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A BASIC GENERAL MODEL OF VECTOR-BORNE DISEASES

S. Y. TCHOUMI[1,*], J. C. KAMGANG[2], D. TIEUDJO[2], G. SALLET[3],

[1]Department of Mathematics and Computer Sciences, Faculty of Sciences, University of Ngaoundere, P. O. Box 454, NGaoundere, Cameroon

[2]Department of Mathematics and Computer Sciences, ENSAI – University of NGaoundere, P. O. Box 455 NGaoundere, Cameroon

[3]INRIA-Lorraine and Laboratoire de Mathmatiques et Applications de Metz UMR CNRS 7122, University of Metz, 57045 Metz Cedex 01, France

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Abstract. We propose a model that can translate the dynamics of vector-borne diseases, for this model we compute the basic reproduction number and show that if $\mathscr{R}_0 < \zeta < 1$ the DFE is globally asymptotically stable. For $\mathscr{R}_0 > 1$ we prove the existence of a unique endemic equilibrium and if $\mathscr{R}_0 \leq 1$ the system can have one or two endemic equilibrium, we also show the existence of a backward bifurcation. By numerical simulations we illustrate with data on malaria all the results including existence, stability and bifurcation.

Keywords: epidemiological model; sensitivity analysis; basic reproduction number; global asymptotic stability; bifurcation; simulation.

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*Corresponding author

E-mail address: sytchoumi83@gmail.com

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1. Introduction

The vector-borne diseases are responsible for more than 17% of infectious diseases, and causes over one million deaths each year [18]. Mathematical models help to better understand and propose solutions to reduce the negative impact of these disease in society, and several studies have already been made on diseases such as malaria, leishmaniasis, trypanosomiasis just to name a few (see [15, 6, 19, 16, 1, 2, 10, 11]). We realise that the behavior of these diseases can be modeled by a generic model. Thus in this first work, we propose a model of one population that can translate the dynamics of several vector-borne diseases.

We take a different approach in modeling vectors, drawing on the work Ngwa, Ngonghala [8, 4, 5] which integrates the three phases of the gonotrophic cycle. We assume as in [13] that rest and laying occur in the same place ie we have a questing phase and another phase resting. In addition to the consideration of the gonotriphic cycle, we integrate the management of the sporogonic cycle, that is to say that we consider the fact that after the first meal infecting the vector can transmit the disease only after a certain number of meals (This number depends on the species). In the population of host we introduce the two parameters $u, v \in \{0,1\}$ opposite to [12] where $u, v \in]0,1[$ thereby controlling the presence or absence of the compartments E of exposed and R of immune. This allows us to place ourselves in one of *SIS*, *SIRS*, *SEIS* or *SEIRS* dynamics, we study the existence and stability of equilibrium and the bifurcation.

2. Model description and mathematical specification

In our model, we consider two populations, namely a population of host which may be humans or animals and a vector population that can be specified according to the disease that one wishes to model.

2.1. Host population structure and dynamics

The host population is subdivided in four compartments: susceptible, infected, infectious and immune as shown in the graph above. The parameters $u, v \in \{0;1\}$ are used to choose

the dynamics in the host population. Thus according to the values of u and v we can have the dynamics *SIS*, *SIRS*, *SEIS* or *SEIRS*.

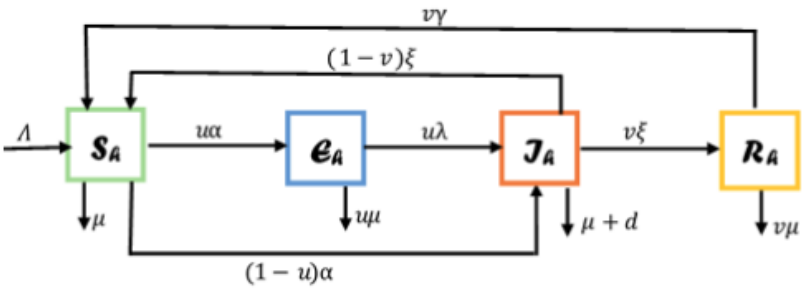

FIGURE 1. Dynamics in the host population

2.2. Mosquito population structure and dynamics

In the vector population we adopt the questing-resting as described in [13], but to simplify the calculations we consider the questing-resting phase number equal to 1. The figure below illustrates the dynamics in Population of vectors.

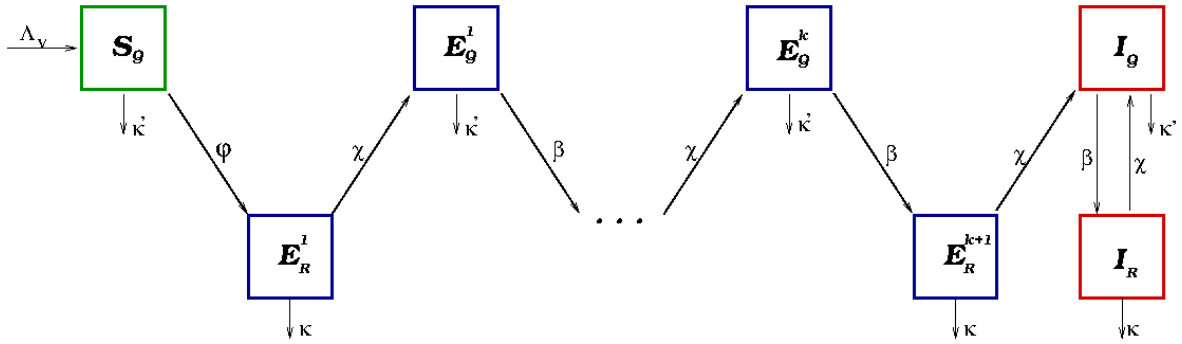

FIGURE 2. Mosquito population dynamics

The variables, parameters and ranges of the values of the model are presented in the following table.

TABLE 1. Variable of model

Variable	Description
humans	
S_h	Number of susceptible humans in the population
E_h	Number of infected humans in the population
I_h	Number of infectious humans in the population
R_h	Number of immune humans in the population
mosquitoes	
S_q	Number of questing susceptible mosquitoes
E_q^i	Number of questing infected mosquitoes in step i
E_r^i	Number of resting infected mosquitoes in step i
I_q	Number of questing infectious mosquitoes
I_r	Number of resting infectious mosquitoes

TABLE 2. fundamental model parameter

Parameter	Description	Unity
human		
Λ_h	immigration in the host population	$h \times j^{-1}$
γ	rate of lost of immunity in the host population	j^{-1}
λ	rate of transition for infected to infectious in the host population	j^{-1}
ξ	rate of recovery in the host population	j^{-1}
μ	death rate in the host population	j^{-1}
d	disease-induced death rate in the host population	j^{-1}
a	number of bites on humans by a single female mosquito per unit time	
m	Infectivity coefficient of hosts due to bite of infectious vector	variable
Mosquitoes		
Λ_v	imigration of vectors	$m \times j^{-1}$
χ	Rate at which resting vectors move to the questing state	j^{-1}
β	Rate at which quessting vectors move to the resting state	j^{-1}
κ	death rate of resting vectors	j^{-1}
$\kappa^{'}$	death rate of questing vectors	j^{-1}
c	Infectivity coefficient of vector due to bite of infectious host .	variable
$\tilde{c}$	Infectivity coefficient of vector due to bite of removed host group.	variable

TABLE 3. Derived model parameters

Param.	Formula	Description
α	$\frac{amI_Q}{N_h}$	incidence rate of susceptible human
φ	$\frac{acI_h}{N_h}+\frac{av\tilde{c}R_h}{N_h}$	incidence rate of susceptible mosquitoes

2.3. Model equation

The diagrams (1) and (2) allow us to have the following system of equations:

$$
(1)\quad \begin{cases}
\dot{S}_h &= \Lambda_h + v\gamma R_h + \tilde{v}\xi I_h - (\mu+\alpha)S_h & \tilde{v}=1-v\\
\dot{E}_h &= u\left[\alpha S_h - (\mu+\lambda)E_h\right] &\\
\dot{I}_h &= u\lambda E_h + \tilde{u}\alpha S_h - [\hat{\mu}+\xi]I_h & \hat{\mu}=\mu+d;\quad \tilde{u}=1-u\\
\dot{R}_h &= v\left[\xi I_h - (\mu+\gamma)R_h\right] &\\
\dot{S}_q &= \Lambda_v - (\kappa^{'}+\varphi)S_q &\\
\dot{E}_r^0 &= \varphi S_q - (\kappa+\chi)E_r^0 &\\
\dot{E}_q^1 &= \chi E_r^0 - (\kappa^{'}+\beta)E_q^1 &\\
\dot{E}_r^1 &= \beta E_q^1 - (\kappa+\chi)E_r^1 &\\
\dot{I}_q &= \chi E_r^1 - (\kappa^{'}+\beta)I_q + \chi I_r &\\
\dot{I}_r &= \beta I_q - (\kappa+\chi)I_r &
\end{cases}
$$

3. Well-posedness, dissipativity

In this section we demonstrate well-posedness of the model by demonstrating invariance of the set of non-negative states, as well as boundedness properties of the solution. We also calculate the equilibria of the system.

3.1. Positive invariance of the non-negative cone in state space

The system (1) can be rewritten in the matrix form as

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{b}(\mathbf{x}) \Longleftrightarrow \begin{cases} \dot{\mathbf{x}}_S &= \mathbf{A}_S(\mathbf{x}).\mathbf{x}_S + \mathbf{A}_{SI}(\mathbf{x})\mathbf{x}_I + \mathbf{b}_S \\ \dot{\mathbf{x}}_I &= \mathbf{A}_I(\mathbf{x}).\mathbf{x}_I \end{cases} \tag{2}$$

Equation (2) is defined for values of the state variable $\mathbf{x} = (\mathbf{x}_S; \mathbf{x}_I)$ lying in the non-negative cone of $\mathbb{R}^{10}_+$. Here $\mathbf{x}_S = \left(S_h; S_q\right)$ represents the naive component and

$\mathbf{x}_I = \left(E_h; E_r^0; E_q^1; E_r^1; I_r; I_q; I_h; R_h\right)$ represents the infected and infectious components of the state of the system.

The matrix $\mathbf{A}_S(\mathbf{x})$, $\mathbf{A}_{SI}(\mathbf{x})$ and $\mathbf{A}_I(\mathbf{x})$ are define as

$$\mathbf{A}_S(\mathbf{x}) = \begin{pmatrix} -(\alpha+\mu) & 0 \\ 0 & -(\varphi+\kappa^{'}) \end{pmatrix}$$

$$\mathbf{A}_{SI}(\mathbf{x}) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \tilde{v}\xi & v_1\gamma_1 \\ & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and

(3)

$$\mathbf{A}_I(\mathbf{x}) = \begin{pmatrix} -u(\lambda+\mu) & 0 & 0 & 0 & 0 & \frac{amu}{N_h}S_h & 0 & 0 \\ 0 & -(\chi+\kappa) & 0 & 0 & 0 & 0 & \frac{ac}{N_h}S_q & \frac{av\tilde{c}}{N_h}S_q \\ 0 & \chi & -(\beta+\kappa') & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta & -(\chi+\kappa') & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -(\chi+\kappa) & \beta & 0 & 0 \\ 0 & 0 & 0 & \chi & \chi & -(\beta+\kappa') & 0 & 0 \\ u\lambda & 0 & 0 & 0 & 0 & \frac{am\tilde{u}}{N_h}S_h & -(\xi+\hat{\mu}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & v\xi & -v(\gamma+\mu) \end{pmatrix}$$

For a given $\mathbf{x} \in \mathbb{R}^{11}_+$, the matrices $\mathbf{A}(\mathbf{x})$, $\mathbf{A}_S(\mathbf{x})$ and $\mathbf{A}_I(\mathbf{x})$ are Metzler matrices. The following proposition establishes that system (1) is epidemiologically well posed.

Proposition 3.1 The non-negative cone $\mathbb{R}^{10}_+$ is positive invariant for the system (1).

3.2. Boundedness and dissipativity of the trajectories

Let $N_h^* = \dfrac{\Lambda_h}{\mu}$, $N_v^* = \dfrac{\Lambda_v}{\kappa}$, $N_h^{\#} = \dfrac{\Lambda_h}{\hat{\mu}}$, $N_v^{\#} = \dfrac{\Lambda_v}{\kappa'}$ $(\kappa' = \kappa + d')$.

proposition 3.2. The set $\mathscr{G}$ defined by

$\mathscr{G} = \left\{ \left(S_h; S_q; E_h; E_r^0; E_q^1; E_r^1; I_r; I_q; I_h; R_h\right) \in \mathbb{R}^{10}_+ \mid N_h^{\#} \leq N_h \leq N_h^*, N_v^{\#} \leq N_v \leq N_v^* \right\}$ is *GAS* for the dynamical system (1) defined on $\mathbb{R}^{10}_+$.

4. Equilibria of the system and Computation of the threshold condition

4.1. Disease free equilibrium

We obtain the disease free equilibrium *DFE* after solve the system $A(\mathbf{x}) \times \left(\mathbf{x}_S^\star, 0\right)^T = 0$

Proposition 4.1 The disease free equilibrium of the system (1) is given by:

$$\mathbf{x}^\star = (\mathbf{x}_S^\star, \mathbf{x}_I^\star) = (\mathbf{x}_S^\star, 0) = \left(\frac{\Lambda_h}{\mu}; \frac{\Lambda_v}{\kappa'}; 0\right) \quad with$$

where $f_q = \dfrac{\beta}{\beta+\kappa'}$ and $f_r = \dfrac{\chi}{\chi+\kappa}$ are respectively the questing and the resting frequencies of mosquitoes.

4.2. Basic reproduction number $\mathscr{R}_0$

Unlike the method proposed in [17] we will use the one given in [14] which is more appropriate for systems like what we describe

Proposition 4.1. The basic reproduction number is given by:

$$\mathscr{R}_0 = \mathscr{R}_0^v \times \mathscr{R}_0^h = \frac{a\Lambda_v \left(f_q f_r\right)^2}{\kappa' \beta (1-f_q f_r)} \times \frac{am\mu\left[\lambda+\mu(1-u)\right]\left[v\xi\tilde{c}+(\mu+\gamma)c\right]}{\Lambda_h(\hat{\mu}+\xi)(\mu+\gamma)(\mu+\lambda)} \tag{4}$$

Proof: The matrix of the infected $\mathbf{A}_I(\mathbf{x}^\star)$ can be written in the form

$$\mathbf{A}_I(\mathbf{x}^\star) = \begin{pmatrix} \mathbf{A}_{I_E}(\mathbf{x}^\star) & \mathbf{A}_{I_{E,I}}(\mathbf{x}^\star) \\ \mathbf{A}_{I_{I,E}}(\mathbf{x}^\star) & \mathbf{A}_{I_I}(\mathbf{x}^\star) \end{pmatrix} \tag{5}$$

with

$$\mathbf{A}_{I_E}(\mathbf{x}^\star) = \begin{pmatrix} -(\lambda+\mu)u & 0 & 0 & 0 \\ 0 & -(\chi+\kappa) & 0 & 0 \\ 0 & \chi & -(\beta+\kappa^{'}) & 0 \\ 0 & 0 & \beta & -(\chi+\kappa) \end{pmatrix}$$

$$\mathbf{A}_{I_{E,I}}(\mathbf{x}^\star) = \begin{pmatrix} 0 & \frac{S_h^\star amu}{N_h^\star} & 0 & 0 \\ 0 & 0 & \frac{S_q^\star ac}{N_h^\star} & \frac{S_q^\star a\tilde{c}v}{N_h^\star} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{A}_{I_{I,E}}(\mathbf{x}^\star) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \chi \\ \lambda u & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{A}_{I_I}(\mathbf{x})^\star = \begin{pmatrix} -(\chi+\kappa) & \beta & 0 & 0 \\ \chi & -(\beta+\kappa') & 0 & 0 \\ 0 & \frac{S_h^\star am(1-u)}{N_h^\star} & -(\xi+\hat{\mu}) & 0 \\ 0 & 0 & v\xi & -(\gamma+\mu)v \end{pmatrix}$$

We apply the algorithm given in the proposition to the matrix $\mathbf{A}_I(\mathbf{x}^\star)$ we have : $\mathbf{A}_I(\mathbf{x}^\star)$ is metzler stable if and only if $\mathbf{A}_{I_E}(\mathbf{x}^\star)$ and $\mathbf{A}_{I_I}(\mathbf{x}^\star) - \mathbf{A}_{I_{I,E}}\mathbf{A}_{I_E}^{-1}(\mathbf{x}^\star)\mathbf{A}_{I_{E,I}}(\mathbf{x}^\star)$ are Metzler stable. The matrix $\mathbf{A}_{I_E}(\mathbf{x}^\star)$ is always a Metzler stable matrix. The condition of being a Metzler stable matrix of $\mathbf{A}_I(\mathbf{x}^\star)$ must be deported on the matrix $\mathbf{A}_{I_I}(\mathbf{x}^\star) - \mathbf{A}_{I_{I,E}}\mathbf{A}_{I_E}^{-1}(\mathbf{x}^\star)\mathbf{A}_{I_{E,I}}(\mathbf{x}^\star)$.

We denote by $\mathbf{N}(\mathbf{x}^\star) = \mathbf{A}_{I_I}(\mathbf{x}^\star) - \mathbf{A}_{I_{I,E}}\mathbf{A}_{I_E}^{-1}(\mathbf{x}^\star)\mathbf{A}_{I_{E,I}}(\mathbf{x}^\star)$

$\mathbf{N}(\mathbf{x}^\star)$ it is a 6×6 square matrix that can be decomposed in the following block matrix form :

$$\mathbf{N}(\mathbf{x}^\star) = \begin{pmatrix} \mathbf{N}_{11}(\mathbf{x}^\star) & \mathbf{N}_{12}(\mathbf{x}^\star) \\ \mathbf{N}_{21}(\mathbf{x}^\star) & \mathbf{N}_{22}(\mathbf{x}^\star) \end{pmatrix}$$

$\mathbf{N}_{11}(\mathbf{x}^\star)$ is a 2×2 square matrix given by

$$\mathbf{N}_{11}(\mathbf{x}^\star) = \begin{pmatrix} -(\chi+\kappa) & \beta \\ \chi & -(\beta+\kappa') \end{pmatrix}$$

$\mathbf{N}_{12}(\mathbf{x}^\star)$ is a 2×2 matrix given by

$$\mathbf{N}_{12}(\mathbf{x}^\star) = \begin{pmatrix} 0 & 0 \\ \frac{S_q^\star a\beta c\chi^2}{N_h^\star(\beta+\kappa')(\chi+\kappa)^2} & \frac{S_q^\star a\beta\tilde{c}\chi^2 v}{N_h^\star(\beta+\kappa')(\chi+\kappa)^2} \end{pmatrix}$$

$\mathbf{N}_{21}(\mathbf{x}^\star)$ is a 2×2 matrix given by

$$\mathbf{N}_{21}(\mathbf{x}^\star) = \begin{pmatrix} 0 & \frac{S_h^\star a\lambda mu}{N_h^\star(\lambda+\mu)} + \frac{S_h^\star am(1-u)}{N_h^\star} \\ 0 & 0 \end{pmatrix}$$

$\mathbf{N}_{22}(\mathbf{x}^\star)$ is a 2×2 square matrix given by:

$$\mathbf{N}_{22}(\mathbf{x}^\star) = \begin{pmatrix} -(\xi+\hat{\mu}) & 0 \\ v\xi & -(\gamma+\mu)v \end{pmatrix}$$

We make another iteration of the algorithm given by the proposition

We denote by $\mathbf{L}(\mathbf{x}^\star) = \mathbf{N}_{22}(\mathbf{x}^\star) - \mathbf{N}_{21}(\mathbf{x}^\star)\mathbf{N}_{11}^{-1}(\mathbf{x}^\star)\mathbf{N}_{12}(\mathbf{x}^\star)$

$$\mathbf{L}(\mathbf{x}^\star) = \begin{pmatrix} -(\kappa+\chi) & \beta \\ \chi & \frac{f_q f_r^2 S_q^\star}{N^{\star 2}} \frac{a^2 m S_h^\star [\lambda+\mu\tilde{u}]\,[v\xi_i\tilde{c}+(\mu+\gamma)c]}{(\mu+\lambda)(\hat{\mu}+\xi)(\mu+\gamma)} - (\kappa^{'}+\beta) \end{pmatrix}$$

The last iteration of the algorithm, since $\mathbf{N}_{22}(\mathbf{x}^\star)$ is negative coefficient leads to the consideration of the matrix $\mathbf{L}(\mathbf{x}^\star)$ and thus $\mathbf{L}(\mathbf{x}^\star)$ of being Meztler stable on the unique condition

$$\mathbf{L}_{22}(\mathbf{x}^\star) - \mathbf{L}_{21}(\mathbf{x}^\star)\mathbf{L}_{11}^{-1}(\mathbf{x}^\star)\mathbf{L}_{12}(\mathbf{x}^\star) \leq 0$$

$$\frac{f_q f_r^2 S_q^\star}{N^{\star 2}} \frac{a^2 m S_h^\star [\lambda+\mu\tilde{u}]\,[v\xi_i\tilde{c}+(\mu+\gamma)c]}{(\mu+\lambda)(\hat{\mu}+\xi)(\mu+\gamma)} - (\kappa^{'}+\beta^\star) + \frac{\beta\chi}{(\kappa+\chi)} \leq 0$$

A simple calculation shows that

$$\frac{\Lambda_v \left(f_q f_r\right)^2}{\kappa^{'}\beta(1-f_q f_r)} \frac{a^2 m\mu\,[\lambda+\mu(1-u)]\,[v\xi\tilde{c}+(\mu+\gamma)c]}{\Lambda_h(\hat{\mu}+\xi)(\mu+\gamma)(\mu+\lambda)} \leq 1$$

4.3. Endemic equilibrium

The system (1) admit two equilibriums, one named Disease Free Equilibrium (DFE) defined in the previous subsection and the other named Endemic Equilibrium (EE).

To determine the endemic equilibrium (EE) we must solve equation $A(\mathbf{x}) \times (\mathbf{x}^*)^T = 0$.

Theorem 4.3.

The model (1) has:

(a) if $\mathscr{R}_0 > 1$, the system has a unique endemic equilibrium

(b) if $\mathscr{R}_0 = 1$ and $\mathscr{R}_c < 1$, the system has a unique endemic equilibrium,

(c) if $\mathscr{R}_c < \mathscr{R}_0 < 1$ and $\left(\mathscr{R}_0 = \mathscr{R}_0^1 \ \ or \ \ \mathscr{R}_0 = \mathscr{R}_0^2\right)$, the system has a unique endemic equilibrium,

(d) If $\mathscr{R}_c < \mathscr{R}_0 < min(1, \mathscr{R}_0^1)$ or $min(\mathscr{R}_c, \mathscr{R}_0^2) < \mathscr{R}_0 < 1$, the system has two endemic equilibrium,

(e) No endemic equilibrium elsewhere.

The proof is given in the appendix A.

5. Global asymptotic stability of the Disease Free Equilibrium (DFE)

In this section we analyze the stability of the system equilibria given in Proposition .
We have the following results for the global asymptotic stability of the disease free equilibrium:

Theorem 5.1 Let $\zeta = \frac{\mu}{\mu+d}$, and $\tilde{\mathscr{G}} = \{\mathbf{x} \in \mathscr{G} : \mathbf{x} \neq 0\}$ a positively invariant space. When $\mathscr{R}_0 \leq \zeta$, then the DFE for system (1) is GAS in the sub–domain $\{\mathbf{x} \in \tilde{\mathscr{G}} : \mathbf{x}_I = 0\}$.

Proof: Our proof is based on Theorem 4.3 of Kamgang & Sallet [14] , which establishes global asymptotic stability for epidemiological systems that can be expressed in the matrix form (2). We need only establish for the system (1) that the five conditions (**h**1–**h**5) required in Theorem 4.3 of Kamgang & Sallet [14] are satisfied when $\mathscr{R}_0 \leq \zeta$.

(**h**1) The system (1) is defined on a positively invariant set $\mathbb{R}^{10}_+$ of the non-negative orthant. The system is dissipative on $\tilde{\mathscr{G}}$.

(**h**2) The sub-system $\dot{\mathbf{x}}_S = \mathbf{A}_S(\mathbf{x}_S, 0)(\mathbf{x} - \mathbf{x}^*_S)$ is express like: $\begin{cases} \dot{S}_h = \Lambda_h - \mu S_h \\ \\ \dot{S}_q = \Lambda_v - \kappa^{'} S_q \end{cases}$ is the linear system which is GAS at the DFE $\left(\frac{\Lambda_h}{\mu}; \frac{\Lambda_v}{\kappa^{'}}; 0\right)$. The DFE, satisfying the hypotheses H_2.

(**h**3) The matrix $\mathbf{A}_I(\mathbf{x})$ given by (4) is Metzler. The graph shown in the figure below, whose nodes represent the various infected disease states is strongly connected, which shows that the matrix $\mathbf{A}_I$ is irreductible. In this case, the two properties required for condition ($\mathbf{h}_3$) follow immediately: off-diagonal terms of the matrix $\mathbf{A}_I(\mathbf{x})$ are non–positive; and Figure (3) shows the associated direct graph $G(\mathbf{A}_I(\mathbf{x}))$, which is evidently connected, thus establishing irreducibility.

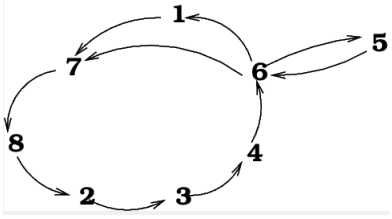

FIGURE 3. graph associated to the matrix $\mathbf{A}_I(\mathbf{x})$

(**h**4) Knowing that $\frac{1}{N_h^{\#}} > \frac{1}{N_h}$, $S_h^* > S_h$ and $S_v^* > S_v$, we obtain the upper bound $\bar{\mathbf{A}}_I$ of $\mathbf{A}_I(\mathbf{x})$ given by:

$$\bar{\mathbf{A}}_I = \begin{pmatrix} M & N \\ P & Q \end{pmatrix}$$

with

$$M = \begin{pmatrix} -(\lambda+\mu)u & 0 & 0 & 0 \\ 0 & -(\chi+\kappa) & 0 & 0 \\ 0 & \chi & -(\beta+\kappa^{'}) & 0 \\ 0 & 0 & \beta & -(\chi+\kappa) \end{pmatrix}$$

$$N = \begin{pmatrix} 0 & \frac{S_h^\star amu}{N_h^{\#}} & 0 & 0 \\ 0 & 0 & \frac{S_q^\star ac}{N_h^{\#}} & \frac{S_q^\star a\tilde{c}v}{N_h^{\#}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$P = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \chi \\ \lambda u & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$Q = \begin{pmatrix} -(\chi+\kappa) & \beta & 0 & 0 \\ \chi & -(\beta+\kappa^{'}) & 0 & 0 \\ 0 & \frac{S_h^\star am(1-u)}{N_h^{\#}} & -(\xi+\hat{\mu}) & 0 \\ 0 & 0 & v\xi & -(\gamma+\mu)v \end{pmatrix}$$

$\mathbf{A}_I(\mathbf{x}) < \bar{\mathbf{A}}_I$ for all $\mathbf{x} \in \mathscr{G}$ and $\mathbf{A}_I(\mathbf{x}^*) = \bar{\mathbf{A}}_I$ for all $\mathbf{x} \in \tilde{\mathscr{G}}$ condition (h_4) is satisfied.

(**h**5) $\alpha(\bar{\mathbf{A}}_I) < 0 \Longleftrightarrow \alpha(Q - PM^{-1}N) < 0$

After tree iterations, we have

$$T = \begin{pmatrix} -(\kappa+\chi) & \beta \\ \chi & \frac{f_q f_r^2 S_q^\star}{N^{\#2}} \frac{a^2 m S_h^\star [\lambda+\mu\tilde{u}][v\xi_i\tilde{c}+(\mu+\gamma)c]}{(\mu+\lambda)(\hat{\mu}+\xi)(\mu+\gamma)} - (\kappa^{'}+\beta) \end{pmatrix}$$

The last iteration gives

$\alpha(\bar{A}_I) < 0 \Longleftrightarrow \mathscr{R}_0 < \dfrac{\mu}{\mu+d}$

Since the five conditions for Theorem 4.3 of Kamgang & Sallet [14] are satisfied, the DFE is GAS when $\mathscr{R}_0 < \dfrac{\mu}{\mu+d}$.

Corollary 5.1 If the disease-induced death rate is 0 ($d = 0$) then, when $\mathscr{R}_0 \leq 1$, then the DFE for system (1) is GAS in the sub–domain $\{\mathbf{x} \in \tilde{\mathscr{G}} : \mathbf{x}_I = 0\}$.

6. Bifurcation analysis

To explore the possibility of bifurcation in our system at critical points, we use the centre manifold theory [3]. A bifurcation parameter $m^\star$ is chosen, by solving $\mathscr{R}_0 = 1$, we have

$$m^\star = \frac{\Lambda_h \kappa^{'} \beta (1 - f_q f_r)(\xi + \hat{\mu})(\lambda + \mu)(\gamma + \mu)}{a^2 \mu \Lambda_v (f_q f_r)^2 (\lambda + \tilde{u}\mu)[c(\gamma + \mu) + c\tilde{c}\xi]}$$

$J_{m^\star}$ is the Jacobian matrix of of system (1) evaluated at the DFE and for $m = m^\star$

$$J_{m^\star} = \begin{pmatrix}
-\mu & 0 & 0 & 0 & 0 & 0 & 0 & -am^\star & \tilde{v}\xi & v\gamma \\
0 & -\kappa^{'} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-acS_q^\star}{N^\star} & \frac{-av\tilde{c}S_q^\star}{N^\star} \\
0 & 0 & -u(\lambda+\mu) & 0 & 0 & 0 & 0 & uam^\star & 0 & 0 \\
0 & 0 & 0 & -(\chi+\kappa) & 0 & 0 & 0 & 0 & \frac{acS_q^\star}{N^\star} & \frac{av\tilde{c}S_q^\star}{N^\star} \\
0 & 0 & 0 & \chi & -(\beta+\kappa^{'}) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \beta & -(\chi+\kappa) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -(\chi+\kappa^{'}) & \beta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \chi & \chi & -(\beta+\kappa^{'}) & 0 & 0 \\
0 & 0 & u\lambda & 0 & 0 & 0 & 0 & \tilde{u}am^\star & -(\xi+\hat{\mu}) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & v\xi & -v(\gamma+\mu)
\end{pmatrix}$$

$$J_{m^\star} = \begin{pmatrix} J^1_{m^\star} & J^3_{m^\star} \\ 0 & J^2_{m^\star} \end{pmatrix}$$

The eigenvalues of this matrix are the eigenvalues of the sub-matrix

$$J^1_{m^\star} = \begin{pmatrix} -\mu & 0 \\ 0 & -\kappa^{'} \end{pmatrix}$$

$$J^2_{m^\star} = \begin{pmatrix} -u(\lambda+\mu) & 0 & 0 & 0 & 0 & uam^\star & 0 & 0 \\ 0 & -(\chi+\kappa) & 0 & 0 & 0 & 0 & \frac{acS^\star_q}{N^\star} & \frac{av\tilde{c}S^\star_q}{N^\star} \\ 0 & \chi & -(\beta+\kappa^{'}) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta & -(\chi+\kappa) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -(\chi+\kappa^{'}) & \beta & 0 & 0 \\ 0 & 0 & 0 & \chi & \chi & -(\beta+\kappa^{'}) & 0 & 0 \\ u\lambda & 0 & 0 & 0 & 0 & \tilde{u}am^\star & -(\xi+\hat{\mu}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & v\xi & -v(\gamma+\mu) \end{pmatrix}$$

The caracteristic polynom of the matrix $J^2_{m^\star}$ is given by:

$$P(x) = x^8 + p_7x^7 + p_6x^6 + p_5x^5 + p_4x^4 + p_3x^3 + p_2x^2 + p_1x + p_0$$

Let $\eta = \hat{\mu} + \xi$, $l_1 = \chi + \kappa$, $l_2 = \beta + \kappa^{'}$, $b_1 = v(\gamma+\mu)$ and $b_2 = u(\lambda+\mu)$

$$p_7 = b_1 + b_2 + \eta + 3l_1 + 3l_2$$

$$p_6 = -\beta\chi + b_1b_2 + (b_1+b_2)(\eta + 3l_1 + 2l_2) + 3l_1(\eta + l_1) + l_2(2\eta + 6l_1 + l_2)$$

$$\begin{aligned} p_5 = \quad & -\beta\chi(\eta + b_1 + b_2 + 2l_1) + b_1b_2\eta + 3l_1(b_2+\eta)(b_1 + l_1 + 2l_2) + l_2(b_1+b_2)(6l_1 + l_2) \\ & + l_1^3 - \beta\chi l_2 + 2l_2(b_1b_2 + \eta(b_1+b_2)) + 3l_1b_2(\eta + l_1) + l_2(6l_1^2 + l_2(\eta + 3l_1)) \end{aligned}$$

$$\begin{aligned} p_4 = \quad & (b_1+b_2+d)(-\beta\chi(2l_1+l_2) + l_1(l_1^2 + 3l_2(l_2+2l_1))) + (b_1b_2 + \eta(b_1+b_2))(-\beta\chi + l_2(l_2+6l_1)) \\ & + 3b_1b_2(l_1(l_1+\eta) + \eta l_2) + l_1^2(3(b_1+b_2) + 2l_1l_2) - \beta\chi l_1(l_1+2l_2) \end{aligned}$$

$$\begin{aligned} p_3 = \quad & -\beta\chi\left[\frac{S^\star_q a^2 cm\tilde{u}}{N} + l_1^2l_2 + 2b_1l_1(b_2+\eta) + b_2\eta(b_1+2l_1) + (b_1+b_2+\eta)(l_1^2+2l_1l_2) + l_2(b_1b_2 + b_1\eta + b_2\eta)\right] \\ & + b_2\eta l_1^2(3b_1+l_1) + b_1l_1^2(b_2+\eta) + b_1b_2\eta l_2^2 + l_1l_2\left[6b_1b_2(\eta+l_1) + 6\eta l_1(b_1+b_2) + (b_1+b_2+\eta)(2l_1^2+3l_1l_2)\right. \\ & \left. + 3l_2(b_1b_2 + b_1\eta + b_2\eta) + l_1^2\right] \end{aligned}$$

$$\begin{aligned} p_2 = \quad & -\frac{S^\star_q a^2\beta\chi^2 m}{N^\star}(\tilde{c}v^2\omega\xi + c\lambda u^2 + c\omega(b_1+l_1+c)) - \beta\chi(b_1b_2\eta(2l_1+l_2) + (b_2+\eta)(b_1l_1^2 + 2b_1l_1l_2 + l_1^2l_2) \\ & + l_1(b_2\eta l_1 + 2b_2\eta l_2 + b_1l_1l_2)) + 3b_1b_2\eta l_1l_2(2l_1+l_2) + l_1^2(b_1+b_2+\eta)(2b_1l_1l_2 + l_2^2) + 3l_1^2l_2^2(b_1b_2 + b_1\eta + b_2\eta) \end{aligned}$$

$$\begin{aligned} p_1 = \quad & l_1^2l_2(b_1b_2(\eta(2+3l_2)+l_2) + \eta l_2(b_1+b_2)) - \beta\chi l_1(b_1b_2(\eta l_1 + 2\eta l_2 + l_1l_2) + l_1l_2\eta(b_1+b_2)) \\ & - \frac{Sq^\star a^2\beta\chi^2 m}{N^\star}\left[\tilde{c}v^2\xi(\lambda u^2 + \tilde{u}(b_2+l_1)) + cu^2\lambda(b_1+l_1) + c\tilde{u}(b_1b_2 + l_1(b_1+b_2))\right] \end{aligned}$$

$$p_0 = \frac{uv\beta\chi(\beta+\kappa^{'})(\chi+\kappa)^2(\hat{\mu}+\xi)(\lambda+\mu)(\gamma+\mu)(1-f_qf_r)}{f_qf_r}\,[1-\mathscr{R}_0]$$

When $\mathscr{R}_0 = 1$, 0 is a eigenvalue of the matrix $J_{m^\star}$

The components of the left eigenvector of $J(\mathbf{x}^\star, m^\star)$ are given by $\mathbf{v} = (v_1, v_2, ..., v_{11})$, where

$$v_1 = v_2 = v_3 = 0; \quad v_6 = \frac{1}{f_r}v_5; \quad v_7 = v_8 = \frac{1}{f_qf_r}v_5; \quad v_9 = \frac{1}{f_q(f_r)^2}$$

$$v_{11} = \frac{S_q^\star a\tilde{c}}{N_h^\star(\gamma+\mu)}v_5; \quad v_{10} = \frac{1}{\xi+\hat{\mu}}\left[\frac{acS_q^\star}{N_h^\star}v_5 + v\xi v_{11}\right]; \quad v_4 = \frac{\lambda u}{\lambda+\mu}v_{10}; \quad v_5 > 0$$

A non-zero components correspond to the infected states.

Similarly, the component of the right eigenvector $\mathbf{w}$ are given by

$$w_6 = \frac{\chi}{\beta}f_qw_5; \quad w_7 = f_qf_rw_6; \quad w_8 = \frac{f_q}{1-f_qf_r}w_7; \quad w_9 = \frac{\chi}{\beta}f_rw_8; \quad w_4 = \frac{am}{\lambda+\mu}w_9; \quad w_{10} = \frac{am\tilde{u}w_9+\lambda uw_4}{\xi+\hat{\mu}}$$

$$w_{11} = \frac{\xi}{\gamma+\mu}w_{10}; \quad w_1 = \frac{\gamma vw_{11}-amw_9+\chi\tilde{v}w_{10}}{\mu}; \quad w_2 = \frac{aS_qf_r}{N_h^\star(1-f_qf_r)}\,[cw_{10}+\tilde{c}vw_{11}]; \quad w_3 = \frac{\chi}{\beta}f_qw_2; \quad w_5 > 0$$

$$\mathscr{A} = \sum_{k,i,j=1}^{11} v_kw_iw_j\frac{\partial^2 f_k}{\partial x_i\partial x_j}(\mathbf{x}^\star, m^\star), \quad \mathscr{B} = \sum_{k,i=1}^{11} v_kw_i\frac{\partial^2 f_k}{\partial x_i\partial m}(\mathbf{x}^\star, m^\star) \tag{6}$$

$$\begin{aligned}
\mathscr{A} =& v_4\sum_{i,j=1}^{11} w_iw_j\frac{\partial f_4(\mathbf{x}^\star, m^\star)}{\partial x_i\partial x_j} + v_5\sum_{i,j=1}^{11} w_iw_j\frac{\partial f_5(\mathbf{x}^\star, m^\star)}{\partial x_i\partial x_j} + v_{10}\sum_{i,j=1}^{11} w_iw_j\frac{\partial f_{10}(\mathbf{x}^\star, m^\star)}{\partial x_i\partial x_j} \\
=& -2v_4w_9\frac{uam^\star}{S_q^\star}(w_4+w_{10}+w_{11}) - 2v_{10}w_9\frac{\tilde{u}am^\star}{S_q^\star}(w_4+w_{10}+w_{11}) + 2v_5w_3\frac{a}{S_q^\star}(cw_{10}+v\tilde{c}w_{11}) \\
=& 2v_5w_3\frac{a}{S_q^\star}(cw_{10}+v\tilde{c}w_{11}) - 2w_9\frac{am^\star}{S_q^\star}(w_4+w_{10}+w_{11})\,[uv_4+\tilde{u}v_{10}] \\
=& \pi_1 - \pi_2
\end{aligned} \tag{7}$$

$$\mathscr{B} = aw_9\,(v_4+\tilde{u}v_{10}) > 0 \tag{8}$$

Theorem 6.1 The model (1) exhibits a backward bifurcation at $\mathscr{R}_0 = 1$ whenever $\mathscr{A} > 0$ $(i.e., \pi_1 > \pi_2)$. If the reversed inequality holds, then the bifurcation at $\mathscr{R}_0 = 1$ is forward.

The proof of the previous theorem is based on Theorem 4.1 in [9],

7. Local sensibility analysis

We have an explicit expression for $\mathscr{R}_0$, we can evaluate the sensitivity index of different parameters intervening in this expression. In addition to the various parameters involved in the model, we also evaluate the sensitivity index for f_q and f_r which are calculated parameters derived from the parameters β and χ.

The formula below proposed in [7] gives us the expression of the index of sensitivity of a parameter q to $\mathscr{R}_0$.

$$\Upsilon_q^{\mathscr{R}_0} = \frac{\partial \mathscr{R}_0}{\partial q} \times \frac{q}{\mathscr{R}_0}$$

Using this expression, and those values $\Lambda_h = \frac{1000}{50\times 365}$; $c = 0.83$; $\tilde{c} = 0.083$; $\xi = 0.017$; $\gamma = 1.4\times 10^{-3}$; $\mu = \frac{1}{50\times 365}$; $d = 4\times 10^{-4}$; $m = 0.27$; $a = 0.56$; $u = 1$; $v = 1$; $\lambda = 0.2$; $\Lambda_v = \frac{10000}{21}$; $\beta = \frac{2}{3}$; $\chi = \frac{1}{5}$; $\kappa = \frac{1}{21}$; $\kappa' = \frac{2}{21}$. we calculated for each of the parameters of the model its sensitivity index. The summary table is given below.

	Parameter	Sensitivity index
1	f_q	4.4098
2	f_r	4.4028
3	a	2
4	Λ_v	1
5	m	1
6	Λ_h	-1
7	μ	-0.9758
8	c	0.4611
9	ξ	-0.4345
10	$\tilde{c}$	0.5388
11	γ	-0.1887
12	d	-0.02291
13	λ	0.0003

TABLE 4. Sensitivity indices of $\mathscr{R}_0$ to parameters for the model

By analyzing the table above we can say that if we do not take into account f_q and f_r the most sensitive parameter of our model is number of bites on humans by a single female mosquito per unit time a As in most works of the literature. On the other hand, by integrating f_q and f_r, these become the most sensitive parameters, ie their modifications can play a large role in reducing the level of infection. In other words, the consideration of the dynamics is questing-resting is important and it is necessary to increase the time from the state questing to the state resting and vis versa. This increase can be made by use of the methods already proposed such as the distance of the gites from the places of dwellings, notably by making cleanliness around the dwellings

8. Numerical Simulation

We performed simulations for the particular case of malaria and for *SEIRS* i.e $u = 1$ and $v = 1$ dynamics in humans. All the results of stability, existence of endemic equilibrium and bifurcation established above are illustrated by graphs.

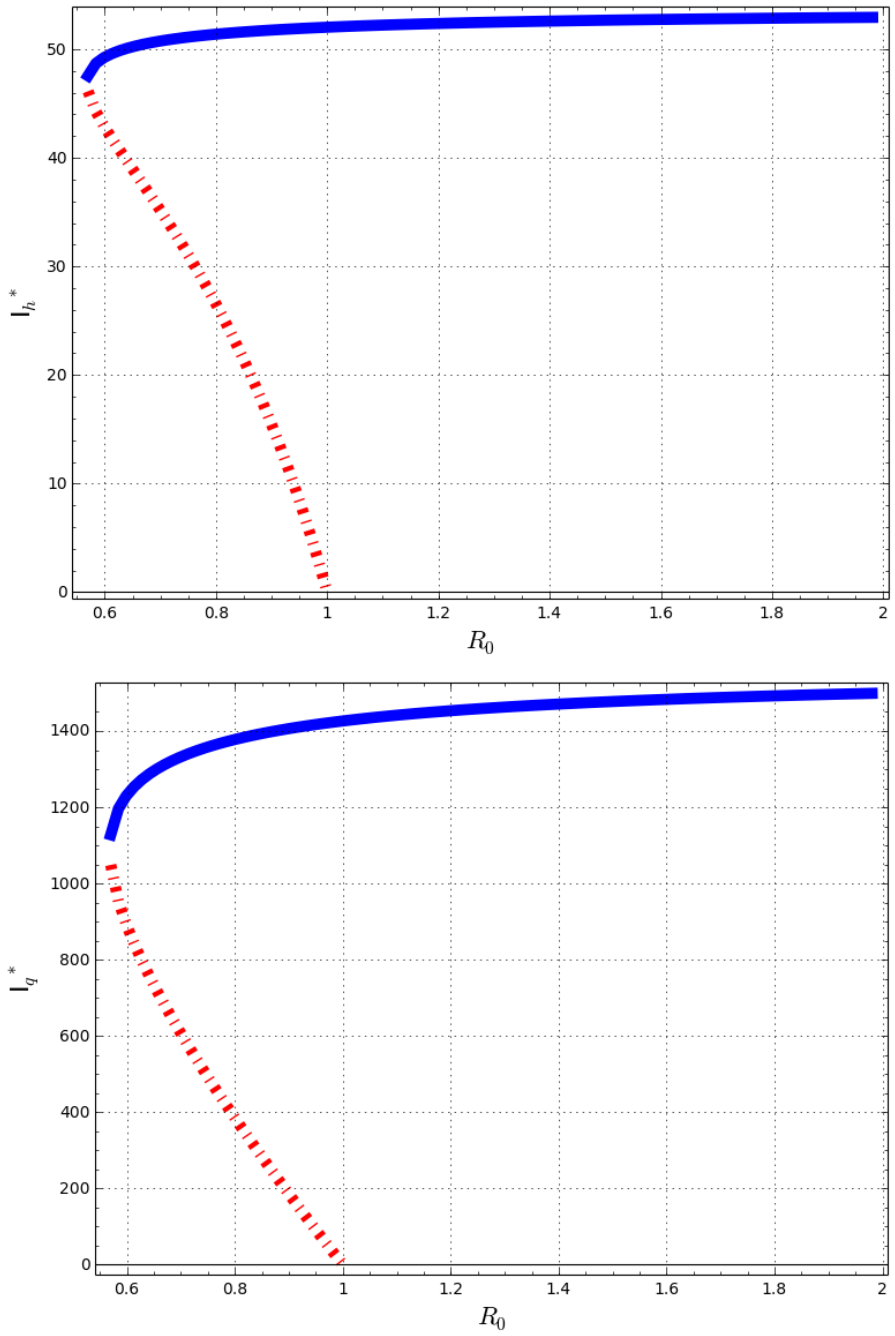

FIGURE 4. Those graphics presents the backward bifurcation for model system (1) curve of I_h^* and I_q^* as a function of $\mathscr{R}_0$ for values of the bifurcation parameter m ranging from 0.0004 to 0.0014 . It illustrates for $\mathscr{R}_0 < 1$ the existence of two endemic equilibria of which, one unstable represented in red and the other stable represented in blue.

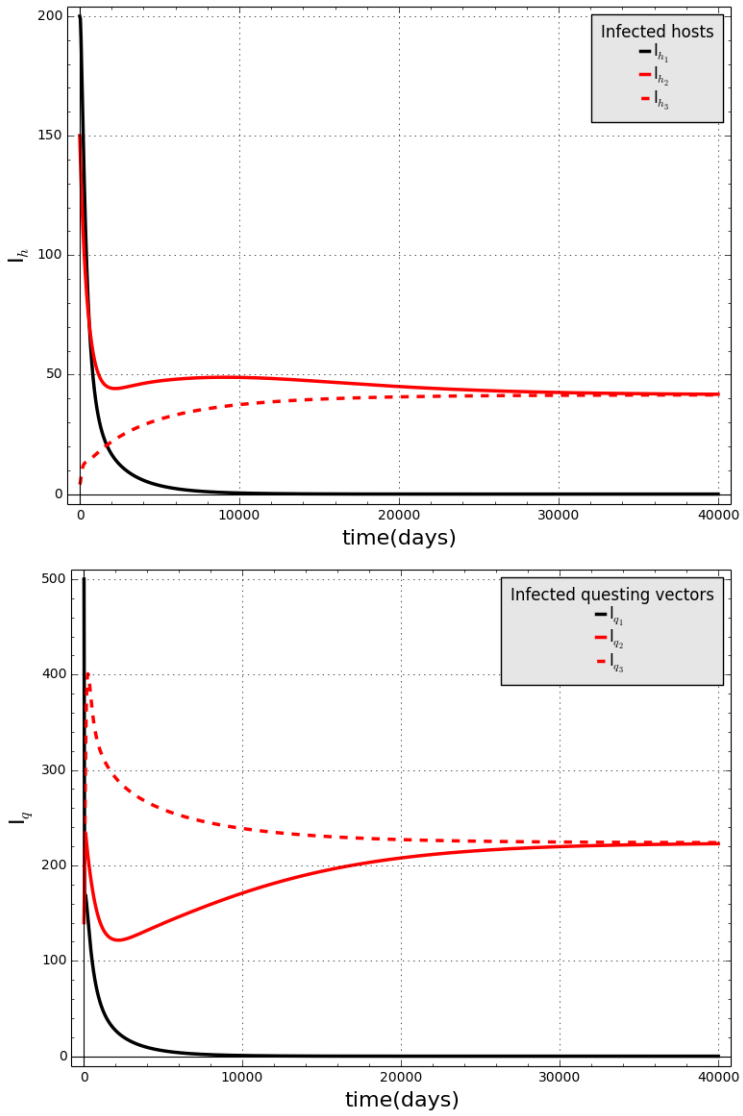

FIGURE 5. Solutions of model (1) of the number of infectious humans, I_h , and the number of infectious vectors, I_v , for $\mathscr{R}_0 = 0.8789171$ and $\zeta = 0.05194$ ($\zeta < \mathscr{R}_0 < 1$) with the parameter $\Lambda_h = \dfrac{1000}{50 \times 365}$; $c = 0.28$; $\tilde{c} = 0.028$; $\xi = 0.0035$; $\gamma = 1.4 \times 10^{-3}$; $\mu = \dfrac{1}{50 \times 365}$; $d = 0.001$; $m = 0.007$; $a = 0.36$; $u = 1$; $v = 1$; $\lambda = 0.01$; $\Lambda_v = \dfrac{10000}{21}$; $\beta = \dfrac{2}{3}$; $\chi = \dfrac{1}{5}$; $\kappa = \dfrac{1}{21}$; $\kappa' = \dfrac{2}{21}$. The solution for initial condition $X1 = [1000, 100, 200, 10, 1250, 150, 300, 200, 500, 200]$ approaches the locally asymptotically stable DFE point, while the solution for initial condition $X2 = [400, 50, 150, 20, 1200, 250, 50, 30, 140, 130]$ and $X3 = [10, 2, 4, 1, 100, 80, 40, 50, 200, 150]$ approaches the locally asymptotically stable endemic equilibrium

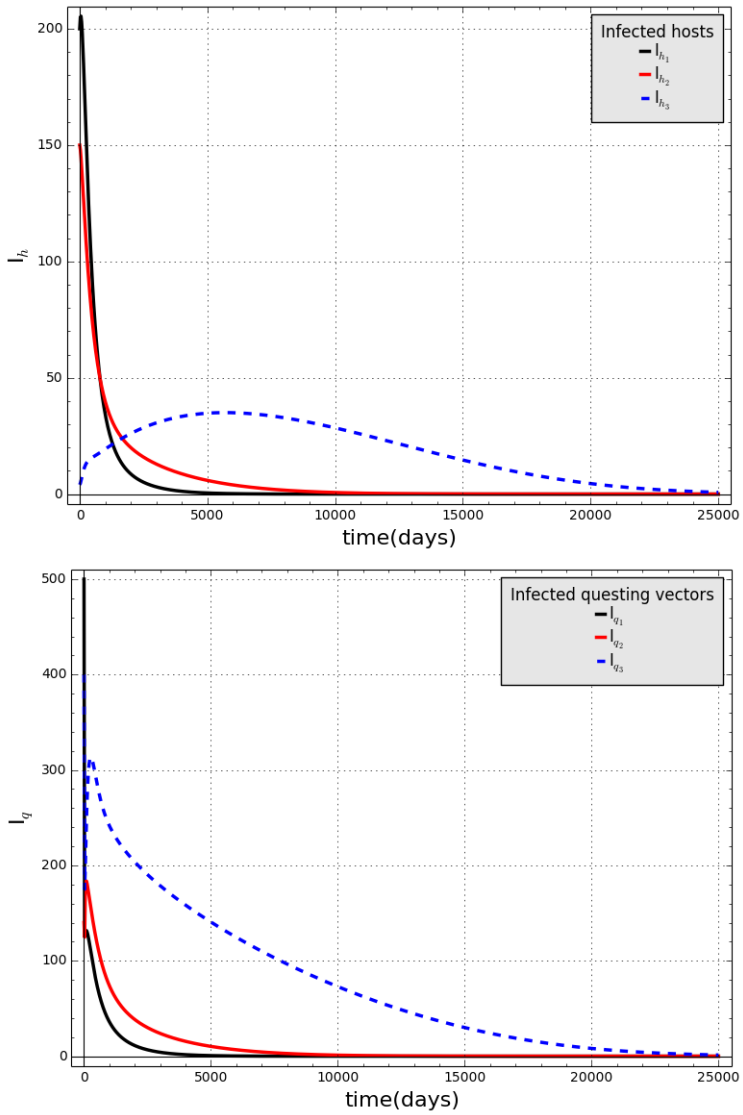

FIGURE 6. Solutions of model (1) of the number of infectious humans, I_h , and the number of infectious vectors, I_v , for $\mathscr{R}_0 = 0.58576$ and $\zeta = 0.84566$ ($\mathscr{R}_0 < \zeta < 1$) with the parameter $\Lambda_h = \dfrac{1000}{50 \times 365}$; $c = 0.28$; $\tilde{c} = 0.028$; $\xi = 0.0035$; $\gamma = 1.4 \times 10^{-3}$; $\mu = \dfrac{1}{50 \times 365}$; $d = 0.0001$; $m = 0.007$; $a = 0.26$; $u = 1$; $v = 1$; $\lambda = 0.01$; $\Lambda_v = \dfrac{10000}{21}$; $\beta = \dfrac{2}{3}$; $\chi = \dfrac{1}{5}$; $\kappa = \dfrac{1}{21}$; $\kappa' = \dfrac{2}{21}$. The solution for initial condition $X1 = [1000, 100, 200, 10, 1250, 150, 300, 200, 500, 200]$, $X2 = [400, 50, 150, 20, 1200, 250, 50, 30, 140, 130]$ and $X3 = [10, 2, 4, 1, 100, 80, 40, 50, 200, 150]$ approaches the global asymptotically stable DFE point

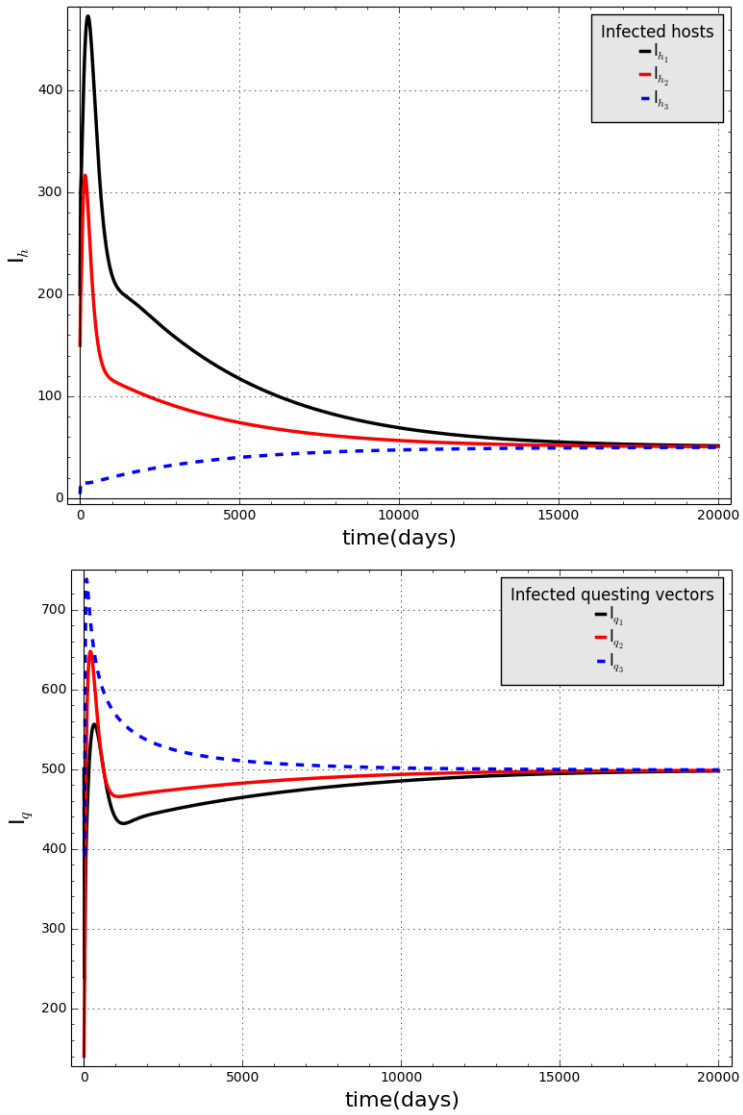

FIGURE 7. Solutions of model (1) of the number of infectious humans, I_h , and the number of infectious vectors, I_v , for $\mathscr{R}_0 = 7.32798126$ with the parameter $\Lambda_h = \frac{1000}{50 \times 365}$; $c = 0.53$; $\tilde{c} = 0.053$; $\xi = 0.0035$; $\gamma = 1.4 \times 10^{-3}$; $\mu = \frac{1}{50 \times 365}$; $d = 0.0009$; $m = 0.03$; $a = 0.36$; $u = 1$; $v = 1$; $\lambda = 0.2$; $\Lambda_v = \frac{10000}{21}$; $\beta = \frac{2}{3}$; $\chi = \frac{1}{5}$; $\kappa = \frac{1}{21}$; $\kappa' = \frac{2}{21}$. The solution for initial condition $X1 = [1000, 100, 200, 10, 1250, 150, 300, 200, 500, 200]$, $X2 = [400, 50, 150, 20, 1200, 250, 50, 30, 140, 130]$ and $X3 = [10, 2, 4, 1, 100, 800, 400, 150, 400, 50]$ approaches the unique stable endemic equilibrium

Conclusion

At the end of this work, we studied a generic model of vector-borne diseases incorporating questing-resting dynamics in vectors. We determined the basic reproduction rate and showed that the DFE is GAS when $\mathscr{R}_0 < \zeta$. We have also shown that there exist two endemic equilibria when $\mathscr{R}_0 < 1$, for $\mathscr{R}_0 > 1$ the system admits a unique endemic equilibrium and that for $\mathscr{R}_0 = 1$ the system admits a backward bifurcation. We also have at the end of a sensitivity analysis show that the rate of transition from the questing state to the resting f_q state and the transition from the resting state to the questing f_r state are the parameters More sensitive, which reflects the importance of considering this dynamic.

Appendix

Appendix A. To facilate writing, let $D = \dfrac{(\lambda+\mu)(\xi+\hat{\mu})}{\lambda+\tilde{u}\mu}$, $F = \dfrac{v\gamma\xi}{\gamma+\mu} + \xi\tilde{v}$, $C = \dfrac{\xi+\mu}{\lambda+\tilde{u}\mu}$, $M = \dfrac{\xi}{\gamma+\mu}$, $P = \dfrac{\gamma+\mu}{\tilde{c}v\xi + c(\gamma+\mu)}$

$$I_h^* = \frac{\alpha^*\Lambda_h}{\alpha^*(D-F)+D\mu} \quad R_h^* = MI_h^* \qquad E_h^* = CI_h^* \qquad S_h^* = \frac{DI_h^*}{\alpha^*}$$

$$S_q^* = \frac{\Lambda_v}{\kappa'+\varphi^*} \qquad E_r^{0*} = \frac{\varphi^* S_q^*}{\kappa+\chi} \qquad E_q^{1*} = \frac{\chi E_r^{0*}}{\kappa'+\beta} \qquad E_r^{1*} = \frac{\beta E_q^{1*}}{\kappa+\chi} \qquad I_q^* = \frac{f_q\chi E_r^{1*}}{\beta(1-f_q f_r)} \qquad I_r^* = \frac{\beta}{\chi} f_r I_q^*$$

where

$$\varphi^* = \frac{a(cI_h^* + \tilde{c}R_h^*)}{N} = \frac{a\alpha^*(M\tilde{c}v+c)}{\alpha^*(Cu+Mv+1)+D}$$

and

$$\alpha^* = \frac{amI_q^*}{N} = \frac{((D-F)\alpha^* + D\mu)DP\beta\chi^2 f_q\kappa'\varphi^*\mathscr{R}_0}{a\left((Cu+Mv+1)\alpha^*+D\right)(f_q f_r)^2(\kappa+\chi)^2(\kappa'+\beta)(\kappa'+\varphi^*)\mu}$$

Substituting φ^* in α^*, we obtain

$$\alpha^*[A_2(\alpha^*)^2 + A_1\alpha^* + A_0] = 0 \Leftrightarrow \alpha^* = 0 \quad or \quad A_2(\alpha^*)^2 + A_1\alpha^* + A_0 = 0 \tag{9}$$

$\alpha^* = 0$ Corresponds to DFE, so we look the solution of

$$A_2(\alpha^*)^2 + A_1\alpha^* + A_0 = 0 \tag{10}$$

$$A_2 = \mu(f_q f_r)^2(\kappa+\chi)^2(\kappa^{'}+\beta)\left[a(M\tilde{c}v+c)+\kappa^{'}(Cu+Mv+1)\right](Cu+Mv+1) > 0 \tag{11}$$

$$A_1 = -D\beta f_q\chi^2 Pc\kappa^{'}(D-F)(Mv+1)\left[\mathscr{R}_0 - \mathscr{R}_c\right] = -A_1^{'}\left[\mathscr{R}_0 - \mathscr{R}_c\right] \tag{12}$$

where $\mathscr{R}_c = \dfrac{(f_q f_r)^2(\kappa+\chi)^2(\kappa^{'}+\beta)\mu\left[a(M\tilde{c}v+c)+2\kappa^{'}(Cu+Mv+1)\right]}{\beta f_q\chi^2 c\kappa^{'}P(D-F)(Mv+1)}$

$$A_0 = \frac{D^2\mu\kappa^{'}}{P\beta\chi^2 f_q(M\tilde{c}v+c)}(1-\mathscr{R}_0) = A_0^{'}(1-\mathscr{R}_0) \tag{13}$$

If $\mathscr{R}_0 > 1$ ($A_0 < 0$) then the discriminant of the equation (10) is positive, if s is the sum of the solutions and p the product of these solutions then: $s = -\dfrac{A_1}{A_2}$ And $p = \dfrac{A_0}{A_2} < 0$ therefore the equation has only one positive solution from which the system admits a unique endemic equilibrium.

If $\mathscr{R}_0 = 1$, then the equation (10) has a unique solution $-\dfrac{A_1}{A_2}$ which is positive if $A_1 < 0$ ie $\mathscr{R}_c < 1$.

If $\mathscr{R}_0 = 1$ ($A_0 = 0$), then the equation has a unique solution which is $\alpha^* = -\dfrac{A_1}{A_2}$, or $A_2 > 0$ so this solution is positive if $A_1 < 0$ ie if $\mathscr{R}_c < \mathscr{R}_0 = 1$

If $\mathscr{R}_0 < 1$ ($A_0 > 0$), the discriminant of (10) is given by:

$$\Delta = A_1^2 - 4A_0A_2 = A_1^{'2}\mathscr{R}_0^2 + \left(4A_2A_0^{'} - 2A_1^{'}\mathscr{R}_c\right)\mathscr{R}_0 - \left[(A_1^{'}\mathscr{R}_c)^2 + 4A_2A_0^{'}\right] \tag{14}$$

Δ is a second degree equation in $\mathscr{R}_0$ and his discriminant Δ_r is define by:

$$\Delta_r = \left[4A_2A_0^{'} - 2A_1^{'}\mathscr{R}_c\right]^2 + 4A_1^{'2}\left[(A_1^{'}\mathscr{R}_c)^2 + 4A_2A_0^{'}\right] > 0$$

So $\Delta = 0$ admits two distinct solutions $\mathscr{R}_0^1$ and $\mathscr{R}_0^2$

If $\mathscr{R}_0 = \mathscr{R}_0^1$ or $\mathscr{R}_0 = \mathscr{R}_0^2$ alors $\Delta = 0$ and equation (10) has a unique solution $\alpha^* = -\dfrac{A_1}{2A_2}$ which is positive if $A_1 < 0$ ie if $\mathscr{R}_c < \mathscr{R}_0$

If $\mathscr{R}_0 \in [0;\mathscr{R}_0^1[\cup]\mathscr{R}_0^1;+\infty[$ then $\Delta > 0$ Hence the equation has two solutions, so the sum and the product are given by $s = -\dfrac{A_1}{A_2}$ and $p = \dfrac{A_0}{A_2} > 0$. These two solutions are positive if $s > 0$ ie $A_1 < 0$ $(\mathscr{R}_0 < \mathscr{R}_c)$.

Conflict of Interests

The authors declare that there is no conflict of interests.

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