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EXPONENTIAL SYNCHRONIZATION OF COHEN-GROSSBERG NEURAL NETWORKS WITH STOCHASTIC PERTURBATION AND REACTION-DIFFUSION TERMS VIA PERIODICALLY INTERMITTENT CONTROL

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Abstract. In this paper, a class of Cohen-Grossberg neural networks with mixed time-varying delays, stochastic perturbation and reaction-diffusion terms is investigated. The exponential synchronization criteria in terms of p-norm are obtained based on periodically intermittent control by means of Lyapunov functional theory, mathematical induction and inequality technique. The influences of stochastic perturbation, spacial diffusion, the control rate and the control strength on the exponential synchronization are discussed according to the obtained synchronization criteria. The proposed criteria improve the previous known results in the literature and remove the restrictions on the mixed time-varying delays. Numerical simulations are carried out to illustrate the feasibility of the results.

Keywords: exponential synchronization; Cohen-Grossberg neural networks; spacial diffusion; stochastic perturbation; periodically intermittent control.

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1. Introduction

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Cohen-Grossberg neural network [1] proposed in 1983 is a typical neural network model. Some models such as Hopfield neural networks and cellular neural networks are special cases of this model. In recent years, Cohen-Grossberg neural networks have attracted the attention of many researchers due to their potential applications in signal and image processing, associative memory, optimization problems, and so on (see, for example, [2-4] and references cited therein).

It has been shown that delays in particular time-varying delays are unavoidable in the information processing of neurons. In fact, time-varying delays occur in the electronic implementation of analog neural networks caused by the finite switching speed of amplifier circuits. In addition, neural networks usually have a spatial property due to the presence of a lot of parallel pathways of a variety of axon sizes and lengths. Hence, the signal propagation is distributed during a certain time period. Such an inherent nature can be modeled by distributed delays such that the distant past has less influence compared to the recent behaviors of the state. Therefore, discrete and distributed time-varying delays should be considered when the dynamical behaviors of neural networks are studied (see, for example, [5-7] and references cited therein).

In real neural networks, synaptic transmission is a noisy process introduced by random fluctuations from the release of neurotransmitters and other probabilistic causes [8]. Besides, diffusion phenomena cannot be ignored in neural networks and electric circuits when electrons are moving in asymmetric electromagnetic fields. For example, the dynamic behavior of multilayer cellular neural networks are dependent on not only the evolution time of each variable and its position, but also its interactions deriving from the space-distributed structure of the whole networks [9]. Hence, more realistic neural networks should consider the effects of stochastic perturbation and reaction-diffusion.

In the past decade, many mathematical researchers have studied the synchronization problems of chaotic neural networks because of their extensive applications in secure communications, information processing, chaos generators design, and so on (see, for example, [10-13] and references cited therein). Up to now, various control methods for the synchronization of chaotic neural networks have been investigated. Periodically intermittent control first proposed in [14] is a kind of discontinuous control method to achieve synchronization of chaos neural networks. In each period of this control method, the controller is activated in the work time and is off in the rest time. It is more economical and efficient in practice than the continuous control methods including state feedback control [15], pinning control [16], adaptive control [17], sliding mode control [18], and so on (see, for example, [19-21] and references cited therein). Hence, periodically intermittent control has been intensively studied due to its important applications in the engineering fields [22-26].

Many results with respect to the synchronization problems of Cohen-Grossberg neural networks have been obtained based on periodically intermittent control (see, for example, [27-32] and references cited therein). In [27-30], the synchronization problems of Cohen-Grossberg neural networks with the constant amplification gains were researched. In [31], Yu et al. studied the exponential synchronization of Cohen-Grossberg neural networks with the general amplification functions. But the combined effects of the mixed time-varying delays, stochastic perturbation and reaction-diffusion terms on the exponential synchronization of Cohen-Grossberg neural networks were not considered in [31]. In [32], sufficient conditions are given to realize the exponential synchronization of stochastic Cohen-Grossberg neural networks with mixed time-varying delays and reaction-diffusion terms via periodically intermittent control based on p-morm by using Lyapunov stability theory with stochastic analysis approaches. However, it was required that the derivative of the mixed time-varying delays was smaller than one in [32], that is, the mixed time-varying delays are slowly varying delays. In fact, the continuous varying of the delay may be slow or fast, so these restrictions are unnecessary and impractical.

From the above discussion, the previous synchronization criteria for Cohen-Grossberg neural networks under periodically intermittent control are somewhat conservative. Hence, motivated by the work of Gan [32], in this paper, we are concerned with the combined effects of mixed time-varying delays, stochastic perturbation and reaction-diffusion terms on the exponential synchronization of Cohen-Grossberg neural networks via periodically intermittent control to improve the previous results. To this end, we consider the following Cohen-Grossberg neural networks

$$\begin{aligned}\frac{\partial u_i(t,x)}{\partial t} = \sum_{k=1}^{l^*}\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial u_i(t,x)}{\partial x_k}\right) - \alpha_i(u_i(t,x))\Bigg[\beta_i(u_i(t,x)) - \sum_{j=1}^{n} a_{ij} f_j(u_j(t,x)) \\ - \sum_{j=1}^{n} b_{ij} g_j(u_j(t-\tau_{ij}(t),x)) - \sum_{j=1}^{n} d_{ij}\int_{t-\tau^*_{ij}(t)}^{t} h_j(u_j(s,x))\mathrm{d}s + J_i\Bigg],\end{aligned} \tag{1.1}$$

where $i \in \ell = \{1,2,\ldots,n\}$, n is the number of neurons in the networks; $x = (x_1,x_2,\ldots,x_{l^*})^T \in \Omega \subset \mathbb{R}^{l^*}$ and $\Omega = \{x = (x_1,x_2,\cdots,x_{l^*})^T || x_k| < m_k, k \in \hbar = \{1,2,\ldots,l^*\}\}$ is a bound compact set with smooth boundary $\partial\Omega$ and $mes\Omega > 0$ in space $\mathbb{R}^{l^*}$, where $mes\Omega$ is the measure of the set Ω; $u(t,x) = (u_1(t,x),u_2(t,x),\ldots,u_n(t,x))^T$ with $u_i(t,x)$ is the state of the ith neuron at time t and in space x; $\alpha_i(\cdot)$ represents an amplification function; $\beta_i(\cdot)$ is an appropriately behaved function; a_{ij}, b_{ij} and d_{ij} denote the connection strength, the discrete time-varying delay connection strength, and the distributed time-varying delay connection strength of the jth neuron on the ith neuron, respectively; $f_j(\cdot), g_j(\cdot)$ and $h_j(\cdot)$ denote the activation functions of the jth neuron in space x; $D_{ik} \geq 0$ represents the transmission diffusion coefficient along the ith neuron; $0 < \tau_{ij}(t) \leq \tau$ and $0 < \tau^*_{ij}(t) \leq \tau^*$ correspond to the discrete time-varying delay and the distributed time-varying delay along the axon of the jth neuron from the ith neuron, respectively; J_i denotes the external inputs on the ith neuron.

The boundary conditions and the initial values of system (1.1) take the form

$$u_i(t,x)|_{\partial\Omega} = 0,\ (t,x) \in [-\bar{\tau},+\infty) \times \partial\Omega,\ i \in \ell, \tag{1.2}$$

and

$$u_i(s,x) = \phi_i(s,x),\ (s,x) \in [-\bar{\tau},0] \times \Omega,\ i \in \ell, \tag{1.3}$$

where $\bar{\tau} = \max\{\tau,\tau^*\}, \phi(s,x) = (\phi_1(s,x),\phi_2(s,x),\ldots,\phi_n(s,x))^T \in \mathscr{C}$ is bounded and continuous and $\mathscr{C} \triangleq \mathscr{C}([-\bar{\tau},0) \times \Omega, \mathbb{R}^n)$ denotes the Banach space of continuous functions which maps $[-\bar{\tau},0) \times \Omega$ into $\mathbb{R}^n$ with p-norm (p is a positive integer) defined by

$$\|\phi\|_p = \left(\int_\Omega \sum_{i=1}^n \sup_{-\bar{\tau}\leq s\leq 0} |\phi_i(s,x)|^p \mathrm{d}x\right)^{\frac{1}{p}}.$$

System (1.1) is called the master system. To observe the exponential synchronization behavior of the master system (1.1), the response system with stochastic perturbation is described by

$$
\begin{aligned}
\mathrm{d}v_i(t,x) =& \Bigg\{ \sum_{k=1}^{l^*} \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial v_i(t,x)}{\partial x_k}\right) - \alpha_i(v_i(t,x))\Bigg[\beta_i(v_i(t,x)) - \sum_{j=1}^{n} a_{ij}f_j(v_j(t,x)) \\
& - \sum_{j=1}^{n} b_{ij}g_j(v_j(t-\tau_{ij}(t),x)) - \sum_{j=1}^{n} d_{ij}\int_{t-\tau_{ij}^*(t)}^{t} h_j(v_j(s,x))\mathrm{d}s + J_i\Bigg] + K_i(t,x)\Bigg\}\mathrm{d}t \quad (1.4) \\
& + \sum_{j=1}^{n} \sigma_{ij}(e_j(t,x), e_j(t-\tau_{ij}(t),x), e_j(t-\tau_{ij}^*(t),x))\mathrm{d}\omega_j(t),
\end{aligned}
$$

where $v(t,x) = (v_1(t,x), v_2(t,x), \ldots, v_n(t,x))^T$ denotes the state of the response system; $e(t,x) = (e_1(t,x), e_2(t,x), \ldots, e_n(t,x))^T = v(t,x) - u(t,x)$ is the synchronization error signal; $\sigma = (\sigma_{ij})_{n\times n}$ is the noise intensity matrix; $\omega(t) = (\omega_1(t), \omega_2(t), \cdots, \omega_n(t))^T \in \mathbb{R}^n$ is the stochastic disturbance which is a Brownian motion defined on $(\Omega, \mathscr{F}, \mathscr{P})$, $(\Omega, \mathscr{F}, \mathscr{P})$ is a complete probability space, Ω is the sample space, $\mathscr{F}$ is the $\sigma-$algebra of subsets of the sample space Ω and $\mathscr{P}$ is the probability measure on $\mathscr{F}$.

The response system (1.4) satisfies the following boundary conditions and initial values

$$
v_i(t,x)|_{\partial\Omega} = 0,\ (t,x) \in [-\bar{\tau}, +\infty) \times \partial\Omega,\ i \in \ell, \tag{1.5}
$$

and

$$
v_i(s,x) = \psi_i(s,x),\ (s,x) \in [-\bar{\tau}, 0] \times \Omega,\ i \in \ell, \tag{1.6}
$$

where $\psi(s,x) = (\psi_1(s,x), \psi_2(s,x), \ldots, \psi_n(s,x)) \in \mathscr{C}$ is bounded and continuous.

Let $K(t,x) = (K_1(t,x), K_2(t,x), \ldots, K_n(t,x))$ be an intermittent controller defined by

$$
K_i(t,x) = \begin{cases} \sum_{j=1}^{n} -k_{ij}(v_j(t,x) - u_j(t,x)), & (t,x) \in [mT, mT+\delta T) \times \Omega, \\ 0, & (t,x) \in [mT+\delta T, (m+1)T) \times \Omega, \end{cases} \tag{1.7}
$$

where $m \in N = \{0, 1, 2, \ldots\}, k_{ij}$ are constants and $k_{ii} > 0$ for all $i, j \in \ell$, which denote the control strength, $T > 0$ denotes the control period and $0 < \delta < 1$ is called the rate of control time.

The main aim of this paper is to design the suitable T, δ and k_{ij} such that systems (1.1) and (1.4) can achieve exponential synchronization under the intermittent controller (1.7). The model is derived under the following assumptions.

(H1) There exist positive constants $L_i, L_i^*, M_i, M_i^*, N_i$, and N_i^* such that

$$|f_i(\hat{v}_i) - f_i(\check{v}_i)| \le L_i|\hat{v}_i - \check{v}_i|, \quad |f_i(\hat{v}_i)| \le L_i^*,$$

$$|g_i(\hat{v}_i) - g_i(\check{v}_i)| \le M_i|\hat{v}_i - \check{v}_i|, \quad |g_i(\hat{v}_i)| \le M_i^*,$$

$$|h_i(\hat{v}_i) - h_i(\check{v}_i)| \le N_i|\hat{v}_i - \check{v}_i|, \quad |h_i(\hat{v}_i)| \le N_i^*$$

for $\hat{v}_i, \check{v}_i \in \mathbb{R}, i \in \ell$.

(H2) There exist positive constants $\bar{\alpha}_i$ and α_i^* such that

$$|\alpha_i(\hat{v}_i) - \alpha_i(\check{v}_i)| \le \bar{\alpha}_i|\hat{v}_i - \check{v}_i|, \quad 0 \le \alpha_i(\hat{v}_i) \le \alpha_i^*$$

for $\hat{v}_i, \check{v}_i \in \mathbb{R}, i \in \ell$.

(H3) There exist positive constants γ_i such that

$$\frac{\alpha_i(\hat{v}_i)\beta_i(\hat{v}_i) - \alpha_i(\check{v}_i)\beta_i(\check{v}_i)}{\hat{v}_i - \check{v}_i} \ge \gamma_i$$

for $\hat{v}_i, \check{v}_i \in \mathbb{R}$, and $\hat{v}_i \neq \check{v}_i, i \in \ell$.

(H4) There exist positive constants η_{ij} such that

$$|\sigma_{ij}(\tilde{v}_1, \hat{v}_1, \check{v}_1) - \sigma_{ij}(\tilde{v}_2, \hat{v}_2, \check{v}_2)|^2 \le \eta_{ij}\left(|\tilde{v}_1 - \tilde{v}_2|^2 + |\hat{v}_1 - \hat{v}_2|^2 + |\check{v}_1 - \check{v}_2|^2\right)$$

for $\tilde{v}_1, \tilde{v}_2, \hat{v}_1, \hat{v}_2, \check{v}_1, \check{v}_2 \in \mathbb{R}$, and $\sigma_{ij}(0,0,0) = 0, i, j \in \ell$.

(H5) There exist positive constants ρ' and ρ'' such that $\dot{\tau}_{ij}(t) \le \rho' < 1$ or $\dot{\tau}_{ij}(t) \ge \rho'' > 1$ for $t,\ i, j \in \ell$.

(H6) There exist positive constants ρ' and ρ'' such that $\dot{\tau}_{ij}^*(t) \le \rho' < 1$ or $\dot{\tau}_{ij}(t) \ge \rho'' > 1$ for all $t,\ i, j \in \ell$.

The paper is organized as follows. In the next section, we introduce some definitions and state several lemmas which will be essential to our proofs. In Section 3, by constructing a suitable Lyapunov functional, some criteria are obtained to ensure the exponential synchronization of Cohen-Grossberg neural networks with stochastic perturbation and reaction-diffusion terms under the periodically intermittent control in terms of $p-$norm. Numerical simulations are carried out in Section 4 to illustrate the feasibility of the main theoretical results. A brief conclusion is given in Section 5.

2. Preliminaries

In this section, we introduce some definitions and lemmas which will be useful in next section.

For any $u(t,x) = (u_1(t,x), u_2(t,x), \ldots, u_n(t,x))^T \in \mathbb{R}^n$, define

$$\|u(t,x)\|_p = \left(\int_\Omega \sum_{i=1}^n |u_i(t,x)|^p \mathrm{d}x \right)^{\frac{1}{p}}.$$

Definition 1.1. The noise-perturbed response system (1.4) and the master system (1.1) can be exponentially synchronized under the intermittent controller (1.7) based on p-norm, if there exist constants $\mu > 0$ and $M \geq 1$ such that

$$\mathbf{E}\left\{\|v(t,x) - u(t,x)\|_p\right\} \leq M\mathbf{E}\left\{\|\psi - \phi\|_p\right\} e^{-\mu t}, \quad (t,x) \in [0,+\infty) \times \Omega,$$

where $u(t,x)$ and $v(t,x)$ are two solutions of systems (1.1) and (1.4) with differential initial functions $\phi, \varphi \in \mathscr{C}$, respectively, and $\mathbf{E}\{\cdot\}$ is the mathematical expectation operator with respect to the given probability measure $\mathscr{P}$.

Lemma 1.1. [33] *Let $p \geq 2$ be a positive integer, $m_k (k \in \hbar)$ a positive integer, X a cube $|x_k| \leq m_k$, and let $h(x)$ be a real-valued function belonging to $\mathscr{C}^1(\Omega)$ which vanish on the boundary $\partial\Omega$ of Ω, i.e., $h(x)|_{\partial\Omega} = 0$. Then*

$$\int_\Omega |h(x)|^p \mathrm{d}x \leq \frac{p^2 m_k^2}{4} \int_\Omega |h(x)|^{p-2} \left| \frac{\partial h}{\partial x_k} \right|^2 \mathrm{d}x.$$

Lemma 1.2.[34] *Let $f(x), g(x) : [a,b] \to \mathbb{R}$ be continuous functions. Suppose that positive constants p and q satisfy*

$$\frac{1}{p} + \frac{1}{q} = 1.$$

Then we have

$$\int_a^b |f(x)g(x)| \mathrm{d}x \leq \left[\int_a^b |f(x)|^p \mathrm{d}x \right]^{\frac{1}{p}} \left[\int_a^b |g(x)|^q \mathrm{d}x \right]^{\frac{1}{q}}.$$

3. Exponential synchronization criterion

In this section, the exponential synchronization criterion of the master system (1.1) and the response system (1.4) is obtained by designing the suitable T,δ and k_{ij}. For convenience, the following denotations are introduced.

Denote

$$
\begin{aligned}
q_{ij} =&\frac{p-1}{2\alpha|1-\rho|}\left(\eta_{ji}^{p\varepsilon^{**}_{(p-1)ji}}+\eta_{ji}^{p\varepsilon^{**}_{pji}}\right),\\
q^*_{ij} =&q_{ij}-\alpha\mathrm{sgn}(1-\rho)q_{ij},\\
q^{**}_{ij} =&(\tau^*)^{p-1}\alpha^*_i|d_{ji}|^{p\xi^{**}_{pji}}N_i^{p\zeta^{**}_{pji}},\\
p_{ij} =&\frac{1}{\alpha|1-\rho|}\left[\alpha^*_i|b_{ji}|^{p\xi^*_{pji}}M_i^{p\zeta^*_{pji}}+\frac{p-1}{2}\left(\eta_{ji}^{p\varepsilon^*_{(p-1)ji}}+\eta_{ji}^{p\varepsilon^*_{pji}}\right)\right],\\
p^*_{ij} =&p_{ij}-\alpha\mathrm{sgn}(1-\rho)p_{ij},\\
w_i =&-pk_{ii}+\sum_{j=1,j\neq i}^{n}\sum_{l=1}^{p-1}|k_{ij}|^{p\varpi_{lij}}+\sum_{j=1,j\neq i}^{n}|k_{ji}|^{p\varpi_{pji}},\\
\lambda_i =&\sum_{k=1}^{l^*}\frac{4(p-1)D_{ik}}{pm_k^2}+p\left[\gamma_i-\alpha^*_i|a_{ii}|L_i-\bar{\alpha}_i\sum_{j=1}^{n}\left[|a_{ij}|L^*_j+|b_{ij}|M^*_j+|d_{ij}|N^*_j\tau^*+|J_i|\right]-\frac{p-1}{2}\eta_{ii}\right]\\
&-\alpha^*_i\sum_{j=1,j\neq i}^{n}\sum_{l=1}^{p-1}|a_{ij}|^{p\xi_{lij}}L_j^{p\zeta_{lij}}-\alpha^*_i\sum_{j=1}^{n}\sum_{l=1}^{p-1}\left[|b_{ij}|^{p\xi^*_{lij}}M_j^{p\zeta^*_{lij}}+|d_{ij}|^{p\xi^{**}_{lij}}N_j^{p\zeta^{**}_{lij}}\right]\\
&-\frac{p-1}{2}\left[\sum_{j=1,j\neq i}^{n}\sum_{l=1}^{p-2}\eta_{ij}^{p\varepsilon_{lij}}+\sum_{j=1}^{n}\sum_{l=1}^{p-2}\left(\eta_{ij}^{p\varepsilon^*_{lij}}+\eta_{ij}^{p\varepsilon^{**}_{lij}}\right)\right]\\
&-\alpha^*_i\sum_{j=1,j\neq i}^{n}|a_{ji}|^{p\xi_{pji}}L_i^{p\zeta_{pji}}-\frac{p-1}{2}\sum_{j=1,j\neq i}^{n}\left(\eta_{ji}^{p\varepsilon_{(p-1)ji}}+\eta_{ji}^{p\varepsilon_{pji}}\right),
\end{aligned}
$$

where $0<\alpha<1$, $|1-\rho|=\max\{|1-\rho'|,|1-\rho''|\}$, $|1-\rho|=\max\{|1-\rho'|,|1-\rho''|\}$, ξ_{lij},ζ_{lij}, $\xi^*_{lij},\zeta^*_{lij},\xi^{**}_{lij},\zeta^{**}_{lij},\varpi_{lij},\varepsilon_{lij},\varepsilon^*_{lij}$, and ε^{**}_{lij} are nonnegative real numbers and satisfy, respectively,

$$
\begin{aligned}
&\sum_{l=1}^{p}\xi_{lij}=1,\quad \sum_{l=1}^{p}\zeta_{lij}=1,\quad \sum_{l=1}^{p}\xi^*_{lij}=1,\quad \sum_{l=1}^{p}\zeta^*_{lij}=1,\quad \sum_{l=1}^{p}\xi^{**}_{lij}=1,\\
&\sum_{l=1}^{p}\zeta^{**}_{lij}=1,\quad \sum_{l=1}^{p}\varpi_{lij}=1,\quad \sum_{l=1}^{p}\varepsilon_{lij}=1,\quad \sum_{l=1}^{p}\varepsilon^*_{lij}=1,\quad \sum_{l=1}^{p}\varepsilon^{**}_{lij}=1.
\end{aligned}
$$

Consider the function

$$F_i(\varepsilon_i) = \lambda_i - w_i - \varepsilon_i - \sum_{j=1}^{n} \left(e^{\varepsilon_i \tau} p_{ij} + e^{\varepsilon_i \tau^*} q_{ij} + e^{\varepsilon_i \tau^*} q_{ij}^{**} \tau^* \right),$$

where $\varepsilon_i \geq 0, i \in \ell$.

If the following assumption holds:

(H7) $\lambda_i - w_i - \sum_{j=1}^{n} \left(p_{ij} + q_{ij} + q_{ij}^{**} \tau^* \right) > 0, \quad i \in \ell,$

then $F_i(0) > 0$, and $F_i(\varepsilon_i) \to -\infty$ as $\varepsilon_i \to +\infty$. Noting that $F_i(\varepsilon_i)$ is continuous on $[0, +\infty)$ and $F_i'(\varepsilon_i) < 0$, using the zero point theorem, it follows that there exists a unique positive number $\bar{\varepsilon}_i$ such that $F_i(\bar{\varepsilon}_i) = 0$ and $F_i(\varepsilon_i) > 0$ for $\varepsilon_i \in (0, \bar{\varepsilon}_i)$.

Denote $\bar{\varepsilon} = \min_{i \in \ell} \{\bar{\varepsilon}_i\}$, then

$$F_i(\bar{\varepsilon}) = \lambda_i - w_i - \bar{\varepsilon} - \sum_{j=1}^{n} \left(e^{\bar{\varepsilon} \tau} p_{ij} + e^{\bar{\varepsilon} \tau^*} q_{ij} + e^{\bar{\varepsilon} \tau^*} q_{ij}^{**} \tau^* \right) \geq 0, \quad i \in \ell. \tag{3.1}$$

Theorem 3.1. *Assume* (H1) – (H7) *hold. If the following condition is also satisfied:*
(H8) $\bar{\varepsilon} - (1 - \delta) w > 0$, *where* $w = \max_{i \in \ell} \{|w_i|\}$,
then the noise-perturbed response system (1.4) and the master system (1.1) can be exponentially synchronized under the intermittent controller (1.7) based on $p-$*norm.*

Proof. Subtracting (1.1) from (1.4), we obtain the error system

$$\begin{aligned} \mathrm{d}e_i(t,x) = & \Bigg\{ \sum_{k=1}^{l^*} \frac{\partial}{\partial x_k} \left(D_{ik} \frac{\partial e_i(t,x)}{\partial x_k} \right) - \Big[\alpha_i(v_i(t,x)) \beta_i(v_i(t,x)) - \alpha_i(u_i(t,x)) \beta_i(u_i(t,x)) \Big] \\ & + \alpha_i(v_i(t,x)) \sum_{j=1}^{n} \left[a_{ij} f_j^*(e_j(t,x)) + b_{ij} g_j^*(e_j(t - \tau_{ij}(t), x) + d_{ij} \int_{t - \tau_{ij}^*(t)}^{t} h_j^*(e_j(s,x)) \mathrm{d}s \right] \\ & + \alpha_i^*(e_i(t,x)) \sum_{j=1}^{n} \Bigg[a_{ij} f_j(u_j(t,x)) + b_{ij} g_j(u_j(t - \tau_{ij}(t), x) + d_{ij} \int_{t - \tau_{ij}^*(t)}^{t} h_j(u_j(s,x)) \mathrm{d}s \\ & - J_i \Bigg] + \sum_{j=1}^{n} k_{ij} e_j(t,x) \Bigg\} \mathrm{d}t + \sum_{j=1}^{n} \sigma_{ij}(e_j(t,x), e_j(t - \tau_{ij}(t), x), e_j(t - \tau_{ij}^*(t), x)) \mathrm{d}\omega_j(t), \\ & (t,x) \in [mT, mT + \delta T) \times \Omega, \end{aligned} \tag{3.2}$$

$$
\begin{aligned}
\mathrm{d}e_i(t,x) =& \Bigg\{ \sum_{k=1}^{l^*} \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial e_i(t,x)}{\partial x_k}\right) - \Big[\alpha_i(v_i(t,x))\beta_i(v_i(t,x)) - \alpha_i(u_i(t,x))\beta_i(u_i(t,x))\Big] \\
&+ \alpha_i(v_i(t,x)) \sum_{j=1}^{n} \left[a_{ij} f_j^*(e_j(t,x)) + b_{ij} g_j^*(e_j(t-\tau_{ij}(t),x) + d_{ij}\int_{t-\tau_{ij}^*(t)}^{t} h_j^*(e_j(s,x))\mathrm{d}s\right] \\
&+ \alpha_i^*(e_i(t,x)) \sum_{j=1}^{n} \Bigg[a_{ij} f_j(u_j(t,x)) + b_{ij} g_j(u_j(t-\tau_{ij}(t),x) + d_{ij}\int_{t-\tau_{ij}^*(t)}^{t} h_j(u_j(s,x))\mathrm{d}s \\
&- J_i\Bigg]\Bigg\}\mathrm{d}t + \sum_{j=1}^{n} \sigma_{ij}(e_j(t,x), e_j(t-\tau_{ij}(t),x), e_j(t-\tau_{ij}^*(t),x))\mathrm{d}\omega_j(t), \\
&\qquad (t,x) \in [mT+\delta T, (m+1)T) \times \Omega,
\end{aligned} \tag{3.3}
$$

where

$$
\begin{aligned}
\alpha_i^*(e_i(\cdot,x)) &= \alpha_i(v_i(\cdot,x)) - \alpha_i(u_i(\cdot,x)), \\
f_j^*(e_j(\cdot,x)) &= f_j(v_j(\cdot,x)) - f_j(u_j(\cdot,x)), \\
g_j^*(e_j(\cdot,x)) &= g_j(v_j(\cdot,x)) - g_j(u_j(\cdot,x)), \\
h_j^*(e_j(\cdot,x)) &= h_j(v_j(\cdot,x)) - h_j(u_j(\cdot,x)).
\end{aligned}
$$

Define

$$
\begin{aligned}
V(t,x) = \int_\Omega \sum_{i=1}^{n} \Bigg[& V_i(t,x) + e^{\bar{\varepsilon}\tau} \sum_{j=1}^{n} p_{ij} \int_{t-\tau_{ij}(t)}^{t} V_i(s,x)\mathrm{d}s + e^{\bar{\varepsilon}\tau} \sum_{j=1}^{n} p_{ij}^* \int_{t-\tau}^{t-\tau_{ij}(t)} V_i(s,x)\mathrm{d}s \\
&+ e^{\bar{\varepsilon}\tau^*} \sum_{j=1}^{n} q_{ij} \int_{t-\tau_{ij}^*(t)}^{t} V_i(s,x)\mathrm{d}s + e^{\bar{\varepsilon}\tau^*} \sum_{j=1}^{n} q_{ij}^* \int_{t-\tau^*}^{t-\tau_{ij}^*(t)} V_i(s,x)\mathrm{d}s \\
&+ e^{\bar{\varepsilon}\tau^*} \sum_{j=1}^{n} q_{ij}^{**} \int_{-\tau^*}^{0} \int_{t+s}^{t} V_i(\eta,x)\mathrm{d}\eta\mathrm{d}s\Bigg]\mathrm{d}x,
\end{aligned} \tag{3.4}
$$

where $V_i(t,x) = e^{\bar{\varepsilon}t}|e_i(t,x)|^p$.

For $(t,x) \in [mT, mT+\delta T) \times \Omega$, calculating the Dini right-upper derivative of $V(t,x)$ along solutions of system (3.2), it follows that

$$
\begin{aligned}
D^{+}\mathbf{E}\{V(t,x)\} \leq & \mathbf{E}\Bigg\{ \int_{\Omega} \sum_{i=1}^{n} \Bigg\{ \bar{\varepsilon} V_i(t,x) + e^{\bar{\varepsilon}\tau} \sum_{j=1}^{n} p_{ij} \Big[V_i(t,x) - (1-\dot{\tau}_{ij}(t)) V_i(t-\tau_{ij}(t),x) \Big] \\
& + e^{\bar{\varepsilon}\tau} \sum_{j=1}^{n} p_{ij}^{*} \Big[(1-\dot{\tau}_{ij}(t)) V_i(t-\tau_{ij}(t),x) - V_i(t-\tau,x) \Big] \\
& + e^{\bar{\varepsilon}\tau^{*}} \sum_{j=1}^{n} q_{ij} \Big[V_i(t,x) - (1-\dot{\tau}_{ij}^{*}(t)) V_i(t-\tau_{ij}^{*}(t),x) \Big] \\
& + e^{\bar{\varepsilon}\tau^{*}} \sum_{j=1}^{n} q_{ij}^{*} \Big[(1-\dot{\tau}_{ij}^{*}(t)) V_i(t-\tau_{ij}^{*}(t),x) - V_i(t-\tau^{*},x) \Big] \\
& + e^{\bar{\varepsilon}\tau^{*}} \sum_{j=1}^{n} q_{ij}^{**} \int_{-\tau^{*}}^{0} \Big[V_i(t,x) - V_i(t+s,x) \Big] \mathrm{d}s \\
& + p e^{\bar{\varepsilon}t} |e_i(t,x)|^{p-1} \Bigg[\sum_{k=1}^{l^{*}} \frac{\partial}{\partial x_k} \left(D_{ik} \frac{\partial |e_i(t,x)|}{\partial x_k} \right) - k_{ii} |e_i(t,x)| \\
& + \sum_{j=1, j\neq i}^{n} |k_{ij}| |e_j(t,x)| - \Big| \alpha_i(v_i(t,x)) \beta_i(v_i(t,x)) - \alpha_i(u_i(t,x)) \beta_i(u_i(t,x)) \Big| \\
& + |\alpha_i(v_i(t,x))| \sum_{j=1}^{n} \Bigg[|a_{ij}| |f_j^{*}(e_j(t,x))| + |b_{ij}| |g_j^{*}(e_j(t-\tau_{ij}(t),x))| \\
& + |d_{ij}| \int_{t-\tau_{ij}^{*}(t)}^{t} |h_j^{*}(e_j(s,x))| \mathrm{d}s \Bigg] \\
& + |\alpha_i^{*}(e_i(t,x))| \sum_{j=1}^{n} \Bigg[|a_{ij}| |f_j(u_j(t,x))| + |b_{ij}| |g_j(u_j(t-\tau_{ij}(t),x))| \\
& + |d_{ij}| \int_{t-\tau_{ij}^{*}(t)}^{t} |h_j^{*}(u_j(s,x))| \mathrm{d}s + |J_i| \Bigg] \Bigg] \\
& + \frac{p(p-1)}{2} e^{\bar{\varepsilon}t} |e_i(t,x)|^{p-2} \sum_{j=1}^{n} \sigma_{ij}^{2}(e_j(t,x), e_j(t-\tau_{ij}(t),x), e_j(t-\tau_{ij}^{*}(t),x)) \Bigg\} \mathrm{d}x \Bigg\}.
\end{aligned}
\tag{3.5}
$$

If (H1) – (H6) hold, it is easy to show that

$$
\begin{aligned}
D^{+}\mathbf{E}\{V(t,x)\} \leq & \mathbf{E}\Bigg\{ \int_{\Omega} \sum_{i=1}^{n} \Bigg\{ \bar{\varepsilon} V_i(t,x) + e^{\bar{\varepsilon}\tau} \sum_{j=1}^{n} p_{ij} \Big[V_i(t,x) - (1-\dot{\tau}_{ij}(t)) V_i(t-\tau_{ij}(t),x) \Big] \\
& + e^{\bar{\varepsilon}\tau} \sum_{j=1}^{n} p_{ij}^{*} \Big[(1-\dot{\tau}_{ij}(t)) V_i(t-\tau_{ij}(t),x) - V_i(t-\tau,x) \Big] \\
& + e^{\bar{\varepsilon}\tau^{*}} \sum_{j=1}^{n} q_{ij} \Big[V_i(t,x) - (1-\dot{\tau}_{ij}^{*}(t)) V_i(t-\tau_{ij}^{*}(t),x) \Big]
\end{aligned}
$$

$$
\begin{aligned}
&+e^{\bar{\varepsilon}\tau^*}\sum_{j=1}^{n} q_{ij}^{*}\Big[(1-\dot{\tau}_{ij}^{*}(t))V_i(t-\tau_{ij}^{*}(t),x)-V_i(t-\tau^*,x)\Big] \\
&+e^{\bar{\varepsilon}\tau^*}\sum_{j=1}^{n} q_{ij}^{**}\tau^* V_i(t,x)-e^{\bar{\varepsilon}\tau^*}\sum_{j=1}^{n} q_{ij}^{**}\int_{t-\tau^*}^{t} V_i(s,x)\mathrm{d}s \\
&+pe^{\bar{\varepsilon}t}|e_i(t,x)|^{p-1}\left[\sum_{k=1}^{l^*}\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial|e_i(t,x)|}{\partial x_k}\right)-\gamma_i|e_i(t,x)|\right. \\
&-k_{ii}|e_i(t,x)|+\sum_{j=1,j\neq i}^{n}|k_{ij}||e_j(t,x)|+\alpha_i^*\sum_{j=1}^{n}\Big[|a_{ij}|L_j|e_j(t,x)| \\
&+|b_{ij}|M_j|e_j(t-\tau_{ij}(t),x)|+|d_{ij}|\int_{t-\tau_{ij}^*(t)}^{t}N_j|e_j(s,x)|\mathrm{d}s\Big] \\
&+\bar{\alpha}_i|e_i(t,x)|\sum_{j=1}^{n}\Big[|a_{ij}|L_j^*+|b_{ij}|M_j^*+|d_{ij}|N_j^*\tau^*+|J_i|\Big]\Big] \\
&+\frac{p(p-1)}{2}e^{\bar{\varepsilon}t}|e_i(t,x)|^{p-2}\sum_{j=1}^{n}\eta_{ij}\Big[|e_j(t,x)|^2+|e_j(t-\tau_{ij}(t))|^2 \\
&+|e_j(t-\tau_{ij}^*(t))|^2\Big]\Big\}\mathrm{d}x\Big\}.
\end{aligned}
\tag{3.6}
$$

By Lemma 2.2, we obtain

$$
\begin{aligned}
\int_{t-\tau^*}^{t} V_i(s,x)\mathrm{d}s &\geq \int_{t-\tau_{ij}^*(t)}^{t} V_i(s,x)\mathrm{d}s \\
&\geq e^{\bar{\varepsilon}(t-\tau_{ij}^*(t))}\int_{t-\tau_{ij}^*(t)}^{t}|e_i(s,x)|^p\mathrm{d}s \\
&\geq e^{\bar{\varepsilon}t}e^{-\bar{\varepsilon}\tau^\star}\frac{\left(\int_{t-\tau_{ij}^*(t)}^{t}|e_i(s,x)|\mathrm{d}s\right)^p}{\left(\tau_{ij}^*(t)\right)^{\frac{p}{q}}} \\
&\geq e^{\bar{\varepsilon}t}e^{-\bar{\varepsilon}\tau^\star}(\tau^*)^{1-p}\left(\int_{t-\tau_{ij}^*(t)}^{t}|e_i(s,x)|\mathrm{d}s\right)^p.
\end{aligned}
\tag{3.7}
$$

It follows from the boundary conditions (1.2), (1.5) and Lemma 2.1 that

$$
p\int_{\Omega}|e_i(t,x)|^{p-1}\sum_{k=1}^{l^*}\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial|e_i(t,x)|}{\partial x_k}\right)\mathrm{d}x \leq -\sum_{k=1}^{l^*}\frac{4(p-1)D_{ik}}{pm_k^2}\int_{\Omega}|e_i(t,x)|^p\mathrm{d}x. \tag{3.8}
$$

Noting that

$$
a_1^p+a_2^p+\cdots+a_p^p \geq pa_1a_2\cdots a_p, \quad a_i \geq 0, \quad i=1,2,\ldots,p,
$$

we have

$$
\begin{aligned}
p|e_i(t,x)|^{p-1}\sum_{j=1,j\neq i}^{n}|a_{ij}|L_j|e_j(t,x)| &= \sum_{j=1,j\neq i}^{n} p\left[\prod_{l=1}^{p-1}|a_{ij}|^{\xi_{lij}}L_j^{\zeta_{lij}}|e_i(t,x)|\right]|a_{ij}|^{\xi_{pij}}L_j^{\zeta_{pij}}|e_j(t,x)| \\
&\leq \sum_{j=1,j\neq i}^{n}\sum_{l=1}^{p-1}|a_{ij}|^{p\xi_{lij}}L_j^{p\zeta_{lij}}|e_i(t,x)|^p \\
&\quad + \sum_{j=1,j\neq i}^{n}|a_{ij}|^{p\xi_{pij}}L_j^{p\zeta_{pij}}|e_j(t,x)|^p.
\end{aligned}
\tag{3.9}
$$

Similarly,

$$
\begin{aligned}
p|e_i(t,x)|^{p-1}\sum_{j=1,j\neq i}^{n}|k_{ij}||e_j(t,x)| &\leq \sum_{j=1,j\neq i}^{n}\sum_{l=1}^{p-1}|k_{ij}|^{p\varpi_{lij}}|e_i(t,x)|^p + \sum_{j=1,j\neq i}^{n}|k_{ij}|^{p\varpi_{pij}}|e_j(t,x)|^p, \\
p|e_i(t,x)|^{p-2}\sum_{j=1,j\neq i}^{n}\eta_{ij}|e_j(t,x)|^2 &\leq \sum_{j=1,j\neq i}^{n}\sum_{l=1}^{p-2}\eta_{ij}^{p\varepsilon_{lij}}|e_i(t,x)|^p \\
&\quad + \sum_{j=1,j\neq i}^{n}\left(\eta_{ij}^{p\varepsilon_{(p-1)ij}}+\eta_{ij}^{p\varepsilon_{pij}}\right)|e_j(t,x)|^p, \\
p|e_i(t,x)|^{p-2}\sum_{j=1}^{n}\eta_{ij}|e_j(t-\tau_{ij}(t),x)|^2 &\leq \sum_{j=1}^{n}\sum_{l=1}^{p-2}\eta_{ij}^{p\varepsilon^*_{lij}}|e_i(t,x)|^p \\
&\quad + \sum_{j=1}^{n}\left(\eta_{ij}^{p\varepsilon^*_{(p-1)ij}}+\eta_{ij}^{p\varepsilon^*_{pij}}\right)|e_j(t-\tau_{ij}(t),x)|^p, \\
p|e_i(t,x)|^{p-2}\sum_{j=1}^{n}\eta_{ij}|e_j(t-\tau^*_{ij}(t),x)|^2 &\leq \sum_{j=1}^{n}\sum_{l=1}^{p-2}\eta_{ij}^{p\varepsilon^{**}_{lij}}|e_i(t,x)|^p \\
&\quad + \sum_{j=1}^{n}\left(\eta_{ij}^{p\varepsilon^{**}_{(p-1)ij}}+\eta_{ij}^{p\varepsilon^{**}_{pij}}\right)|e_j(t-\tau^*_{ij}(t),x)|^p, \\
p|e_i(t,x)|^{p-1}\sum_{j=1}^{n}|b_{ij}|M_j|(e_j(t-\tau_{ij}(t),x)| &\leq \sum_{j=1}^{n}\sum_{l=1}^{p-1}|b_{ij}|^{p\xi^*_{lij}}M_j^{p\zeta^*_{lij}}|e_i(t,x)|^p \\
&\quad + \sum_{j=1}^{n}|b_{ij}|^{p\xi^*_{pij}}M_j^{p\zeta^*_{pij}}|e_j(t-\tau_{ij}(t),x)|^p, \\
p|e_i(t,x)|^{p-1}\sum_{j=1}^{n}|d_{ij}|\int_{t-\tau^*_{ij}(t)}^{t}N_j|e_j(s,x)|\mathrm{d}s &\leq \sum_{j=1}^{n}\sum_{l=1}^{p-1}|d_{ij}|^{p\xi^{**}_{lij}}N_j^{p\zeta^{**}_{lij}}|e_i(t,x)|^p \\
&\quad + \sum_{j=1}^{n}|d_{ij}|^{p\xi^{**}_{pij}}N_j^{p\zeta^{**}_{pij}}\left(\int_{t-\tau^*_{ij}(t)}^{t}|e_j(s,x)|\mathrm{d}s\right)^p.
\end{aligned}
\tag{3.10}
$$

Substituting (3.7)-(3.10) into (3.6), we get

$$
\begin{aligned}
D^+\mathbf{E}\{V(t,x)\} \leq & \mathbf{E}\bigg\{\int_\Omega \sum_{i=1}^{n}\bigg\{\bigg[\bar{\varepsilon} - \sum_{k=1}^{l^*}\frac{4(p-1)D_{ik}}{pm_k^2} - p\gamma_i - pk_{ii} + \sum_{j=1,j\neq i}^{n}\sum_{l=1}^{p-1}|k_{ij}|^{p\varpi_{lij}} \\
& + \alpha_i^* p|a_{ii}|L_i + \alpha_i^*\sum_{j=1,j\neq i}^{n}\sum_{l=1}^{p-1}|a_{ij}|^{p\xi_{lij}}L_j^{p\zeta_{lij}} + \alpha_i^*\sum_{j=1}^{n}\sum_{l=1}^{p-1}|b_{ij}|^{p\xi^*_{lij}}M_j^{p\zeta^*_{lij}} \\
& + \alpha_i^*\sum_{j=1}^{n}\sum_{l=1}^{p-1}|d_{ij}|^{p\xi^{**}_{lij}}N_j^{p\zeta^{**}_{lij}} + \bar{\alpha}_i\sum_{j=1}^{n}\bigg[|a_{ij}|L_j^*p + |b_{ij}|M_j^*p + |d_{ij}|N_j^*\tau^*p + |J_i|p\bigg] \\
& + \frac{p-1}{2}\bigg[p\eta_{ii} + \sum_{j=1,j\neq i}^{n}\sum_{l=1}^{p-2}\eta_{ij}^{p\varepsilon_{lij}} + \sum_{j=1}^{n}\sum_{l=1}^{p-2}\eta_{ij}^{p\varepsilon^*_{lij}} + \sum_{j=1}^{n}\sum_{l=1}^{p-2}\eta_{ij}^{p\varepsilon^{**}_{lij}}\bigg] \\
& + \sum_{j=1,j\neq i}^{n}|k_{ji}|^{p\varpi_{pji}} + \frac{p-1}{2}\sum_{j=1,j\neq i}^{n}\left(\eta_{ji}^{p\varepsilon_{(p-1)ji}} + \eta_{ji}^{p\varepsilon_{pji}}\right) \\
& + \alpha_i^*\sum_{j=1,j\neq i}^{n}|a_{ji}|^{p\xi_{pji}}L_i^{p\zeta_{pji}} + e^{\bar{\varepsilon}\tau}\sum_{j=1}^{n}p_{ij} + e^{\bar{\varepsilon}\tau^*}\sum_{j=1}^{n}q_{ij} + e^{\bar{\varepsilon}\tau^*}\sum_{j=1}^{n}q_{ij}^{**}\tau^*\bigg]V_i(t,x) \\
& + e^{\bar{\varepsilon}\tau}\sum_{j=1}^{n}\bigg[-\alpha|1-\rho|p_{ij} + \alpha_i^*|b_{ji}|^{p\xi^*_{pji}}M_i^{p\zeta^*_{pji}} \\
& + \frac{p-1}{2}\left(\eta_{ji}^{p\varepsilon^*_{(p-1)ji}} + \eta_{ji}^{p\varepsilon^*_{pji}}\right)\bigg]V_i(t-\tau_{ij}(t),x) \\
& + e^{\bar{\varepsilon}\tau^*}\sum_{j=1}^{n}\bigg[-\alpha|1-\rho|q_{ij} + \frac{p-1}{2}\left(\eta_{ji}^{p\varepsilon^{**}_{(p-1)ji}} + \eta_{ji}^{p\varepsilon^{**}_{pji}}\right)\bigg]V_i(t-\tau^*_{ij}(t),x) \\
& + e^{\bar{\varepsilon}t}\sum_{j=1}^{n}\bigg[\alpha_i^*|d_{ji}|^{p\xi^{**}_{pji}}N_i^{p\zeta^{**}_{pji}} - (\tau^*)^{1-p}q^{**}_{ij}\bigg]\left(\int_{t-\tau^*_{ij}(t)}^{t}|e_i(s,x)|\mathrm{d}s\right)^p\bigg\}\mathrm{d}x\bigg\} \\
= & \mathbf{E}\bigg\{\int_\Omega \sum_{i=1}^{n}\bigg\{\bigg[\bar{\varepsilon} - \sum_{k=1}^{l^*}\frac{4(p-1)D_{ik}}{pm_k^2} - p\gamma_i - pk_{ii} + \sum_{j=1,j\neq i}^{n}\sum_{l=1}^{p-1}|k_{ij}|^{p\varpi_{lij}} \\
& + \alpha_i^* p|a_{ii}|L_i + \alpha_i^*\sum_{j=1,j\neq i}^{n}\sum_{l=1}^{p-1}|a_{ij}|^{p\xi_{lij}}L_j^{p\zeta_{lij}} + \alpha_i^*\sum_{j=1}^{n}\sum_{l=1}^{p-1}|b_{ij}|^{p\xi^*_{lij}}M_j^{p\zeta^*_{lij}} \\
& + \alpha_i^*\sum_{j=1}^{n}\sum_{l=1}^{p-1}|d_{ij}|^{p\xi^{**}_{lij}}N_j^{p\zeta^{**}_{lij}} + \bar{\alpha}_i\sum_{j=1}^{n}\bigg[|a_{ij}|L_j^*p + |b_{ij}|M_j^*p + |d_{ij}|N_j^*\tau^*p + |J_i|p\bigg] \\
& + \frac{p-1}{2}\bigg[p\eta_{ii} + \sum_{j=1,j\neq i}^{n}\sum_{l=1}^{p-2}\eta_{ij}^{p\varepsilon_{lij}} + \sum_{j=1}^{n}\sum_{l=1}^{p-2}\eta_{ij}^{p\varepsilon^*_{lij}} + \sum_{j=1}^{n}\sum_{l=1}^{p-2}\eta_{ij}^{p\varepsilon^{**}_{lij}}\bigg] \\
& + \alpha_i^*\sum_{j=1,j\neq i}^{n}|a_{ji}|^{p\xi_{pji}}L_i^{p\zeta_{pji}} + \frac{p-1}{2}\sum_{j=1,j\neq i}^{n}\left(\eta_{ji}^{p\varepsilon_{(p-1)ji}} + \eta_{ji}^{p\varepsilon_{pji}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{j=1,j\neq i}^{n}|k_{ji}|^{p\varpi_{pji}}+e^{\bar{\varepsilon}\tau}\sum_{j=1}^{n}p_{ij}+e^{\bar{\varepsilon}\tau^*}\sum_{j=1}^{n}q_{ij}+e^{\bar{\varepsilon}\tau^*}\sum_{j=1}^{n}q_{ij}^{**}\tau^*\Bigg]V_i(t,x)\Bigg\}\mathrm{d}x\Bigg\}\\
=&-\mathbf{E}\Bigg\{\int_\Omega\sum_{i=1}^{n}\Bigg[\lambda_i-w_i-\bar{\varepsilon}-\sum_{j=1}^{n}(e^{\bar{\varepsilon}\tau}p_{ij}+e^{\bar{\varepsilon}\tau^*}q_{ij}+e^{\bar{\varepsilon}\tau^*}q_{ij}^{**}\tau^*)\Bigg]V_i(t,x)\mathrm{d}x\Bigg\}\\
\leq&0,
\end{aligned}
$$

which implies that

$$\mathbf{E}\{V(t,x)\}\leq\mathbf{E}\{V(mT,x)\},\quad (t,x)\in[mT,mT+\delta T)\times\Omega. \tag{3.11}$$

Similarly, for $(t,x)\in[mT+\delta T,(m+1)T)\times\Omega$, we can derive

$$
\begin{aligned}
D^+\mathbf{E}\{V(t,x)\}\leq&\mathbf{E}\Bigg\{-\int_\Omega\sum_{i=1}^{n}\Bigg[\lambda_i+|w_i|-\bar{\varepsilon}-\sum_{j=1}^{n}\bigg(e^{\bar{\varepsilon}\tau}p_{ij}+e^{\bar{\varepsilon}\tau^*}q_{ij}+e^{\bar{\varepsilon}\tau^*}q_{ij}^{**}\tau^*\bigg)\Bigg]V_i(t,x)\mathrm{d}x\\
&+\int_\Omega\sum_{i=1}^{n}|w_i|V_i(t,x)\mathrm{d}x\Bigg\}\\
\leq&\mathbf{E}\Bigg\{wV(t,x)\Bigg\},
\end{aligned}
\tag{3.12}
$$

which leads to

$$\mathbf{E}\{V(t,x)\}\leq\mathbf{E}\{V(mT+\delta T,x)\exp\{w(t-mT-\delta T)\}\},\quad (t,x)\in[mT+\delta T,(m+1)T)\times\Omega. \tag{3.13}$$

Now, the following inequalities will be proved by mathematical induction:

$$
\begin{aligned}
&\mathbf{E}\{V(t,x)\}\leq\mathbf{E}\{V(0,x)\exp\{mw(1-\delta)T\}\}, && (t,x)\in[mT,mT+\delta T)\times\Omega,\\
&\mathbf{E}\{V(t,x)\}\leq\mathbf{E}\{V(0,x)\exp\{w(t-(m+1)\delta T)\}\}, && (t,x)\in[mT+\delta T,(m+1)T)\times\Omega.
\end{aligned}
\tag{3.14}
$$

(1) For $m=0$.

If $(t,x)\in[0,\delta T)\times\Omega$, it follows from (3.11) that

$$\mathbf{E}\{V(t,x)\}\leq\mathbf{E}\{V(0,x)\}.$$

If $(t,x)\in[\delta T,T)\times\Omega$, we derive from (3.13) that

$$\mathbf{E}\{V(t,x)\}\leq\mathbf{E}\{V(\delta T,x)\exp\{w(t-\delta T)\}\}\leq\mathbf{E}\{V(0,x)\exp\{w(t-\delta T)\}\}.$$

(2) Assume that (3.14) is true for all $m\leq l-1$.

(3) In the following, we will prove (3.14) is also true when $m=l$.

If $(t,x)\in[lT,lT+\delta T)\times\Omega$, we see that

$\mathbf{E}\{V(t,x)\}\le\mathbf{E}\{V(lT,x)\}\le\mathbf{E}\{V(0,x)\exp\{w(lT-l\delta T)\}\}=\mathbf{E}\{V(0,x)\exp\{lw(1-\delta)T\}\}$.

If $(t,x)\in[lT+\delta T,(l+1)T)\times\Omega$, we get

$$\begin{aligned}\mathbf{E}\{V(t,x)\}&\le\mathbf{E}\{V(lT+\delta T,x)\exp\{w(t-lT-\delta T)\}\}\\&\le\mathbf{E}\{V(0,x)\exp\{lw(1-\delta)T\}\exp\{w(t-lT-\delta T)\}\}\\&=\mathbf{E}\{V(0,x)\exp\{w(t-(l+1)\delta T)\}\}.\end{aligned}$$

Therefore, by mathematical induction, we know that (3.14) is true for any positive integer.

If $(t,x)\in[mT,mT+\delta T)\times\Omega$, then $m\le t/T$, we conclude from (3.14) that

$$\mathbf{E}\{V(t,x)\}\le\mathbf{E}\left\{V(0,x)\exp\left\{\frac{t}{T}w(1-\delta)T\right\}\right\}=\mathbf{E}\left\{V(0,x)\exp\left\{(1-\delta)wt\right\}\right\}. \tag{3.15}$$

Similarly, if $(t,x)\in[mT+\delta T,(m+1)T)\times\Omega$, then $t/T<m+1$, we derive from (3.14) that

$$\mathbf{E}\{V(t,x)\}\le\mathbf{E}\left\{V(0,x)\exp\left\{w\left(t-\frac{t}{T}\delta T\right)\right\}\right\}=\mathbf{E}\left\{V(0,x)\exp\left\{(1-\delta)wt\right\}\right\}. \tag{3.16}$$

Hence, for any $(t,x)\in[0,+\infty)\times\Omega$, we always have

$$\mathbf{E}\{V(t,x)\}\le\mathbf{E}\{V(0,x)\exp\{(1-\delta)wt\}\}=\exp\{(1-\delta)wt\}\mathbf{E}\{V(0,x)\}. \tag{3.17}$$

Note that

$$\mathbf{E}\{V(0,x)\}=\mathbf{E}\bigg\{\int_\Omega\sum_{i=1}^n\bigg[V_i(0,x)+e^{\bar{\varepsilon}\tau}\sum_{j=1}^n p_{ij}\int_{-\tau_{ij}(0)}^0 V_i(s,x)\mathrm{d}s+e^{\bar{\varepsilon}\tau}\sum_{j=1}^n p_{ij}^*\int_{-\tau}^{-\tau_{ij}(0)}V_i(s,x)\mathrm{d}s$$

$$
\begin{aligned}
&+e^{\bar{\varepsilon}\tau^*}\sum_{j=1}^{n} q_{ij}\int_{-\tau_{ij}^*(0)}^{0} V_i(s,x)\mathrm{d}s + e^{\bar{\varepsilon}\tau^*}\sum_{j=1}^{n} q_{ij}^*\int_{-\tau^*}^{-\tau_{ij}^*(0)} V_i(s,x)\mathrm{d}s \\
&+e^{\bar{\varepsilon}\tau^*}\sum_{j=1}^{n} q_{ij}^{**}\int_{-\tau^*}^{0}\int_{s}^{0} V_i(\eta,x)\mathrm{d}\eta\mathrm{d}s\Bigg]\mathrm{d}x\Bigg\} \\
\le&\mathbf{E}\Bigg\{\int_\Omega\sum_{i=1}^{n}\Bigg[|e_i(0,x)|^p + e^{\bar{\varepsilon}\tau}\sum_{j=1}^{n} p_{ij}\int_{-\tau_{ij}(0)}^{0} e^{\bar{\varepsilon}s}|e_i(s,x)|^p\mathrm{d}s \\
&+e^{\bar{\varepsilon}\tau}\sum_{j=1}^{n} p_{ij}^*\int_{-\tau}^{-\tau_{ij}(0)} e^{\bar{\varepsilon}s}|e_i(s,x)|^p\mathrm{d}s + e^{\bar{\varepsilon}\tau^*}\sum_{j=1}^{n} q_{ij}\int_{-\tau_{ij}^*(0)}^{0} e^{\bar{\varepsilon}s}|e_i(s,x)|^p\mathrm{d}s \\
&+e^{\bar{\varepsilon}\tau^*}\sum_{j=1}^{n} q_{ij}^*\int_{-\tau^*}^{-\tau_{ij}^*(0)} e^{\bar{\varepsilon}s}|e_i(s,x)|^p\mathrm{d}s + e^{\bar{\varepsilon}\tau^*}\sum_{j=1}^{n} q_{ij}^{**}\int_{-\tau^*}^{0}\int_{s}^{0} e^{\bar{\varepsilon}s}|e_i(s,x)|^p\mathrm{d}\eta\mathrm{d}s\Bigg]\mathrm{d}x\Bigg\} \\
\le&\mathbf{E}\Bigg\{\int_\Omega\sum_{i=1}^{n}\Bigg[|e_i(0,x)|^p + e^{\bar{\varepsilon}\tau}\sum_{j=1}^{n} p_{ij}\int_{-\tau_{ij}(0)}^{0} e^{\bar{\varepsilon}s}|e_i(s,x)|^p\mathrm{d}s \\
&+e^{\bar{\varepsilon}\tau}\sum_{j=1}^{n} p_{ij}^*\int_{-\tau}^{-\tau_{ij}(0)} e^{\bar{\varepsilon}s}|e_i(s,x)|^p\mathrm{d}s + e^{\bar{\varepsilon}\tau^*}\sum_{j=1}^{n} q_{ij}\int_{-\tau_{ij}^*(0)}^{0} e^{\bar{\varepsilon}s}|e_i(s,x)|^p\mathrm{d}s \\
&+e^{\bar{\varepsilon}\tau^*}\sum_{j=1}^{n} q_{ij}^*\int_{-\tau^*}^{-\tau_{ij}^*(0)} e^{\bar{\varepsilon}s}|e_i(s,x)|^p\mathrm{d}s + e^{\bar{\varepsilon}\tau^*}\sum_{j=1}^{n} q_{ij}^{**}\tau^*\int_{-\tau^*}^{0} e^{\bar{\varepsilon}s}|e_i(s,x)|^p\mathrm{d}s\Bigg]\mathrm{d}x\Bigg\} \\
\le&\mathbf{E}\Bigg\{\int_\Omega\sum_{i=1}^{n}\Bigg[|e_i(0,x)|^p + e^{\bar{\varepsilon}\tau}\max_{i\in\ell}\Bigg\{\sum_{j=1}^{n} p_{ij}\Bigg\}\sum_{j=1}^{n}\int_{-\tau_{ij}(0)}^{0} e^{\bar{\varepsilon}s}|e_i(s,x)|^p\mathrm{d}s \\
&+e^{\bar{\varepsilon}\tau}\max_{i\in\ell}\Bigg\{\sum_{j=1}^{n} p_{ij}^*\Bigg\}\sum_{j=1}^{n}\int_{-\tau}^{-\tau_{ij}(0)} e^{\bar{\varepsilon}s}|e_i(s,x)|^p\mathrm{d}s \\
&+e^{\bar{\varepsilon}\tau^*}\max_{i\in\ell}\Bigg\{\sum_{j=1}^{n} q_{ij}\Bigg\}\sum_{j=1}^{n}\int_{-\tau_{ij}^*(0)}^{0} e^{\bar{\varepsilon}s}|e_i(s,x)|^p\mathrm{d}s \\
&+e^{\bar{\varepsilon}\tau^*}\max_{i\in\ell}\Bigg\{\sum_{j=1}^{n} q_{ij}^*\Bigg\}\sum_{j=1}^{n}\int_{-\tau^*}^{-\tau_{ij}^*(0)} e^{\bar{\varepsilon}s}|e_i(s,x)|^p\mathrm{d}s \\
&+e^{\bar{\varepsilon}\tau^*}\max_{i\in\ell}\Bigg\{\sum_{j=1}^{n} q_{ij}^{**}\tau^*\Bigg\}\int_{-\tau^*}^{0} e^{\bar{\varepsilon}s}|e_i(s,x)|^p\mathrm{d}s\Bigg]\mathrm{d}x\Bigg\}.
\end{aligned}
$$

Since

$$\|\psi-\phi\|_p^p = \int_\Omega\sum_{i=1}^{n}\sup_{-\bar{\tau}\le s\le 0}|\psi(s,x)-\phi(s,x)|^p\mathrm{d}x,$$

then

$$
\begin{aligned}
\int_\Omega\sum_{i=1}^{n}\int_{-\tau_{ij}(0)}^{0} e^{\bar{\varepsilon}s}|e_i(s,x)|^p\mathrm{d}s\mathrm{d}x &\le \int_\Omega\sum_{i=1}^{n}\tau\sup_{-\bar{\tau}\le s\le 0}|e_i(s,x)|^p\mathrm{d}x = \tau\|\psi-\phi\|_p^p, \\
\int_\Omega\sum_{i=1}^{n}\int_{-\tau}^{-\tau_{ij}(0)} e^{\bar{\varepsilon}s}|e_i(s,x)|^p\mathrm{d}s\mathrm{d}x &\le \tau\int_\Omega\sum_{i=1}^{n}\sup_{-\bar{\tau}\le s\le 0}|e_i(s,x)|^p\mathrm{d}x = \tau\|\psi-\phi\|_p^p,
\end{aligned}
$$

$$\int_\Omega \sum_{i=1}^n \int_{-\tau_{ij}^*(0)}^0 e^{\bar{\varepsilon}s}|e_i(s,x)|^p \mathrm{d}s\mathrm{d}x \le \tau^* \int_\Omega \sum_{i=1}^n \sup_{-\bar{\tau}\le s\le 0} |e_i(s,x)|^p \mathrm{d}x = \tau^* \|\psi-\phi\|_p^p,$$
$$\int_\Omega \sum_{i=1}^n \int_{-\tau^*}^{-\tau_{ij}^*(0)} e^{\bar{\varepsilon}s}|e_i(s,x)|^p \mathrm{d}s\mathrm{d}x \le \tau^* \int_\Omega \sum_{i=1}^n \sup_{-\bar{\tau}\le s\le 0} |e_i(s,x)|^p \mathrm{d}x = \tau^* \|\psi-\phi\|_p^p,$$
$$\int_\Omega \sum_{i=1}^n \int_{-\tau^*}^0 e^{\bar{\varepsilon}s}|e_i(s,x)|^p \mathrm{d}s\mathrm{d}x \le \tau^* \int_\Omega \sum_{i=1}^n \sup_{-\bar{\tau}\le s\le 0} |e_i(s,x)|^p \mathrm{d}x = \tau^* \|\psi-\phi\|_p^p.$$

Therefore,

$$\mathbf{E}\Big\{V(0,x)\Big\} \le \Big[1 + n\tau e^{\bar{\varepsilon}\tau} \max_{i\in\ell} \sum_{j=1}^n (p_{ij}+p_{ij}^*) + n\tau^* e^{\bar{\varepsilon}\tau^*} \max_{i\in\ell} \sum_{j=1}^n (q_{ij}+q_{ij}^*)$$
$$+ \tau^{*2} e^{\bar{\varepsilon}\tau^*} \max_{i\in\ell} \Big\{ \sum_{j=1}^n q_{ij}^{**} \Big\}\Big] \mathbf{E}\{\|\psi-\phi\|_p^p\}.$$

Let

$$M = \Big[1 + n\tau e^{\bar{\varepsilon}\tau} \max_{i\in\ell} \sum_{j=1}^n (p_{ij}+p_{ij}^*) + n\tau^* e^{\bar{\varepsilon}\tau^*} \max_{i\in\ell} \sum_{j=1}^n (q_{ij}+q_{ij}^*)$$
$$+ \tau^{*2} e^{\bar{\varepsilon}\tau^*} \max_{i\in\ell} \Big\{ \sum_{j=1}^n q_{ij}^{**} \Big\}\Big]^{\frac{1}{p}} > 1.$$

Then

$$\mathbf{E}\{V(0,x)\} \le M^p \mathbf{E}\{\|\psi-\phi\|_p^p\}.$$

Further, from (3.17), we obtain

$$\mathbf{E}\{V(t,x)\} \le \exp\{(1-\delta)wt\} M^p \mathbf{E}\{\|\psi-\phi\|_p^p\}. \tag{3.18}$$

In addition,

$$\mathbf{E}\{V(t,x)\} \ge \mathbf{E}\Big\{ \int_\Omega \sum_{i=1}^n e^{\bar{\varepsilon}t}|e_i(t,x)|^p \mathrm{d}x \Big\} = e^{\bar{\varepsilon}t}\mathbf{E}\{\|v(t,x)-u(t,x)\|_p^p\}. \tag{3.19}$$

From (3.18) and (3.19), we have

$$\mathbf{E}\{\|v(t,x)-u(t,x)\|_p\} \le \exp\Big\{\frac{1}{p}\Big((1-\delta)w-\bar{\varepsilon}\Big)t\Big\} M \mathbf{E}\{\|\psi-\phi\|_p\}. \tag{3.20}$$

Let

$$\mu = \frac{1}{p}\Big[\bar{\varepsilon} - (1-\delta)w\Big].$$

Then

$$\mathbf{E}\{\|v(t,x)-u(t,x)\|_p\} \le M\mathbf{E}\{\|\psi-\phi\|_p\} e^{-\mu t}.$$

Hence, the noise-perturbed response system (1.4) and the master system (1.1) can be exponentially synchronized under the intermittent controller (1.7) based on $p-$norm. The proof of Theorem 3.1 is complete.

Remark 1. Hu et al.[29] investigated the exponential synchronization for the following reaction-diffusion neural networks with mixed delays in terms of p-norm based on periodically intermittent control

$$\begin{aligned}
\frac{\partial u_i(t,x)}{\partial t} =&\sum_{l=1}^{m}\frac{\partial}{\partial x_l}\left(D_{il}\frac{\partial u_i(t,x)}{\partial x_l}\right)-c_iu_i(t,x)+\sum_{j=1}^{n}h_{ij}f_j(u_j(t,x))\\
&+\sum_{j=1}^{n}a_{ij}f_j(u_j(t-\tau_j(t),x)+\sum_{j=1}^{n}b_{ij}\int_{t-\sigma_j}^{t}f_j(u_j(s,x))\mathrm{d}s+J_i,\\
\frac{\partial v_i(t,x)}{\partial t} =&\sum_{l=1}^{m}\frac{\partial}{\partial x_l}\left(D_{il}\frac{\partial v_i(t,x)}{\partial x_l}\right)-c_iv_i(t,x)+\sum_{j=1}^{n}h_{ij}f_j(v_j(t,x))\\
&+\sum_{j=1}^{n}a_{ij}f_j(v_j(t-\tau_j(t),x)+\sum_{j=1}^{n}b_{ij}\int_{t-\sigma_j}^{t}f_j(v_j(s,x))\mathrm{d}s+J_i+K_i(t).
\end{aligned}\tag{3.21}$$

Gan et al.[30] dealed with the exponential synchronization problem for the following reaction-diffusion neural networks with mixed time-varying delays and stochastic disturbance in terms of $p-$norm via periodically intermittent control

$$\begin{aligned}
\frac{\partial u_i(t,x)}{\partial t} =&\sum_{k=1}^{l^*}\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial u_i(t,x)}{\partial x_k}\right)-c_iu_i(t,x)+\sum_{j=1}^{n}a_{ij}f_j(u_j(t,x))\\
&+\sum_{j=1}^{n}b_{ij}g_j(u_j(t-\tau_{ij}(t),x)+\sum_{j=1}^{n}d_{ij}\int_{t-\tau_{ij}^*(t)}^{t}h_j(u_j(s,x))\mathrm{d}s+J_i,\\
\mathrm{d}v_i(t,x) =&\Bigg\{\sum_{k=1}^{l^*}\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial v_i(t,x)}{\partial x_k}\right)-c_iv_i(t,x)+\sum_{j=1}^{n}a_{ij}f_j(v_j(t,x))\\
&+\sum_{j=1}^{n}b_{ij}g_j(v_j(t-\tau_{ij}(t),x)+\sum_{j=1}^{n}d_{ij}\int_{t-\tau_{ij}^*(t)}^{t}h_j(v_j(s,x))\mathrm{d}s+J_i+K_i(t,x)\Bigg\}\mathrm{d}t\\
&+\sum_{j=1}^{n}\sigma_{ij}(e_j(t,x),e_j(t-\tau_{ij}(t),x))\mathrm{d}\omega_j(t),
\end{aligned}\tag{3.22}$$

It is evident that system (3.21) and system (3.22) are the special cases of system (1.1) in this paper. The corresponding exponential synchronization criteria obtained in [29,30] are included to Theorem 3.1 in this paper. From this point, our results are more general.

Remark 2. In this paper, the issue of exponential synchronization for Cohen-Grossberg neural networks with mixed time-varying delays, stochastic noise disturbance and reaction-diffusion effects was investigated. The same model was researched in [32]. However, in [32], the author obtained the exponential synchronization criteria for neural networks by assuming that discrete time-varying delays $\tau_{ij}(t)$ and distributed time-varying delays $\tau_{ij}^*(t)$ satisfy $\dot{\tau}_{ij}(t) \leq \rho < 1$ and $\dot{\tau}_{ij}^*(t) \leq \rho^* < 1$ for all t. These restrictions in [32] are removed in this paper. Therefore, the synchronization criteria derived in this paper are less conservative.

4. Numerical simulations

In this section, some numerical simulations are presented to show the feasibility of our results.

Example In system (1.1), we choose $n = 2, k = 1$. Then system (1.1) takes the form

$$
\begin{aligned}
\frac{\partial u_i(t,x)}{\partial t} = D_i \frac{\partial^2 u_i(t,x)}{\partial x^2} - \alpha_i(u_i(t,x)) \Bigg[\beta_i(u_i(t,x)) - \sum_{j=1}^{2} a_{ij} f_j(u_j(t,x)) \\
- \sum_{j=1}^{2} b_{ij} g_j(u_j(t-\tau(t),x) - \sum_{j=1}^{2} d_{ij} \int_{t-\tau^*(t)}^{t} h_j(u_j(s,x)) \mathrm{d}s \Bigg],
\end{aligned}
\tag{4.1}
$$

where $i = 1,2,\ \alpha_1(u_1(t,x)) = 0.7 + \dfrac{0.2}{1+u_1^2(t,x)}, \alpha_2(u_2(t,x)) = 1 + \dfrac{0.1}{1+u_2^2(t,x)},\ \beta_1(u_1(t,x)) = 1.4u_1(t,x), \beta_2(u_2(t,x)) = 1.6u_2(t,x),\ f_j(u_j(t,x)) = g_j(u_j(t,x)) = h_j(u_j(t,x)) = \tanh(u_j(t,x))$, $\tau(t) = \begin{cases} 1.1t, & t < \pi/2, \\ 0.55\pi + 0.1\pi\cos t & t \geq \pi/2 \end{cases}, \tau^*(t) = \begin{cases} 1.02t, & t < 0.1, \\ 0.102 + 0.01\sin(t-0.1), & t \geq 0.1 \end{cases}$. The parameters of system (4.1) are assumed that $D_1 = 0.1, D_2 = 0.1, a_{11} = 1.5, a_{12} = -0.25, a_{21} = 3.2, a_{22} = 1.9, b_{11} = -1.8, b_{12} = -1.3, b_{21} = -0.2, b_{22} = 2.2, d_{11} = 0.9, d_{12} = -0.15, d_{21} = 0.2, d_{22} = -0.2, x \in \Omega = [-5,5]$. The initial condition of the master system (4.1) is chosen as

$$
u_1(s,x) = 0.1\sin\left(\frac{x+5}{10}\pi\right), \quad u_2(s,x) = 0.2\sin\left(\frac{x+5}{10}\pi\right), \tag{4.2}
$$

where $(s,x) \in [-0.65\pi, 0] \times \Omega$. Numerical simulation illustrates that the reaction-diffusion neural network (4.1) with boundary condition (1.2) and the initial condition (4.2) exhibits a chaotic behavior (see Fig.1).

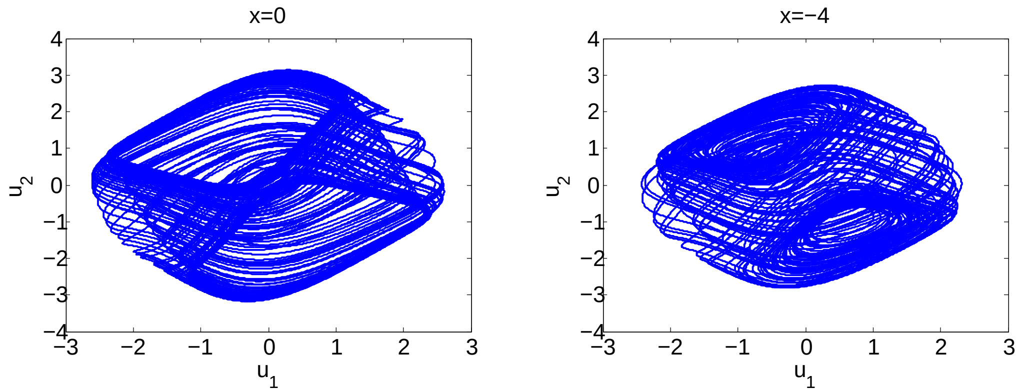

Fig.1 Chaotic behaviors of Cohen-Grossberg neural networks (4.1).

The noise-perturbed response system is described by

$$\begin{aligned} \mathrm{d}v_i(t,x) = \Big\{ & D_i \frac{\partial^2 v_i(t,x)}{\partial x^2} - \alpha_i(v_i(t,x)) \Big[\beta_i(v_i(t,x)) - \sum_{j=1}^{2} a_{ij} f_j(v_j(t,x)) \\ & - \sum_{j=1}^{2} b_{ij} g_j(v_j(t-\tau(t),x)) - \sum_{j=1}^{2} d_{ij} \int_{t-\tau^*(t)}^{t} h_j(v_j(s,x))\mathrm{d}s \Big] + K_i(t,x) \Big\}\mathrm{d}t \\ & + \sum_{j=1}^{2} \sigma_{ij}(e_j(t,x), e_j(t-\tau(t),x), e_j(t-\tau^*(t),x))\mathrm{d}\omega_j(t), \end{aligned} \tag{4.3}$$

where

$$\sigma_{11} = 0.1e_1(t,x) + 0.2e_1(t-\tau(t),x) + 0.1e_1(t-\tau^*(t),x), \quad \sigma_{12} = 0,$$

$$\sigma_{21} = 0, \quad \sigma_{22} = 0.1e_2(t,x) + 0.1e_2(t-\tau(t),x) + 0.1e_2(t-\tau^*(t),x).$$

The initial condition for the response system (4.3) is chosen as

$$v_1(s,x) = 0.5\sin\left(\frac{x+5}{10}\pi\right), \quad v_2(s,x) = 0.6\sin\left(\frac{x+5}{10}\pi\right),$$

where $(s,x) \in [-0.65\pi, 0] \times \Omega$.

By simple computation, we obtain that $L_i^* = M_i^* = N_i^* = L_i = M_i = N_i = 1, i = 1,2, \bar{\alpha}_1 = 0.2, \bar{\alpha}_2 = 0.1, \alpha_1^* = 0.9, \alpha_2^* = 1.1, \gamma_1 = 0.84, \gamma_2 = 1.52, \eta_{11} = 0.12, \eta_{12} = 0, \eta_{21} = 0, \eta_{22} = 0.03, \rho' = 0.314 < 1, \rho'' = 1.1 > 1, \rho' = 0.01 < 1, \rho'' = 1.02 > 1, \tau = 0.65\pi, \tau^* = 0.112$. Therefore, assumptions $(\mathrm{H1}) - (\mathrm{H6})$ hold for systems (4.1) and (4.3).

Let $\alpha = 0.95, p = 2, \xi_{lij} = \zeta_{lij} = \xi_{lij}^* = \zeta_{lij}^* = \xi_{lij}^{**} = \zeta_{lij}^{**} = \varpi_{lij} = \varepsilon_{lij} = \varepsilon_{lij}^* = \varepsilon_{lij}^{**} = 1/2$ for $i,j, = 1,2, l = 1,2$, and choose the control parameters $k_{11} = 20, k_{12} = 0, k_{21} = 0, k_{22} = 20, \delta =$

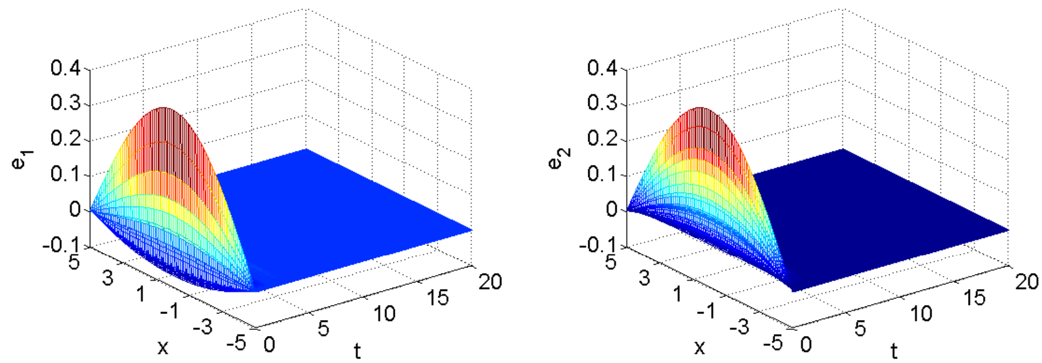

Fig.2 Asymptotical behaviors of the synchronization errors.

$0.981, T = 10$, then

$$
\begin{array}{llll}
\lambda_1 = -10.0790, & \lambda_2 = -9.5760, & w_1 = -40.0000, & w_2 = -40.0000, \\
p_{11} = 2.6706, & p_{12} = 0.2763, & p_{21} = 2.1948, & p_{22} = 3.7603, \\
q_{11} = 0.1276 & q_{12} = 0, & q_{21} = 0, & q_{22} = 0.0319, \\
q_{11}^{**} = 0.0907, & q_{12}^{**} = 0.0202, & q_{21}^{**} = 0.0185, & q_{22}^{**} = 0.0246.
\end{array}
$$

Then, $\bar{\varepsilon}_1 = 1.1138, \bar{\varepsilon}_2 = 0.7852$. Therefore, $\bar{\varepsilon} = 0.7852, w = 40.0000$. It is easy to verify that assumptions $(\mathrm{H7}) - (\mathrm{H8})$ are satisfied. According to Theorem 3.1, the master system (4.1) and the response system (4.3) are exponential synchronized based on $p-$norm. Numerical simulation illustrates our results (see Fig.2).

Remark 3. In example, $\dot{\tau}(t) = 1.1 > 1$ for $t < \pi/2$ and $\dot{\tau}^*(t) = 1.02 > 1$ for $t < 0.1$. It is evident that the synchronization criteria obtained in [32] do not succeed. However, the numerical simulations clearly illustrate the effectiveness of the exponential synchronization criteria in this paper.

Remark 4. The influences of reaction-diffusion on the exponential synchronization of Cohen-Grossberg neural networks can be discussed from the synchronization criteria obtained in this paper. Evidently, it is beneficial for reaction-diffusion Cohen-Grossberg neural networks to achieve the synchronization by increasing diffusion coefficients D_i or reducing diffusion space

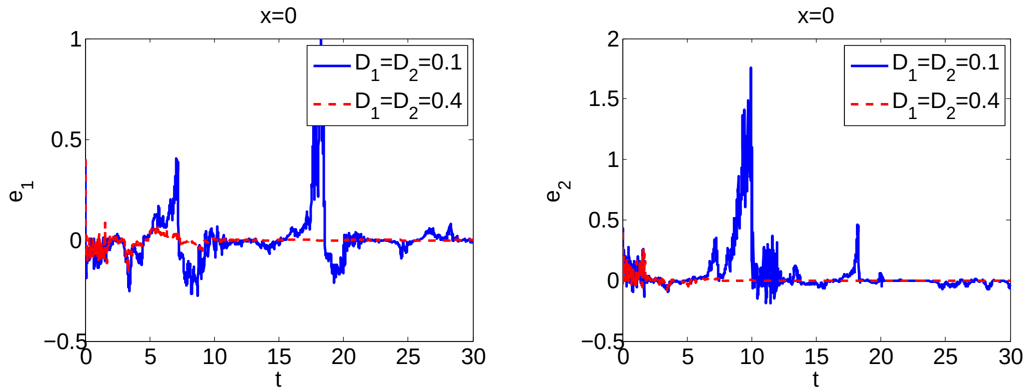

Fig.3 Asymptotical behaviors of the synchronization errors with different diffusion coefficients.

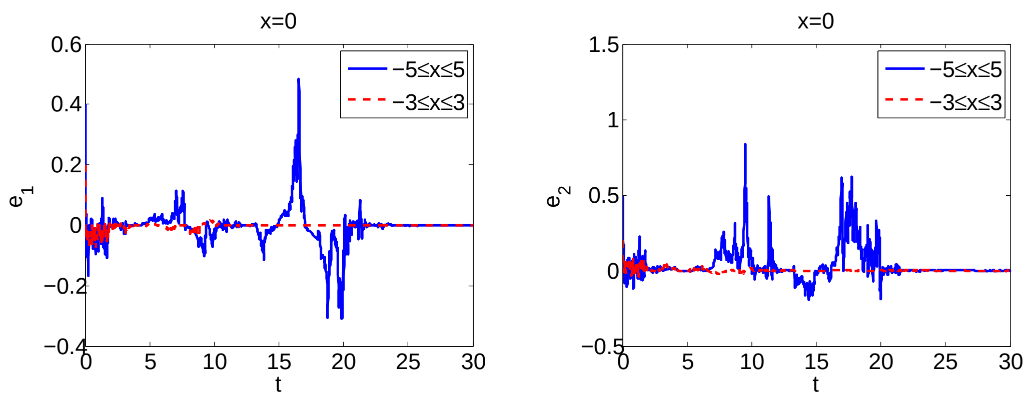

Fig.4 Asymptotical behaviors of the synchronization errors with different diffusion space.

x_k, respectively. Dynamical behaviors of the error systems with differential diffusion coefficients or differential diffusion space, respectively, are shown in Fig.3 and Fig.4.

Remark 5. Clearly, the larger stochastic perturbation is, the more difficult (H7) is satisfied. Hence, the exponential synchronization of neural networks with the smaller stochastic perturbation is more easily realized. Dynamical behaviors of the error systems with differential stochastic perturbation are shown in Fig.5.

Remark 6. Obviously, if the control rate δ or the control strength k_{ii} increase, respectively, assumptions $(H7)-(H8)$ can be satisfied more easily. Hence, the exponential synchronization

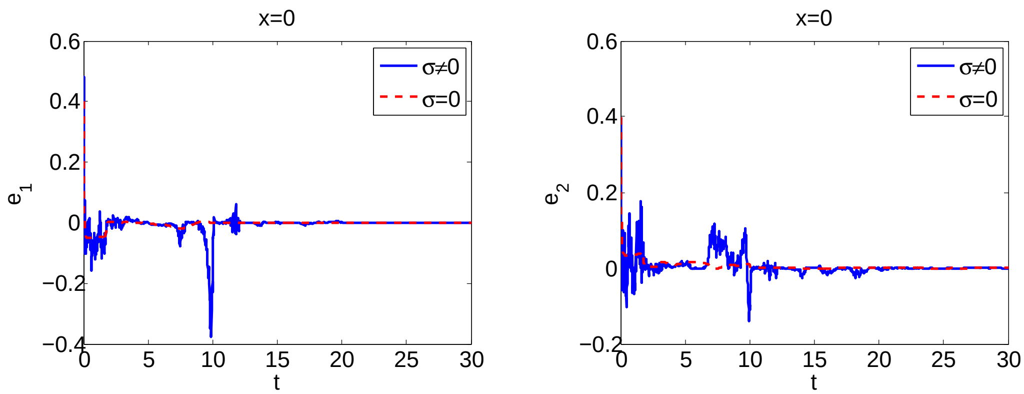

Fig.5 Asymptotical behaviors of the synchronization errors with different stochastic perturbation.

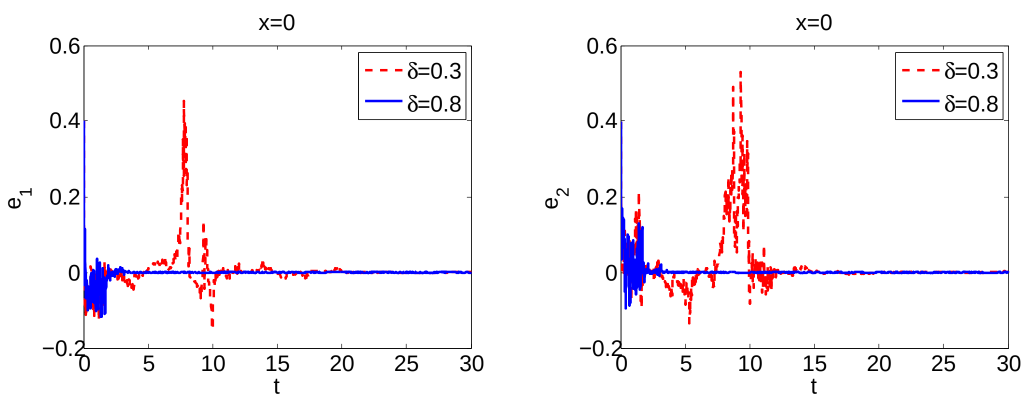

Fig.6 Asymptotical behaviors of the synchronization errors with different control rate.

of neural networks is more easily realized under the larger control rate or the larger control strength of the intermittent controller, respectively. Dynamical behaviors of the synchronization errors with differential control rate or the control strength, respectively, are shown in Fig.6 and Fig.7.

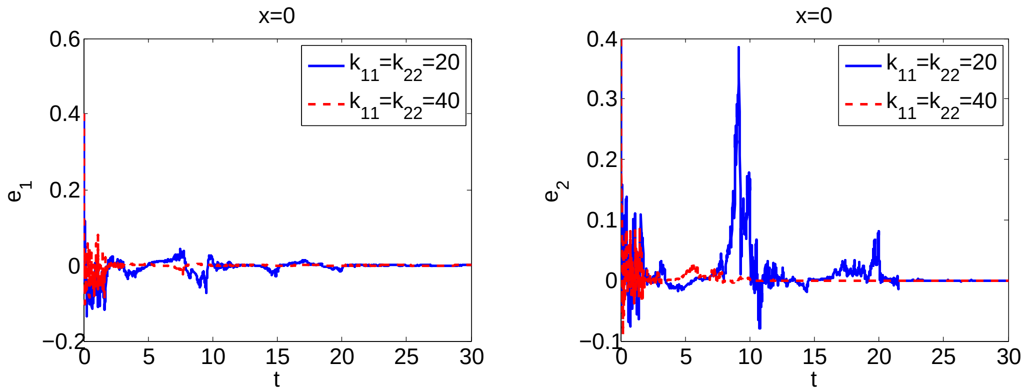

Fig.7 Asymptotical behaviors of the synchronization errors with different control strength.

5. Conclusion

In this paper, periodically intermittent controller was designed to achieve the exponential synchronization for a class of stochastic Cohen-Grossgerg neural networks with mixed time-varying delays and reaction-diffusion in terms of $p-$norm. By constructing the Lyapunov functional, the exponential synchronization criteria were obtained. The influences of stochastic perturbation, spacial diffusion, the control rate and the control strength on the exponential synchronization of Cohen-Grossgerg neural networks were discussed by the synchronization criteria. A chaotic Cohen-Grossberg neural network was proposed to verify the feasibility of our results. Compared with the previous works([27-32]), the model in this paper is more general, and the obtained conditions are less conservative.

It is shown that (H7) can be satisfied as long as feedback strengh parameter $k_{ii}, i \in \ell$ is small enough. Furthermore, the upper bound of $|k_{ii}|, (i \in \ell)$ is given in (H8). So, the results in this paper should provide some guidelines for designing the suitable periodically intermittent controller in the practical applications.

Conflict of Interests

The authors declare that there is no conflict of interests.

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