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DYNAMIC BEHAVIORS OF A COMMENSAL SYMBIOSIS MODEL WITH NON-MONOTONIC FUNCTIONAL RESPONSE AND NON-SELECTIVE HARVESTING IN A PARTIAL CLOSURE

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Abstract. A two species commensal symbiosis model with non-monotonic functional response and non-selective harvesting in a partial closure takes the form

$$\begin{aligned}\frac{dx}{dt} &= x\Big(a_1 - b_1 x + \frac{c_1 y}{d_1 + y^2}\Big) - q_1 Emx,\\ \frac{dy}{dt} &= y(a_2 - b_2 y) - q_2 Emy\end{aligned}$$

is proposed and studied, where $a_i, b_i, q_i, i = 1,2$ c_1, E, $m(0 < m < 1)$ and d_1 are all positive constants. Depending on the range of the parameter m, the system may be collapse, or partial survival, or the two species could be coexist in a stable state. We also show that if the system admits a unique positive equilibrium, then it is globally asymptotically stable. By introducing the harvesting term and the reserve area, the system exhibit rich dynamic behaviors. Our results generalize the main results of Chen and Wu (A commensal symbiosis model with non-monotonic functional response, Commun. Math. Biol. Neurosci. 2017 (2017), Article ID 5).

Keywords: commensal symbiosis model; non-monotonic; stability; harvesting; partial closoure.

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1. Introduction

The aim of this paper is to investigate the dynamic behaviors of the following two species commensal symbiosis model with non-monotonic functional response and non-selective harvesting in a partial closure

$$\begin{aligned}\frac{dx}{dt} &= x\Big(a_1 - b_1 x + \frac{c_1 y}{d_1 + y^2}\Big) - q_1 Emx,\\ \frac{dy}{dt} &= y(a_2 - b_2 y) - q_2 Emy,\end{aligned} \tag{1.1}$$

where $a_i, b_i, q_i, i = 1,2$ c_1, E, $m(0 < m < 1)$ and d_1 are all positive constants, where E is the combined fishing effort used to harvest and $m(0 < m < 1)$ is the fraction of the stock available for harvesting. Here we make the following assumption:

(1) Two species obey the Logistic type growing;

(2) The commensal of the second species to the first one obey the non-monotonic functional response, i.e., $\frac{y}{1+y^2}$;

(3) Two species are harvested but harvesting is limited to a suitable area $(0 < m < 1)$.

During the last decade, many scholars investigated the dynamic behaviors of the mutualism model ([1]-[12]), however, only recently did scholars pay attention to the commensal symbiosis mode, a model describes a relationship which is only favorable to the one side and have no influence to the other side([14]-[23]), commensal symbiosis is one of the relationship which could be observed in nature for many cases, for example: A squirrel in an oak tree gets a place to live and food for its survival, while the tree remains neither benefited nor harmed. As was pointed out by Georgescu and Maxin[21] “One would think that the stability of the coexisting equilibria for two-species models of commensalism would follow immediately from the corresponding results for models of mutualism, when these results are available. After all, commensalism can be thought as mutualism in which one of the two interspecies interaction terms is zero, so at a glance everything should be simpler. However, this is not actually the case”. Hence, it’s very necessary to investigate the dynamic behaviors of the commensalism model.

Sun and Wei[15] first time proposed a intraspecific commensal model:

$$\begin{aligned} \frac{dx}{dt} &= r_1x\Big(\frac{k_1-x+ay}{k_1}\Big),\\ \frac{dy}{dt} &= r_2y\Big(\frac{k_2-y}{k_2}\Big). \end{aligned} \tag{1.2}$$

They investigated the local stability of all equilibrium points.

Zhu et al.[16] proposed and studied the following commensalism system

$$\begin{aligned} \frac{dx}{dt} &= x(a_1+b_1x+c_1y),\\ \frac{dy}{dt} &= y(a_1+c_2y), \end{aligned} \tag{1.3}$$

where $a_1<0, a_2>0, c_1>0, c_2<0, b_1<0$. The authors had assumed that species x is the benefit population and x will be driven to extinction without the help of the species y. By giving the phase trajectories analysis of the above system, they are able to analysis the stability property of the positive equilibrium and boundary equilibrium.

Xie et al. [22] proposed the following discrete commensal symbiosis model

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\big\{a_1(k)-b_1(k)x_1(k)+c_1(k)x_2(k)\big\},\\ x_2(k+1) &= x_2(k)\exp\big\{a_2(k)-b_2(k)x_2(k)\big\}, \end{aligned} \tag{1.4}$$

They showed that the system (1.4) admits at least one positive ω-periodic solution. For the autonomous case, Xue et al[19] investigated the local stability property of the equilibria, and they also gave a set of sufficient conditions which ensure the global stability of the positive equilibrium; Xue et al[22] further incorporate the delay to system (1.4), and they investigated the almost periodic solution of the system.

Noting that all of the above model are based on the traditional Lotka-Volterra model, which suppose that the influence of the second species to the first one is linearize. Recently, Chen and Wu[13] argued that this may not be suitable since the commensal between two species become infinity as the density of the species become infinity. They proposed the following two species commensal symbiosis model with non-monotonic functional response

$$\begin{aligned} \frac{dx}{dt} &= x\Big(a_1-b_1x+\frac{c_1y}{d_1+y^2}\Big),\\ \frac{dy}{dt} &= y(a_2-b_2y). \end{aligned} \tag{1.5}$$

Their study shows that the system admits a unique positive equilibrium, which is globally asymptotically stable.

On the other hand, to obtain the resource for the development of the human being, harvesting of the species is necessary. Recently, Chakraborty, Das and Kar[24] argued that it is necessary to harvest the population but harvesting should be regulated, such that both the ecological sustainability and conservation of the species can be implemented in a long run. To the best of the authors knowledge, to this day, still no scholar consider the influence of harvesting to the commensalism model, this stimulated us propose and study the dynamic behaviors of the system (1.1).

The aim of this paper is to investigate the local and global stability property of the possible equilibria of system (1.1). We arrange the paper as follows: In the next section, we will investigate the existence and local stability property of the equilibria of system (1.1). In Section 3, we will investigate the global stability property of the equilibria; In Section 4, an example together with its numeric simulations is presented to show the feasibility of our main results.

2. The existence and local stability of the equilibria

The equilibria of system (1.1) is determined by the system

$$x\Big(a_1 - b_1 x + \frac{c_1 y}{d_1 + y^2}\Big) - q_1 E m x = 0,$$

$$y(a_2 - b_2 y) - q_2 E m y = 0.$$

The system always admits the boundary equilibrium $A_1(0,0)$.

If $a_1 > Emq_1$ holds, the system admits the boundary equilibrium $A_2(x_0, 0)$, where $x_0 = \dfrac{a_1 - Emq_1}{b_1}$.

If $a_2 > Emq_2$ holds, the system admits the boundary equilibrium $A_3(0, y_0)$, where $y_0 = \dfrac{a_2 - Emq_2}{b_2}$.

If $a_1 + \dfrac{c_1 y^*}{(y^*)^2 + d_1} > q_1 Em$ and $a_2 > Emq_2$ hold, then the system admits a unique positive equilibrium

$$A_4(x^*, y^*) = \Bigg(\frac{a_1 - q_1 Em + \dfrac{c_1 y^*}{(y^*)^2 + d_1}}{b_1},\ y^*\Bigg),$$

where $y^* = \dfrac{a_2 - Emq_2}{b_2}$.

Concerned with the local stability property of the above four equilibria, we have

Theorem 2.1

(1) Assume that

$$m > \max\left\{\frac{a_1}{Eq_1}, \frac{a_2}{Eq_2}\right\} \tag{2.1}$$

hold, then $A_1(0,0)$ is locally asymptotically stable, otherwise, it is unstable;

(2) Assume that

$$\frac{a_2}{Eq_2} < m < \frac{a_1}{Eq_1} \tag{2.2}$$

hold, then $A_2(x_0,0)$ is locally asymptotically stable, otherwise, it is unstable;

(3) Assume that

$$a_1 + \frac{c_1 y_0}{d_1 + (y_0)^2} < q_1 Em \tag{2.3}$$

and

$$m < \frac{a_2}{Eq_2} \tag{2.4}$$

hold, then $A_3(0,y_0)$ is locally asymptotically stable, otherwise, it is unstable;

(4) Assume that

$$a_1 + \frac{c_1 y^*}{(y^*)^2 + d_1} > q_1 Em \tag{2.5}$$

and

$$m < \frac{a_2}{Eq_2} \tag{2.6}$$

hold, then $E_4(x^*,y^*)$ is locally asymptotically stable, otherwise, it is unstable.

Proof. The Jacobian matrix of the system (1.1) is calculated as

$$J(x,y) = \begin{pmatrix} a_1 - 2b_1 x + \dfrac{c_1 y}{1+y^2} - q_1 Em & \dfrac{c_1 x(d_1 - y^2)}{(d_1+y^2)^2} \\ 0 & -2b_2 y + a_2 - q_2 Em \end{pmatrix}. \tag{2.7}$$

Then the Jacobian matrix of the system (1.1) about the equilibrium $A_1(0,0)$ is given by

$$\begin{pmatrix} a_1 - q_1 Em & 0 \\ 0 & a_2 - q_2 Em \end{pmatrix}. \tag{2.8}$$

The eigenvalues of the matrix are $\lambda_1 = a_1 - q_1 Em, \lambda_2 = a_2 - q_2 Em$. Hence, if $a_1 < Emq_1$ and $a_2 < Emq_2$ holds, then $\lambda_1 < 0, \lambda_2 < 0$, consequently $A_1(0,0)$ is locally stable, otherwise, it is unstable.

The Jacobian matrix of the system (1.1) about the equilibrium $A_2(x_0,0)$ is given by

$$\begin{pmatrix} Emq_1-a_1 & \dfrac{c_1(a_1-Emq_1)}{b_1d_1} \\ 0 & a_2-Emq_2 \end{pmatrix}. \tag{2.9}$$

The eigenvalues of the matrix are $\lambda_1=Emq_1-a_1, \lambda_2=a_2-Emq_2$. Hence, if $a_1>Emq_1$ and $a_2<Emq_2$ hold, then $\lambda_1<0, \lambda_2<0$, consequently $A_2(x_0,0)$ is locally stable, otherwise, it is unstable.

The Jacobian of the system about the equilibrium point $A_3(0,y_0)$ is given by

$$\begin{pmatrix} a_1-q_1Em+\dfrac{c_1y_0}{d_1+(y_0)^2} & 0 \\ 0 & Emq_2-a_2 \end{pmatrix}. \tag{2.10}$$

Under the assumption (2.3) and (2.4), The two eigenvalues of the matrix satisfies

$$\lambda_1=a_1-q_1Em+\frac{c_1y_0}{d_1+(y_0)^2}<0,\ \lambda_2=Emq_2-a_2<0.$$

Consequently $A_3(0,y_0)$ is locally stable, otherwise, it is unstable.

Under the assumption (2.5) and (2.6), system (1.1) admits unique positive equilibrium $A_4(x^*,y^*)$. Also, (x^*,y^*) satisfies the equation

$$\begin{aligned} &\left(a_1-b_1x^*+\frac{c_1y^*}{d_1+(y^*)^2}\right)-q_1Em=0, \\ &(a_2-b_2y^*)-q_2Em=0. \end{aligned} \tag{2.11}$$

By using (2.11), the Jacobian matrix about the equilibrium $A_4(x^*,y^*)$ is given by

$$\begin{pmatrix} -b_1x^* & * \\ 0 & -b_2y^* \end{pmatrix}, \tag{2.12}$$

The eigenvalues of the above matrix are $\lambda_1=-b_1x^*<0, \lambda_2=-b_2y^*<0$. Hence, $A_4(x^*,y^*)$ is locally stable.

This ends the proof of Theorem 2.1.

3. Global stability of the equilibria

This section try to obtain some sufficient conditions which could ensure the global asymptotical stability of the equilibria.

As a direct corollary of Lemma 2.2 of Chen[26], we have

Lemma 3.1. *If $a > 0, b > 0$ and $\dot{x} \geq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have*

$$\liminf_{t \to +\infty} x(t) \geq \frac{b}{a}.$$

If $a > 0, b > 0$ and $\dot{x} \leq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have

$$\limsup_{t \to +\infty} x(t) \leq \frac{b}{a}.$$

Lemma 3.2.[25] System

$$\frac{dy}{dt} = y(a - by) \tag{3.1}$$

has a unique globally attractive positive equilibrium $y^* = \frac{a}{b}$.

Theorem 3.1

(1) Assume that

$$m > \max\left\{\frac{a_1}{Eq_1}, \frac{a_2}{Eq_2}\right\} \tag{3.2}$$

hold, then $A_1(0,0)$ is globally asymptotically stable;

(2) Assume that

$$\frac{a_2}{Eq_2} < m < \frac{a_1}{Eq_1} \tag{3.3}$$

hold, then $A_2(x_0,0)$ is globally asymptotically stable;

(3) Assume that

$$a_1 + \frac{c_1 y_0}{d_1 + (y_0)^2} < q_1 Em \tag{3.4}$$

and

$$m < \frac{a_2}{Eq_2} \tag{3.5}$$

holds, then $A_3(0,y_0)$ is globally asymptotically stable;

(4) Assume that

$$a_1 + \frac{c_1 y^*}{(y^*)^2 + d_1} > q_1 Em \tag{3.6}$$

and

$$m < \frac{a_2}{Eq_2} \tag{3.7}$$

hold, then $A_4(x^*, y^*)$ is globally asymptotically stable.

Proof.

(1) From $a_1 < Eq_1m$ there exists enough small $\varepsilon > 0$ such that

$$a_1 + \frac{c_1\varepsilon}{d_1} - Eq_1m < -\varepsilon.$$

From the second equation of (1.1) we have

$$\frac{dy}{dt} = y\Big(a_2 - Eq_2m - b_2y\Big) < (a_2 - Eq_2m)y. \tag{3.8}$$

Hence

$$y(t) < y(0)\exp\{(r_2 - Eq_2m)t\} \to 0 \text{ as } t \to +\infty.$$

For above $\varepsilon > 0$, there exists a $T_1 > 0$, such that

$$y(t) < \varepsilon \text{ for all } t > T_1. \tag{3.9}$$

For $t > T_1$, from the first equation of system (1.1), we have

$$\begin{aligned} \frac{dx}{dt} &< x\Big(a_1 - b_1x + \frac{c_1\varepsilon}{d_1}\Big) - q_1Emx \\ &= x\Big(a_1 + \frac{c_1\varepsilon}{d_1} - q_1Em - b_1x\Big) \\ &< -\varepsilon x, \end{aligned}$$

Hence

$$x(t) < x(T_1)\exp\{-\varepsilon(t - T_1)\} \to 0 \text{ as } t \to +\infty. \tag{3.10}$$

(2) Similarly to the analysis of (3.8)-(3.9), for arbitrary enough small $\varepsilon > 0$, there exists a $T_2 > 0$, such that

$$y(t) < \varepsilon \text{ as } t > T_2.$$

For $t > T_2$, from the first equation of system (1.1), we have

$$\begin{aligned} \frac{dx}{dt} &< x\Big(a_1 - b_1x + \frac{c_1\varepsilon}{d_1}\Big) - q_1Emx \\ &= x\Big(a_1 + \frac{c_1\varepsilon}{d_1} - q_1Em - b_1x\Big). \end{aligned} \tag{3.11}$$

It follows from Lemma 3.1 that

$$\limsup_{t\to+\infty} x(t) \le \frac{a_1 + \frac{c_1\varepsilon}{d_1} - q_1Em}{b_1}. \tag{3.12}$$

On the other hand, from the first equation of system (1.1), we also have

$$\begin{aligned} \frac{dx}{dt} &> x\Big(a_1 - b_1 x\Big) - q_1 E m x \\ &= x\Big(a_1 - q_1 E m - b_1 x\Big). \end{aligned} \tag{3.13}$$

It follows from Lemma 3.1 that

$$\liminf_{t\to+\infty} x(t) \geq \frac{a_1 - q_1 E m}{b_1}. \tag{3.14}$$

It follows from (3.12) and (3.14) that

$$\frac{a_1 - q_1 E m}{b_1} \leq \liminf_{t\to+\infty} x(t) \leq \limsup_{t\to+\infty} x(t) \leq \frac{a_1 + \frac{c_1 \varepsilon}{d_1} - q_1 E m}{b_1}. \tag{3.15}$$

Since ε is any arbitrary small positive constants, setting $\varepsilon \to 0$ in (3.15) leads to

$$\lim_{t\to+\infty} x(t) = \frac{a_1 - q_1 E m}{b_1}.$$

(3) For $\varepsilon > 0$ enough small, without loss of generality, we may assume that $\varepsilon < \frac{1}{2} y_0$, from (3.4), we have

$$a_1 + \frac{c_1 (y_0 + \varepsilon)}{d_1 + (y_0 - \varepsilon)^2} - q_1 E m < -\varepsilon \tag{3.16}$$

From the second equation of (1.1) we have

$$\frac{dy}{dt} = y\Big(a_2 - E q_2 m - b_2 y\Big). \tag{3.17}$$

It follows from Lemma 3.2 that

$$\lim_{t\to+\infty} y(t) = \frac{a_2 - E q_2 m}{b_2} = y_0 > 0.$$

For $\varepsilon > 0$ enough small, there exists an enough large $T_3 > 0$ such that

$$y_0 - \varepsilon < y(t) < y_0 + \varepsilon \ \text{ for all } \ t \geq T_3. \tag{3.18}$$

For $t > T_3$, from the first equation of system (1.1), we have

$$\begin{aligned} \frac{dx}{dt} &< x\Big(a_1 - b_1 x + \frac{c_1 (y_0 + \varepsilon)}{d_1 + (y_0 - \varepsilon)^2}\Big) - q_1 E m x \\ &= x\Big(a_1 + \frac{c_1 (y_0 + \varepsilon)}{d_1 + (y_0 - \varepsilon)^2} - q_1 E m - b_1 x\Big) \\ &< -\varepsilon x. \end{aligned} \tag{3.19}$$

Hence

$$x(t) < x(T_3)\exp\{-\varepsilon(t-T_3)\} \to 0 \text{ as } t \to +\infty. \tag{3.20}$$

(4) Firstly we proof that every solution of system (1.1) that starts in R_+^2 is uniformly bounded. Similarly to the analysis of (3.17), we have

$$\lim_{t\to+\infty} y(t) = \frac{a_2 - Eq_2m}{b_2} = y^* > 0.$$

Hence, for arbitrary small positive constant $\varepsilon > 0$, there exists a $T_4 > 0$ such that

$$y(t) < y^* + \varepsilon \text{ for all } t \geq T_4. \tag{3.21}$$

Similarly to the analysis of (3.18)-(3.19), For $t > T_4$, from the first equation of system (1.1), we have

$$\begin{aligned} \frac{dx}{dt} &< x\Big(a_1 - b_1x + \frac{c_1(y^*+\varepsilon)}{d_1+(y^*-\varepsilon)^2}\Big) - q_1Emx \\ &= x\Big(a_1 + \frac{c_1(y^*+\varepsilon)}{d_1+(y^*-\varepsilon)^2} - q_1Em - b_1x\Big). \end{aligned} \tag{3.22}$$

It follows from Lemma 3.1 that

$$\limsup_{t\to+\infty} x(t) \leq \frac{a_1 + \frac{c_1(y^*+\varepsilon)}{d_1+(y^*-\varepsilon)^2} - q_1Em}{b_1}. \tag{3.23}$$

Hence, there exists a $T_5 > T_4$ such that

$$x(t) < \frac{a_1 + \frac{c_1(y^*+\varepsilon)}{d_1+(y^*-\varepsilon)^2} - q_1Em}{b_1} + \varepsilon \text{ for all } t \geq T_5. \tag{3.24}$$

Let

$$D = \left\{(x,y) \in R_+^2 : x < \frac{a_1 + \frac{c_1(y^*+\varepsilon)}{d_1+(y^*-\varepsilon)^2} - q_1Em}{b_1} + \varepsilon,\ y < y^* + \varepsilon\right\}.$$

Then every solution of system (1.1) starts in R_+^2 is uniformly bounded on D. Also, from Theorem 2.1 there is a unique local stable positive equilibrium $A_4(x^*, y^*)$, all the other three boundary equilibrium are unstable. To show that $A_4(x^*, y^*)$ is globally stable, it's enough to show that the system admits no limit cycle in the area D, Let's consider the Dulac function $u(x,y) = x^{-1}y^{-1}$, then

$$\frac{\partial(uF_1)}{\partial x} + \frac{\partial(uF_2)}{\partial y} = -\frac{b_1x + b_2y}{xy} < 0,$$

where

$$\begin{aligned} F_1(x,y) &= x\Big(a_1 - b_1x + \frac{c_1 y}{d_1 + y^2}\Big) - q_1Emx, \\ F_2(x,y) &= y(a_2 - b_2y) - q_2Emy. \end{aligned}$$

By Dulac Theorem[27], there is no closed orbit in area D. Consequently, $A_4(x^*, y^*)$ is globally asymptotically stable. This completes the proof of Theorem 3.1.

4. Numeric simulations

Now let us consider the following example.

Example 4.1. Consider the following system

$$\begin{aligned} \frac{dx}{dt} &= x\Big(3 - 6x + \frac{2y}{10 + y^2}\Big) - 4mx, \\ \frac{dy}{dt} &= y(1 - 2y) - 4my. \end{aligned} \tag{4.1}$$

In this system, corresponding to system (1.1), we take $a_1 = 3, b_1 = 6, c_1 = 2, d_1 = 10, a_2 = 1, b_2 = 2, q_1 = q_2 = 1, E = 4$. For the system without harvesting, i. e., $m = 0$, from Chen and Wu[13], the system admits a unique positive equilibrium $\big(\frac{127}{246}, \frac{1}{2}\big)$, which is globally asymptotically stable.

(1) Take $m = 0.8$, then

$$m > \max\Big\{\frac{a_1}{Eq_1}, \frac{a_2}{Eq_2}\Big\} = 0.75,$$

and so, from Theorem 3.1, $A_1(0,0)$ is globally asymptotically stable, see Fig.1;

(2) Take $m = 0.5$, then

$$\frac{1}{4} = \frac{a_2}{Eq_2} < m < \frac{a_1}{Eq_1} = \frac{3}{4}$$

hold, and $A_2(\frac{1}{6}, 0)$ is globally asymptotically stable, see Fig.2;

(3) Take $m = 0.1$, then

$$a_1 + \frac{c_1 y^*}{(y^*)^2 + d_1} > a_1 = 3 > q_1Em = \frac{4}{10}$$

and

$$m < \frac{a_2}{Eq_2} = \frac{1}{4}$$

hold, then $A_4(0.4432441361, 0.3)$ is globally asymptotically stable, see Fig.3;

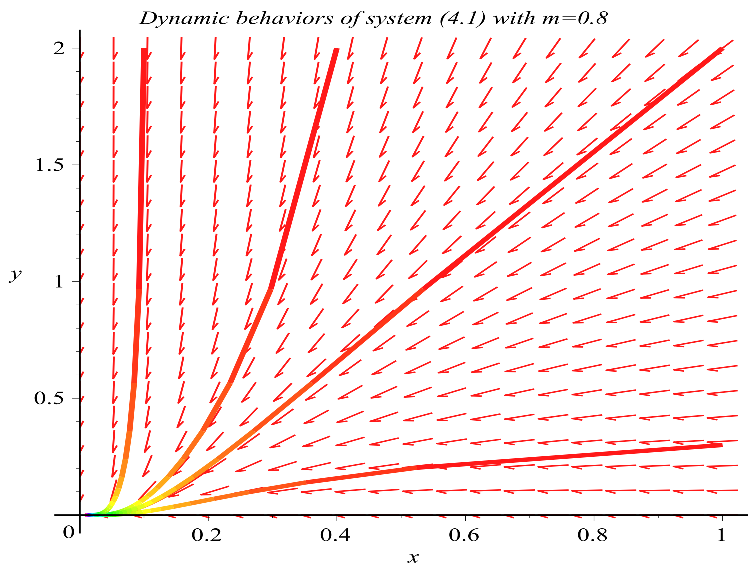

FIGURE 1. Numeric simulations of system (4.1) with $m = 0.8$, the initial conditions $(x(0), y(0)) = (0.4, 2), (1, 0.3), (0.2, 0.2), (1, 2)$ and $(0.1, 2)$, respectively.

5. Conclusion

Chen and Wu[13] proposed a two species commensal symbiosis model with non-monotonic functional response, they showed that the unique positive equilibrium of the system is globally asymptotically stable. Stimulated by the work of Chakraborty, Das and Kar[24], we further incorporate the harvesting term to the system, however, the harvesting is restricted to a limited area. Our study shows that the harvesting effort and the area for harvesting plays essential factor on the dynamic behaviors of the system. Indeed, depending the parameter m, both species maybe driven to extinction, or one of the species will be driven to extinction, while the other one is permanent, or both species could be coexist in a stable state.

It's well known that due to the over exploitation of the resources in the world, more and more species become endangered, to avoid the extinction of the species, more and more natural reserves area are established. However, our study shows that if the reserve area is limited, the

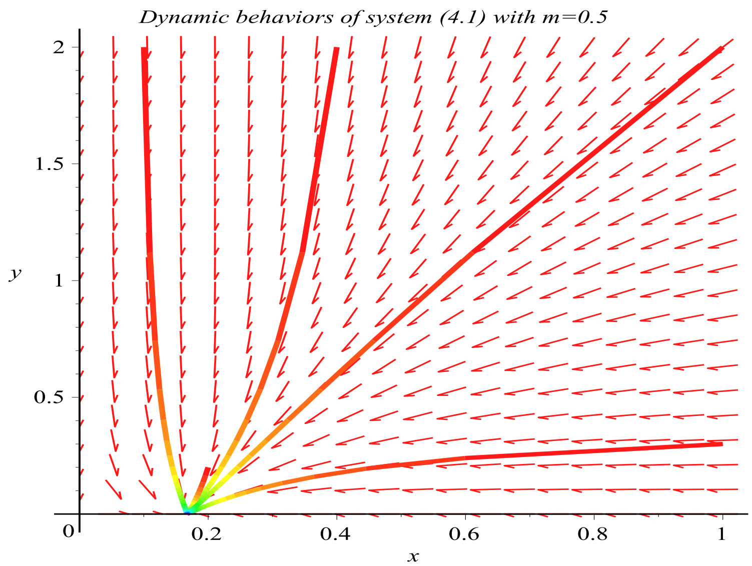

FIGURE 2. Numeric simulations of system (4.1) with $m = 0.5$, the initial conditions $(x(0), y(0)) = (0.4, 2), (1, 0.3), (0.2, 0.2), (1, 2)$ and $(0.1, 2)$, respectively.

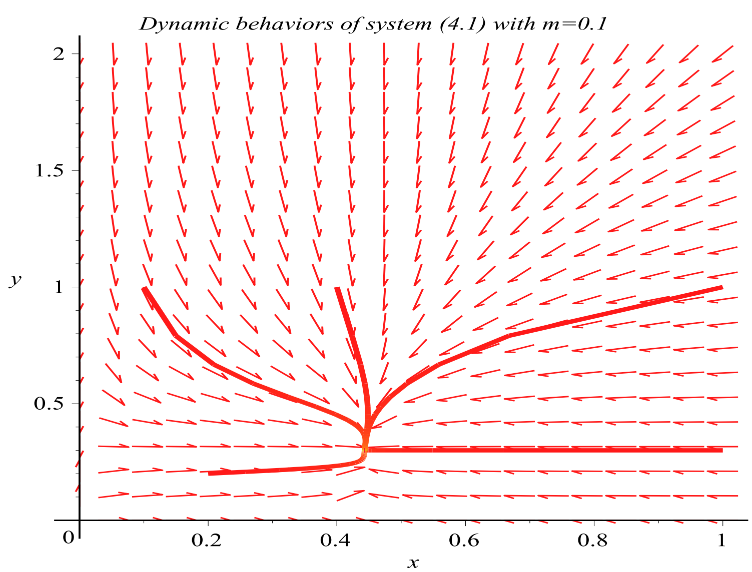

FIGURE 3. Numeric simulations of system (4.1) with $m = 0.1$, the initial conditions $(x(0), y(0)) = (0.4, 2), (1, 0.3), (0.2, 0.2), (1, 2)$ and $(0.1, 2)$, respectively.

species may still be driven to the extinction due to the harvesting outside the reserve area, such an finding seems interesting and maybe useful in advising the protect of wild animals.

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Conflict of Interests

The authors declare that there is no conflict of interests.

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