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DYNAMICS OF A DELAYED SEIR EPIDEMIC MODEL WITH PULSE VACCINATION AND RESTRICTING THE INFECTED DISPERSAL

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Abstract. In this work, we propose a delayed SEIR epidemic model with pulse vaccination and restricting the infected dispersal. By the stroboscopic map of the discrete dynamical system, we obtain infection-free boundary periodic solution. Further, we prove that the infection-free boundary periodic solution is globally attractive. By the theory on the delay and impulsive differential equation, we prove that the investigated system is permanent. Our results indicate that the time delay, pulse vaccination and impulsive dispersal have influence to the dynamical behaviors of the investigated system.

Keywords: delay; SEIR epidemic model; pulse vaccination; infection-free; restricting the infected dispersal.

2010 AMS Subject Classification: 34D23, 92B05.

1. Introduction

The mathematical epidemiologists[$1-8$] have recently been attracted by epidemic models. An SVEIR epidemic model was studied by Wang et al.[9]. Sun and Shi [10] considered the global stability of an SEIR

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model with nonlinear removal functions between compartments. To understand the effect of transport-related infection on disease spread, Cui et al. [11] investigated the spreading disease with transport-related infection. Takeuchi et al. [12] proposed an SIS models with transport-related infection. Liu et al. [13] considered the global stability of an SEIR epidemic model with age-dependent latency and relapse. Bai and Zhou [14] investigated the global dynamics of an SEIRS epidemic model with periodic vaccination and seasonal contact rate Quarantine and isolation measures $[15-21]$ have been widely used to control the spread of diseases such as yellow fever, smallpox, measles, ebola, pandemic influenza, diphtheria, plague, cholera, and, more recently, severe acute respiratory syndrome (SARS). Xie et al.[22] simultaneously use two kinds of measures: expand the treatment ranges of suspected case and limit population flows freely to suppress the diffusion of SARS effectively. Gong et al. [23] showed that the SARS may fluctuate with import of SARS infectiousness from outside Beijing, weakenness of quarantine, more social activities and so on.

Different types of vaccination policies and strategies combining pulse vaccination policy, treatment, pre-outbreak vaccination or isolation have already been introduced by many referees $[24-29]$. The pulse vaccination strategy $[24-26]$ consists of repeated application of vaccine at discrete time with equal interval in a population in contrast to the traditional constant vaccination. Nokes and Swinton [27] discussed the control of childhood viral infections by pulse vaccination strategy. Stone et al. [28] presented a theoretical examination of the pulse vaccination strategy in the SIR epidemic model. d'Onofrio [29] investigated the application of the pulse vaccination policy to eradicate infectious disease for SIR and SEIR epidemic models.

The dispersal is a ubiquitous phenomenon in the natural world. It is important for us to understand the ecological and evolutionary dynamics of populations mirrored by the large number of mathematical models devoting to it in the scientific literatures $[30-38]$. If the population dynamics with the effects of spatial heterogeneity is modeled by a diffusion process, most previous papers focused on the population dynamical system modeled by the ordinary differential equations. But in practice, it is often the case that diffusion occurs in regular pulse. For example, when winter comes, birds will migrate between patches in search for a better environment,whereas they do not diffuse in other seasons, and the excursion of foliage seeds occurs at fixed period of time every year. Thus impulsive diffusion provides a more natural description. Lately theories of impulsive differential equations [31] has been introduced into population dynamics. Impulsive differential equations are found in almost domain of applied science $[31-32,36-37,39-41]$.

The organization of this paper is as follows. In the next section, we introduce the model and background concepts. In Section 3, some important lemmas are presented. In Section 4, we give the conditions of global attractivity and permanence for system (2.4). In Section 5, A brief discussion is given in the last section to conclude this work.

2. The model

Gao et al.[42] investigated an SEIR model with time delay and pulse vaccination

$$(1)\quad \begin{cases} \left.\begin{aligned} \frac{dS(t)}{dt} &= -\beta S(t)I(t) - \mu(1-S(t)),\\ \frac{dE(t)}{dt} &= \beta S(t)I(t) - \beta e^{-\mu\tau_1}S(t-\tau_1)I(t-\tau_1) - \mu E(t),\\ \frac{dI(t)}{dt} &= \beta e^{-\mu\tau_1}S(t-\tau_1)I(t-\tau_1) - (r+\mu)I(t),\\ \frac{dR(t)}{dt} &= rI(t) - \mu R(t), \end{aligned}\right\} t \neq n\tau,\\ \left.\begin{aligned} \triangle S(t) &= -\theta S(t),\\ \triangle E(t) &= 0,\\ \triangle I(t) &= 0,\\ \triangle R(t) &= \theta S(t), \end{aligned}\right\} t = n\tau, n = 1,2\cdots, \end{cases}$$

where $S(t)$, $I(t)$ and $R(t)$ represent the number of susceptible, infected, recovered individuals respectively. The meanings of parameters in system (2.2) can be seen in reference [42].

Wang and Chen [38] considered the following model

$$(2)\quad \begin{cases} \left.\begin{aligned} \frac{dN_1(t)}{dt} &= r_1 N_1(t)\ln\frac{k_1}{N_1(t)},\\ \frac{dN_2(t)}{dt} &= r_2 N_2(t)\ln\frac{k_2}{N_2(t)}, \end{aligned}\right\} t \neq n\tau,\\ \left.\begin{aligned} \triangle N_1(t) &= d_1(N_2(t)-N_1(t)),\\ \triangle N_2(t) &= d_2(N_1(t)-N_2(t)), \end{aligned}\right\} t = n\tau, n = 1,2\cdots, \end{cases}$$

where we suppose that the system is composed of two patches connected by diffusion; $N_i(i=1,2)$ is the density of species in the ith patch. Intrinsic rate of natural increase of population in the ith habitat is denoted by $r_i(i=1,2)$; $k_i(i=1,2)$ denotes the carrying capacity in the ith patch, $d_i(i=1,2)$ is dispersal rate in the ith patch. It is assumed here that the net exchange from the jth patch to ith patch is proportional to the difference $N_j - N_i$ of population densities. The pulse diffusion occurs every τ period (τ is a positive constant), the system evolves from its initial state without being further affected by diffusion until the next pulse appears; $\triangle N_i = N_i(n\tau^+) - N_i(n\tau)$, and $N_i(n\tau^+)$ represents the density of population in the ith patch immediately after the nth diffusion pulse at time $t = n\tau$, while $N_i(n\tau)$ represents the density of population in the ith patch before the nth diffusion pulse at time $t = n\tau, n = 0, 1, 2, \cdots$; r_i, k_i and $d_i (i = 1, 2)$ are positive constants.

Inspired by the above discussion, we establish a delayed SEIR epidemic model with pulse vaccination and restricting the infected dispersal.

$$
\left\{
\begin{array}{l}
\left.
\begin{array}{l}
\frac{dS_1(t)}{dt} = \lambda_1 - d_1 S_1(t) - \beta_1 S_1(t) I_1(t), \\
\frac{dE_1(t)}{dt} = \beta_1 S_1(t) I_1(t) - \beta_1 e^{-d_1\tau_1} S_1(t-\tau_1) I_1(t-\tau_1) - d_1 E_1(t), \\
\frac{dI_1(t)}{dt} = \beta_1 e^{-d_1\tau_1} S_1(t-\tau_1) I_1(t-\tau_1) - (r_1 + d_1 + b_1) I_1(t), \\
\frac{dR_1(t)}{dt} = r_1 I_1(t) - d_1 R_1(t), \\
\frac{dS_2(t)}{dt} = \lambda_2 - d_2 S_2(t) - \beta_2 S_2(t) I_2(t), \\
\frac{dE_2(t)}{dt} = \beta_2 S_2(t) I_2(t) - \beta_2 e^{-d_2\tau_2} S_2(t-\tau_2) I_2(t-\tau_2) - d_2 E_2(t), \\
\frac{dI_2(t)}{dt} = \beta_2 e^{-d_2\tau_2} S_2(t-\tau_2) I_2(t-\tau_2) - (r_2 + d_2 + b_2) I_2(t), \\
\frac{dR_2(t)}{dt} = r_2 I_2(t) - d_2 R_2(t),
\end{array}
\right\} t \neq (n+l)\tau, t \neq (n+1)\tau, \\
\left.
\begin{array}{l}
\triangle S_1(t) = D(S_2(t) - S_1(t)), \\
\triangle E_1(t) = D(E_2(t) - E_1(t)), \\
\triangle I_1(t) = 0, \\
\triangle R_1(t) = D(R_2(t) - R_1(t)), \\
\triangle S_2(t) = D(S_1(t) - S_2(t)), \\
\triangle E_2(t) = D(E_1(t) - E_2(t)), \\
\triangle I_2(t) = 0, \\
\triangle R_2(t) = D(R_1(t) - R_2(t)),
\end{array}
\right\} t = (n+l)\tau, n \in Z^+, \\
\left.
\begin{array}{l}
\triangle S_1(t) = -\mu_1 S_1(t), \\
\triangle E_1(t) = 0, \\
\triangle I_1(t) = 0, \\
\triangle R_1(t) = \mu_1 S_1(t), \\
\triangle S_2(t) = -\mu_2 S_2(t), \\
\triangle E_2(t) = 0, \\
\triangle I_2(t) = 0, \\
\triangle R_2(t) = \mu_2 S_2(t),
\end{array}
\right\} t = (n+1)\tau, n \in Z^+,
\end{array}
\right.
\tag{3}
$$

with initial condition

$$(\varphi_1(\zeta),\varphi_2(\zeta),\varphi_3(\zeta),\varphi_4(\zeta),\varphi_5(\zeta),\varphi_6(\zeta),\varphi_7(\zeta),\varphi_8(\zeta)) \in C_+ = C([-\tau_1,0],R^8_+),$$

and

$$\varphi_i(0) > 0, i = 1,2,3,4,5,6,7,8.$$

where system (3) is constructed of two cities or regions. $S_i(t)$, $E_i(t)$, $I_i(t)$ and $R_i(t)$ represent the number of susceptible, exposed, infected, recovered individuals in city or region $i(i=1,2)$ at time t. It is assumed that we adopt the fixed number of offspring, denoted by $\lambda_i(i=1,2)$, joins into the susceptible class per unit time in city or region $i(i=1,2)$. The natural death rate is assumed as the same constant $d_i(i=1,2)$ for the susceptible, exposed, infected, recovered individuals in city $i(i=1,2)$. Disease is transmitted with the incidence rate, that is , the number of new cases of infection per unit time $\beta_i S_i I_i$ with city or regions $i(i=1,2)$. The transmission rate with city i is a constant $\beta_i(i=1,2)$. The time delay τ_i is the latent period of the disease in city or region $i(i=1,2)$. The infected individuals in city or regions $i(i=1,2)$ suffer an extra disease-related death with constant rate $b_i(i=1,2)$. $r_i(i=1,2)$ is the recovery rate of the infected individuals in city or regions $i(i=1,2)$. By boarding transports, the susceptible and recovered individuals of city or regions i leave to city or regions $j(i\neq j,i,j=1,2)$ with a dispersal rate $D(0<D<1)$ at moment $t=(n+l)\tau, n\in Z_+$. The susceptible is successfully vaccinated with μ_i in city or regions $i(i=1,2)$ at moment $t=(n+1)\tau, n\in Z_+$.

Because $E_i(t)(i=1,2)$ and $R_i(t)(i=1,2)$ do not affect the other equations of (3), we can simplify system (3) and restrict our attention to the following system

$$\left\{\begin{array}{l}
\left.\begin{array}{l}
\frac{dS_1(t)}{dt} = \lambda_1 - d_1S_1(t) - \beta_1S_1(t)I_1(t),\\
\frac{dI_1(t)}{dt} = \beta_1 e^{-d_1\tau_1}S_1(t-\tau_1)I_1(t-\tau_1) - (r_1+d_1+b_1)I_1(t),\\
\frac{dS_2(t)}{dt} = \lambda_2 - d_2S_2(t) - \beta_2S_2(t)I_2(t),\\
\frac{dI_2(t)}{dt} = \beta_2 e^{-d_2\tau_2}S_2(t-\tau_2)I_2(t-\tau_2) - (r_2+d_2+b_2)I_2(t),
\end{array}\right\} t\neq(n+l)\tau, t\neq(n+1)\tau,\\
\left.\begin{array}{l}
\triangle S_1(t) = D(S_2(t)-S_1(t)),\\
\triangle I_1(t) = 0,\\
\triangle S_2(t) = D(S_1(t)-S_2(t)),\\
\triangle I_2(t) = 0,
\end{array}\right\} t=(n+l)\tau, n\in Z^+,\\
\left.\begin{array}{l}
\triangle S_1(t) = -\mu_1S_1(t),\\
\triangle I_1(t) = 0,\\
\triangle S_2(t) = -\mu_2S_2(t),\\
\triangle I_2(t) = 0,
\end{array}\right\} t=(n+1)\tau, n\in Z^+,
\end{array}\right. \tag{4}$$

with initial condition

$$(\varphi_1(\zeta),\varphi_3(\zeta),\varphi_5(\zeta),\varphi_7(\zeta)) \in C_+ = C([-\tau_1,0],R^4_+),$$

and

$$\varphi_i(0) > 0, i = 1,3,5,7.$$

3. The lemmas

The solution of (3), denote by $X(t) = (S_1(t), E_1(t), I_1(t), R_1(t), S_2(t), E_2(t), I_2(t),\ R_2(t))^T$, is a piecewise continuous function $X : R_+ \to R_+^8$, $X(t)$ is continuous on $(n\tau,(n+l)\tau]$, $((n+l)\tau,(n+1)\tau]$, $n \in Z_+$ and $X(n\tau^+) = \lim_{t\to n\tau^+} X(t)$, $X((n+l)\tau^+) = \lim_{t\to(n+l)\tau^+} X(t)$ exist. Obviously the global existence and uniqueness of solutions of (3) are guaranteed by the smoothness properties of f, which denotes the mapping defined by right-side of system (3) (see Lakshmikantham,[27]). Before we have the the main results. we need give some lemmas which will be used in the next.

According to the biological meanings, it is assumed that $S_i(t) \geq 0, E_i(t) \geq 0, I_i(t) \geq 0$, and $R_i(t) \geq 0 (i = 1,2)$.

Let $V : R_+ \times R_+^8 \to R_+$, then V is said to belong to class V_0, if

i) V is continuous in $(n\tau,(n+l)\tau] \times R_+^8$ and $((n+l)\tau,(n+1)\tau] \times R_+^8$, for each $z \in R_+^8, n \in Z_+$, $V(n\tau^+, z) = \lim_{(t,y)\to(n\tau^+,z)} V(t,y)$, $V((n+l)\tau^+, z) = \lim_{(t,y)\to((n+l)\tau^+,y)} V(t,y)$ exist.

ii) V is locally Lipschitzian in z.

Definition 3.1. $V \in V_0$, then, for $(t,z) \in (n\tau,(n+l)\tau] \times R_+^6$ and $((n+l)\tau,(n+1)\tau] \times R_+^6$, the upper right derivative of $V(t,z)$ with respect to the impulsive differential system (3) is defined as

$$D^+V(t,z) = \limsup_{h\to 0} \frac{1}{h}[V(t+h, z+hf(t,z)) - V(t,z)].$$

Lemma 3.2. [31] Let the function$m \in PC'[R^+, R]$satisfies the inequalities

$$\begin{cases} m'(t) \leq p(t)m(t) + q(t), \\ t \geq t_0, t \neq t_k, k = 1,2,\cdots, \\ m(t_k^+) \leq d_k m(t_k)) + b_k, t = t_k, \end{cases} \tag{5}$$

where $p,q \in PC[R^+, R]$ and $d_k \geq 0, b_k$ are constants ,then

$$m(t) \leq m(t_0) \prod_{t_0<t_k<t} d_k exp(\int_{t_0}^t p(s)ds)$$

$$+ \sum_{t_0<t_k<t} (\prod_{t_k<t_j<t} d_j exp(\int_{t_0}^t p(s)ds)) b_k$$

$$+\int_{t_0}^{t}\prod_{s<t_k<t} d_k exp(\int_{s}^{t} p(\sigma)d\sigma)q(s)ds, t\geq t_0.$$

Now, we show that all solutions of (3) are uniformly ultimately bounded.

Lemma 3.3. There exists a constant $M>0$ such that $S_i(t)\leq M, E_i(t)\leq M, I_i(t)\leq M, R_i(t)\leq M (i=1,2)$ for each solution $(S_1(t),E_1(t),I_1(t),R_1(t),S_2(t),E_2(t),I_2(t),R_2(t))$ of (3) with all t large enough.

Proof. Define

$$V(t)=\sum_{i=1}^{2}[S_i(t)+E_i(t)+I_i(t)+R_i(t)],$$

and $d=\min\{d_1,d_2\}$, then $t\neq n\tau, t\neq (n+l)\tau$, we have

$$D^+V(t)+dV(t)=\lambda_1+\lambda_2-\sum_{i=1}^{2}[(d_i-d)S_i(t)+(d_i-d)I_i(t)+(d_i-d)R_i(t)]-\sum_{i=1}^{2}b_iI_i(t)$$

$$\leq\lambda_1+\lambda_2.$$

When $t=n\tau$,

$$V(n\tau^+)=\sum_{i=1}^{2}[S_i(n\tau^+)+I_i(n\tau^+)+R_i(n\tau^+)]$$

$$=\sum_{i=1}^{2}[S_i(n\tau)+E_i(n\tau)+I_i(n\tau)+R_i(n\tau)]=V(n\tau).$$

When $t=(n+l)\tau$,

$$V((n+l)\tau^+)=\sum_{i=1}^{2}[S_i((n+l)\tau^+)+E_i((n+l)\tau^+)+I_i((n+l)\tau^+)+R_i((n+l)\tau^+)]$$

$$=\sum_{i=1}^{2}[S_i((n+l)\tau)+E_i((n+l)\tau)+I_i((n+l)\tau)+R_i((n+l)\tau)]=V((n+l)\tau).$$

By lemma 3.2, for $t\in(n\tau,(n+1)\tau]$, we have

$$V(t)\leq V(0)\exp(-dt)+\int_{0}^{t}(\lambda_1+\lambda_2)\exp(-d(t-s))ds$$

$$=V(0)\exp(-dt)+\frac{\lambda_1+\lambda_2}{d}(1-\exp(-dt))$$

$$\to\frac{\lambda_1+\lambda_2}{d},\quad as\quad t\to\infty.$$

So $V(t)$ is uniformly ultimately bounded. Hence, by the definition of $V(t)$, we have there exists a constant $M>0$ such that $S_i(t)\leq M, E_i(t)\leq M, I_i(t)\leq M, R_i(t)\leq M (i=1,2)$ for t large enough. The proof is complete.

If $I_i(t)=0 (i=1,2)$, we have the following subsystem of (4)

$$
\left\{
\begin{array}{l}
\left.\begin{array}{l}
\dfrac{dS_1(t)}{dt}=\lambda_1-d_1S_1(t),\\
\dfrac{dS_2(t)}{dt}=\lambda_2-d_2S_2(t),
\end{array}\right\} t\neq(n+l)\tau, t\neq(n+1)\tau,\\
\left.\begin{array}{l}
\triangle S_1(t)=D(S_2(t)-S_1(t)),\\
\triangle S_2(t)=D(S_1(t)-S_2(t)),
\end{array}\right\} t=(n+l)\tau,\\
\left.\begin{array}{l}
\triangle S_1(t)=-\mu_1S_1(t),\\
\triangle S_2(t)=-\mu_2S_2(t),
\end{array}\right\} t=(n+1)\tau, n=1,2\cdots.
\end{array}
\right. \tag{6}
$$

We can easily obtain the analytic solution of (6) between pulses as following

$$
\left\{
\begin{array}{l}
S_1(t)=\left\{\begin{array}{l}
\dfrac{1}{d_1}[\lambda_1-(\lambda_1-d_1S_1(n\tau^+))e^{-d_1(t-n\tau)}], t\in[n\tau,(n+l)\tau),\\
\dfrac{1}{d_1}[\lambda_1-(\lambda_1-d_1S_1((n+l)\tau^+))e^{-d_1(t-(n+l)\tau)}], t\in[(n+l)\tau,(n+1)\tau),
\end{array}\right.\\
S_2(t)=\left\{\begin{array}{l}
\dfrac{1}{d_2}[\lambda_2-(\lambda_2-d_2S_2(n\tau^+))e^{-d_2(t-n\tau)}], t\in[n\tau,(n+l)\tau).\\
\dfrac{1}{d_2}[\lambda_2-(\lambda_2-d_2S_2((n+l)\tau^+))e^{-d_2(t-(n+l)\tau)}], t\in[(n+l)\tau,(n+1)\tau).
\end{array}\right.
\end{array}
\right. \tag{7}
$$

Considering the third and fourth equations of (6), we have

$$
\left\{
\begin{array}{l}
S_1((n+l)\tau^+)=\dfrac{1-D}{d_1}[\lambda_1-(\lambda_1-d_1S_1(n\tau^+))e^{-d_1l\tau)}]\\
\qquad\qquad\qquad+\dfrac{D}{d_2}[\lambda_2-(\lambda_2-d_2S_2(n\tau^+))e^{-d_2l\tau)}],\\
S_2((n+l)\tau^+)=\dfrac{D}{d_1}[\lambda_1-(\lambda_1-d_1S_1(n\tau^+))e^{-d_1l\tau)}]\\
\qquad\qquad\qquad+\dfrac{1-D}{d_2}[\lambda_2-(\lambda_2-d_2S_2(n\tau^+))e^{-d_2l\tau)}].
\end{array}
\right. \tag{8}
$$

Considering the fifth and sixth equations of (6), we also have

$$
\left\{
\begin{array}{l}
S_1((n+1)\tau^+)=\dfrac{1-\mu_1}{d_1}[\lambda_1-(\lambda_1-d_1S_1((n+l)\tau^+))e^{-d_1(1-l)\tau)}],\\
S_2((n+1)\tau^+)=\dfrac{1-\mu_2}{d_2}[\lambda_2-(\lambda_2-d_2S_2((n+l)\tau^+))e^{-d_2(1-l)\tau)}].
\end{array}
\right. \tag{9}
$$

Substituting (8) into (7), we have the stroboscopic map of (6)

$$
\left\{
\begin{array}{l}
S_1((n+1)\tau^+)=(1-\mu_1)(1-D)e^{-d_1\tau}S_1(n\tau^+)+(1-\mu_1)De^{-[d_1(1-l)+d_2l]\tau}S_2(n\tau^+)\\
\qquad+(1-\mu_1)\times[\dfrac{\lambda_1(1-e^{-d_1l\tau})(1-(1-D)e^{-d_1(1-l)\tau})}{d_1}+\dfrac{D\lambda_2(1-e^{-d_2l\tau})e^{-d_1(1-l)\tau}}{d_2}],\\
S_2((n+1)\tau^+)=(1-\mu_2)De^{-[d_1l+d_2(1-l)]\tau}S_1(n\tau^+)+(1-\mu_2)(1-D)e^{-d_2\tau}S_2(n\tau^+)\\
\qquad+(1-\mu_2)\times[\dfrac{D\lambda_1(1-e^{-d_1l\tau})e^{-d_2(1-l)\tau}}{d_1}+\dfrac{\lambda_2(1-e^{-d_2l\tau})(1-(1-D)e^{-d_2(1-l)\tau})}{d_2}].
\end{array}
\right. \tag{10}
$$

(10) has one fixed point as

$$
\begin{cases} S_1^* = \dfrac{(1-A_1)B - AA_2}{(1-A_1)(1-B_2)-A_2B_1} > 0, \\ S_2^* = \dfrac{B_1B - A(1-B_2)}{(1-A_1)(1-B_2)-A_2B_1} > 0, \end{cases} \tag{11}
$$

where

$$A_1 = (1-\mu_1)(1-D)e^{-d_1\tau}(0 < A_1 < 1),$$

$$B_1 = (1-\mu_1)De^{-[d_1(1-l)+d_2 l]\tau}(0 < B_1 < 1),$$

$$A_2 = (1-\mu_2)De^{-[d_1 l+d_2(1-l)]\tau}(0 < A_2 < 1),$$

$$B_2 = (1-\mu_2)(1-D)e^{-d_2\tau}(0 < B_2 < 1),$$

$$A = (1-\mu_1)\times[\frac{\lambda_1(1-e^{-d_1 l\tau})(1-(1-D)e^{-d_1(1-l)\tau})}{d_1} + \frac{D\lambda_2(1-e^{-d_2 l\tau})e^{-d_1(1-l)\tau}}{d_2}] > 0,$$

$$B = (1-\mu_2)\times[\frac{D\lambda_1(1-e^{-d_1 l\tau})e^{-d_2(1-l)\tau}}{d_1} + \frac{\lambda_2(1-e^{-d_2 l\tau})(1-(1-D)e^{-d_2(1-l)\tau})}{d_2}] > 0.$$

Lemma 3.4. The unique fixed point (S_1^*, S_2^*) of (10) is globally asymptotically stable.

Proof. For convenience, we make a notation as $(S_1^n, S_2^n) = (S_1(n\tau^+), S_2(n\tau^+))$. The linear form of (10) can be written as

$$
\begin{pmatrix} S_1^{n+1} \\ S_2^{n+1} \end{pmatrix} = M \begin{pmatrix} S_1^n \\ S_2^n \end{pmatrix}. \tag{12}
$$

Obviously, the near dynamics of (S_1^*, S_2^*) is determined by linear system (10). The stabilities of (S_1^*, S_2^*) is determined by the eigenvalue of M less than 1. If M satisfies the *Jury* criteria[43], we can know the eigenvalue of M less than 1,

$$1 - trM + \det M > 0. \tag{13}$$

We can easily know that (S_1^*, S_2^*) is unique fixed point of (10), and

$$
M = \begin{pmatrix} A_1 & B_1 \\ A_2 & B_2 \end{pmatrix}. \tag{14}
$$

For

$$
\begin{aligned}
1 - trM + \det M &= 1 - (A_1 + B_2) + (A_1B_2 - A_2B_1) \\
&= (1-A_1)(1-B_2) - A_2B_1 \\
&= [(1-(1-\mu_1)e^{-d_1\tau}) + (1-\mu_1)De^{-d_1\tau}][(1-(1-\mu_2)e^{-d_2\tau})
\end{aligned}
$$

$$+(1-\mu_2)De^{-d_2\tau}]-(1-\mu_1)(1-\mu_2)D^2e^{-(d_1+d_2)\tau}$$

$$=[1-(1-\mu_1)e^{-d_1\tau}][1-(1-\mu_2)e^{-d_2\tau}]$$

$$+[1-(1-\mu_1)e^{-d_1\tau}](1-\mu_2)De^{-d_2\tau}+[1-(1-\mu_2)e^{-d_2\tau}](1-\mu_1)De^{-d_1\tau}$$

$$>0.$$

From *Jury* criteria, (S_1^*, S_2^*) is locally stable. Because the fixed point (S_1^*, S_2^*) of (10) is unique, then, it is globally asymptotically stable. This completes the proof.

Lemma 3.5. The periodic solution $(\widetilde{S_1(t)}, \widetilde{S_2(t)})$ of System (6) is globally asymptotically stable, where

$$(15)\qquad \begin{cases} \widetilde{S_1(t)} = \begin{cases} \dfrac{1}{d_1}[\lambda_1-(\lambda_1-d_1S_1^*)e^{-d_1(t-n\tau)}], t\in[n\tau,(n+l)\tau), \\ \dfrac{1}{d_1}[\lambda_1-(\lambda_1-d_1S_1^{**})e^{-d_1(t-(n+l)\tau)}], t\in[(n+l)\tau,(n+1)\tau), \end{cases} \\ \widetilde{S_2(t)} = \begin{cases} \dfrac{1}{d_2}[\lambda_2-(\lambda_2-d_2S_2^*)e^{-d_2(t-n\tau)}], t\in[n\tau,(n+l)\tau). \\ \dfrac{1}{d_2}[\lambda_2-(\lambda_2-d_2S_2^{**})e^{-d_2(t-(n+l)\tau)}], t\in[(n+l)\tau,(n+1)\tau), \end{cases} \end{cases}$$

here S_1^* and S_2^* are determined as (11), S_1^{**} and S_2^{**} are defined as

$$(16)\qquad \begin{cases} S_1^{**} = \dfrac{1-D}{d_1}[\lambda_1-(\lambda_1-d_1S_1^*)e^{-d_1l\tau)}] \\ \qquad\qquad +\dfrac{D}{d_2}[\lambda_2-(\lambda_2-d_2S_2^*)e^{-d_2l\tau)}], \\ S_2^{**} = \dfrac{D}{d_1}[\lambda_1-(\lambda_1-d_1S_1^*)e^{-d_1l\tau)}] \\ \qquad\qquad +\dfrac{1-D}{d_2}[\lambda_2-(\lambda_2-d_2S_2^*)e^{-d_2l\tau)}]. \end{cases}$$

Lemma 3.6.[42] Consider the following equation

$$\frac{dx(t)}{dt} = a_1x(t-\omega)-a_2x(t),$$

where $a_1, a_2, \omega > 0$; $x(t) > 0$ for $-\omega \le t \le 0$, we have

(i) if $a_1 < a_2$, then, $\lim_{t\to\infty} x(t) = 0$,

(ii) if $a_1 > a_2$, then, $\lim_{t\to\infty} x(t) = +\infty$.

4. The dynamics

From the above discussion, we know there exists a infection-free boundary periodic solution $(\widetilde{S_1(t)}, 0, \widetilde{S_2(t)}, 0)$ of system (4). In this section, we will prove that the infection-free boundary periodic solution $(\widetilde{S_1(t)}, 0, \widetilde{S_2(t)}, 0)$ of system (4) is globally attractive.

Theorem 4.1. If

$$\max_{i=1,2}\{\beta_i e^{-d_i\tau_i}[\frac{2\lambda_i}{d_i}+(S_i^*+S_i^{**})]-(r_i+d_i+b_i)\}<0(i=1,2), \tag{17}$$

holds, the infection-free boundary periodic solution $(\widetilde{S_1(t)},0,\widetilde{S_2(t)},0)$ of (4) is globally attractive, where $S_i^*(i=1,2)$ is determined as (11), $S_i^{**}(i=1,2)$ is defined as (16).

Proof. From (17), we can obtain

$$\beta_i e^{-d_i\tau_i}[\frac{2\lambda_i}{d_i}+(S_i^*+S_i^{**})]<(r_i+d_i+b_i)(i=1,2). \tag{18}$$

Then, we can choose ε_0 sufficiently small such that

$$\beta_i e^{-d_i\tau_i}\{[\frac{2\lambda_i}{d_i}+(S_i^*+S_i^{**})]+\varepsilon_0\}<(r_i+d_i+b_i)(i=1,2). \tag{19}$$

From the first and third equations of system (4), we obtain that $\frac{dS_i(t)}{dt}\leq\lambda_i-d_1S_i(t)(i=1,2)$. So we consider the following comparison impulsive differential system

$$\begin{cases}\left.\begin{array}{l}\dfrac{dx_1(t)}{dt}=\lambda_1-d_1x_1(t),\\ \dfrac{dx_2(t)}{dt}=\lambda_2-d_2x_2(t),\end{array}\right\}t\neq(n+l)\tau,t\neq(n+1)\tau,\\ \left.\begin{array}{l}\triangle x_1(t)=D(x_2(t)-x_1(t)),\\ \triangle x_2(t)=D(x_1(t)-x_2(t)),\end{array}\right\}t=(n+l)\tau,\\ \left.\begin{array}{l}\triangle x_1(t)=-\mu_1x_1(t),\\ \triangle x_2(t)=-\mu_2x_2(t),\end{array}\right\}t=(n+1)\tau,n=1,2\cdots.\end{cases} \tag{20}$$

In view of lemma 3.4. and (15), we obtain that the boundary periodic solution of system (20)

$$\begin{cases}\widetilde{x_1(t)}=\begin{cases}\dfrac{1}{d_1}[\lambda_1-(\lambda_1-d_1S_1^*)e^{-d_1(t-n\tau)}],t\in[n\tau,(n+l)\tau),\\ \dfrac{1}{d_1}[\lambda_1-(\lambda_1-d_1S_1^{**})e^{-d_1(t-(n+l)\tau)}],t\in[(n+l)\tau,(n+1)\tau),\end{cases}\\ \widetilde{x_2(t)}=\begin{cases}\dfrac{1}{d_2}[\lambda_2-(\lambda_2-d_2S_2^*)e^{-d_2(t-n\tau)}],t\in[n\tau,(n+l)\tau).\\ \dfrac{1}{d_2}[\lambda_2-(\lambda_2-d_2S_2^{**})e^{-d_2(t-(n+l)\tau)}],t\in[(n+l)\tau,(n+1)\tau),\end{cases}\end{cases} \tag{21}$$

is globally asymptotically stable, where S_1^* and S_2^* are determined as (11), S_1^{**} and S_2^{**} are defined as (16).

From lemma 3.5. and comparison theorem of impulsive equation [2], we have $S_i(t) \le x_i(t)(i=1,2)$ and $x_i(t) \to \widetilde{S_i(t)}$ as $t \to \infty$. Then there exists an integer $k_2 > k_1, t > k_2$ such that

$$S_i(t) \le x_i(t) \le \widetilde{S_i(t)} + \varepsilon_0 (i=1,2), n\tau < t \le (n+1)\tau, n > k_2,$$

that is

$$S_i(t) < \widetilde{S_i(t)} + \varepsilon_0 \le [\frac{2\lambda_i}{d_i} + (S_i^* + S_i^{**})] + \varepsilon_0 \stackrel{\Delta}{=} \rho (i=1,2), n\tau < t \le (n+1)\tau, n > k_2.$$

From (4), we get

$$\frac{dI_i(t)}{dt} \le \beta_i e^{-d_i\tau_i} \rho I_i(t-\tau_1) - (r_i + d_i + b_i) I_i(t)(i=1,2)), t > n\tau + \tau_1, n > k_2, \tag{22}$$

Consider the following comparison differential system referring to (20)

$$\frac{dy_i(t)}{dt} = \beta_i e^{-d_i\tau_i} \rho y_i(t-\tau_1) - (r_i + d_i + b_i) y_i(t)(i=1,2)), t > n\tau + \tau_1, n > k_2, \tag{23}$$

From (19) and Lemma 3.6., we have $\lim_{t\to\infty} y_i(t) = 0$.

Let $(S_1(t), I_1(t), S_2(t), I_2(t))$ be the solution of system (20) with initial conditions and $I_1(\zeta) = \varphi_3(\zeta)(\zeta \in [-\tau_1, 0]), I_2(\zeta) = \varphi_7(\zeta)(\zeta \in [-\tau_1, 0])$. $y_i(t)(i=1,2)$ is the solution of system (23) with initial conditions $y_1(\zeta) = \varphi_3(\zeta)(\zeta \in [-\tau_1, 0])$, $y_2(\zeta) = \varphi_7(\zeta)(\zeta \in [-\tau_1, 0])$. By the comparison theorem, we have

$$\lim_{t\to\infty} I_i(t) < \lim_{t\to\infty} y_i(t) = 0.$$

Incorporating into the positivity of $I_i(t)$, we know that $\lim_{t\to\infty} I_I(t) = 0$, Therefore, for any $\varepsilon_1 > 0$ (sufficiently small), there exists an integer $k_3 (k_3\tau > k_2\tau + \tau_1)$ such that $I_i(t) < \varepsilon_1 (i=1,2)$ for all $t > k_3\tau$.

For system (4), we have

$$\lambda_i - (d_i + \beta_i \varepsilon_1) S_i(t) \le \frac{dS_i(t)}{dt} \le \lambda_i - d_i S_i(t), \tag{24}$$

Then we have $z_i(t) \leq S_i(t) \leq z_i'(t)$ and $z_i(t) \to \widetilde{z_i(t)}, z_i'(t) \to \widetilde{S_i(t)}$ as $t \to \infty$. While $(z_1(t), z_2(t))$ and $(z_1'(t), z_2'(t)$ are the solutions of

$$\left\{\begin{array}{l} \left.\begin{array}{l} \dfrac{dz_1(t)}{dt} = \lambda_1 - (d_1 + \beta_1\varepsilon_1)z_1(t), \\ \dfrac{dz_2(t)}{dt} = \lambda_2 - (d_2 + \beta_2\varepsilon_1)z_2(t), \end{array}\right\} t \neq (n+l)\tau, t \neq (n+1)\tau, \\ \left.\begin{array}{l} \triangle z_1(t) = D(z_2(t) - z_1(t)), \\ \triangle z_2(t) = D(z_1(t) - z_2(t)), \end{array}\right\} t = (n+l)\tau, \\ \left.\begin{array}{l} \triangle z_1(t) = -\mu_1 z_1(t), \\ \triangle z_2(t) = -\mu_2 z_2(t), \end{array}\right\} t = (n+1)\tau, n = 1, 2\cdots. \end{array}\right. \tag{25}$$

and

$$\left\{\begin{array}{l} \left.\begin{array}{l} \dfrac{dz_1'(t)}{dt} = \lambda_1 - d_1 z_1'(t), \\ \dfrac{dz_2'(t)}{dt} = \lambda_2 - d_2 z_2'(t), \end{array}\right\} t \neq (n+l)\tau, t \neq (n+1)\tau, \\ \left.\begin{array}{l} \triangle z_1'(t) = D(z_2'(t) - z_1'(t)), \\ \triangle z_2'(t) = D(z_1'(t) - z_2'(t)), \end{array}\right\} t = (n+l)\tau, \\ \left.\begin{array}{l} \triangle z_1'(t) = -\mu_1 z_1'(t), \\ \triangle z_2'(t) = -\mu_2 z_2'(t), \end{array}\right\} t = (n+1)\tau, n = 1, 2\cdots. \end{array}\right. \tag{26}$$

respectively. Where

(27)

$$\left\{\begin{array}{l} \widetilde{z_1(t)} = \left\{\begin{array}{l} \dfrac{1}{(d_1+\beta_1\varepsilon_1)}[\lambda_1 - (\lambda_1 - (d_1+\beta_1\varepsilon_1)z_1^*)e^{-(d_1+\beta_1\varepsilon_1)(t-n\tau)}], t \in [n\tau, (n+l)\tau), \\ \dfrac{1}{(d_1+\beta_1\varepsilon_1)}[\lambda_1 - (\lambda_1 - (d_1+\beta_1\varepsilon_1)z_1^{**})e^{-(d_1+\beta_1\varepsilon_1)(t-(n+l)\tau)}], t \in [(n+l)\tau, (n+1)\tau), \end{array}\right. \\ \widetilde{z_2(t)} = \left\{\begin{array}{l} \dfrac{1}{(d_2+\beta_2\varepsilon_1)}[\lambda_2 - (\lambda_2 - (d_2+\beta_2\varepsilon_1)z_2^*)e^{-(d_2+\beta_2\varepsilon_1)(t-n\tau)}], t \in [n\tau, (n+l)\tau). \\ \dfrac{1}{(d_2+\beta_2\varepsilon_1)}[\lambda_2 - (\lambda_2 - (d_2+\beta_2\varepsilon_1)z_2^{**})e^{-(d_2+\beta_2\varepsilon_1)(t-(n+l)\tau)}], t \in [(n+l)\tau, (n+1)\tau), \end{array}\right. \end{array}\right.$$

here

$$\left\{\begin{array}{l} z_1^* = \dfrac{(1-A_1')B' - A'A_2'}{(1-A_1')(1-B_2') - A_2'B_1'} > 0, \\ z_2^* = \dfrac{B_1'B' - A'(1-B_2')}{(1-A_1')(1-B_2') - A_2'B_1'} > 0, \end{array}\right. \tag{28}$$

and

$$(29)\qquad \begin{cases} z_1^{**} = \dfrac{1-D}{(d_1+\beta_1\varepsilon_1)}[\lambda_1 - (\lambda_1 - (d_1+\beta_1\varepsilon_1)z_1^*)e^{-(d_1+\beta_1\varepsilon_1)l\tau)}] \\ \qquad\qquad + \dfrac{D}{(d_2+\beta_2\varepsilon_1)}[\lambda_2 - (\lambda_2 - (d_2+\beta_2\varepsilon_1)z_2^*)e^{-(d_2+\beta_2\varepsilon_1)l\tau)}], \\ z_2^{**} = \dfrac{D}{(d_1+\beta_1\varepsilon_1)}[\lambda_1 - (\lambda_1 - (d_1+\beta_1\varepsilon_1)z_1^*)e^{-(d_1+\beta_1\varepsilon_1)l\tau)}] \\ \qquad\qquad + \dfrac{1-D}{(d_2+\beta_2\varepsilon_1)}[\lambda_2 - (\lambda_2 - (d_2+\beta_2\varepsilon_1)z_2^*)e^{-(d_2+\beta_2\varepsilon_1)l\tau)}]. \end{cases}$$

and

$$A_1' = (1-\mu_1)(1-D)e^{-(d_1+\beta_1\varepsilon_1)\tau}(0 < A_1' < 1),$$

$$B_1' = (1-\mu_1)De^{-[(d_1+\beta_1\varepsilon_1)(1-l)+(d_2+\beta_2\varepsilon_1)l]\tau}(0 < B_1' < 1),$$

$$A_2' = (1-\mu_2)De^{-[(d_1+\beta_1\varepsilon_1)l+(d_2+\beta_2\varepsilon_1)(1-l)]\tau}(0 < A_2' < 1),$$

$$B_2' = (1-\mu_2)(1-D)e^{-(d_2+\beta_2\varepsilon_1)\tau}(0 < B_2' < 1),$$

$$A' = (1-\mu_1)\times[\frac{\lambda_1(1-e^{-(d_1+\beta_1\varepsilon_1)l\tau})(1-(1-D)e^{-(d_1+\beta_1\varepsilon_1)(1-l)\tau})}{(d_1+\beta_1\varepsilon_1)} + \frac{D\lambda_2(1-e^{-(d_2+\beta_2\varepsilon_1)l\tau})e^{-(d_1+\beta_1\varepsilon_1)(1-l)\tau}}{(d_2+\beta_2\varepsilon_1)}] > 0,$$

$$B' = (1-\mu_2)\times[\frac{D\lambda_1(1-e^{-(d_1+\beta_1\varepsilon_1)l\tau})e^{-(d_2+\beta_2\varepsilon_1)(1-l)\tau}}{(d_1+\beta_1\varepsilon_1)} + \frac{\lambda_2(1-e^{-(d_2+\beta_2\varepsilon_1)l\tau})(1-(1-D)e^{-(d_2+\beta_2\varepsilon_1)(1-l)\tau})}{(d_2+\beta_2\varepsilon_1)}] > 0.$$

Therefore, for any $\varepsilon_2 > 0$. there exists a integer $k_4, n > k_4$ such that $\widetilde{z_i(t)} - \varepsilon_2 < S_i(t) < \widetilde{z_i'(t)} + \varepsilon_2 (i = 1,2)$. Let $\varepsilon_1 \to 0$, so we have $\widetilde{S_i(t)} - \varepsilon_2 < S_i(t) < \widetilde{S_i(t)} + \varepsilon_2 (i = 1,2)$, for t large enough. Which implies $S_i(t) \to \widetilde{S_i(t)} (i = 1,2)$ as $t \to \infty$. This completes the proof.

The next work is to investigate the permanence of the system(3). Before starting our theorem, we give the following definition.

Definition 4.2. System (4) is said to be permanent if there are constants $m, M > 0$ (independent of initial value) and a finite time T_0 such that for all solutions $(S_1(t), I_1(t), S_2(t), I_2(t))$ with all initial values $S_1(0^+) > 0, I_1(0^+) > 0, S_2(0^+) > 0, I_2(0^+) > 0$, $m \le S_1(t) \le M, m \le I_1(t) \le M, m \le S_2(t) \le M, m \le I_2(t) \le M$ holds for all $t \ge T_0$. Here T_0 may depend on the initial values $(S_1(0^+), I_1(0^+), S_2(0^+), I_2(0^+))$.

Theorem 4.3. If

$$\min_{i=1,2}\{\beta_i e^{-d_i\tau_i}[v_i^* e^{-(d_i+\beta_i I_i^*)l\tau} + v_i^{**} e^{-(d_i+\beta_i I_i^*)(1-l)\tau}] - (r_i + d_i + b_i)\} > 0,$$

there is a positive constant q such that each positive solution $(S_1(t), I_1(t), S_2(t), I_2(t))$ of (2.4) satisfies $I_i(t) \geq q$, for t large enough, where $I_i^*(i=1,2)$ is decided by

$$\beta_i e^{-d_i\tau_i}[v_i^* e^{-(d_i+\beta_i I_i^*)l\tau} + v_i^{**} e^{-(d_i+\beta_i I_i^*)(1-l)\tau}] = (r_i + d_i + b_i)(i=1,2),$$

here $v_i^*(i=1,2)$ and $v_i^{**}(i=1,2)$ are defined as (35) and (36) respectively.

Proof. The second and fourth equations of (4) can be rewritten as

$$\begin{aligned} \frac{dI_i(t)}{dt} &= [\beta_i e^{-d_i\tau_i} S_i(t) - (r_i + d_i + b_i] I_i(t) \\ &\quad - \beta_i e^{-d_i\tau_i} \frac{d}{dt} \int_{t-\tau_i}^{t} S_i(u) I_i(u) du (i=1,2). \end{aligned} \tag{30}$$

According to(30), $Q_i(t)(i=1,2)$ is defined as

$$Q_i(t) = I_i(t) + \beta_i e^{-d_i\tau_i} \int_{t-\tau_1}^{t} S_i(u) I_i(u) du (i=1,2).$$

We calculate the derivative of $Q_i(t)(i=1,2)$ along the solution of (4)

$$\frac{dQ_i(t)}{dt} = [\beta_i e^{-d_i\tau_i} S_i(t) - (r_i + d_i + b_i] I_i(t)(i=1,2). \tag{31}$$

Since

$$\beta_i e^{-d_i\tau_i}[v_i^* e^{-(d_i+\beta_i I_i^*)l\tau} + v_i^{**} e^{-(d_i+\beta_i I_i^*)(1-l)\tau}] > r_i + d_i + b_i(i=1,2),$$

we can easily know that there exists sufficiently small $\varepsilon > 0$ such that

$$\beta_i e^{-d_i\tau_i}\{[v_i^* e^{-(d_i+\beta_i I_i^*)l\tau} + v_i^{**} e^{-(d_i+\beta_i I_i^*)(1-l)\tau}] - \varepsilon\} > r_i + d_i + b_i(i=1,2),$$

We claim that for any $t_0 > 0$, it is impossible that $I_i(t) < I_i^*(i=1,2)$ for all $t > t_0$. Suppose that the claim is not valid. Then there is a $t_0 > 0$ such that $I_i(t) < I_i^*(i=1,2)$ for all $t > t_0$. It follows from the first and third equations of (4) that for all $t > t_0$

$$\frac{dS_i(t)}{dt} > \lambda_i - (d_i + \beta_i I_i^*) S_i(t)(i=1,2). \tag{32}$$

Consider the following comparison impulsive system for all $t > t_0$

$$(33)\qquad \begin{cases} \left.\begin{aligned} \frac{dv_1(t)}{dt} &= \lambda_1 - (d_1+\beta_1 I_1^*)v_1(t), \\ \frac{dv_2(t)}{dt} &= \lambda_2 - (d_2+\beta_2 I_2^*)v_2(t), \end{aligned}\right\} t \neq (n+l)\tau, t \neq (n+1)\tau, \\ \left.\begin{aligned} \triangle v_1(t) &= D(v_2(t)-v_1(t)), \\ \triangle v_2(t) &= D(v_1(t)-v_2(t)), \end{aligned}\right\} t = (n+l)\tau, \\ \left.\begin{aligned} \triangle v_1(t) &= -\mu_1 v_1(t), \\ \triangle v_2(t) &= -\mu_2 v_2(t), \end{aligned}\right\} t = (n+1)\tau, n = 1,2\cdots. \end{cases}$$

By lemma 3.5., we obtain

$$(34)\qquad \begin{cases} \widetilde{v_1(t)} = \begin{cases} \frac{1}{(d_1+\beta_1 I_1^*)}[\lambda_1 - (\lambda_1 - (d_1+\beta_1 I_1^*)v_1^*)e^{-(d_1+\beta_1 I_1^*)(t-n\tau)}], t \in [n\tau, (n+l)\tau), \\ \frac{1}{(d_1+\beta_1 I_1^*)}[\lambda_1 - (\lambda_1 - (d_1+\beta_1 I_1^*)v_1^{**})e^{-(d_1+\beta_1 I_1^*)(t-(n+l)\tau)}], t \in [(n+l)\tau, (n+1)\tau), \end{cases} \\ \widetilde{v_2(t)} = \begin{cases} \frac{1}{(d_2+\beta_2 I_2^*)}[\lambda_2 - (\lambda_2 - (d_2+\beta_2 I_2^*)v_2^*)e^{-(d_2+\beta_2 I_2^*)(t-n\tau)}], t \in [n\tau, (n+l)\tau). \\ \frac{1}{(d_2+\beta_2 I_2^*)}[\lambda_2 - (\lambda_2 - (d_2+\beta_2 I_2^*)v_2^{**})e^{-(d_2+\beta_2 I_2^*)(t-(n+l)\tau)}], t \in [(n+l)\tau, (n+1)\tau), \end{cases} \end{cases}$$

is the unique positive periodic solution of (34). Here

$$(35)\qquad \begin{cases} v_1^* = \frac{(1-A_1'')B'' - A''A_2''}{(1-A_1'')(1-B_2'') - A_2''B_1''} > 0, \\ v_2^* = \frac{B_1''B'' - A''(1-B_2'')}{(1-A_1'')(1-B_2'') - A_2''B_1''} > 0, \end{cases}$$

and

$$(36)\qquad \begin{cases} v_1^{**} = \frac{1-D}{(d_1+\beta_1 I_1^*)}[\lambda_1 - (\lambda_1 - (d_1+\beta_1 I_1^*)v_1^*)e^{-(d_1+\beta_1 I_1^*)l\tau)}] \\ \qquad\qquad + \frac{D}{(d_2+\beta_2 I_2^*)}[\lambda_2 - (\lambda_2 - (d_2+\beta_2 I_2^*)v_2^*)e^{-(d_2+\beta_2 I_2^*)l\tau)}], \\ v_2^{**} = \frac{D}{(d_1+\beta_1 I_1^*)}[\lambda_1 - (\lambda_1 - (d_1+\beta_1 I_1^*)v_1^*)e^{-(d_1+\beta_1 I_1^*)l\tau)}] \\ \qquad\qquad + \frac{1-D}{(d_2+\beta_2 I_2^*)}[\lambda_2 - (\lambda_2 - (d_2+\beta_2 I_2^*)v_2^*)e^{-(d_2+\beta_2 I_2^*)l\tau)}]. \end{cases}$$

and

$$A_1'' = (1-\mu_1)(1-D)e^{-(d_1+\beta_1 I_1^*)\tau}(0 < A_1'' < 1),$$

$$B_1'' = (1-\mu_1)De^{-[(d_1+\beta_1 I_1^*)(1-l)+(d_2+\beta_2 I_2^*)l]\tau}(0 < B_1'' < 1),$$

$$A_2'' = (1-\mu_2)De^{-[(d_1+\beta_1 I_1^*)l+(d_2+\beta_2 I_1^*)(1-l)]\tau}(0 < A_2'' < 1),$$

$$B_2'' = (1-\mu_2)(1-D)e^{-(d_2+\beta_2 I_1^*)\tau}(0 < B_2'' < 1),$$

$$A'' = (1-\mu_1)\times[\frac{\lambda_1(1-e^{-(d_1+\beta_1 I_1^*)l\tau})(1-(1-D)e^{-(d_1+\beta_1 I_1^*)(1-l)\tau})}{(d_1+\beta_1 I_1^*)}$$
$$+\frac{D\lambda_2(1-e^{-(d_2+\beta_2 I_2^*)l\tau})e^{-(d_1+\beta_1 I_1^*)(1-l)\tau}}{(d_2+\beta_2 I_2^*)}] > 0,$$
$$B'' = (1-\mu_2)\times[\frac{D\lambda_1(1-e^{-(d_1+\beta_1 I_1^*)l\tau})e^{-(d_2+\beta_2 I_1^*)(1-l)\tau}}{(d_1+\beta_1 I_1^*)}$$
$$+\frac{\lambda_2(1-e^{-(d_2+\beta_2 I_2^*)l\tau})(1-(1-D)e^{-(d_2+\beta_2 I_2^*)(1-l)\tau})}{(d_2+\beta_2 I_2^*)}] > 0.$$

By the comparison theorem for impulsive differential equation [28], we know that there exists sufficient small $\varepsilon > 0$ and $t_1(> t_0+\tau_1)$ such that the inequality $S_i(t) \geq \widetilde{v_i(t)} - \varepsilon(i=1,2)$ holds for $t \geq t_1$, thus $S_i(t) \geq [v_i^* e^{-(d_i+\beta_i I_i^*)l\tau} + v_i^{**} e^{-(d_i+\beta_i I_i^*)(1-l)\tau}] - \varepsilon$ for all $t \geq t_1$. We make notation as $\sigma_i \stackrel{\Delta}{=} [v_i^* e^{-(d_i+\beta_i I_i^*)l\tau} + v_i^{**} e^{-(d_i+\beta_i I_i^*)(1-l)\tau}] - \varepsilon(i=1,2))$ for convenience. So we have

$$\beta_i e^{-d_i\tau_i}\sigma > r_i + d_i + b_i(i=1,2),$$

then we have

$$Q_i'(t) > y_2(t)[\beta_i e^{-d_i\tau_i}\sigma - (r_i+d_i+b_i)](i=1,2),$$

for all $t > t_1$. Set $I_i^m = \min_{t\in[t_1,t_1+\tau_1]} I_i(t)$, we will show that $I_i(t) \geq I_i^m$ for all $t \geq t_1$. Suppose the contrary, then there is a $T_0 > 0$ such that $I_i(t) \geq I_i^m$ for $t_1 \leq t \leq t_1+\tau_1+T_0, I_i(t_1+\tau_1+T_0) = I_i^m$ and $I_i'(t_1+\tau_1+T_0) < 0$. Hence, the second and fourth equations of system (4) imply that

$$I_i'(t_1+\tau_1+T_0) = \beta_i e^{-d_i\tau_i} S_i(t_1+\tau_1+T_0) I_i(t_1+\tau_1+T_0) - (r_i+d_i+b_i) I_i(t_1+\tau_1+T_0),$$
$$\geq [\beta_i e^{-d_i\tau_i}\sigma - (r_i+d_i+b_i)] I_i^m > 0,$$

This is a contradiction. Thus, $I_i(t) \geq I_i^m$ for all $t > t_1$. As a consequence, Then $Q_i'(t) > I_i^m(\beta_i e^{-d_i\tau_i}\sigma - (r_i+d_i+b_i) > 0$ for all $t > t_1$. This implies that as $t \to \infty$, $Q_i(t) \to \infty$. It is a contradiction to $Q_i(t) \leq M(1+\tau_1\beta_i e^{-d_i\tau_i} M)$. Hence, the claim is complete.

By the claim, we are left to consider two case. First, $I_i(t) \geq I_i^*(i=1,2)$ for all t large enough. Second, $I_i(t)(i=1,2)$ oscillates about $I_i^*(i=1,2)$ for t large enough.

Define

$$q = \min\{\frac{I_1^*}{2}, \frac{I_2^*}{2}, q_1, q_2\}, \tag{37}$$

where $q_i = I_i^* e^{-(r_i+d_i+b_i)\tau_i}(i=1,2)$. We hope to show that $I_i(t) \geq q(i=1,2)$ for all t large enough. The conclusion is evident in first case. For the second case, let $t^* > 0$ and $\xi > 0$ satisfy $I_i(t^*) = I_i(t^*+\xi) = I_i^*(i=1,2)$ and $I_i(t) < I_i^*(i=1,2)$ for all $t^* < t < t^*+\xi$ where t^* is sufficiently large such that

$I_i(t) > \sigma(i=1,2)$ for $t^* < t < t^*+\xi$, $I_i(t)(i=1,2)$ is uniformly continuous. The positive solutions of (4) are ultimately bounded and $I_i(t)(i=1,2)$ is not affected by impulses. Hence, there is a $T(0<t<\tau_1$ and T is dependent of the choice of t^* such that $I_i(t^*) > \frac{I_i^*}{2}(i=1,2)$ for $t^* < t < t^*+T$. If $\xi < T$, there is nothing to prove. Let us consider the case $T < \xi < \tau_1$. Since $I_i'(t) > -(r_i+d_i+b_i)I_i(t)(i=1,2)$ and $I_i(t^*) = I_i^*(i=1,2)$, it is clear that $I_i(t) \geq q_i(i=1,2)$ for $t \in [t^*, t^*+\tau_1]$. Then, proceeding exactly as the proof for the above claim. We see that $I_i(t) \geq q_i$ for $t \in [t^*+\tau_1, t^*+\xi]$. Because the kind of interval $t \in [t^*, t^*+\xi]$ is chosen in an arbitrary way (we only need t^* to be large). We concluded $I_i(t) \geq q$ for all large t. In the second case. In view of our above discussion, the choice of q is independent of the positive solution, and we proved that any positive solution of (4) satisfies $I_i(t) \geq q$ for all sufficiently large t. This completes the proof of the theorem.

From theorem 4.3., we can easily obtain the following two corollaries.

Corollary 4.4. If

$$\min_{i=1,2}\{\beta_i e^{-d_i\tau_i} v_i^* e^{-(d_i+\beta_i I_i^*)l\tau} - (r_i+d_i+b_i)\} > 0,$$

there is a positive constant q such that each positive solution $(S_1(t), I_1(t), S_2(t), I_2(t))$ of (4) satisfies $I_i(t) \geq q$, for t large enough, where $I_i^*(i=1,2)$ is decided by

$$\beta_i e^{-d_i\tau_i}[v_i^* e^{-(d_i+\beta_i I_i^*)l\tau} + v_i^{**} e^{-(d_i+\beta_i I_i^*)(1-l)\tau}] = (r_i+d_i+b_i)(i=1,2),$$

here $v_i^*(i=1,2)$ is defined as (35).

Corollary 4.5. If

$$\min_{i=1,2}\{\beta_i e^{-d_i\tau_i} v_i^{**} e^{-(d_i+\beta_i I_i^*)(1-l)\tau} - (r_i+d_i+b_i)\} > 0,$$

there is a positive constant q such that each positive solution $(S_1(t), I_1(t), S_2(t), I_2(t))$ of (4) satisfies $I_i(t) \geq q$, for t large enough, where $I_i^*(i=1,2)$ is decided by

$$\beta_i e^{-d_i\tau_i} v_i^{**} e^{-(d_i+\beta_i I_i^*)(1-l)\tau} = (r_i+d_i+b_i)(i=1,2),$$

here $v_i^{**}(i=1,2)$ is defined as (36).

Theorem 4.6. If

$$\min_{i=1,2}\{\beta_i e^{-d_i\tau_i}[v_i^* e^{-(d_i+\beta_i I_i^*)l\tau} + v_i^{**} e^{-(d_i+\beta_i I_i^*)(1-l)\tau}] - (r_i+d_i+b_i)\} > 0,$$

system (4) is permanent.

Proof. Denote $(S_1(t), I_1(t), S_2(t), I_2(t))$ be any solution of system (4). From system (4) and lemma 3.3., we can easily obtain

$$\frac{dS_i(t)}{dt} > \lambda_i - (d_i + \beta_i M) S_i(t) (i = 1, 2). \tag{38}$$

Consider the following comparison impulsive system for all $t > t_0$

$$\begin{cases} \left.\begin{aligned} \frac{du_1(t)}{dt} &= \lambda_1 - (d_1 + \beta_1 M) u_1(t), \\ \frac{du_2(t)}{dt} &= \lambda_2 - (d_2 + \beta_2 M) u_2(t), \end{aligned}\right\} t \neq (n+l)\tau, t \neq (n+1)\tau, \\ \left.\begin{aligned} \triangle u_1(t) &= D(u_2(t) - u_1(t)), \\ \triangle u_2(t) &= D(u_1(t) - u_2(t)), \end{aligned}\right\} t = (n+l)\tau, \\ \left.\begin{aligned} \triangle u_1(t) &= -\mu_1 u_1(t), \\ \triangle u_2(t) &= -\mu_2 u_2(t), \end{aligned}\right\} t = (n+1)\tau, n = 1, 2 \cdots. \end{cases} \tag{39}$$

By lemma 3.5., we obtain

$$\begin{cases} \widetilde{u_1(t)} = \begin{cases} \frac{1}{(d_1 + \beta_1 M)} [\lambda_1 - (\lambda_1 - (d_1 + \beta_1 M) u_1^*) e^{-(d_1 + \beta_1 M)(t - n\tau)}], t \in [n\tau, (n+l)\tau), \\ \frac{1}{(d_1 + \beta_1 M)} [\lambda_1 - (\lambda_1 - (d_1 + \beta_1 M) u_1^{**}) e^{-(d_1 + \beta_1 M)(t - (n+l)\tau)}], t \in [(n+l)\tau, (n+1)\tau), \end{cases} \\ \widetilde{u_2(t)} = \begin{cases} \frac{1}{(d_2 + \beta_2 M)} [\lambda_2 - (\lambda_2 - (d_2 + \beta_2 M) u_2^*) e^{-(d_2 + \beta_2 M)(t - n\tau)}], t \in [n\tau, (n+l)\tau). \\ \frac{1}{(d_2 + \beta_2 M)} [\lambda_2 - (\lambda_2 - (d_2 + \beta_2 M) u_2^{**}) e^{-(d_2 + \beta_2 M)(t - (n+l)\tau)}], t \in [(n+l)\tau, (n+1)\tau), \end{cases} \end{cases} \tag{40}$$

is the unique positive periodic solution of (39). Here

$$\begin{cases} u_1^* = \frac{(1 - A_1''')B''' - A'''A_2'''}{(1 - A_1''')(1 - B_2''') - A_2'''B_1'''} > 0, \\ u_2^* = \frac{B_1'''B''' - A'''(1 - B_2''')}{(1 - A_1''')(1 - B_2''') - A_2'''B_1'''} > 0, \end{cases} \tag{41}$$

and

$$\begin{cases} u_1^{**} = \frac{1 - D}{(d_1 + \beta_1 M)} [\lambda_1 - (\lambda_1 - (d_1 + \beta_1 M) u_1^*) e^{-(d_1 + \beta_1 M) l\tau}] \\ \qquad + \frac{D}{(d_2 + \beta_2 M)} [\lambda_2 - (\lambda_2 - (d_2 + \beta_2 M) u_2^*) e^{-(d_2 + \beta_2 M) l\tau}], \\ u_2^{**} = \frac{D}{(d_1 + \beta_1 M)} [\lambda_1 - (\lambda_1 - (d_1 + \beta_1 M) u_1^*) e^{-(d_1 + \beta_1 M) l\tau}] \\ \qquad + \frac{1 - D}{(d_2 + \beta_2 M)} [\lambda_2 - (\lambda_2 - (d_2 + \beta_2 M) u_2^*) e^{-(d_2 + \beta_2 M) l\tau}]. \end{cases} \tag{42}$$

and

$$A_1''' = (1-\mu_1)(1-D)e^{-(d_1+\beta_1 M)\tau}(0 < A_1''' < 1),$$

$$B_1''' = (1-\mu_1)De^{-[(d_1+\beta_1 M)(1-l)+(d_2+\beta_2 M)l]\tau}(0 < B_1''' < 1),$$

$$A_2''' = (1-\mu_2)De^{-[(d_1+\beta_1 M)l+(d_2+\beta_2 M)(1-l)]\tau}(0 < A_2''' < 1),$$

$$B_2''' = (1-\mu_2)(1-D)e^{-(d_2+\beta_2 M)\tau}(0 < B_2''' < 1),$$

$$A''' = (1-\mu_1)\times[\frac{\lambda_1(1-e^{-(d_1+\beta_1 M)l\tau})(1-(1-D)e^{-(d_1+\beta_1 M)(1-l)\tau})}{(d_1+\beta_1 M)} + \frac{D\lambda_2(1-e^{-(d_2+\beta_2 M)l\tau})e^{-(d_1+\beta_1 M)(1-l)\tau}}{(d_2+\beta_2 M)}] > 0,$$

$$B''' = (1-\mu_2)\times[\frac{D\lambda_1(1-e^{-(d_1+\beta_1 M)l\tau})e^{-(d_2+\beta_2 M)(1-l)\tau}}{(d_1+\beta_1 M)} + \frac{\lambda_2(1-e^{-(d_2+\beta_2 M)l\tau})(1-(1-D)e^{-(d_2+\beta_2 M)(1-l)\tau})}{(d_2+\beta_2 M)}] > 0.$$

By the comparison theorem for impulsive differential equation [28], we know that there exists sufficient small $\varepsilon > 0$ and $t_1(> t_0+\tau_1)$ such that the inequality $S_i(t) \geq \widetilde{u_i(t)} - \varepsilon(i=1,2)$ holds for $t \geq t_1$, thus $S_i(t) \geq [u_i^* e^{-(d_i+\beta_i M)l\tau} + u_i^{**}e^{-(d_i+\beta_i M)(1-l)\tau}] - \varepsilon \triangleq p_i$ for all $t \geq t_1$. By theorem 4.3. and lemma3.3. and the above discussion, system (4) is permanent. The proof of theorem 4.6. is complete.

From theorem 4.6., we can also easily obtain the following two corollaries.

Corollary 4.7. If

$$\min_{i=1,2}\{\beta_i e^{-d_i\tau_i} v_i^* e^{-(d_i+\beta_i I_i^*)l\tau} - (r_i+d_i+b_i)\} > 0,$$

system (4) is permanent.

Corollary 4.8. If

$$\min_{i=1,2}\{\beta_i e^{-d_i\tau_i} v_i^{**} e^{-(d_i+\beta_i I_i^*)(1-l)\tau} - (r_i+d_i+b_i)\} > 0,$$

system (4) is permanent.

5. Discussion

In this paper, we investigate a delayed SEIR epidemic model with pulse vaccination and restricting the infected dispersal. We analyze that the infection-free boundary periodic solution of system (4) is globally attractive, and we also obtain the permanent condition of system (4). From theorem 4.1. and theorem 4.6., we can easily guess that there must exist a threshold μ^*. If $\mu > \tau^*$, the infection-free boundary periodic solution $(\widetilde{S_1(t)}, 0, \widetilde{S_2(t)}, 0)$ of (4) is globally attractive. If $\mu < \mu^*$, system (4) is permanent. From theorem 4.1. and theorem 4.6., we can also easily guess that there must exist a threshold $D^*(0 < D^* < 1)$.

If $D < D^*$, the infection-free boundary periodic solution $(\widetilde{S_1(t)}, 0, \widetilde{S_2(t)}, 0)$ of (4) is globally attractive. If $D > D^*$, system (4) is permanent. This indicates that restricting the pulse vaccination and dispersal amount of population can affect the eliminating disease. That is to say, pulse vaccination and restricting the dispersal amount of population play important roles for eliminating disease of system (4). The parameters as $\tau_i (i = 1, 2)$ and τ can also be discussed, its change also affect the dynamical system of (4). The results of this paper provide tactical basis for eliminating disease.

Conflict of Interests

The authors declare that there is no conflict of interests.

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