and Mawhin([30]).

Lemma 2.1 (Continuation Theorem) *Let L be a Fredholm mapping of index zero and let N be L-compact on $\bar{\Omega}$. Suppose*

(a).For each $\lambda \in (0,1)$, every solution x of $Lx = \lambda Nx$ is such that $x \notin \partial\Omega$;

(b).$QNx \neq 0$ for each $x \in \partial\Omega \cap KerL$ and

$$deg\{JQN, \Omega \cap KerL, 0\} \neq 0.$$

Then the equation $Lx = Nx$ has at least one solution lying in $DomL \cap \bar{\Omega}$.

Let Z, Z^+, R and R^+ denote the sets of all integers, nonnegative integers, real unumbers, and nonnegative real numbers, respectively. For convenience, in the following discussion, we will use the notation below throughout this paper:

$$I_\omega = \{0,1,...,\omega-1\},\ \overline{g} = \frac{1}{\omega}\sum_{k=0}^{\omega-1} g(k),\ \ g^u = \max_{k\in I_\omega} g(k),\ \ g^l = \min_{k\in I_\omega} g(k),$$

where $\{g(k)\}$ ł is an ω-periodic sequence of real numbers defined for $k \in Z$.

Lemma 2.2[16] *Let $g: Z \to R$ be ω-periodic, i. e., $g(k+\omega) = g(k)$. Then for any fixed $k_1, k_2 \in I_\omega$,* *and any $k \in Z$, one has*

$$g(k) \leq g(k_1) + \sum_{s=0}^{\omega-1} |g(s+1) - g(s)|,$$

$$g(k) \geq g(k_2) - \sum_{s=0}^{\omega-1} |g(s+1) - g(s)|.$$

We now reach the position to establish our main result.

Theorem 2.1 *System (1.3) admits at least one positive ω-periodic solution.*

Proof. Let

$$x_i(k) = \exp\{u_i(k)\}, \quad i = 1, 2,$$

so that system (1.3) becomes

$$\begin{aligned} u_1(k+1) - u_1(k) = & \quad a_1(k) - b_1(k)\exp\{u_1(k)\} + \frac{c_1(k)\exp\{u_2(k)\}}{e_1(k) + f_1(k)\exp\{u_2(k)\}}, \\ u_2(k+1) - u_2(k) = & \quad a_2(k) - b_2(k)\exp\{u_2(k)\}. \end{aligned} \tag{2.1}$$

Define

$$l_2 = \Big\{ y = \{y(k)\}, y(k) = (y_1(k), y_2(k))^T \in R^2 \Big\}.$$

For $a = (a_1, a_2)^T \in R^2$, define $|a| = \max\{|a_1|, |a_2|\}$. Let $l^\omega \subset l_2$ denote the subspace of all ω sequences equipped with the usual normal form $\|y\| = \max_{k \in I_\omega} |y(k)|$. It is not difficult to show that l^ω is a finite-dimensional Banach space. Let

$$l_0^\omega = \{y = \{y(k)\} \in l^\omega : \sum_{k=0}^{\omega-1} y(k) = 0\},\ l_c^\omega = \{y = \{y(k)\} \in l^\omega : y(k) = h \in R^2, k \in Z\},$$

then l_0^ω and l_c^ω are both closed linear subspace of l^ω, and

$$l^\omega = l_0^\omega \oplus l_c^\omega, \ \ dim l_c^\omega = 2.$$

Now let us define $X = Y = l^\omega$, $(Ly)(k) = y(k+1) - y(k)$. It is trivial to see that L is a bounded linear operator and

$$KerL = l_c^\omega, \ \ ImL = l_0^\omega, \ \ dimKerL = 2 = CodimImL.$$

Then it follows that L is a Fredholm mapping of index zero. Let

$$N(u_1, u_2)^T = (N_1, N_2)^T := N(u, k),$$

where

$$\begin{cases} N_1 &= a_1(k) - b_1(k)\exp\{u_1(k)\} + \dfrac{c_1(k)\exp\{u_2(k)\}}{e_1(k)+f_1(k)\exp\{u_2(k)\}}, \\ N_2 &= a_2(k) - b_2(k)\exp\{u_2(k)\}. \end{cases}$$

$$Px = \frac{1}{\omega}\sum_{s=0}^{\omega-1} x(s), x \in X, \ \ Qy = \frac{1}{\omega}\sum_{s=0}^{\omega-1} y(s), y \in Y.$$

It is not difficult to show that P and Q are two continuous projectors such that

$$ImP = KerL \quad and \quad ImL = KerQ = Im(I-Q).$$

Furthermore, the generalized inverse (to L) K_p: ImL $\rightarrow$Ker$P\cap$Dom$L$ exists and is given by

$$K_p(z) = \sum_{s=0}^{k-1} z(s) - \frac{1}{\omega}\sum_{s=0}^{\omega-1}(\omega - s)z(s).$$

Thus

$$QNx = \frac{1}{\omega}\sum_{k=0}^{\omega-1} N(x,k),$$

$$Kp(I-Q)Nx = \sum_{s=0}^{k-1} N(x,s) + \frac{1}{\omega}\sum_{s=0}^{\omega-1} sN(x,s) - \Big(\frac{k}{\omega} + \frac{\omega-1}{2\omega}\Big)\sum_{s=0}^{\omega-1} N(x,s).$$

Obviously, QN and $K_p(I-Q)N$ are continuous. Since X is a finite-dimensional Banach space, it is not difficult to show that $\overline{K_p(I-Q)N(\overline{\Omega})}$ is compact for any open bounded set $\Omega \subset X$. Moreover, $QN(\overline{\Omega})$ is bounded. Thus, N is L-compact on any open bounded set $\Omega \subset X$. The isomorphism J of ImQ onto Ker$L$ can be the identity mapping, since ImQ=Ker$L$.

Now we are at the point to search for an appropriate open, bounded subset Ω in X for the application of the continuation theorem. Corresponding to the operator equation $Lx = \lambda Nx, \lambda \in (0,1)$, we have

$$\begin{aligned} u_1(k+1) - u_1(k) = &\ \lambda\Big[a_1(k) - b_1(k)\exp\{u_1(k)\} + \frac{c_1(k)\exp\{u_2(k)\}}{e_1(k)+f_1(k)\exp\{u_2(k)\}}\Big], \\ u_2(k+1) - u_2(k) = &\ \lambda[a_2(k) - b_2(k)\exp\{u_2(k)\}]. \end{aligned} \tag{2.2}$$

Suppose that $y = (y_1(k), y_2(k))^T \in X$ is an arbitrary solution of system (2.2) for a certain $\lambda \in (0,1)$. Summing on both sides of (2.2) from 0 to $\omega - 1$ with respect to k, we reach

$$\begin{aligned} &\sum_{k=0}^{\omega-1}\Big[a_1(k) - b_1(k)\exp\{u_1(k)\} + \frac{c_1(k)\exp\{u_2(k)\}}{e_1(k)+f_1(k)\exp\{u_2(k)\}}\Big] = 0, \\ &\sum_{k=0}^{\omega-1}[a_2(k) - b_2(k)\exp\{u_2(k)\}] = 0. \end{aligned}$$

That is,

$$\sum_{k=0}^{\omega-1} b_1(k)\exp\{u_1(k)\} = \bar{a}_1\omega + \sum_{k=0}^{\omega-1} \frac{c_1(k)\exp\{u_2(k)\}}{e_1(k)+f_1(k)\exp\{u_2(k)\}}, \tag{2.3}$$

$$\sum_{k=0}^{\omega-1} b_2(k)\exp\{u_2(k)\} = \bar{a}_2\omega. \tag{2.4}$$

From (2.3) and (2.4), we have

$$\begin{aligned}
&\sum_{k=0}^{\omega-1} |u_1(k+1)-u_1(k)| \\
= \ & \lambda\sum_{k=0}^{\omega-1} |a_1(k) - b_1(k)\exp\{u_1(k)\} + \frac{c_1(k)\exp\{u_2(k)\}}{e_1(k)+f_1(k)\exp\{u_2(k)\}}| \\
\leq \ & \sum_{k=0}^{\omega-1} |a_1(k)| + \sum_{k=0}^{\omega-1}\Big(b_1(k)\exp\{u_1(k)\} + \frac{c_1(k)\exp\{u_2(k)\}}{e_1(k)+f_1(k)\exp\{u_2(k)\}}\Big) \\
= \ & \sum_{k=0}^{\omega-1} |a_1(k)| + \bar{a}_1\omega + 2\sum_{k=0}^{\omega-1} \frac{c_1(k)\exp\{u_2(k)\}}{e_1(k)+f_1(k)\exp\{u_2(k)\}} \\
= \ & (\bar{A}_1+\bar{a}_1)\omega + 2\sum_{k=0}^{\omega-1} \frac{c_1(k)\exp\{u_2(k)\}}{e_1(k)+f_1(k)\exp\{u_2(k)\}} \\
\leq \ & (\bar{A}_1+\bar{a}_1)\omega + 2\sum_{k=0}^{\omega-1} \frac{c_1(k)}{f_1(k)} \\
= \ & (\bar{A}_1+\bar{a}_1)\omega + 2\overline{\Big(\frac{c_1}{f_1}\Big)}\omega, \\
&\sum_{k=0}^{\omega-1} |u_2(k+1)-u_2(k)| \\
= \ & \lambda\sum_{k=0}^{\omega-1} |a_2(k) - b_2(k)\exp\{u_2(k)\}| \\
\leq \ & (\bar{A}_2+\bar{a}_2)\omega.
\end{aligned} \tag{2.5}$$

where $\bar{A}_1 = \frac{1}{\omega}\sum_{k=0}^{\omega-1}|a_1(k)|,\ \bar{A}_2 = \frac{1}{\omega}\sum_{k=0}^{\omega-1}|a_2(k)|,\ \overline{(\frac{c_1}{f_1})} = \frac{1}{\omega}\sum_{k=0}^{\omega-1}\frac{c_1(k)}{f_1(k)}$.

Since $\{u(k)\} = \{(u_1(k),u_2(k))^T\} \in X$, there exist $\eta_i, \delta_i, i=1,2$ such that

$$u_i(\eta_i) = \min_{k\in I_\omega} u_i(k),\ u_i(\delta_i) = \max_{k\in I_\omega} u_i(k). \tag{2.6}$$

By (2.4), one could easily obtain

$$u_2(\eta_2) \leq \ln\frac{\bar{a}_2}{\bar{b}_2},\ u_2(\delta_2) \geq \ln\frac{\bar{a}_2}{\bar{b}_2}. \tag{2.7}$$

Similarly to the analysis of (2.7)-(2.11) in [17], by using (2.5) and (2.7), we could obtain

$$u_2(k) \le \ln \frac{\bar{a}_2}{\bar{b}_2} + (\bar{A}_2 + \bar{a}_2)\omega,\ u_2(k) \ge \ln \frac{\bar{a}_2}{\bar{b}_2} - (\bar{A}_2 + \bar{a}_2)\omega, \tag{2.8}$$

$$|u_2(k)| \le \max\Big\{ |\ln \frac{\bar{a}_2}{\bar{b}_2} + (\bar{A}_2 + \bar{a}_2)\omega|, |\ln \frac{\bar{a}_2}{\bar{b}_2} - (\bar{A}_2 + \bar{a}_2)\omega| \Big\} \stackrel{\text{def}}{=} H_2. \tag{2.9}$$

It follows from (2.3) that

$$\sum_{k=0}^{\omega-1} b_1(k) \exp\{u_1(\eta_1)\} \quad \le \quad \bar{a}_1 \omega + \overline{(\frac{c_1}{f_1})}\omega,$$

and so,

$$u_1(\eta_1) \le \ln \frac{\Delta_1}{\bar{b}_1}, \tag{2.10}$$

where

$$\Delta_1 = \bar{a}_1 + \overline{(\frac{c_1}{f_1})}.$$

It follows from Lemma 2.2, (2.5) and (2.10) that

$$\begin{aligned} u_1(k) &\le u_1(\eta_1) + \sum_{k=0}^{\omega-1} |u_1(k+1) - u_1(k)| \\ &\le \ln \tfrac{\Delta_1}{\bar{b}_1} + (\bar{A}_1 + \bar{a}_1)\omega + 2\overline{(\frac{c_1}{f_1})}\omega \stackrel{\text{def}}{=} M_1. \end{aligned} \tag{2.11}$$

It follows from (2.3) and (2.8) that

$$\begin{aligned} \sum_{k=0}^{\omega-1} b_1(k) \exp\{u_1(\delta_1)\} &= \bar{a}_1 \omega + \sum_{k=0}^{\omega-1} \frac{c_1(k) \exp\{u_2(k)\}}{e_1(k) + f_1(k) \exp\{u_2(k)\}} \\ &\ge \bar{a}_1 \omega + \sum_{k=0}^{\omega-1} \frac{c_1(k) \exp\{\ln \frac{\bar{a}_2}{\bar{b}_2} - (\bar{A}_2 + \bar{a}_2)\omega\}}{e_1(k) + f_1(k) \exp\{\ln \frac{\bar{a}_2}{\bar{b}_2} - (\bar{A}_2 + \bar{a}_2)\omega\}} \\ &\ge \bar{a}_1 \omega, \end{aligned}$$

and so,

$$u_1(\delta_1) \ge \ln \frac{\bar{a}_1}{\bar{b}_1}, \tag{2.12}$$

It follows from Lemma 2.2, (2.6) and (2.12) that

$$\begin{aligned} u_1(k) &\ge u_1(\delta_1) - \sum_{k=0}^{\omega-1} |u_1(k+1) - u_1(k)| \\ &\ge \ln \tfrac{\bar{a}_1}{\bar{b}_1} - (\bar{A}_1 + \bar{a}_1)\omega - 2\overline{(\frac{c_1}{f_1})}\omega \stackrel{\text{def}}{=} M_2. \end{aligned} \tag{2.13}$$

It follows from (2.11) and (2.13) that

$$|u_1(k)| \le \max\Big\{ |M_1|, |M_2| \Big\} \stackrel{\text{def}}{=} H_1. \tag{2.14}$$

Clearly, H_1 and H_2 are independent on the choice of λ. Obviously, the system of algebraic equations

$$\bar{a}_1 - \bar{b}_1 x_1 + \frac{\bar{c}_1 x_2}{\bar{e}_1 + \bar{f}_1 x_2} = 0, \ \ \bar{a}_2 - \bar{b}_2 x_2 = 0 \tag{2.15}$$

has a unique positive solution $(x_1^*, x_2^*) \in R_2^+$, where

$$x_1^* = \frac{\bar{a}_1 + \Delta_3}{\bar{b}_1}, \ x_2^* = \frac{\bar{a}_2}{\bar{b}_2}.$$

where $\Delta_3 = \dfrac{\bar{c}_1 x_2^*}{\bar{e}_1 + \bar{f}_1 x_2^*}$.

Let $H = H_1 + H_2 + H_3$, where $H_3 > 0$ is taken sufficiently enough large such that $||(\ln\{x_1^*\}, \ \ln\{x_2^*\})^T|| = |\ln\{x_1^*\}| + |\ln\{x_2^*\}| < H_3$.

Let $H = H_1 + H_2 + H_3$, and define

$$\Omega = \left\{ u(t) = (u_1(k), u_2(k))^T \in X : \|u\| < H \right\}.$$

It is clear that Ω verifies requirement (a) in Lemma 2.1. When $u \in \partial\Omega \cap KerL = \partial\Omega \cap R^2$, u is constant vector in R^2 with $||u|| = B$. Then

$$QNu = \begin{pmatrix} \bar{a}_1 - \bar{b}_1 \exp\{u_1\} + \dfrac{\bar{c}_1 \exp\{u_2\}}{\bar{e}_1 + \bar{f}_1 \exp\{u_2\}} \\ \bar{a}_2 - \bar{b}_2 \exp\{u_2\} \end{pmatrix} \neq 0.$$

Moreover, direct calculation shows that

$$deg\{JQN, \Omega \cap KerL, 0\} = \mathrm{sgn}\Big(\bar{b}_1 \bar{b}_2 \exp\{x_1^*\} \exp\{x_2^*\}\Big) = 1 \neq 0.$$

where $deg(.)$ is the Brouwer degree and the J is the identity mapping since $ImQ = KerL$.

By now we have proved that Ω verifies all the requirements in Lemma 2.1. Hence (2.1) has at least one solution $(u_1^*(k), u_2^*(k))^T$ in $DomL \cap \bar{\Omega}$. And so, system (1.3) admits a positive periodic solution $(x_1^*(k), x_2^*(k))^T$, where $x_i^*(k) = \exp\{u_i^*(k)\}, i = 1, 2$. This completes the proof of the claim.□

By applying the analysis technique of this paper, one could easily establish sufficient conditions for the following two species discrete commensal symbiosis model with Holling type functional response

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\Big\{a_1(k)-b_1(k)x_1(k)+\frac{c_1(k)(x_2(k))^p}{e_1(k)+f_1(k)(x_2(k))^p}\Big\}, \\ x_2(k+1) &= x_2(k)\exp\big\{a_2(k)-b_2(k)x_2(k)\big\}, \end{aligned} \tag{2.16}$$

where $\{b_i(k)\}, i=1,2, \{c_1(k)\}\{e_1(k)\}, \{f_1(k)\}$ are all positive ω-periodic sequences, ω is a fixed positive integer, $\{a_i(k)\}$ are ω-periodic sequences, which satisfies $\overline{a}_i = \frac{1}{\omega}\sum\limits_{k=0}^{\omega-1} a_i(k) > 0, i=1,2$. p is positive constant.

Concerned with the existence of positive periodic solution of the system (2.16), we have

Theorem 2.2 *System (2.16) admits at least one positive ω-periodic solution.*

Conflict of Interests

The authors declare that there is no conflict of interests.

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