Available online at http://scik.org
Commun. Math. Biol. Neurosci. 2018, 2018:10
https://doi.org/10.28919/cmbn/3674
ISSN: 2052-2541

THE INFLUENCE OF PARTIAL CLOSURE FOR THE POPULATIONS TO A HARVESTING LOTKA-VOLTERRA COMMENSALISM MODEL

HANG DENG*, XIAOYAN HUANG

College of Mathematics and Computer Science, Fuzhou University, Fuzhou, Fujian, 350116, China

Abstract. The aim of this paper is to investigate the dynamic behaviors of a harvesting Lotka-Volterra commensalism model incorporating partial closure for the populations. By analyzing the characteristic equation of the variational matrix, sufficient conditions which ensure the local stability of the equilibria are obtained; By applying the differential inequality theory and the Dulac criterion, sufficient conditions which ensure the globally asymptotical stability of the equilibria are obtained; Our study shows that depending on the fraction of the stock available for harvesting, the system maybe extinction, partial survival or two species coexist in a stable state. The dynamic behaviors of the system becomes complicated compared with the non-harvesting system. Numeric simulations are carried out to show the feasibility of the main results.

Keywords: commensalism model; different inequality; variational matrix; global stability.

2010 Mathematics Subject Classification: 34C25, 92D25, 34D20, 34D40.

1. Introduction

Commensalism is a symbiotic interaction between two populations where one population gets benefit from while the other is neither harmed nor benefited due to the interaction with

*Corresponding author
E-mail addresses: 2451871575@qq.com
Received February 11, 2018

the previous species[1]. There are many real-life examples of commensalism, for example, the relationship between squirrel and the oak, the clownfish and the sea anemone, etc, see [1]-[12] and the references cited therein.

During the last decade, many scholars ([13]-[25]) investigated the dynamic behaviors of the mutualism model, where the interaction of two species is to the advantage of both side. Many interesting results are obtained, for example, Chen et al [13] showed that stage structure of the species could have great influence on the stability property of the mutualism model; Chen et al([20]-[21]) showed that feedback control variables have no influence on the persistent property of the system; Xie et al[25] proved that if the mutualism system admits an unique positive equilibrium, it is globally asymptotically stable. However, there are still not so much works on commensalism model (see[1]-[12]).

Recently, Sun et al.[8] proposed the following commensalism system

$$\begin{aligned} \frac{dx}{dt} &= r_1 x\Big(1-\frac{x}{K_1}+\alpha\frac{y}{K_1}\Big), \\ \frac{dy}{dt} &= r_2 y\Big(1-\frac{y}{K_2}\Big), \end{aligned} \tag{1.1}$$

where r_1, r_2, K_1, K_2, α are all positive constants. The system admits four equilibria:

$$E_1(0,0), E_2(K_1,0), E_3(0,K_2), E_4(K_1+\alpha K_2, K_2).$$

Concern with the stability property of above equilibria,by linearizing the system at equilibrium,the author obtained the following results.

Theorem A.

(1) $E_1(0,0)$ is unstable node;

(2) $E_2(K_1,0)$ is a saddle point;

(3) $E_3(0,K_2)$ is a saddle point;

(4) $E_4(K_1+\alpha K_2,K_2)$ is a stable node;

It bring to our attention that the author gave no information about the global stability property of the equilibrium.

On the other hand, to obtain the resource for the development of the human being, harvest of the species is necessary, Chakraborty, Das and Kar[22] argued that it is necessary to harvest the population but harvesting should be regulated, such that both the ecological sustainability and conservation of the species can be implemented in a long run.

Though there are many papers concerned with the harvesting of the predatorprey system ([26]-[30]), however, to this day, seldom did scholars consider the influence of harvesting to the commensalism model. Stimulated by the works of Sun et al[8] and Chakraborty, Das and Kar[27], in this paper, we propose the following non-selective harvesting Lotka-Volterra amensalism model incorporating partial closure for the populations:

$$
\begin{aligned}
\frac{dx}{dt} &= r_1 x\Big(1-\frac{x}{K_1}+\alpha\frac{y}{K_1}\Big)-q_1Emx,\\
\frac{dy}{dt} &= r_2 y\Big(1-\frac{y}{K_2}\Big)-q_2Emy,
\end{aligned}
\tag{1.2}
$$

where r_1, r_2, K_1, K_2, α are all positive constants, and have the same meaning as that of the system (1.1). E is the combined fishing effort used to harvest and $m(0<m<1)$ is the fraction of the stock available for harvesting.

We will try to investigate the dynamic behaviors of the system (1.2), and to find out the influence of the harvesting and the fraction of the stock.

The paper is arranged as follows. We will investigate the local and global stability property of the equilibria of system (1.2) in section 2 and 3, respectively. Some examples together with their numeric simulations are present in Section 5 to show the feasibility of the main results. We end this paper by a briefly discussion.

2. Local stability of the equilibria

The system always admits the boundary equilibrium $E_1(0,0)$.

If $r_1 > Emq_1$ holds, the system admits the boundary equilibrium $E_2(x_0,0)$, where $x_0 = \dfrac{K_1(r_1-Emq_1)}{r_1}$.

If $r_2 > Emq_2$ holds, the system admits the boundary equilibrium $E_3(0,y_0)$, where $y_0 = \dfrac{K_2(r_2-Emq_2)}{r_2}$.

If $r_1r_2K_1+r_1r_2\alpha K_2 > r_1q_2m\alpha K_2E+r_2mq_1EK_1$ and $r_2 > Emq_2$ hold, then the system admits a unique positive equilibrium

$$(x^*,y^*)=\Big(\frac{r_1r_2K_1+r_1r_2\alpha K_2-r_1q_2m\alpha K_2E-r_2mq_1EK_1}{r_1r_2},\ \frac{K_2(r_2-Emq_2)}{r_2}\Big).$$

We shall now investigate the local stability property of the above equilibria.

Theorem 2.1

(1) Assume that

$$m>\max\Big\{\frac{r_1}{Eq_1},\frac{r_2}{Eq_2}\Big\} \tag{2.1}$$

hold, then $E_1(0,0)$ is locally asymptotically stable, otherwise, it is unstable;

(2) Assume that

$$\frac{r_2}{Eq_2}<m<\frac{r_1}{Eq_1} \tag{2.2}$$

hold, then $E_2(x_0,0)$ is locally asymptotically stable, otherwise, it is unstable;

(3) Assume that

$$\frac{r_1r_2(K_1+\alpha K_2)}{r_1\alpha EK_2q_2+r_2q_1EK_1}<m<\frac{r_2}{Eq_2} \tag{2.3}$$

holds, then $E_3(0,y_0)$ is locally asymptotically stable, otherwise, it is unstable;

(4) Assume that

$$m<\min\Big\{\frac{r_2}{Eq_2},\frac{r_1r_2(K_1+\alpha K_2)}{r_1\alpha EK_2q_2+r_2q_1EK_1}\Big\} \tag{2.4}$$

hold, then $E_4(x^*,y^*)$ is locally asymptotically stable.

Proof. The variational matrix of the system of Eq. (1.2) at (x,y) is

$$J(x,y)=\begin{pmatrix} r_1(1-\frac{x}{K_1}+\alpha\frac{y}{K_1})-\frac{r_1x}{K_1}-q_1Em & \dfrac{xr_1\alpha}{K_1} \\ 0 & r_2(1-\frac{y}{K_2})-\frac{r_2y}{K_2}-q_2Em \end{pmatrix}. \tag{2.5}$$

The characteristic equation of the variational matrix is

$$\lambda^2-tr(J)\lambda+det(J)=0. \tag{2.6}$$

Obviously, if $tr(J)<0$ and $det(J)>0$, both eigenvalues of (2.1) have negative real parts, and the corresponding equilibrium solution is asymptotically stable.

(1) For the steady-state solution $E_1(0,0)$, $tr(J(0,0))=r_1+r_2-Emq_1-Emq_2$, $det(J(0,0))=(r_1-Emq_1)(r_2-Emq_2)$. Obviously, under the assumption (2.1), $tr(J(0,0))<0, det(J(0,0))>0$, and so, $E_1(0,0)$ is locally asymptotically stable, otherwise, it is unstable;

(2) For the steady-state solution $E_2(x_0,0)$, $tr(J(x_0,0))=-r_1+r_2+Emq_1-Emq_2$, $det(J(x_0,0))=$

$(Emq_1 - r_1)(r_2 - Emq_2)$. Obviously, under the assumption (2.2), $tr(J(x_0,0)) < 0, det(J(x_0,0)) > 0$, and so, $E_2(x_0,0)$ is locally asymptotically stable, otherwise, it is unstable;

(3) The Jacobian of the system about the equilibrium point $E_3(0,y_0)$ is given by

$$\begin{pmatrix} \dfrac{r_1r_2K_1 + r_1r_2\alpha K_2 - r_1\alpha mEK_2q_2 - r_2mq_1EK_1}{r_2K_1} & 0 \\ 0 & Emq_2 - r_2 \end{pmatrix}. \tag{2.7}$$

Under the assumption (2.4), the two eigenvalues of the matrix satisfies

$$\lambda_1 = \frac{r_1r_2K_1 + r_1r_2\alpha K_2 - r_1\alpha mEK_2q_2 - r_2mq_1EK_1}{r_2K_1} < 0$$

and

$$\lambda_2 = Emq_2 - r_2 < 0.$$

Consequently, $E_3(0,y_0)$ is locally stable, otherwise, it is unstable;

(4)Noting that the positive equilibrium $E_4(x^*,y^*)$ satisfies

$$\begin{aligned} r_1\Big(1 - \frac{x^*}{K_1} + \alpha\frac{y^*}{K_1}\Big) - q_1Em = 0, \\ r_2\Big(1 - \frac{y^*}{K_2}\Big) - q_2Em = 0. \end{aligned} \tag{2.8}$$

By using (2.8), the Jacobian of the system about the equilibrium point $E_4(x^*,y^*)$ is given by

$$J(x^*,y^*) = \begin{pmatrix} -\dfrac{r_1x^*}{K_1} & \dfrac{r_1x^*\alpha}{K_1} \\ 0 & -\dfrac{r_2y^*}{K_2} \end{pmatrix}. \tag{2.9}$$

Since $tr(J(x^*,y^*)) < 0, det(J(x^*,y^*)) > 0$, and so, $E_4(x^*,y^*)$ is locally asymptotically stable.

The proof of Theorem 2.1 is finished.

3. Global asymptotical stability

This section try to obtain some sufficient conditions which could ensure the global asymptotical stability of the equilibria.

Lemma 3.1.[31] System

$$\frac{dy}{dt} = y(a - by) \tag{3.1}$$

has a unique globally attractive positive equilibrium $y^* = \frac{a}{b}$.

Theorem 3.1

(1) Assume that

$$m > \max\left\{\frac{r_1}{Eq_1}, \frac{r_2}{Eq_2}\right\} \tag{3.1}$$

hold, then $E_1(0,0)$ is globally asymptotically stable;

(2) Assume that

$$\frac{r_2}{Eq_2} < m < \frac{r_1}{Eq_1} \tag{3.2}$$

hold, then $E_2(x_0,0)$ is globally asymptotically stable;

(3) Assume that

$$\frac{r_1 r_2 (K_1 + \alpha K_2)}{r_1 \alpha E K_2 q_2 + r_2 q_1 E K_1} < m < \frac{r_2}{Eq_2} \tag{3.3}$$

holds, then $E_3(0,y_0)$ is globally asymptotically stable;

(4) Assume that

$$m < \min\left\{\frac{r_2}{Eq_2}, \frac{r_1 r_2 (K_1 + \alpha K_2)}{r_1 \alpha E K_2 q_2 + r_2 q_1 E K_1}\right\} \tag{3.4}$$

holds, then $E_4(x^*,y^*)$ is globally asymptotically stable.

Proof.

(1)From $r_1 < Eq_1 m$ there exists enough small $\varepsilon > 0$ such that

$$r_1 + \frac{r_1 \alpha \varepsilon}{K_1} - Eq_1 m < -\varepsilon.$$

From the second equation of (1.2) we have

$$\frac{dy}{dt} = y\Big(r_2 - Eq_2 m - \frac{r_2 y}{K_2}\Big) < (r_2 - Eq_2 m)y. \tag{3.5}$$

Hence

$$y(t) < y(0)\exp\{(r_2 - Eq_2 m)t\} \to 0 \text{ as } t \to +\infty. \tag{3.6}$$

For above $\varepsilon > 0$, there exists a $T_1 > 0$, such that

$$y(t) < \varepsilon \text{ as } t > T_1. \tag{3.7}$$

For $t > T_1$, from the first equation of system (1.2), we have

$$\begin{aligned}\frac{dx}{dt} &< r_1 x\Big(1-\frac{x}{K_1}+\alpha\frac{\varepsilon}{K_1}\Big)-q_1Emx\\ &= x\Big(r_1+\frac{r_1\alpha\varepsilon}{K_1}-q_1Em-\frac{x}{K_1}\Big)\\ &< -\varepsilon x,\end{aligned}$$

Hence

$$x(t) < x(T_1)\exp\{-\varepsilon(t-T_1)\}\to 0 \text{ as } t\to+\infty. \tag{3.8}$$

(2)Similarly to the analysis of (3.5)-(3.8), for arbitrary enough small $\varepsilon > 0$, there exists a $T_2 > 0$, such that

$$y(t) < \varepsilon \text{ as } t > T_2.$$

For $t > T_2$, from the first equation of system (1.2), we have

$$\begin{aligned}\frac{dx}{dt} &< r_1 x\Big(1-\frac{x}{K_1}+\alpha\frac{\varepsilon}{K_1}\Big)-q_1Emx\\ &= x\Big(r_1+\frac{r_1\alpha\varepsilon}{K_1}-q_1Em-\frac{r_1x}{K_1}\Big).\end{aligned} \tag{3.9}$$

Consider the equation

$$\frac{du}{dt}=u\Big(r_1+\frac{r_1\alpha\varepsilon}{K_1}-q_1Em-\frac{r_1u}{K_1}\Big).$$

It follows from Lemma 3.1 that

$$\lim_{t\to+\infty}u(t)=\frac{(r_1+\frac{r_1\alpha\varepsilon}{K_1}-q_1Em)K_1}{r_1}.$$

By using the comparison theorem of differential equation, it follows from (3.6) that

$$\limsup_{t\to+\infty}x(t)\le\frac{(r_1+\frac{r_1\alpha\varepsilon}{K_1}-q_1Em)K_1}{r_1}. \tag{3.10}$$

On the other hand, from the first equation of system (1.2), we also have

$$\begin{aligned}\frac{dx}{dt} &> r_1 x\Big(1-\frac{x}{K_1}\Big)-q_1Emx\\ &= x\Big(r_1-q_1Em-\frac{r_1x}{K_1}\Big)\end{aligned} \tag{3.11}$$

Consider the equation

$$\frac{dv}{dt}=v\Big(r_1-q_1Em-\frac{r_1v}{K_1}\Big).$$

It follows from Lemma 3.1 that

$$\lim_{t\to+\infty} v(t) = \frac{(r_1 - q_1Em)K_1}{r_1}.$$

By using the comparison theorem of differential equation, it follows from (3.11) that

$$\liminf_{t\to+\infty} x(t) \geq \frac{(r_1 - q_1Em)K_1}{r_1}. \tag{3.12}$$

It follows from (3.10) and (3.12) that

$$\frac{(r_1 - q_1Em)K_1}{r_1} \leq \liminf_{t\to+\infty} x(t) \leq \limsup_{t\to+\infty} x(t) \leq \frac{(r_1 + \frac{r_1\alpha\varepsilon}{K_1} - q_1Em)K_1}{r_1}. \tag{3.13}$$

Since ε is any arbitrary small positive constants, setting $\varepsilon \to 0$ in (3.10) leads to

$$\lim_{t\to+\infty} x(t) = \frac{(r_1 - q_1Em)K_1}{r_1}.$$

(3)The left hand side of (3.3) is equal to the inequality

$$r_1r_2K_1 + r_1r_2\alpha K_2 < r_1q_2m\alpha K_2E + r_2mq_1EK_1,$$

hence, for arbitrary enough small $\varepsilon > 0$, the following inequality holds

$$\frac{r_1r_2K_1 + r_1r_2\alpha K_2 - r_1q_2m\alpha K_2E - r_2mq_1EK_1}{K_1r_2} + \frac{\alpha r_1}{K_1}\varepsilon < -\varepsilon. \tag{3.14}$$

From the second equation of (1.2) we have

$$\frac{dy}{dt} = y\Big(r_2 - Eq_2m - \frac{r_2y}{K_2}\Big). \tag{3.15}$$

It follows from Lemma 3.1 that

$$\lim_{t\to+\infty} y(t) = \frac{K_2(r_2 - Eq_2m)}{r_2}.$$

For above $\varepsilon > 0$, there exists an enough large $T_3 > 0$ such that

$$y(t) < \frac{K_2(r_2 - Eq_2m)}{r_2} + \varepsilon \ \text{ for all } \ t \geq T_3. \tag{3.16}$$

For $t > T_3$, from the first equation of system (1.2), we have

$$\begin{aligned} \frac{dx}{dt} &< r_1x\Big(1-\frac{x}{K_1}+\alpha\Big(\frac{K_2(r_2-Eq_2m)}{r_2}+\varepsilon\Big)\Big)-q_1Emx \\ &= x\Big(r_1+\alpha\Big(\frac{K_2(r_2-Eq_2m)}{r_2}+\varepsilon\Big)-q_1Em-\frac{r_1x}{K_1}\Big) \\ &= x\Big(\frac{r_1r_2K_1+r_1r_2\alpha K_2-r_1q_2m\alpha K_2E-r_2mq_1EK_1}{K_1r_2}+\frac{\alpha r_1}{K_1}\varepsilon-\frac{r_1x}{K_1}\Big) \\ &< -\varepsilon x. \end{aligned} \tag{3.17}$$

Hence

$$x(t) < x(T_3)\exp\{-\varepsilon(t-T_3)\} \to 0 \text{ as } t \to +\infty. \tag{3.18}$$

(4)Firstly we proof that every solution of system (1.2)that starts in R_+^2 is uniformly bounded. Similarly to the analysis of (3.15), we have

$$\lim_{t\to+\infty} y(t) = \frac{K_2(r_2-Eq_2m)}{r_2}.$$

Hence, for arbitrary small positive constant $\varepsilon > 0$, there exists a $T_4 > 0$ such that

$$y(t) < \frac{K_2(r_2-Eq_2m)}{r_2}+\varepsilon \text{ for all } t \geq T_4. \tag{3.19}$$

Similarly to the analysis of (3.17), For $t > T_4$, from the first equation of system (1.2), we have

$$\frac{dx}{dt} < x\Big(\frac{r_1r_2K_1+r_1r_2\alpha K_2-r_1q_2m\alpha K_2E-r_2mq_1EK_1}{K_1r_2}+\frac{\alpha r_1}{K_1}\varepsilon-\frac{r_1x}{K_1}\Big) \tag{3.20}$$

Now consider the equation

$$\frac{du}{dt} = u\Big(\frac{r_1r_2K_1+r_1r_2\alpha K_2-r_1q_2m\alpha K_2E-r_2mq_1EK_1}{K_1r_2}+\frac{\alpha r_1}{K_1}\varepsilon-\frac{r_1u}{K_1}\Big) \tag{3.21}$$

It follows from Lemma 3.1 that

$$\lim_{t\to+\infty} u(t) = \frac{r_1r_2K_1+r_1r_2\alpha K_2-r_1q_2m\alpha K_2E-r_2mq_1EK_1}{r_1r_2}+\alpha\varepsilon. \tag{3.22}$$

From (3.20) and (3.22), by applying the differential inequality theory, we have

$$\limsup_{t\to+\infty} x(t) \leq \frac{r_1r_2K_1+r_1r_2\alpha K_2-r_1q_2m\alpha K_2E-r_2mq_1EK_1}{r_1r_2}+\alpha\varepsilon. \tag{3.23}$$

Hence, there exists a $T_5 > T_4$ such that

$$x(t) < \frac{r_1r_2K_1+r_1r_2\alpha K_2-r_1q_2m\alpha K_2E-r_2mq_1EK_1}{r_1r_2}+(\alpha+1)\varepsilon \text{ for all } t \geq T_5. \tag{3.24}$$

Let

$$D=\Big\{(x,y)\in R_+^2 : x<\Gamma_1(\varepsilon),\ y<\frac{K_2(r_2-Eq_2m)}{r_2}+\varepsilon\Big\},$$

where

$$\Gamma_1(\varepsilon)=\frac{r_1r_2K_1+r_1r_2\alpha K_2-r_1q_2m\alpha K_2E-r_2mq_1EK_1}{r_1r_2}+(\alpha+1)\varepsilon.$$

Then every solution of system (1.2) starts in R_+^2 is uniformly bounded on D. Also, from Theorem 2.1 there is a unique local stable positive equilibrium $E_4(x^*,y^*)$. To show that $E_4(x^*,y^*)$ is globally stable, it's enough to show that the system admits no limit cycle in the area D, Let's consider the Dulac function $u(x,y)=x^{-1}y^{-1}$, then

$$\frac{\partial(uF_1)}{\partial x}+\frac{\partial(uF_2)}{\partial y}=-\frac{K_1r_2y+K_2r_1x}{xyK_1K_2}<0,$$

where

$$P(x,y)=r_1x\Big(1-\frac{x}{K_1}+\alpha\frac{y}{K_1}\Big)-q_1Emx,$$
$$Q(x,y)=r_2y\Big(1-\frac{y}{K_2}\Big)-q_2Emy.$$

By Dulac Theorem[32], there is no closed orbit in area D. Consequently, $E_4(x^*,y^*)$ is globally asymptotically stable. This completes the proof of Theorem 3.1.

Remark 3.1. Theorem 2.1 and 3.1 show that if the system (1.2) admits the unique positive equilibrium, then the positive equilibrium is globally asymptotically stable.

Remark 3.2. It follows from Theorem 2.1 and 3.1 that the local stability of the equilibrium also implies the global one.

Remark 3.3. Since

$$\frac{dx^*}{dm}=-\frac{E(K_2q_2r_1\alpha+K_1q_1r_2)}{r_1r_2}<0,$$
$$\frac{dy^*}{dm}=-\frac{EK_2q_2}{r_2}<0,$$

thus both x^* and y^* are the strictly decreasing function of m. This means that with the increasing of the fraction of the stock afford for harvesting, both species reduce their final density. Therefore, to ensure the coexistence of the both species, the area for the harvesting should be restricted to the limited case

$$m<\min\Big\{\frac{r_2}{Eq_2},\frac{r_1r_2(K_1+\alpha K_2)}{r_1\alpha EK_2q_2+r_2q_1EK_1}\Big\}.$$

Otherwise, at least one of the species will be driven to extinction.

4. Numerical simulations

Example 4.1. Let's take $r_1 = 1, E = 4, q_1 = \frac{1}{2}, q_2 = 2, \alpha = \frac{1}{3}, r_2 = 2, K_1 = 1, K_2 = 1$. In this case, by simple computation, one could easily see that

$$\frac{r_1}{Eq_1} = \frac{1}{2}, \frac{r_2}{Eq_2} = \frac{1}{4},$$

$$\frac{r_1 r_2 (K_1 + \alpha K_2)}{r_1 \alpha E K_2 q_2 + r_2 q_1 E K_1} = \frac{2}{5}.$$

Corresponding to Theorem 3.1, we have

(1) For $m > \frac{1}{2}$, $E_1(0,0)$ is the globally asymptotically stable equilibrium, Fig.1, Fig. 2 is the case of $m = 0.75$;

(2) For $\frac{1}{4} < m < \frac{1}{2}$, the boundary equilibrium $E_2(x_{0m}, 0)$ is globally asymptotically stable, Fig. 3, Fig.4 is the case of $m = 0.3$;

(3) For $m < \frac{1}{4}$, the positive equilibrium $E_4(x^*(m), y^*(m))$ is globally asymptotically stable, Fig. 5, Fig.6 is the case of $m = 0.1$.

Above numeric simulations show that if the fraction of stock is too large, the system will be collapse, while if the fraction of stock is limited, the two species could be coexist in a stable state.

5. Discussion

With the aim of the ecological sustainability and conservation of the species can be implemented in a long run, in this paper, we propose a non-selective harvesting Lotka-Volterra commensalism model incorporating partial closure for the populations, i. e., system (1.2), which can be seen as the generalization of the system (1.1).

For the system without harvesting, it follows from Theorem A that the positive equilibrium is locally asymptotically stable, while the other three equilibrium are all unstable. However, by introducing the harvesting, the dynamic behaviors of the system changes greatly. Depending

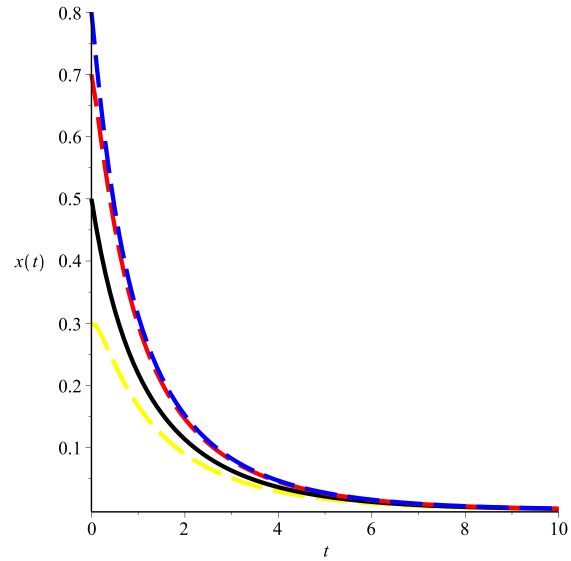

FIGURE 1. Numeric simulations of system (4.1) with $m = 0.75$, the initial conditions $(x(0), y(0)) = (0.5, 0.1), (0.8, 1), (0.3, 3)$, and $(0.7, 2)$, respectively.

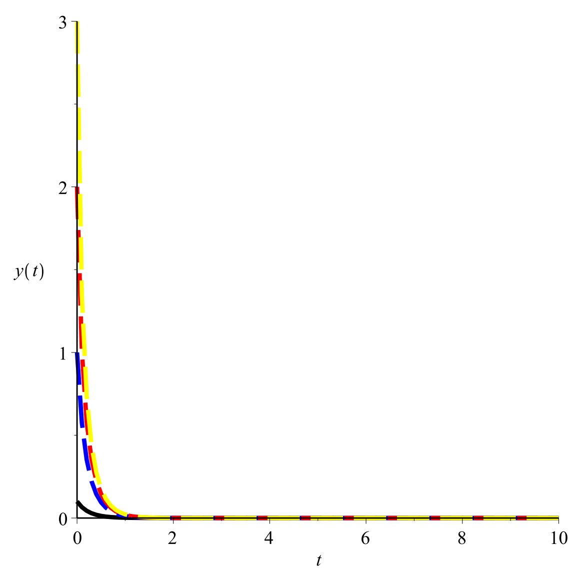

FIGURE 2. Numeric simulations of system (4.1) with $m = 0.75$, the initial conditions $(x(0), y(0)) = (0.5, 0.1), (0.8, 1), (0.3, 3)$, and $(0.7, 2)$, respectively.

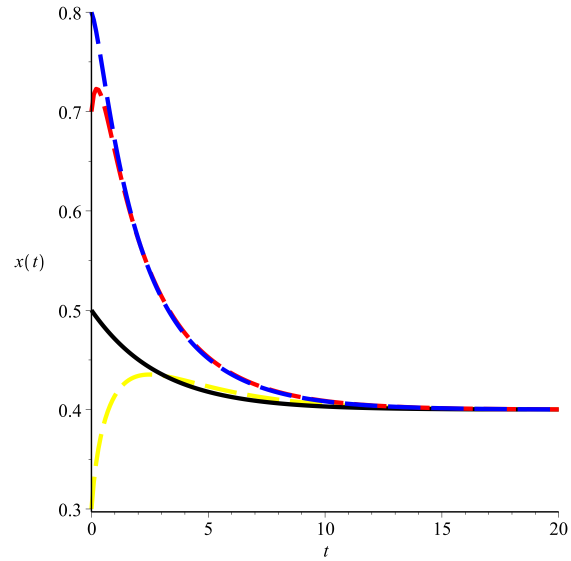

FIGURE 3. Numeric simulations of system (4.1) with $m = 0.3$, the initial conditions $(x(0), y(0)) = (0.5, 0.1), (0.8, 1), (0.3, 3),$ and $(0.7, 2)$, respectively.

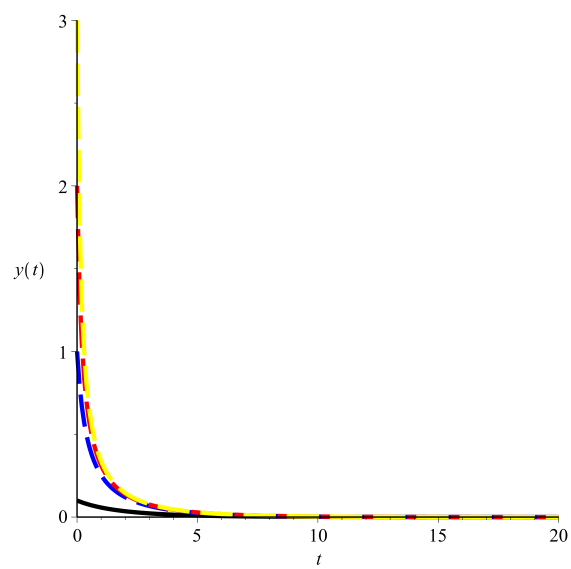

FIGURE 4. Numeric simulations of system (4.1) with $m = 0.3$, the initial conditions $(x(0), y(0)) = (0.5, 0.1), (0.8, 1), (0.3, 3),$ and $(0.7, 2)$, respectively.

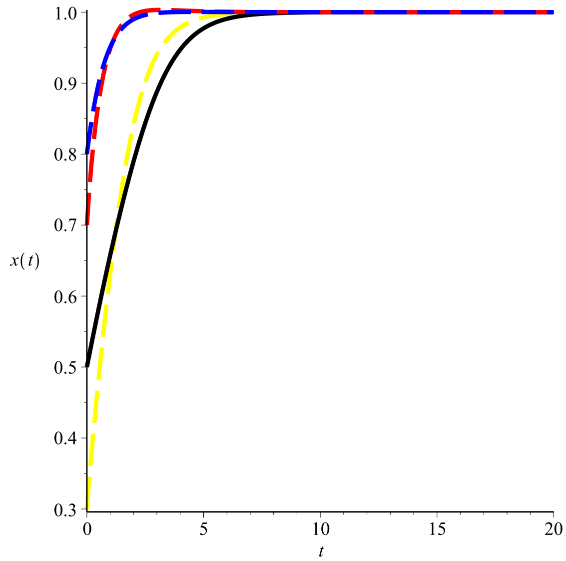

FIGURE 5. Numeric simulations of system (4.1) with $m = 0.1$, the initial conditions $(x(0), y(0)) = (0.5, 0.1), (0.8, 1), (0.3, 3)$, and $(0.7, 2)$, respectively.

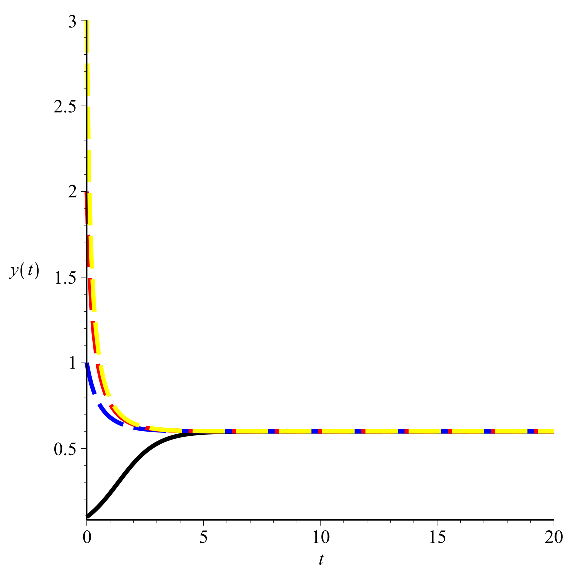

FIGURE 6. Numeric simulations of system (4.1) with $m = 0.1$, the initial conditions $(x(0), y(0)) = (0.5, 0.1), (0.8, 1), (0.3, 3)$, and $(0.7, 2)$, respectively.

on the fraction of the stock and the harvesting effort, the system may be collapse in the sense that both species will be driven to extinction, or partial survival, in the sense that one of the species will be driven to extinction, while the other one is permanent, or both species could be coexisted in a stable state, despite the initial state of the both species.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Authors contributions

All authors contribute equally to the paper.

Acknowledgements

This work is supported by the National Natural Science Foundation of China under Grant (11601085) and the Natural Science Foundation of Fujian Province (2017J01400).

REFERENCES

[1] B. Hari Prasad, N. Ch. Pattabhi Ramacharyulu, Discrete model of commensalism between two species, Int. J. Mod. Educ. Comput. Sci., 8 (2012), 40-46.

[2] J. H. Chen, R. X. Wu, A commensal symbiosis model with non-monotonic functional response, Commun. Math. Biol. Neurosci., 2017 (2017), Article ID 5.

[3] T. T. Li, Q. X. Lin, J. H. Chen, Positive periodic solution of a discrete commensal symbiosis model with Holling II functional response, Commun. Math. Biol. Neurosci., 2016 (2016), Article ID 22.

[4] Y. L. Xue, X. D. Xie, F. D. Chen, et al. Almost periodic solution of a discrete commensalism system, Discrete Dyn. Nat. Soc., 2015 (2015), Article ID 295483.

[5] Z. S. Miao, X. D. Xie, L. Q. Pu, Dynamic behaviors of a periodic Lotka-Volterra commensal symbiosis model with impulsive, Commun. Math. Biol. Neurosci., 2015 (2015), Article ID 3.

[6] R. X. Wu, L. Lin, X. Y. Zhou, A commensal symbiosis model with Holling type functional response, J. Math. Comput. Sci., 16 (2016), 364-371.

[7] X. D. Xie, Z. S. Miao, Y. L. Xue, Positive periodic solution of a discrete Lotka-Volterra commensal symbiosis model, Commun. Math. Biol. Neurosci., 2015 (2015), Article ID 2.

[8] G. C. Sun, H. Sun, Analysis on symbiosis model of two populations, J. Weinan Normal Univ., 28(9)(2013), 6-8.

[9] Q. F. Lin, Dynamic behaviors of a commensal symbiosis model with non-monotonic functional response and non-selective harvesting in a partial closure, Commun. Math. Biol. Neurosci., 2018 (2018), Article ID 4.

[10] R. X. Wu, L. Li, Q. F. Lin, A Holling type commensal symbiosis model involving Allee effect, Commun. Math. Biol. Neurosci., 2018 (2018), Article ID 6.

[11] R. Y. Han, F. D. Chen, Global stability of a commensal symbiosis model with feedback controls, Commun. Math. Biol. Neurosci., 2015 (2015), Article ID 15.

[12] Z. F. Zhu, Q. L. Chen, Mathematic analysis on commensalism Lotka-Volterra model of populations, J. Jixi Univ., 8(5)(2008), 100-101.

[13] F. D. Chen, X. D. Xie, X. F. Chen, Dynamic behaviors of a stage-structured cooperation model, Commun. Math. Biol. Neurosci., 2015 (2015), Article ID 4.

[14] L. Y. Yang, X. D. Xie, C. Q. Wu, Periodic solution of a periodic predator-prey-mutualist system, Commun. Math. Biol. Neurosci., 2015 (2015), Article ID 7.

[15] L. Y. Yang, X. D. Xie, F. D. Chen, Dynamic behaviors of a discrete periodic predator-prey-mutualist system, Discrete Dyn. Nat. Soc., 2015 (2015), Article ID 247269.

[16] K. Yang, Z. S. Miao, F. D. Chen, X. D. Xie, Influence of single feedback control variable on an autonomous Holling-II type cooperative system, J. Math. Anal. Appl., 435(1)(2016), 874-888.

[17] K. Yang, X. D. Xie, F. D. Chen, Global stability of a discrete mutualism model, Abstr. Appl. Anal., 2014 (2014), Article ID 709124.

[18] J. Y. Xu, F. D. Chen, Permanence of a Lotka-Volterra cooperative system with time delays and feedback controls, Commun. Math. Biol. Neurosci., 2015 (2015), Article ID 18.

[19] F. D. Chen, X. D. Xie, Study on the dynamic behaviors of cooperation population modeling, (Chinese) Science Press, Beijing, (2014).

[20] L. J. Chen, L. J. Chen, Z. Li, Permanence of a delayed discrete mutualism model with feedback controls, Math. Comput. Modelling, 50 (2009), 1083-1089.

[21] L. J. Chen, X. D. Xie, Permanence of an N-species cooperation system with continuous time delays and feedback controls, Nonlinear Anal., Real World Appl., 12 (2011), 34-38.

[22] Y. K. Li, T. W. Zhang, Permanence of a discrete n-species cooperation system with time-varying delays and feedback controls, Math. Comput. Modelling, 53 (2011), 1320-1330.

[23] Z. J. Liu, J. H. Wu, R. H. Tan, Y. P. Chen, Modeling and analysis of a periodic delayed two-species model of facultative mutualism, Appl. Math. Comput., 217 (2010), 893–903.

[24] X. D. Xie, F. D. Chen, Y. L. Xue, Note on the stability property of a cooperative system incorporating harvesting, Discrete Dyn. Nat. Soc., 2014 (2014), 5 pages.

[25] X. D. Xie, F. D. Chen, K. Yang, Y. L. Xue, Global attractivity of an integrodifferential model of mutualism, Abstr. Appl. Anal., 2014 (2014), Article ID 928726.

[26] L. Chen, F. Chen, Global analysis of a harvested predator-prey model incorporating a constant prey refuge, Int. J. Biomath., 3 (2010), 177-189.

[27] K. Chakraborty, S. Das, T. K. Kar, On non-selective harvesting of a multispecies fishery incorporating partial closure for the populations, Appl. Math. Comput., 221 (2013), 581-597.

[28] T.K. Kar, K.S. Chaudhuri, On non-selective harvesting of two competing fish species in the presence of toxicity, Ecol. Modell. 161 (2003), 125-137.

[29] B. Leard, J. Rebaza, Analysis of predator-prey models with continuous threshold harvesting, Appl. Math. Comput., 217 (12) (2011), 5265-5278.

[30] K. Chakraborty, S. Jana, T.K. Kar, Global dynamics and bifurcation in a stage structured prey-predator fishery model with harvesting, Appl. Math.Comput. 218 (18) (2012), 9271-9290.

[31] L. S. Chen, X. Y. Song, Z. Y. Lu, Mathematical models and methods in Ecology, Shichuan Science and Technology Press, 2002.

[32] Y. C. Zhou, Z. Jin, J. L. Qin, Ordinary differential equaiton and its application, Beijing, Science Press, 2003.