Available online at http://scik.org
Commun. Math. Biol. Neurosci. 2017, 2017:3
ISSN: 2052-2541

ON THE EXISTENCE OF POSITIVE PERIODIC SOLUTION OF A AMENSALISM MODEL WITH HOLLING II FUNCTIONAL RESPONSE

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Communicated by S. Shen

Abstract. Sufficient conditions are obtained for the existence of positive periodic solution of the following discrete amensalism model with Holling II functional response

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\left\{a_1(k) - b_1(k)x_1(k) - \frac{c_1(k)x_2(k)}{e_1(k)+f_1(k)x_2(k)}\right\}, \\ x_2(k+1) &= x_2(k)\exp\left\{a_2(k) - b_2(k)x_2(k)\right\}, \end{aligned}$$

where $\{b_i(k)\}, i = 1, 2, \{c_1(k)\}\{e_1(k)\}, \{f_1(k)\}$ are all positive ω-periodic sequences, ω is a fixed positive integer, $\{a_i(k)\}$ are ω-periodic sequences, which satisfies $\overline{a}_i = \frac{1}{\omega}\sum_{k=0}^{\omega-1} a_i(k) > 0, i = 1, 2.$

Keywords: amensalism model; positive periodic solution; functional response.

2010 AMS Subject Classification: 34C25, 92D25, 34D20, 34D40.

1. Introduction

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Received May 25, 2016

Amensalism and commensalism are two common relationship between the species, here, amensalism is an interaction where an organism inflicts harm to another organism without any costs or benefits received by the other. And commensalism describe a relationship which is only favorable to the one side and have no influence to the other side.

In the past decade, numerous works on the mutualism model ([1]-[14]) or the commensalism model has been published([15]-[20]). However, only recently did scholars paid attention to the amensalism model([21]-[26]).

Sun [21] first time proposed a amensalism model:

$$\begin{aligned} \frac{dx}{dt} &= r_1 x\Big(\frac{k_1 - x - ay}{k_1}\Big), \\ \frac{dy}{dt} &= r_2 y\Big(\frac{k_2 - y}{k_2}\Big), \end{aligned} \tag{1.1}$$

where all the parameters $r_i, k_i, i = 1,2$ and a are positive constants. They investigated the local stability of all equilibrium points. The model is then generalized by Zhu and Chen[22] to the following more general case

$$\begin{aligned} \frac{dx}{dt} &= x\Big(a_1 + b_1 x + c_1 y\Big), \\ \frac{dy}{dt} &= y\Big(a_2 + c_2 y\Big), \end{aligned} \tag{1.2}$$

where $a_i > 0, c_i < 0, i = 1,2, b_1 < 0$. The qualitative property of the system (1.2) is investigated.

Stimulated by the works of Sun[21] and Zhu and Chen[22], Zhang[23] proposed the following delay amensalism model

$$\begin{aligned} \frac{dx}{dt} &= x\Big(r_1 - a_{11} x(t-\tau)\Big), \\ \frac{dy}{dt} &= y\Big(r_2 - a_{21}\int_{-\infty}^{t} f(t-s)x(s)ds - a_{22} y\Big). \end{aligned} \tag{1.3}$$

By taking τ as parameter, the author investigated the local stability property of the positive equilibrium and found the Hopf bifurcation phenomenon of the system.

All the works of [21]-[23] are autonomous ones and recently, Han et al[28] proposed the following non-autonomous amensalism model:

$$\begin{aligned}\frac{dx_1}{dt} &= x_1\Big(a_1(t)-b_1(t)x_1-c_1(t)x_2\Big),\\ \frac{dx_2}{dt} &= x_2\Big(a_2(t)-b_2(t)x_2\Big).\end{aligned} \tag{1.4}$$

By using a continuation theorem based on Gaines and Mawhin's coincidence degree, a set of easily verified sufficient conditions which guarantee the global existence of positive periodic solutions of above system is established. Chen et al[25, 26] argued that the discrete time models governed by difference equations are more appropriate than the continuous ones when the populations have non-overlapping generations, and they proposed the following discrete non-autonomous amensalism model:

$$\begin{aligned}x_1(k+1) &= x_1(k)\exp\big\{a_1(k)-b_1(k)x_1(k)-c_1(k)x_2(k)\big\},\\ x_2(k+1) &= x_2(k)\exp\big\{a_2(k)-b_2(k)x_2(k)\big\}.\end{aligned} \tag{1.5}$$

In [25], they investigated the persistent, extinction and stability property of the system, and in [26], they established a set of easily verified sufficient conditions which guarantee the global existence of positive periodic solutions of above system.

In system (1.1)-(1.5), the authors made the assumption that the influence of the second species to the first one is linearize, none of them consider the functional response of the second species. Now, by adapting the Holling II functional response to system (1.5), we could establish the following two species discrete amensalism model with Holling II functional response

$$\begin{aligned}x_1(k+1) &= x_1(k)\exp\Big\{a_1(k)-b_1(k)x_1(k)-\frac{c_1(k)x_2(k)}{e_1(k)+f_1(k)x_2(k)}\Big\},\\ x_2(k+1) &= x_2(k)\exp\big\{a_2(k)-b_2(k)x_2(k)\big\},\end{aligned} \tag{1.6}$$

where $\{b_i(k)\}, i=1,2, \{c_1(k)\}\{e_1(k)\}, \{f_1(k)\}$ are all positive ω-periodic sequences, ω is a fixed positive integer, $\{a_i(k)\}$ are ω-periodic sequences, which satisfies $\overline{a}_i = \frac{1}{\omega}\sum\limits_{k=0}^{\omega-1} a_i(k) > 0, i=1,2$. Here we assume that the coefficients of the system (1.6) are all periodic sequences which having a common integer period. Such an assumption seems reasonable in view of seasonal factors, e.g., mating habits, availability of food, weather conditions, harvesting, and hunting, etc.

The aim of this paper is to obtain a set of sufficient conditions which ensure the existence of positive periodic solution of system (1.6).

2. Main results

In the proof of our existence theorem below, we will use the continuation theorem of Gaines and Mawhin([27]).

Lemma 2.1 (Continuation Theorem) *Let L be a Fredholm mapping of index zero and let N be L-compact on $\bar{\Omega}$. Suppose*

(a).For each $\lambda \in (0,1)$, every solution x of $Lx = \lambda Nx$ is such that $x \notin \partial\Omega$;

(b).$QNx \neq 0$ for each $x \in \partial\Omega \cap KerL$ and

$$deg\{JQN, \Omega \cap KerL, 0\} \neq 0.$$

Then the equation $Lx = Nx$ has at least one solution lying in $DomL \cap \bar{\Omega}$.

Let Z, Z^+, R and R^+ denote the sets of all integers, nonnegative integers, real unumbers, and nonnegative real numbers, respectively. For convenience, in the following discussion, we will use the notation below throughout this paper:

$$I_\omega = \{0, 1, ..., \omega - 1\},\ \overline{g} = \frac{1}{\omega}\sum_{k=0}^{\omega-1} g(k),\ \ g^u = \max_{k\in I_\omega} g(k),\ \ g^l = \min_{k\in I_\omega} g(k),$$

where $\{g(k)\}$ is an ω-periodic sequence of real numbers defined for $k \in Z$.

Lemma 2.2[28] *Let $g : Z \to R$ be ω-periodic, i. e., $g(k+\omega) = g(k)$. Then for any fixed $k_1, k_2 \in I_\omega$, and any $k \in Z$, one has*

$$g(k) \leq g(k_1) + \sum_{s=0}^{\omega-1} |g(s+1) - g(s)|,$$

$$g(k) \geq g(k_2) - \sum_{s=0}^{\omega-1} |g(s+1) - g(s)|.$$

We now reach the position to establish our main result.

Theorem 2.1 *Assume that* $\bar{a}_1 > \overline{(\frac{c_1}{f_1})}$ *holds, then system (1.6) admits at least one positive* ω*-periodic solution.*

Proof. Let

$$x_i(k) = \exp\{u_i(k)\}, \;\; i = 1, 2,$$

so that system (1.3) becomes

$$\begin{aligned} u_1(k+1) - u_1(k) = &\;\; a_1(k) - b_1(k)\exp\{u_1(k)\} - \frac{c_1(k)\exp\{u_2(k)\}}{e_1(k) + f_1(k)\exp\{u_2(k)\}}, \\ u_2(k+1) - u_2(k) = &\;\; a_2(k) - b_2(k)\exp\{u_2(k)\}. \end{aligned} \tag{2.1}$$

Define

$$l_2 = \Big\{ y = \{y(k)\}, y(k) = (y_1(k), y_2(k))^T \in R^2 \Big\}.$$

For $a = (a_1, a_2)^T \in R^2$, define $|a| = \max\{|a_1|, |a_2|\}$. Let $l^\omega \subset l_2$ denote the subspace of all ω sequences equipped with the usual normal form $\|y\| = \max\limits_{k \in I_\omega} |y(k)|$. It is not difficult to show that l^ω is a finite-dimensional Banach space. Let

$$l_0^\omega = \{y = \{y(k)\} \in l^\omega : \sum_{k=0}^{\omega-1} y(k) = 0\}, \; l_c^\omega = \{y = \{y(k)\} \in l^\omega : y(k) = h \in R^2, k \in Z\},$$

then l_0^ω and l_c^ω are both closed linear subspace of l^ω, and

$$l^\omega = l_0^\omega \oplus l_c^\omega, \;\; diml_c^\omega = 2.$$

Now let us define $X = Y = l^\omega$, $(Ly)(k) = y(k+1) - y(k)$. It is trivial to see that L is a bounded linear operator and

$$KerL = l_c^\omega, \;\; ImL = l_0^\omega, \;\; dimKerL = 2 = CodimImL.$$

Then it follows that L is a Fredholm mapping of index zero. Let

$$N(u_1, u_2)^T = (N_1, N_2)^T := N(u, k),$$

where

$$\begin{cases} N_1 = a_1(k) - b_1(k)\exp\{u_1(k)\} - \dfrac{c_1(k)\exp\{u_2(k)\}}{e_1(k) + f_1(k)\exp\{u_2(k)\}}, \\ N_2 = a_2(k) - b_2(k)\exp\{u_2(k)\}. \end{cases}$$

$$Px = \frac{1}{\omega}\sum_{s=0}^{\omega-1} x(s), x \in X, \;\; Qy = \frac{1}{\omega}\sum_{s=0}^{\omega-1} y(s), y \in Y.$$

It is not difficult to show that P and Q are two continuous projectors such that

$$ImP = KerL \quad and \quad ImL = KerQ = Im(I-Q).$$

Furthermore, the generalized inverse (to L) K_p: ImL →Ker$P$∩Dom$L$ exists and is given by

$$K_p(z) = \sum_{s=0}^{k-1} z(s) - \frac{1}{\omega}\sum_{s=0}^{\omega-1}(\omega - s)z(s).$$

Thus

$$QNx = \frac{1}{\omega}\sum_{k=0}^{\omega-1} N(x,k),$$

$$Kp(I-Q)Nx = \sum_{s=0}^{k-1} N(x,s) + \frac{1}{\omega}\sum_{s=0}^{\omega-1} sN(x,s) - \Big(\frac{k}{\omega} + \frac{\omega-1}{2\omega}\Big)\sum_{s=0}^{\omega-1} N(x,s).$$

Obviously, QN and $K_p(I-Q)N$ are continuous. Since X is a finite-dimensional Banach space, it is not difficult to show that $\overline{K_p(I-Q)N(\overline{\Omega})}$ is compact for any open bounded set $\Omega \subset X$. Moreover, $QN(\overline{\Omega})$ is bounded. Thus, N is L-compact on any open bounded set $\Omega \subset X$. The isomorphism J of ImQ onto Ker$L$ can be the identity mapping, since ImQ=Ker$L$.

Now we are at the point to search for an appropriate open, bounded subset Ω in X for the application of the continuation theorem. Corresponding to the operator equation $Lx = \lambda Nx, \lambda \in (0,1)$, we have

$$\begin{aligned} u_1(k+1) - u_1(k) = &\quad \lambda\Big[a_1(k) - b_1(k)\exp\{u_1(k)\} - \frac{c_1(k)\exp\{u_2(k)\}}{e_1(k) + f_1(k)\exp\{u_2(k)\}}\Big], \\ u_2(k+1) - u_2(k) = &\quad \lambda[a_2(k) - b_2(k)\exp\{u_2(k)\}]. \end{aligned} \tag{2.2}$$

Suppose that $y = (y_1(k), y_2(k))^T \in X$ is an arbitrary solution of system (2.2) for a certain $\lambda \in (0,1)$. Summing on both sides of (2.2) from 0 to $\omega - 1$ with respect to k, we reach

$$\begin{aligned} &\sum_{k=0}^{\omega-1}\Big[a_1(k) - b_1(k)\exp\{u_1(k)\} - \frac{c_1(k)\exp\{u_2(k)\}}{e_1(k) + f_1(k)\exp\{u_2(k)\}}\Big] = 0, \\ &\sum_{k=0}^{\omega-1}[a_2(k) - b_2(k)\exp\{u_2(k)\}] = 0. \end{aligned}$$

That is,

$$\sum_{k=0}^{\omega-1} b_1(k)\exp\{u_1(k)\} + \sum_{k=0}^{\omega-1}\frac{c_1(k)\exp\{u_2(k)\}}{e_1(k) + f_1(k)\exp\{u_2(k)\}} = \bar{a}_1\omega, \tag{2.3}$$

$$\sum_{k=0}^{\omega-1} b_2(k)\exp\{u_2(k)\} = \bar{a}_2\omega. \tag{2.4}$$

From (2.3) and (2.4), we have

$$\begin{aligned}
&\sum_{k=0}^{\omega-1}|u_1(k+1)-u_1(k)| \\
=\ &\lambda\sum_{k=0}^{\omega-1}|a_1(k)-b_1(k)\exp\{u_1(k)\}-\frac{c_1(k)\exp\{u_2(k)\}}{e_1(k)+f_1(k)\exp\{u_2(k)\}}| \\
\leq\ &\sum_{k=0}^{\omega-1}|a_1(k)|+\sum_{k=0}^{\omega-1}\Big(b_1(k)\exp\{u_1(k)\}+\frac{c_1(k)\exp\{u_2(k)\}}{e_1(k)+f_1(k)\exp\{u_2(k)\}}\Big) \qquad (2.5)\\
=\ &\sum_{k=0}^{\omega-1}|a_1(k)|+\bar{a}_1\omega \\
=\ &(\bar{A}_1+\bar{a}_1)\omega,
\end{aligned}$$

$$\begin{aligned}
&\sum_{k=0}^{\omega-1}|u_2(k+1)-u_2(k)| \\
=\ &\lambda\sum_{k=0}^{\omega-1}|a_2(k)-b_2(k)\exp\{u_2(k)\}| \qquad (2.6)\\
\leq\ &(\bar{A}_2+\bar{a}_2)\omega.
\end{aligned}$$

where $\bar{A}_1=\frac{1}{\omega}\sum_{k=0}^{\omega-1}|a_1(k)|,\ \bar{A}_2=\frac{1}{\omega}\sum_{k=0}^{\omega-1}|a_2(k).$

Since $\{u(k)\}=\{(u_1(k),u_2(k))^T\}\in X$, there exist $\eta_i,\delta_i,i=1,2$ such that

$$u_i(\eta_i)=\min_{k\in I_\omega}u_i(k),\ u_i(\delta_i)=\max_{k\in I_\omega}u_i(k). \qquad (2.6)$$

By (2.4), one could easily obtain

$$u_2(\eta_2)\leq\ln\frac{\bar{a}_2}{\bar{b}_2},\ u_2(\delta_2)\geq\ln\frac{\bar{a}_2}{\bar{b}_2}. \qquad (2.7)$$

Similarly to the analysis of (11)-(15) in [26], by using (2.5) and (2.7), we could obtain

$$u_2(k)\leq\ln\frac{\bar{a}_2}{\bar{b}_2}+(\bar{A}_2+\bar{a}_2)\omega,\ u_2(k)\geq\ln\frac{\bar{a}_2}{\bar{b}_2}-(\bar{A}_2+\bar{a}_2)\omega, \qquad (2.8)$$

$$|u_2(k)|\leq\max\Big\{|\ln\frac{\bar{a}_2}{\bar{b}_2}+(\bar{A}_2+\bar{a}_2)\omega|,|\ln\frac{\bar{a}_2}{\bar{b}_2}-(\bar{A}_2+\bar{a}_2)\omega|\Big\}\stackrel{\text{def}}{=}H_2. \qquad (2.9)$$

It follows from (2.3) that

$$\sum_{k=0}^{\omega-1}b_1(k)\exp\{u_1(\eta_1)\}\ \leq\ \bar{a}_1\omega,$$

and so,

$$u_1(\eta_1)\leq\ln\frac{\bar{a}_1}{\bar{b}_1}. \qquad (2.10)$$

It follows from Lemma 2.2, (2.5) and (2.10) that

$$\begin{aligned} u_1(k) &\le u_1(\eta_1)+\sum_{k=0}^{\omega-1}|u_1(k+1)-u_1(k)| \\ &\le \ln\tfrac{\bar a_1}{\bar b_1}+(\bar A_1+\bar a_1)\omega \overset{\text{def}}{=} M_1. \end{aligned} \tag{2.11}$$

It follows from (2.3) and (2.8) that

$$\begin{aligned} \sum_{k=0}^{\omega-1} b_1(k)\exp\{u_1(\delta_1)\} &= \bar a_1\omega-\sum_{k=0}^{\omega-1}\frac{c_1(k)\exp\{u_2(k)\}}{e_1(k)+f_1(k)\exp\{u_2(k)\}} \\ &\ge \bar a_1\omega-\sum_{k=0}^{\omega-1}\frac{c_1(k)\exp\{\ln\frac{\bar a_2}{\bar b_2}+(\bar A_2+\bar a_2)\omega\}}{e_1(k)+f_1(k)\exp\{\ln\frac{\bar a_2}{\bar b_2}+(\bar A_2+\bar a_2)\omega\}} \\ &\ge \bar a_1\omega-\sum_{k=0}^{\omega-1}\frac{c_1(k)}{f_1(k)} \\ &\ge \bar a_1\omega-\overline{(\frac{c_1}{f_1})}\omega, \end{aligned}$$

where $\overline{(\frac{c_1}{f_1})}=\frac{1}{\omega}\sum_{k=0}^{\omega-1}\frac{c_1(k)}{f_1(k)}$. And so,

$$u_1(\delta_1)\ge \ln\frac{\bar a_1-\overline{(\frac{c_1}{f_1})}}{\bar b_1}, \tag{2.12}$$

It follows from Lemma 2.2, (2.6) and (2.12) that

$$\begin{aligned} u_1(k) &\ge u_1(\delta_1)-\sum_{k=0}^{\omega-1}|u_1(k+1)-u_1(k)| \\ &\ge \ln\frac{\bar a_1-\overline{(\frac{c_1}{f_1})}}{\bar b_1}-(\bar A_1+\bar a_1)\omega \overset{\text{def}}{=} M_2. \end{aligned} \tag{2.13}$$

It follows from (2.11) and (2.13) that

$$|u_1(k)|\le \max\Big\{|M_1|,|M_2|\Big\} \overset{\text{def}}{=} H_1. \tag{2.14}$$

Clearly, H_1 and H_2 are independent on the choice of λ. Obviously, the system of algebraic equations

$$\bar a_1-\bar b_1x_1-\frac{1}{\omega}\sum_{k=0}^{\omega-1}\frac{c_1(k)x_2}{e_1(k)+f_1(k)x_2}=0,\ \ \bar a_2-\bar b_2x_2=0 \tag{2.15}$$

has a unique positive solution $(x_1^*,x_2^*)\in R_2^+$, where

$$x_1^*=\frac{\bar a_1-\frac{1}{\omega}\sum_{k=0}^{\omega-1}\frac{c_1(k)x_2^*}{e_1(k)+f_1(k)x_2^*}}{\bar b_1}>\frac{\bar a_1-\overline{(\frac{c_1}{f_1})}}{\bar b_1}>0,\ x_2^*=\frac{\bar a_2}{\bar b_2}>0.$$

Let $H=H_1+H_2+H_3$, where $H_3>0$ is taken sufficiently enough large such that $||(\ln\{x_1^*\},\ \ln\{x_2^*\})^T||=|\ln\{x_1^*\}|+|\ln\{x_2^*\}|<H_3$.

Let $H=H_1+H_2+H_3$, and define

$$\Omega=\left\{u(t)=(u_1(k),u_2(k))^T\in X:\|u\|<H\right\}.$$

It is clear that Ω verifies requirement (a) in Lemma 2.1. When $u\in\partial\Omega\cap KerL=\partial\Omega\cap R^2$, u is constant vector in R^2 with $||u||=B$. Then

$$QNu=\begin{pmatrix}\bar{a}_1-\bar{b}_1\exp\{u_1\}-\dfrac{1}{\omega}\displaystyle\sum_{k=0}^{\omega-1}\dfrac{c_1(k)\exp\{u_2\}}{e_1(k)+f_1(k)\exp\{u_2\}}\\ \bar{a}_2-\bar{b}_2\exp\{u_2\}\end{pmatrix}\neq 0.$$

Moreover, direct calculation shows that

$$deg\{JQN,\Omega\cap KerL,0\}=\text{sgn}\Big(\bar{b}_1\bar{b}_2\exp\{x_1^*\}\exp\{x_2^*\}\Big)=1\neq 0.$$

where $deg(.)$ is the Brouwer degree and the J is the identity mapping since $ImQ=KerL$.

By now we have proved that Ω verifies all the requirements in Lemma 2.1. Hence (2.1) has at least one solution $(u_1^*(k),u_2^*(k))^T$ in $DomL\cap\bar{\Omega}$. And so, system (1.3) admits a positive periodic solution $(x_1^*(k),x_2^*(k))^T$, where $x_i^*(k)=\exp\{u_i^*(k)\},i=1,2$, This completes the proof of the claim.

3. Numeric simulation

Now let us consider the following example.

Example 3.1.

$$\begin{aligned}x_1(k+1)&=x_1(k)\exp\Big\{2+0.3\sin(\pi k)-(1+0.3\sin(\pi k))x_1(k)-\frac{(2+0.5\sin(\pi k))x_2(k)}{1+2x_2(k)}\Big\},\\ x_2(k+1)&=x_2(k)\exp\big\{0.6+0.3\sin(\pi n)-(3+2\cos(\pi k))x_2(k)\big\},\end{aligned}\tag{3.1}$$

Corresponding to system (1.6), here we choose $a_1(k)=2+0.3\sin(\pi k),b_1(k)=1+0.3\sin(\pi k)$, $c_1(k)=2+0.5\sin(\pi k),e_1(k)=1,f_1(k)=2$. $a_2(k)=0.6+0.3\sin(\pi k),b_2(k)=3+2\cos(\pi k)$. One could easily check that the condition of Theorem 2.1 holds, and consequently, system (3.1) admits at least one positive 2-period solution. Numeric simulation (Fig.1, Fig. 2)also support this assertion.

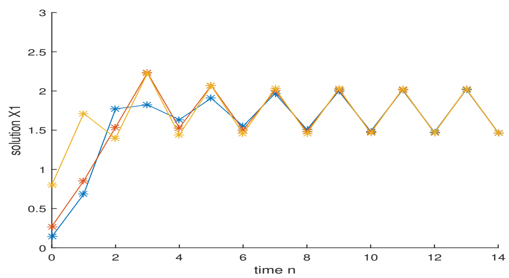

FIGURE 1. Dynamic behavior of the first component $x1$ in system (3.1) with the initial condition $(x(0), y(0)) = (0.14, 0.19)$, $(0.27, 0.69)$ and $(0.80, 0.39)$, respectively.

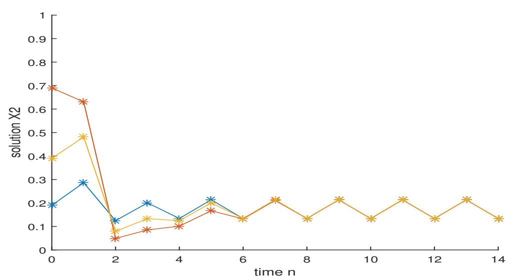

FIGURE 2. Dynamic behavior of the second component $x1$ in system (3.1) with the initial condition $(x(0), y(0)) = (0.14, 0.19)$, $(0.27, 0.69)$ and $(0.80, 0.39)$, respectively.

4. Discussion

In this paper, we propose a discrete ammensilism model with Holling II functional response, by using the coincidence degree theory, sufficient conditions which ensure the existence of positive periodic sequences solution are established.

We mention here that as far as system (1.6) is concerned, such topic as persistent, extinction and stability property of the system is very important, indeed, from Figure 1 and Figure 2, one could see that the periodic solution of the system (3.1) is stable, however, such a conclusion could not be obtained from our Theorem 2.1. We will investigate the stability property of the system (1.6) in the future.

Conflict of Interests

The authors declare that there is no conflict of interests.

Acknowledgements

The research was supported by the Natural Science Foundation of Fujian Province (2015J01012, 2015J01019, 2015J05006) and the Scientific Research Foundation of Fuzhou University (XRC-1438).

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