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PEST INTERACTION ON SEXUAL REPRODUCTIVE SYSTEM OF FOREST SEED DYNAMICS

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Abstract. A dynamical system is framed by considering pest interaction on sexual reproductive system of mono species forest in this work. Analytic approximation has been obtained to the nonlinear dynamical system and impact of endogenous and exogenous parameters have been discussed in detail.

Keywords: forest boundary dynamics; Homotopy perturbation method; pest interaction.

2010 AMS Subject Classification: 37N25, 35Q92, 92D40, 92D25, 35B20.

1. Introduction

Forest and afforestation activities are the major carbon sinks that influence climate by absorbing carbon dioxide from atmosphere, the most prevalent and important green house gas, and stored carbon in wood, litter, leaves, roots and soil. Forest planning is one of the most

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important aspects of ecosystem, yet often neglected. Deforestation affects climate change because it releases the carbon stored in plants and soils, and alters the physical properties of the surface. Tropical ecosystems are the major contributors to manage climate changes and altering them are likely to have the greatest impact on this system. Forests also influence climate locally by altering the surface energy balance through reflection of sunlight, friction caused by surface roughness and evaporation of soil water. The rates at which these effects on climate occur depend on the species of tree present and its relative abundance. These two biological factors in turn depend on the state of the environment along with the history of the forest. Scientists and forest managers would like to understand how changing disturbance regimes and how interactions among disturbances will influence forest successional trajectories.

Plant propagation is both a science and an art. The science of plant propagation requires a knowledge of plant physiology, nursery cultural practices, and characteristics of the particular plant that you want to grow. The art of plant propagation cannot be taught in a book or classroom. It requires key technical skills that must be acquired through innate ability or experience with complete involvement. In past years, reforestation practices have relied heavily on the use of natural seeding, direct seeding, and nursery-grown stock. Maintenance of genetic diversity is so important in ecosystem management and restoration projects, therefore, seed propagation is encouraged whenever possible. It is easier to capture and preserve bio diversity with seeds than with vegetative propagation. As for economics, seed propagation is almost always less expensive than vegetative propagation. The pest interaction on seeds effects the rate of growth of forest propagation. Experimental study of this is a time consuming process hence necessitating a theoretical study.

This research work undertakes to study the approximate solution of a nonlinear mathematical model of age structure dynamics by giving preference to the growth rate due to the interaction of pests on seeds.

2. Mathematical model

Antonwisky and Korzukhin [4] formed a simple model of age structure dynamics of mono species systems. Antonwisky et. al [1] qualitatively described a spatially distributed mono species mixed age forest. Antonvsky et. al [2, 3] discussed the influence of insect pests on the age structure dynamics. A real forest has age structure that varies from one gap to another [17] and the local gaps are integrated into a forest ecosystem by seed dispersion mechanisms and vegetative propagation. By considering seed dynamics into account, Kuznetsov et. al [13] introduced a mathematical model and Wu [18] discussed the asymptotic stability of travelling waves of system in Kuznetsov et. al [13]. Yuangu [19] proved the existence of a bounded global attractor for a cross-diffusion model of forest with homogeneous Dirchlet boundary condition. Yagi et. al [7] described the asymptotic behaviours of numerical solution of the system in Kuznetsov et. al [13]. Yagi et. al [10] also proved the global existence of the system in Kuznetsov et. al [13] and attempts were successfully made with Homotopy perturbation method by Rajasingh et al. [14]. An improved mathematical model has been attempted with Homotopy perturbation by Rajasingh et al. [15]. L. H. Chuan et. al. [8] introduced three kinds of ω-limit sets upon [13], investigated basic properties of these ω-limit sets and arrived at stationary solutions exclusively. The existence of a bounded global attractor for a cross-diffusion model of forest with homogeneous Dirichlet boundary condition has been proved under some condition on the parameters by L. Quanqua [14].

Plants have two ways of reproduction, sexual by means of seeds, and asexually or vegetatively by means of vegetative tissue. Both ways occur in living plants in nature. In nature, some plants reproduce vegetatively while others rely almost totally on sexual reproduction. Genetically the two ways of reproduction differ. Seeds contain genes from the female parent (where we collect the seeds) and the male parent (which contribute the pollen and which is often unknown). Vegetative material is genetically identical to the mother plant from where it was collected. Kuznetsov et. al. [13] considered the sexual reproduction and arrived a mathematical model

$$u_t = \delta\beta w - \gamma(v)u - fu, \tag{1}$$

$$v_t = fu - hv, \tag{2}$$

$$w_t = \alpha v - \beta w + Dw_{xx}. \tag{3}$$

The influence of the pests on seeds with biological interference type II response [12] has been attempted and this alters the mathematical system (1) - (3) as follows,

$$u_t = \delta\beta w - (v-1)^2 u - fu, \tag{4}$$

$$w_t = r_x w\left(1-\frac{w}{k}\right) - \frac{aws}{w+b} + \alpha v - \beta w + Dw_{xx}, \tag{5}$$

$$v_t = fu - hv, \tag{6}$$

$$s_t = \frac{aws}{w+b} - \rho s + Ds_{xx}. \tag{7}$$

where $\frac{1}{a}$ denotes the yield coefficient of young tree to pests [12], b is the protection to young tree offered by the environment [12], $r_x = (e+gx)$ denotes the intrinsic growth rate, k represents carrying capacity and ρ represents coefficients of mortality rate of pests.

In this paper homotopy perturbation method is employed to obtain an approximate solution for the above nonlinear partial differential equations.

3. Homotopy perturbation method

The familiar homotopy perturbation method has been introduced by the Chinese Mathematician J. H. He [11], which is an effective mathematical tool to find the analytic solutions for non linear partial differential equations. It is handled to solve the fin type problems[5, 9] and heat transfer equation[6].

The basic concepts of the homotopy perturbation methods are illustrated for the following nonlinear partial differential equation

$$v_t(x,t) = Nv(x,t) + Lv(x,t) - u(x,t) \tag{8}$$

subject to the initial and boundary conditions $v(0,0) = c,\ B(v, v_t, v_x) = 0$, where L is the linear operator and N is the nonlinear operator and $u(x,t)$ is known analytical function. This results in the construction of the homotopy as follows,

$$(1-p)v_t(x,t) + p(v_t(x,t) + Nv(x,t) + Lv(x,t) - u(x,t)) = 0, p \in [0,1]$$

where p is the homotopy parameter.

As per the homotopy perturbation method approximate solutions of (8) can be uttered as a series of powers of p

$$v(x,t) = v_0(x,t) + pv_1(x,t) + p^2 v_2(x,t) + \ldots,$$

$$v(x,t) = \lim_{p \to 1} v(x,t) = v_0(x,t) + v_1(x,t) + v_2(x,t) + \ldots \tag{9}$$

4. Approximate solution to the forest boundary model

In this section, homotopy perturbation method is applied to construct approximate solution for the system (4) - (7) with initial conditions $u(x,0) = e^{lx},\ v(x,0) = e^{mx},\ w(x,0) = e^{nx},\ s(x,0) = e^{cx}$.

The following homotopy has been constructed $v(r,p) : \Omega[0,1] \to \Re,\ u(r,p) : \Omega[0,1] \to \Re,\ w(r,p) : \Omega[0,1] \to \Re,\ s(r,p) : \Omega[0,1] \to \Re$, which satisfies

$$\begin{aligned}
H(u,p) &= (1-p)u_t + p[u_t - \delta\beta w + (v-1)^2 u + fu] = 0,\ r \in \Omega \\
H(w,p) &= (1-p)w_t + p[w_t - r_x w\left(1 - \frac{w}{k}\right) + \frac{aws}{w+b} - \alpha v + \beta w - Dw_{xx}] = 0,\ r \in \Omega \\
H(v,p) &= (1-p)v_t + p[v_t - fu + hv] = 0,\ r \in \Omega, \\
H(s,p) &= (1-p)s_t + p[s_t - \frac{asv}{v+b} + \rho s - Ds_{xx}] = 0,\ r \in \Omega
\end{aligned}$$

Approximate solution for (4) - (7) has been expressed as a series of powers of p

$$u(x,t) = u_0(x,t) + pu_1(x,t) + p^2 u_2(x,t) + \ldots, \tag{10}$$

$$w(x,t) = w_0(x,t) + pw_1(x,t) + p^2 w_2(x,t) + \ldots, \tag{11}$$

$$v(x,t) = v_0(x,t) + pv_1(x,t) + p^2 v_2(x,t) + \ldots, \tag{12}$$

$$s(x,t) = s_0(x,t) + ps_1(x,t) + p^2 s_2(x,t) + \ldots \tag{13}$$

Substituting (10) - (13) in the above homotopy and arranging the coefficient of powers of p,

$$p^0 : \frac{\partial u_0}{\partial t} = 0; \frac{\partial v_0}{\partial t} = 0; \frac{\partial w_0}{\partial t} = 0; \frac{\partial s_0}{\partial t} = 0,$$

$$
\begin{aligned}
p^1 : \frac{\partial u_1}{\partial t} &= \delta\beta w_0 - v_0(v_0-1)^2 - f u_0, \\
\frac{\partial v_1}{\partial t} &= f u_0 - h v_0,
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial w_1}{\partial t} &= r_x w_0 \left(1 - \frac{w_0}{k}\right) - \frac{a}{b} w_0 s_0 \left(1 + \frac{w_0}{b}\right) + \alpha\, v_0 - \beta w_0 + D\frac{\partial^2 w_0}{\partial x^2}, \\
\frac{\partial s_1}{\partial t} &= \frac{a}{b} w_0 s_0 (1 + \frac{w_0}{b}) - \rho s_0 + D\frac{\partial^2 s_0}{\partial x^2}, \\
p^2 : \frac{\partial u_2}{\partial t} &= \delta\beta w_1 - u_1(v_0-1)^2 + 2u_0 v_1(v_0-1) - f u_1, \\
\frac{\partial v_2}{\partial t} &= f u_1 - h v_1, \\
\frac{\partial w_2}{\partial t} &= r_x w_1 \left(1 - \frac{2w_0}{k}\right) - \frac{a}{b} w_1 s_0 \left(1 + \frac{2w_0}{b}\right) + \alpha\, v_1 - \beta w_1 + D\frac{\partial^2 w_1}{\partial x^2}, \\
\frac{\partial s_2}{\partial t} &= \frac{a}{b} w_1 s_0 \left(1 + \frac{2w_0}{b}\right) - \rho s_1 + D\frac{\partial^2 s_1}{\partial x^2}.
\end{aligned}
$$

Solving the above equations

$$u_0(x,t) = e^{lx}, \tag{14}$$

$$v_0(x,t) = e^{mx}, \tag{15}$$

$$w_0(x,t) = e^{nx}, \tag{16}$$

$$s_0(x,t) = e^{cx}, \tag{17}$$

$$u_1(x,t) = \xi_1(x)t + e^{lx}, \tag{18}$$

$$v_1(x,t) = \xi_2(x)t + e^{mx}, \tag{19}$$

$$w_1(x,t) = \xi_3(x)t + e^{nx}, \tag{20}$$

$$s_1(x,t) = \xi_4(x)t + e^{cx}, \tag{21}$$

$$u_2(x,t) = \xi_5(x)\frac{t^2}{2} + \xi_6(x)t + e^{lx}, \tag{22}$$

$$v_2(x,t) = \xi_7(x)\frac{t^2}{2} + \xi_8(x)t + e^{mx}, \tag{23}$$

$$w_2(x,t) = \xi_{11}(x)\frac{t^2}{2} + \xi_{12}(x)t + e^{nx}, \tag{24}$$

$$s_2(x,t) = \xi_{13}(x)\frac{t^2}{2} + \xi_{14}(x)t + e^{cx}. \tag{25}$$

Incorporating (9) with (10) - (13) the following equation are tailored

$$u(x,t) = \lim_{p\to 1} u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + \ldots, \tag{26}$$

$$w(x,t) = \lim_{p\to 1} w(x,t) = w_0(x,t) + w_1(x,t) + w_2(x,t) + \ldots, \tag{27}$$

$$v(x,t) = \lim_{p\to 1} v(x,t) = v_0(x,t) + v_1(x,t) + v_2(x,t) + \ldots, \tag{28}$$

$$s(x,t) = \lim_{p\to 1} s(x,t) = s_0(x,t) + s_1(x,t) + s_2(x,t) + \ldots \tag{29}$$

Substituting (14)-(25) to (26)-(29) we could get the solution for the system (4)-(7) as follows

$$\begin{aligned}
u(x,t) &= 3e^{lx} + (\xi_1(x) + \xi_6(x))t + \xi_5(x)\frac{t^2}{2},\\
v(x,t) &= 3e^{mx} + (\xi_2(x) + \xi_8(x))t + \xi_7(x)\frac{t^2}{2},\\
w(x,t) &= 3e^{nx} + (\xi_3(x) + \xi_{12}(x))t + \xi_{11}(x)\frac{t^2}{2},\\
s(x,t) &= 3e^{cx} + (\xi_4(x) + \xi_{14}(x))t + \xi_{13}(x)\frac{t^2}{2}.
\end{aligned}$$

where

$$\begin{aligned}
\xi_1(x) &= \delta\beta\ e^{nx} - e^{lx}((e^{mx}-1)^2 - f),\\
\xi_2(x) &= fe^{lx} - he^{mx},\\
\xi_3(x) &= \alpha e^{mx} + e^{nx}\left(r_x\left(1 - \frac{e^{nx}}{k}\right) - \frac{a}{b}e^{nx}\left(1 + \frac{e^{nx}}{b}\right) - \beta + Dn^2e^{nx}\right),\\
\xi_4(x) &= e^{cx}(\frac{a}{b}e^{nx}(1 + \frac{e^{nx}}{b}) - \rho + Dc^2),\\
\xi_5(x) &= \delta\beta\xi_3(x) - \xi_1(x)(e^{mx}-1)^2 + 2e^{lx}\xi_2(x)(e^{mx}-1) - f\xi_1(x),\\
\xi_6(x) &= \delta\beta e^{nx} - e^{lx}(e^{mx}-1)^2 + 2e^{(m+c)x}(e^{mx}-1) - fe^{lx},
\end{aligned}$$

$$\begin{aligned}
\xi_7(x) &= f\xi_1(x) - h\xi_2(x),\\
\xi_8(x) &= fe^{lx} - he^{mx},\\
\xi_9(x) &= \alpha f l^2 e^{lx} - hm^2 e^{mx} + Dr_x n^2\left(e^{nx} - \frac{4e^{2nx}}{k}\right) + Dng\left(e^{nx} - \frac{e^{2nx}}{k}\right),\\
\xi_{10}(x) &= \xi_3(x)e^{nx}\left[r_x\left(1 - \frac{2e^{nx}}{k} - \beta\right) - \frac{a}{b}e^{(n+c)x}\left(1 + \frac{2e^{nx}}{b}\right) + \alpha\right],
\end{aligned}$$

$$\begin{aligned}
\xi_{11}(x) &= \xi_9(x) + \xi_{10}(x) - \frac{a}{b}((n+c)^2 e^{(n+c)x} + \frac{(2n+c)^2}{b}e^{(2n+c)x} - \beta n^2 e^{nx}),\\
\xi_{12}(x) &= r_x\left(1 - \frac{2e^{nx}}{k} - \beta\right) - \frac{a}{b}e^{(n+c)x}\left(1 + \frac{2e^{nx}}{b}\right) + Dn^2c^2e^{cx}\left(1 - \frac{aD}{b}\right),\\
\xi_{13}(x) &= e^{cx}\left[\frac{a}{b}\left(1 + \frac{2e^{nx}}{b}\right)\xi_3(x) + \frac{Da}{b}(n+c)^2e^{nx} + \frac{Da}{b^2}(2n+c)^2e^{2nx} - D\rho c^2 + D^2c^4\right],\\
\xi_{14}(x) &= e^{cx}\left[\frac{a}{b}\left(1 + \frac{2e^{nx}}{b}\right)e^{nx} - \rho + c^2\right].
\end{aligned}$$

5. Results and discussion

In order to have physical significant points of the systems (4) - (7), numerical calculations are attempted with different values of a (($\frac{1}{a}$) denotes the yield coefficient of seed to pest), b (protection to seed offered by the environment), α (seed production), β (seed deposition) and δ (seed establishment rate) by taking diffusion coefficient D=1.

The contour graphs in Fig.1, Fig.2, Fig.3 and Fig.4 shows the densities of young trees, old trees, seeds and pests. The parametric plot between the densities of pests and seeds is described in Fig5. As the densities of pests increase, the densities of seeds decreased. The parametric plot between the densities of pests and young trees is described in Fig.6. As the densities of pests increase, the densities of young trees decreased. Fig.7 and 8 reveal that if a is increased, w is decreased and p is increased. If the factor a is increased, then $\frac{1}{a}$ is decreased. This shows that the yield coefficient decreases with respect to the densities of seeds.

Fig.10 obviously indicates that the density of seed is increased with the protection to the young trees offered by the environment–the factor. Fig.9 illustrates that the environmental protection of seeds is increased which results in the decrease of the densities of pests. Fig.11

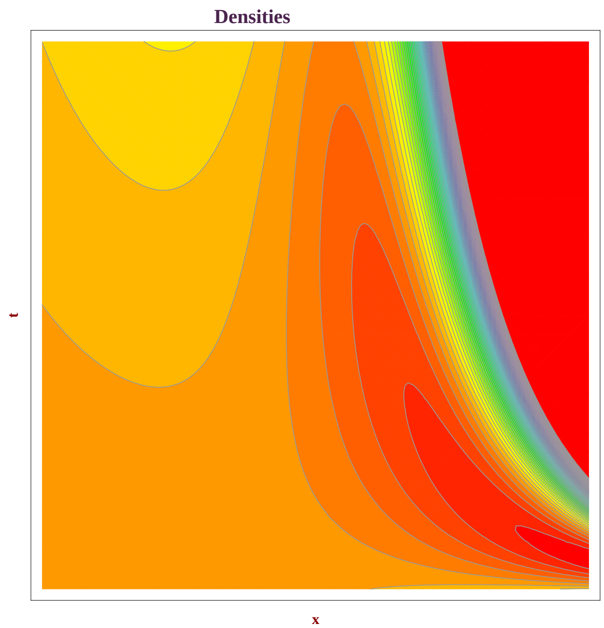

FIGURE 1. The contour plot of young trees when $l = 1,\ m = 1,\ n = 1,\ \alpha = 0.5,\ \beta = 0.6,\ \delta = 2,\ h = 0.8,\ f = 0.75,\ b = 5,\ \rho = 0.75,\ a = 2,\ e = 1,\ g = 5$ and $k = 3$.

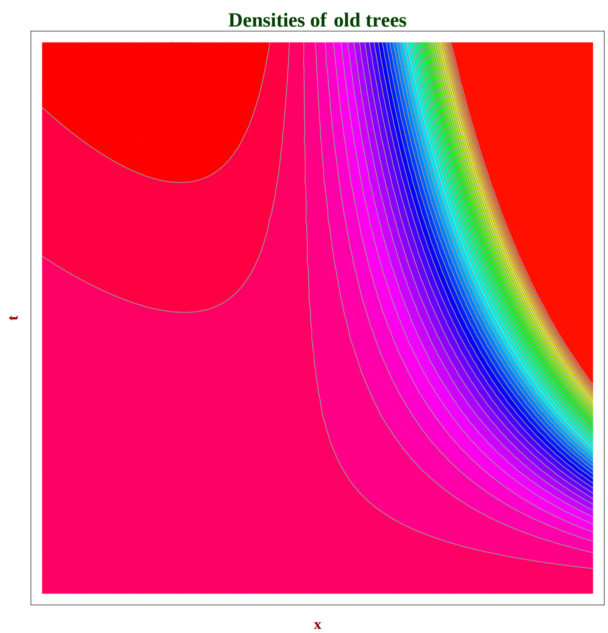

FIGURE 2. The contour plot of old trees when $l = 1,\ m = 1,\ n = 1,\ \alpha = 0.5,\ \beta = 0.6,\ \delta = 2,\ h = 0.8,\ f = 0.75,\ b = 5,\ \rho = 0.75,\ a = 2,\ e = 1,\ g = 5$ and $k = 3$.

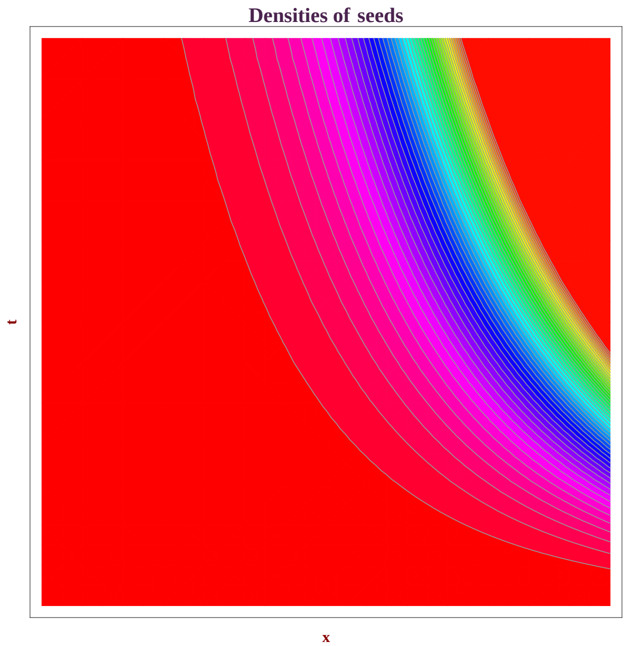

FIGURE 3. The contour plot of seeds when $l = 1,\ m = 1,\ n = 1,\ \alpha = 0.5,\ \beta = 0.6,\ \delta = 2,\ h = 0.8,\ f = 0.75,\ b = 5,\ \rho = 0.75,\ a = 2,\ e = 1,\ g = 5$ and $k = 3$.

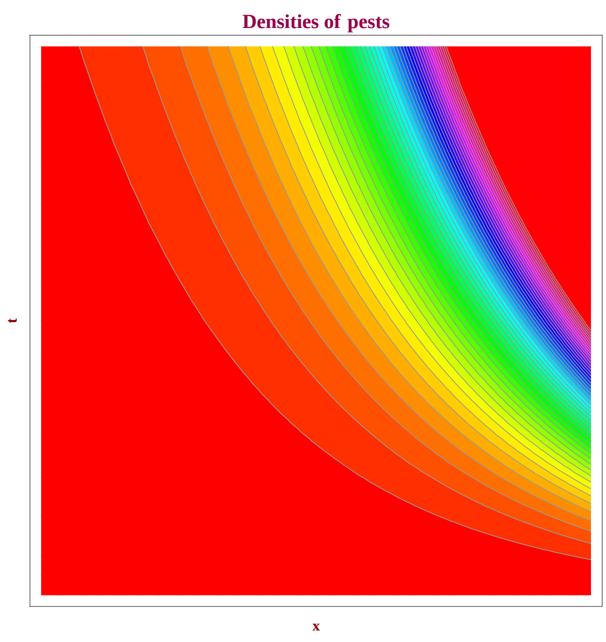

FIGURE 4. The contour plot of pests when $l = 1,\ m = 1,\ n = 1,\ \alpha = 0.5,\ \beta = 0.6,\ \delta = 2,\ h = 0.8,\ f = 0.75,\ b = 5,\ \rho = 0.75,\ a = 2,\ e = 1,\ g = 5$ and $k = 3$.

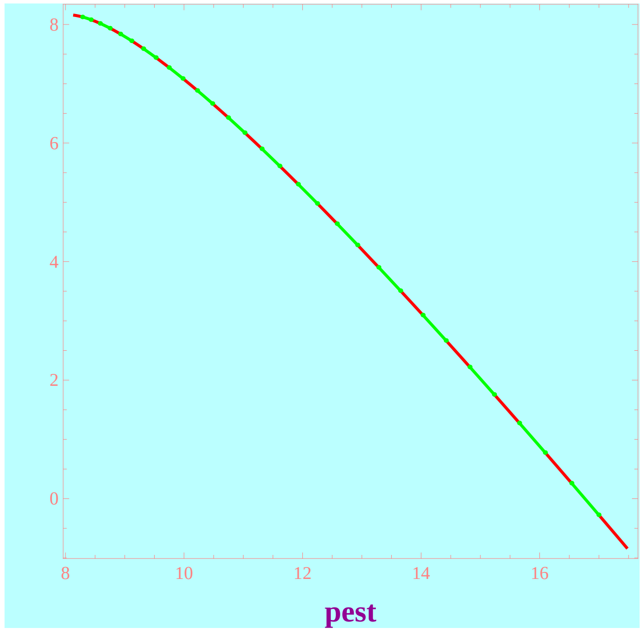

FIGURE 5. The density of pests and seeds when $l = 1,\ m = 1,\ n = 1,\ \alpha = 0.5,\ \beta = 0.6,\ \delta = 2,\ h = 0.8,\ f = 0.75,\ b = 5,\ \rho = 0.75,\ a = 2,\ e = 1,\ g = 5$ and $k = 3$.

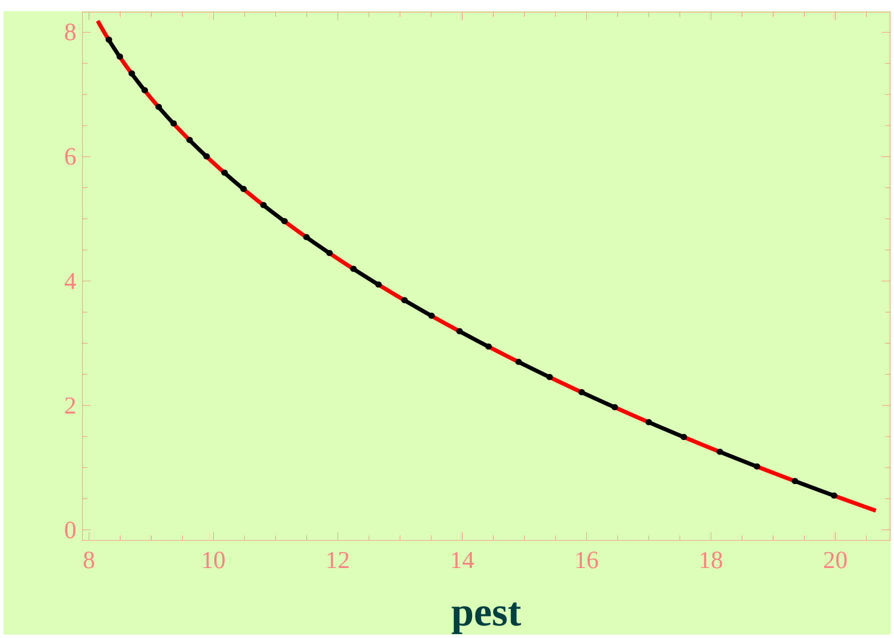

FIGURE 6. The density of pest and young trees when $l = 1,\ m = 1,\ n = 1,\ \alpha = 0.5,\ \beta = 0.6,\ \delta = 2,\ h = 0.8,\ f = 0.75,\ b = 5,\ \rho = 0.75,\ a = 2,\ e = 1,\ g = 5$ and $k = 3$.

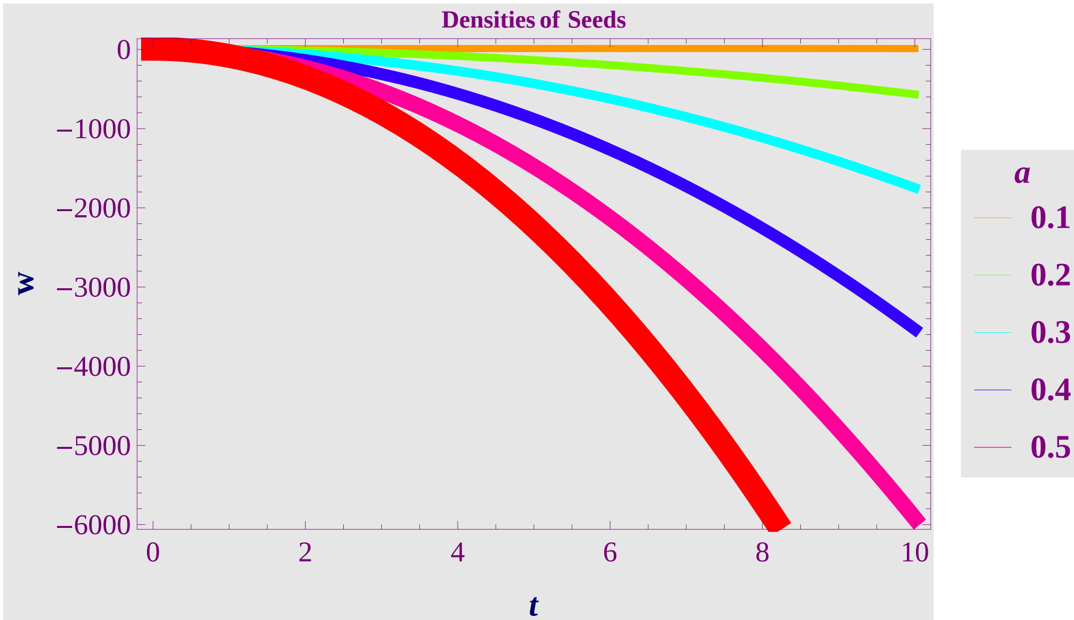

FIGURE 7. The density of seeds when $l = 1$, $m = 1$, $n = 1$, $\alpha = 0.03$, $\beta = 0.08$, $\delta = 0.75$, $h = 0.04$, $f = 0.014$, $\rho = 0.75$, $b = 1$, $x = 1$, and a varies from 0.1 to 0.5.

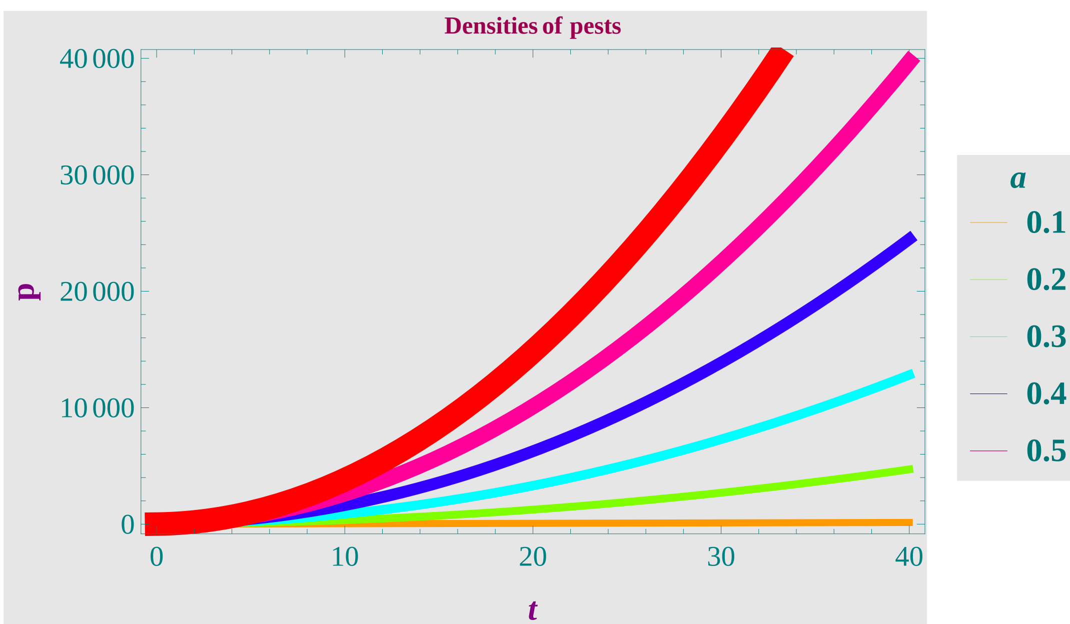

FIGURE 8. The density of seeds when $l = 1$, $m = 1$, $n = 1$, $\alpha = 0.03$, $\beta = 0.08$, $\delta = 0.75$, $h = 0.04$, $f = 0.014$, $\rho = 0.75$, $b = 1$, $x = 1$, and a varies from 0.1 to 0.5.

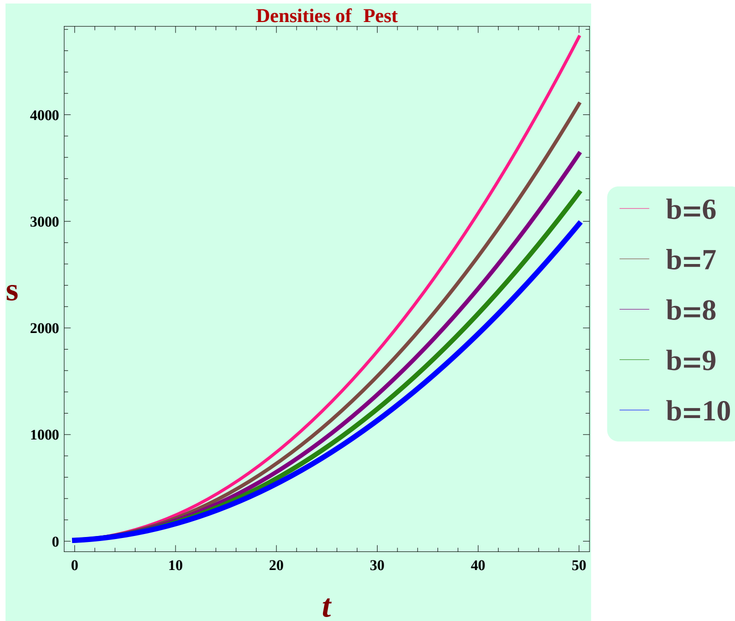

FIGURE 9. The density of pests when $l = 1$, $m = 1$, $n = 1$, $\alpha = 0.03$, $\beta = 0.08$, $\delta = 0.75$, $h = 0.04$, $f = 0.014$, $\rho = 0.75$, $a = 1$, and b varies from 6 to 10.

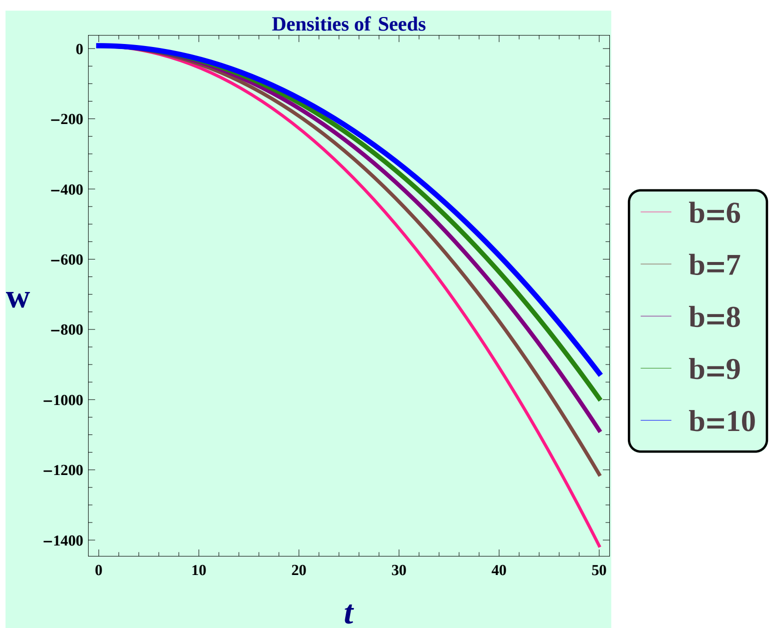

FIGURE 10. The density of seeds when $l = 1$, $m = 1$, $n = 1$, $\alpha = 0.03$, $\beta = 0.08$, $\delta = 0.75$, $h = 0.04$, $f = 0.014$, $\rho = 0.75$, $a = 1$, and b varies from 6 to 10.

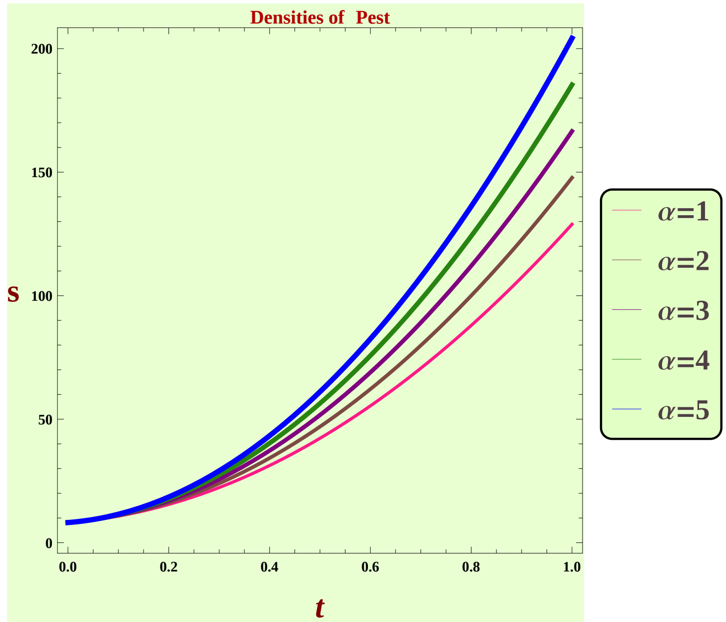

FIGURE 11. The density of pests when $l=1,\ m=1,\ n=1,\ k=3,\ \beta=0.6,\ \delta=2,\ h=0.8,\ f=0.75,\ b=5,\ \rho=0.75,\ a=2,\ e=1,\ g=5$ and α varies from $1\ to\ 5$.

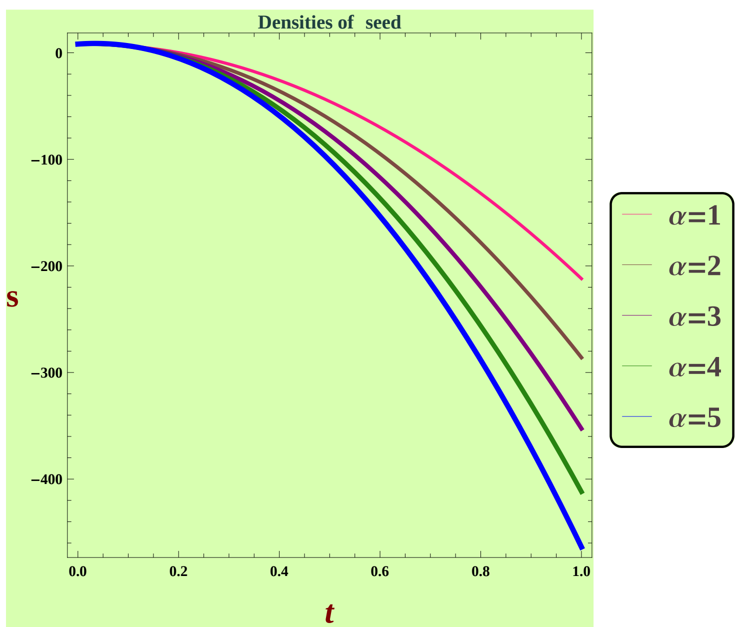

FIGURE 12. The density of seeds when $l=1,\ m=1,\ n=1,\ k=3,\ \beta=0.6,\ \delta=2,\ h=0.8,\ f=0.75,\ b=5,\ \rho=0.75,\ a=2,\ e=1,\ g=5$ and α varies from $1\ to\ 5$.

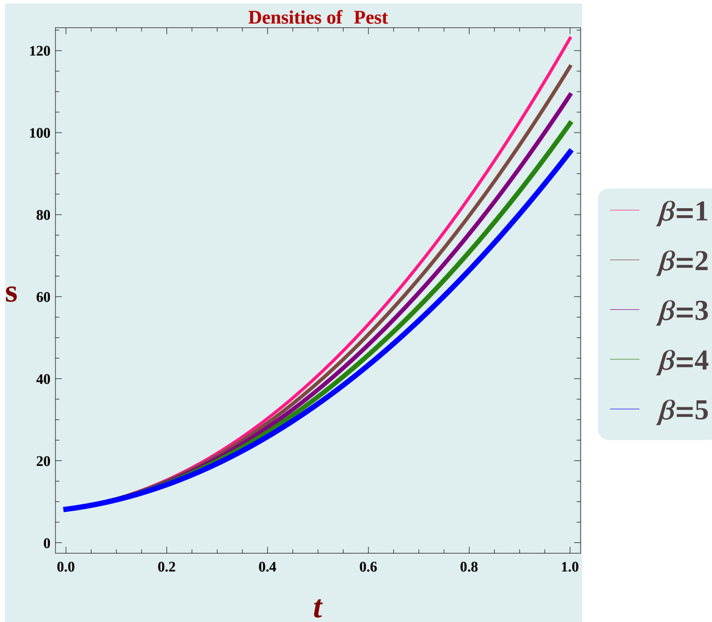

FIGURE 13. The density of pests when $l = 1,\ m = 1,\ n = 1,\ k = 3,\ \alpha = 0.75,\ \delta = 2,\ h = 0.8,\ f = 0.75,\ b = 5,\ \rho = 0.75,\ a = 2,\ e = 1,\ g = 5$ and β varies from 1 *to* 5.

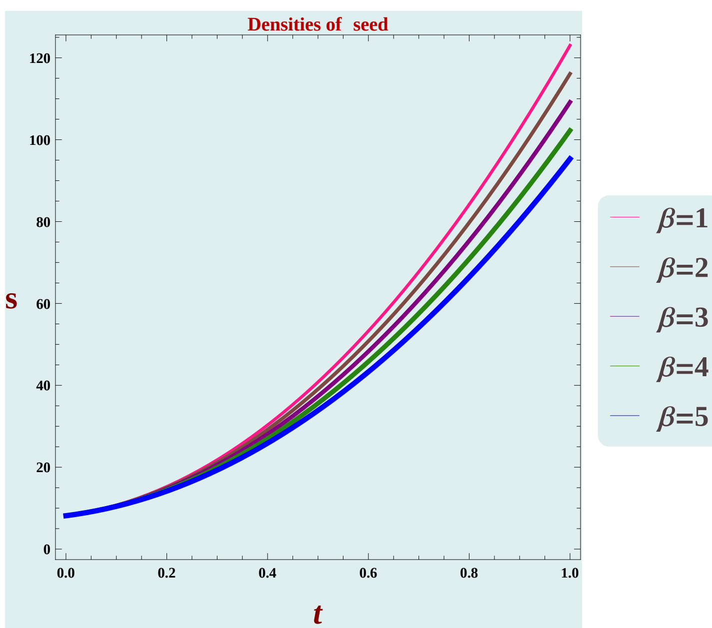

FIGURE 14. The density of seed when $l = 1,\ m = 1,\ n = 1,\ k = 3,\ \alpha = 0.75,\ \delta = 2,\ h = 0.8,\ f = 0.75,\ b = 5,\ \rho = 0.75,\ a = 2,\ e = 1,\ g = 5$ and β varies from 1 *to* 5.

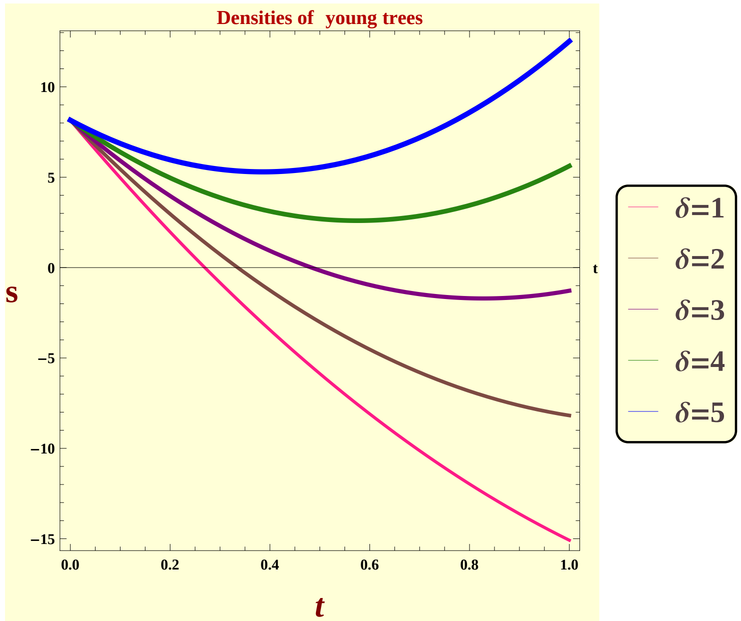

FIGURE 15. The density of young trees when $l = 1,\ m = 1,\ n = 1,\ k = 3,\ \alpha = 0.75,\ \beta = 0.83\ ,\ h = 0.8,\ f = 0.75,\ b = 5,\ \rho = 0.75,\ a = 2,\ e = 1,\ g = 5$ and δ varies from 1 *to* 5.

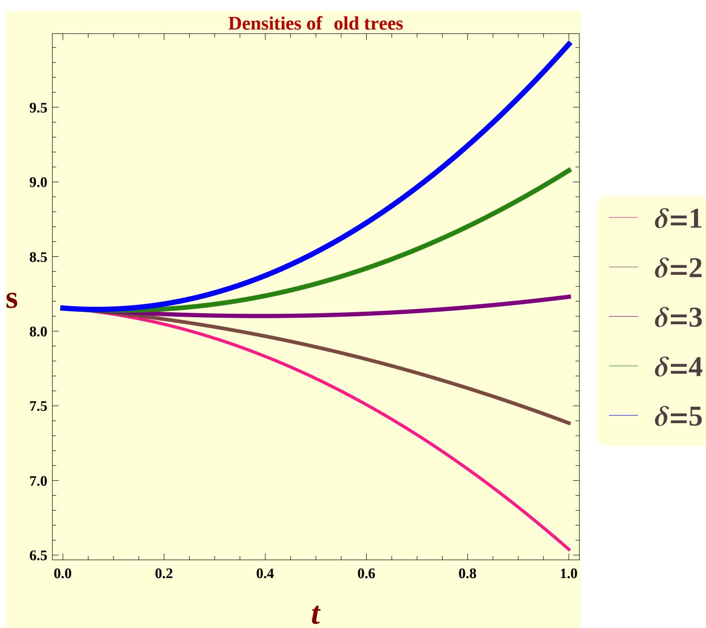

FIGURE 16. The density of old trees when $l = 1,\ m = 1,\ n = 1,\ k = 3,\ \alpha = 0.75,\ \beta = 0.83\ ,\ h = 0.8,\ f = 0.75,\ b = 5,\ \rho = 0.75,\ a = 2,\ e = 1,\ g = 5$ and δ varies from 1 *to* 5.

indicates that the densities of the pest are increased when the reproduction coefficient is increased. Fig.12 demonstrates that if the environmental factor α increases, then the density of seeds decreased due to the interaction between seeds and that of pest and seeds.

The coefficient of deposition (β) and its implication on the density of pests and seeds by decreasing its concentration is graphically represented (Fig.13 and Fig.14). It has also been revealed that the increase of density of the young, old trees and the increase of deposition of seeds are as represented in the Fig.15 and Fig.16.

6. Conclusion

The pest interaction of seed dynamics by giving importance to the temporal effects has been carried over. These parameters namely a ($\frac{1}{a}$ denotes the yield coefficient of seed to pest), b (protection to seed offered by the environment), α (seed production), β (seed deposition) and δ (seed establishment rate) has been discussed in detail in reference to the influence of pests on seeds. The ecotone dynamics of forest boundary surfaces under pest interaction on seed dynamics are elaborately discussed quantitatively, which enables one to demonstrate and illustrate the figures and numerical values successfully.

The mathematical model has been approached with reference to variable parameters to establish its merits over the forest ecosystem. This paper exclusively discusses the temporal effects of the forest dynamics in a useful and valuable informative facts. Incorporation and substantiation of this model for further studies could be applied in terms of spatial effects. This model would serve as an indispensable tool to the forest managers and scientists to successfully implement it in real time management practices of the forest ecosystem.

Conflict of Interests

The authors declare that there is no conflict of interests.

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