Available online at http://scik.org
Commun. Math. Biol. Neurosci. 2019, 2019:8
https://doi.org/10.28919/cmbn/3707
ISSN: 2052-2541

FINITE ELEMENT MODEL TO STUDY POTASSIUM DYNAMICS IN THE RHIZOSPHERE OF A WHEAT ROOT DUE TO PRESENCE OF BIO-PHYSICAL SOURCE

RATNAMALA BADGE[1,*], NEERU ADLAKHA[2]

[1]Department of Mathematics,
Maulana Azad National Institute of Technology, Bhopal, Madhya Pradesh 462003, India
[2]Department of Mathematics and Humanities,
Sardar Vallabhbhai National Institute of Technology, Surat, Gujarat 395007, India

Abstract. Potassium is a crucial nutrient required for maintaining the health of a plant. It increases the resistance power of the plant against the diseases. Apart from this potassium plays important role in various other physical and physiological functions involved in the growth of a plant. Uptake of potassium by the root of crop plant depends upon its availability in the rhizosphere of the root. The potassium dynamics in the soil is not well understood till date. In this paper an attempt has been made to study the potassium dynamics in the rhizosphere of the root of wheat crop in the presence and absence of bio-physical source. A finite element model is proposed for a two dimensional unsteady state case. A bio-physical source consisting of rock powder and yeast torulaspora globosa is assumed to be present in the soil. The rock powder contains insoluble potassium which is converted into soluble form of potassium by yeast torulaspora globosa. The model incorporates the term water flux, diffusion coefficient, soil porosity, soil buffer power, uptake coefficient of root etc. The results have been computed using MATLAB. The numerical results have been used to analyze the impact of bio-physical source on the potassium dynamics in the rhizosphere of a root of wheat. The presence of bio-fertilizer leads to significant changes in potassium concentration in the rhizosphere of the wheat root and therefore the bio-physical source consisting of rock powder and yeast torulaspora globosa is recommended as a bio-fertilizer for maintenance of health and growth of wheat crop.

Keywords: Bio-physical source; Potassium; Alkaline ultramafic rock powder; Yeast torulaspora globosa; Wheat root; Mathematical model; Galerkin finite element method.

2010 AMS Subject Classification: 92B05.

---

[*]Corresponding author

E-mail address: ratnamala.badge@gmail.com.

Received March 26, 2018

1. Introduction

The economy of the several countries of the world is largely based on agriculture. The success of agriculture depends on the proper nutrition and health care of plants. The various factors like the fertility of soil, types of soil, environment, availability of water and crop genotype etc. plays a significant role in the growth of the plants and yield of a crop. The roots of the plant uptake nutrients which may be available in the soil by natural or artificial sources. The three most essential nutrients are nitrogen (N), phosphorus (P) and potassium (K). The inadequate amount of these nutrients in the soil are detrimental to the health of the plant and cause reduction in the yield of the crop [6, 9, 10, 16]. Due to repeated agriculture practices the nutrients are depleted there by making the soil infertile. The chemical fertilizers are being used widely as sources of nutrients for growth and development of crops in order to get better yield in terms of quantity. But these chemical fertilizers have adverse effect on the quality of the crop which in turn is detrimental to the health of the human beings and animals consuming these crops [3, 16, 42]. Therefore efforts have been made by research workers to explore natural biological resources and natural processes to enrich the soil with the nutrients.

The rhizosphere contains many micro organisms which serve as natural biological source for supply of various nutrients in the rhizosphere and are known as plant growth promoting rhizobacteria (PGPR) [8, 36]. These micro organisms influence the health and productivity of plants by a variety of mechanisms like nutrient supplementation, root growth, stimulation, suppression of root diseases etc. They also act as the agents of bio-chemical changes in soil and repository of plant nutrients [17, 41]. The use of PGPR as bio fertilizers is one of the desirable solutions to improve the availability of nutrients in the soil, growth and health care of plants and crops. Also, the yield of the plants by using bio fertilizers is free from side effects of chemical fertilizers and hence beneficial for human beings and animals consuming these crops [5, 11, 19, 29, 40]. The present study is focused on potassium dynamics in the rhizosphere of root of wheat crop involving bio-fertilizers.

Potassium solubilising bacteria are able to solubilise potassium mineral powder such as micas, illite and orthoclases and increases K availability in soils and mineral content in plants. Integrated application of potassium rock with bacteria provides a faster and more continuous supply of K for growth of crops [1, 15, 30, 34]. Some potassium solubilising micro organisms are Bacillus, mucilaginosus, Acidothiobacillicus, ferro oxidans, B. Edaplicus, B. Megaterium etc. [18]. Wheat crop has proved to have a higher agronomic K efficiency. Better growth and yield of wheat crop has been observed with the addition of K. Wheat yield increased upto 30% with Azobacter inoculation and upto 43% with Bacillus inoculants [13, 20, 24].

Potassium is one of the three essential nutrients for proper crop growth which plant uptakes in K+ ion form. It is an extremely dynamic ion in plant and soil system. It is involved in enzyme activation, protein synthesis, photosynthesis and assimilation transport. Potassium plays an important role in growth, metabolism, osmoregulation, energy transfer, phloem transport, cation-anion balance, controls stomata opening, food formation, carbohydrate metabolism breakdown, translocation of starch, increases disease and stress resistance and development of plants. Potassium increases drought resistance and aids in reducing plant water loss. Under unfavourable environmental conditions like drought, salinity this nutrient plays a vital role. Potassium is absorbed by plants in large amount than any other mineral elements. The availability of soil potassium level drops due to rapid agriculture development without replenishment of potassium [12, 23, 32, 35, 36, 37]. The form of potassium present in soil are water soluble (solution K), exchangeable, non-exchangeable and structural/ mineral forms. Plants uptake potassium in water-soluble and exchangeable form [4, 21, 39].

Without adequate potassium, the plant shows poor development, slow growth, produce small seeds and poor yield. Deficiency of K in crop shows scorching in leaf margin, reduction in number of leaves produced and the size of individual leaves also reduces. Deficiency in plant available potassium is a major limiting factor for food production in agricultural soils [35, 38]. K deficiencies become a problem because K decreases easily in soils. Potassium concentration in the soil decreases rapidly within one day in the rhizosphere of crops [12]. Quality of crop can be increased by increasing potassium supply. Potassium removal through the crops should be replenished with balanced and adequate K fertilization [22, 31].

In 1970 Nye-Tinker-Barber developed a model for nutrient uptake by a single cylindrical root in an infinite extent of the soil. In this model they neglected the effect of root hair, root exudates, microbial activities etc. Roose et al. extended this classical model of nutrient uptake by the root of a plant. They provided an explicit closed formula for the uptake and effects of root branching and growth [26]. Roose et al. developed a mathematical model to estimate the rate of nutrient uptake by a plant root system in variable soil moisture conditions [27]. Comerford et al. developed a soil supply and nutrient demand (SSAND) model. This model was a steady state, mechanistic nutrient uptake simulation model based on mass flow and diffusion coefficient of nutrients to the roots [7]. Simunek et al. presented a compensated root water and nutrient uptake model and implemented the model in HYDRUS [33]. Leitner et al. developed a mathematical model of nutrient transport and uptake in the root hair zone of single root growing in soil or solution culture. They derived effective equations for the cumulative effect of root hair surfaces on uptake using the method of homogenization [14]. Zygalakis et al. developed a mathematical model for the nutrient transport and uptake in the root hair zone of a single root in the soil and using homogenization techniques they derived a macroscopic dual porosity model for nutrient diffusion and reaction in the soil which includes the effect of all root hair surfaces [42]. Badge et al. proposed a finite element model to study the effect of water flux on nitrate dynamics in the rhizosphere of a maize root [2]. From the literature survey it is observed that no model is reported for the study of bio-physical source on the dynamics of nutrients especially potassium in the rhizosphere of the root of a crop.

In this paper a model is proposed to study the potassium dynamics in the rhizosphere of the root of a wheat crop involving bio-physical source of potassium by incorporating the important parameters like water flux, diffusion coefficient, soil porosity, soil buffer power, uptake coefficient of root etc. The effect of biological source rock powder with yeast torulaspora globosa on potassium dynamics in the rhizosphere of the root of the wheat crop has been analysed. The mathematical model is presented in the next section.

2. Mathematical Formulation

To study the dynamics of potassium in the rhizosphere of wheat root, the nutrient uptake model given by Roose et al. [26] is employed which is given below

$$(\phi+b)\frac{\partial c}{\partial t}+\nabla.(uc)=\nabla.(D\phi\nabla c) \tag{1}$$

where ϕ is the soil porosity.

When soil is saturated ϕ is equivalent to water content in the soil. b is the soil buffer power, D is the diffusion coefficient of nutrient in the soil. The diffusion coefficient in the soil is equal to the diffusion coefficient in free liquid (D_L) times a tortuosity factor (f) i.e. $D = D_L f$. c is the nutrient concentration in the soil, t is the time taken and u is the darcy water flux in the soil which is given by

$$u=-\frac{aV}{r} \tag{2}$$

where a is the radius of the root, V is the water flux into the root and r is the radial distance.

By the law of mass conservation of water

$$\nabla.u=0 \tag{3}$$

Using Eq. (3) in Eq. (1), Eq. (1) becomes

$$(\phi+b)\frac{\partial c}{\partial t}+u.\nabla c=\nabla.(D\phi\nabla c) \tag{4}$$

On substituting u from Eq. (2) in Eq. (4), Eq. (4) becomes

$$(\phi+b)\frac{\partial c}{\partial t}-\frac{aV}{r}\nabla c=\nabla.(D\phi\nabla c) \tag{5}$$

A bio-physical source of potassium is assumed to be present in the rhizosphere of the root. Thus incorporating the source S due to bio-fertilizer in Eq. (5) we get

$$(\phi+b)\frac{\partial c}{\partial t}-\frac{aV}{r}\nabla c=\nabla.(D\phi\nabla c)+S \tag{6}$$

In this study root is assumed to be cylindrical in shape. Thus Eq. (6) in polar coordinates is given by

$$(7)\quad (\phi+b)\frac{\partial c}{\partial t}-\frac{aV}{r}\left(\frac{\partial c}{\partial r}+\frac{1}{r}\frac{\partial c}{\partial \theta}\right)=D\phi\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 c}{\partial \theta^2}\right)+S \quad in \quad \Omega=[a,\infty)\times[0,2\pi], \quad 0\le t<\infty$$

Here θ represents the angle and r is the radial distance from the surface of the root.

It is the tendency of the root to uptake water and nutrients due to requirement for growth of the plant hence the boundary condition at the surface of the root is represented in the form of enzyme kinetic equation given by Michaelis-Menten and is given by Roose et al. [26]

$$(8)\qquad D\phi\frac{\partial c}{\partial n}-c(n.u)=\frac{V_{max}c}{K_m+c}-E \quad at \quad \Gamma_1 \quad i.e. \quad r=a, \quad 0\le\theta\le 2\pi, \quad t>0$$

where Γ_1 represents the surface boundary of the root, n is the unit normal vector perpendicular to the surface of the root, V_{max} is the maximum flux of nutrient, K_m is the Michaelis constant, a is the radius of the root and E is Efflux of nutrient when there is no influx of nutrient in the root of plant which is given by

$$(9)\qquad E=\frac{V_{max}c_{min}}{K_m+c_{min}}$$

where c_{min} is the minimum concentration when there is no nutrient uptake by the root.

Far Field boundary condition is taken as [26]

$$(10)\qquad D\phi\frac{\partial c}{\partial n}-c(n.u)=0 \quad at \quad \Gamma_2 \quad i.e. \quad r\to\infty, \quad 0\le\theta\le 2\pi, \quad t>0$$

A uniform initial concentration of nutrient is taken throughout the domain Ω of the model and is given by

$$(11)\qquad c=c_0 \quad when \quad t=0, \quad a\le r<\infty \quad and \quad 0\le\theta\le 2\pi$$

where c_0 is the initial concentration of potassium nutrient

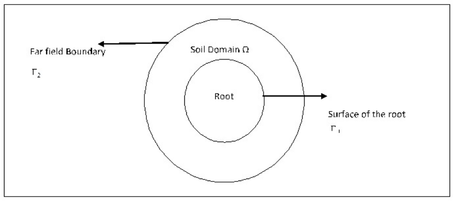


FIGURE 1. Region of the K dynamics

The region of the study Ω is shown in Fig.1 which is enclosed by root surface boundary Γ_1 and far field boundary Γ_2

Following scaling of parameters are used to make model non-dimensional [26]

$$t = \frac{a^2(\phi + b)}{\phi D} T, \quad r = aR, \quad \theta = 2\pi\Psi \quad and \quad c = K_m C \tag{12}$$

The non-dimensional form of Eq. (7) is given by

$$\frac{\partial C}{\partial T} - \frac{P_e}{R}\frac{\partial C}{\partial R} - \frac{P_e}{2\pi R^2}\frac{\partial C}{\partial \Psi} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial C}{\partial R}\right) + \frac{1}{4\pi^2 R^2}\frac{\partial}{\partial \Psi}\left(\frac{\partial C}{\partial \Psi}\right) + \eta \quad in \quad \Omega = [1,\infty) \times [0,1], \quad 0 \le T \le \infty \tag{13}$$

where η is the non-dimensional rate of providing potassium from the source into the rhizosphere and P_e is the peclet number.

Boundary and initial conditions (8), (10) and (11) in non-dimensional form are given by

$$\frac{\partial C}{\partial n} + nP_e C = \frac{\lambda C}{1+C} - \varepsilon \quad at \quad \Gamma_1 \quad i.e. \quad R = 1, \quad 0 \le \Psi \le 1, \quad T > 0 \tag{14}$$

$$\frac{\partial C}{\partial n} + n\frac{P_e}{R}C = 0 \quad at \quad \Gamma_2 \quad i.e. \quad R \to \infty, \quad 0 \le \Psi \le 1, \quad T > 0 \tag{15}$$

$$C = C_\infty \quad when \quad T = 0, \quad 1 \leq R < \infty \quad and \quad 0 \leq \Psi \leq 1 \tag{16}$$

where

$$Pe = \frac{aV}{\phi D}, \quad \lambda = \frac{V_{max} a}{D K_m \phi}, \quad \varepsilon = \frac{Ea}{D K_m \phi}, \quad C_\infty = \frac{c_0}{K_m} \quad and \quad \eta = \frac{Sa^2}{K_m \phi D} \tag{17}$$

P_e is the Peclet number, λ is the uptake parameter, ε is the parameter showing the minimum concentration of nutrient in the soil when there is no uptake of nutrients by the root and C_∞ is the dimensionless nutrient far-field concentration. Here η is a non-dimensional parameter for the source of bio-fertilizer.

The transformation $\xi = \frac{R-1}{R}$ is employed to transform the model from domain $[1,\infty) \times [0,1]$ into $[0,1) \times [0,1]$. Eqs. (13), (14), (15) and (16) are given by

$$\begin{aligned} &\frac{\partial}{\partial \xi}\left((1-\xi)\frac{\partial C}{\partial \xi}\right) + \frac{1}{4\pi^2(1-\xi)}\frac{\partial^2 C}{\partial \Psi^2} + P_e \frac{\partial C}{\partial \xi} + \frac{P_e}{2\pi(1-\xi)}\frac{\partial C}{\partial \Psi} - \\ &\frac{1}{(1-\xi)^3}\frac{\partial C}{\partial T} + \frac{1}{(1-\xi)^3}\eta = 0 \quad in \quad \Omega = [0,1) \times [0,1], \quad 0 \leq T \leq \infty \end{aligned} \tag{18}$$

$$\frac{\partial C}{\partial n} + nP_e C = \frac{\lambda C}{1+C} - \varepsilon \quad at \quad \Gamma_1 \quad i.e. \quad \xi = 0, \quad 0 \leq \Psi \leq 1, \quad T > 0 \tag{19}$$

$$\frac{\partial C}{\partial n} + nP_e(1-\xi)C = 0 \quad at \quad \Gamma_2 \quad i.e. \quad \xi \to 1, \quad 0 \leq \Psi \leq 1, \quad T > 0 \tag{20}$$

$$C = C_\infty \quad when \quad T = 0, \quad 0 \leq \xi < 1 \quad and \quad 0 \leq \Psi \leq 1 \tag{21}$$

Since $\frac{C}{1+C}$ is less than or equal to $\frac{C_\infty}{1+C_\infty}$, hence in order to linearize Eq. (19) upper bound of $\frac{C}{1+C}$ is taken which makes Eq. (22) as given below

$$\frac{\partial C}{\partial n} + nP_e C = \frac{\lambda C_\infty}{1+C_\infty} - \varepsilon \quad i.e. \quad \xi = 0, \quad 0 \leq \Psi \leq 1, \quad T > 0 \tag{22}$$

3. Solution Strategy

Test function $w(X,\Psi,T)$ is taken for the weak formulation of the Eq. (18) which is given as

$$\begin{aligned}&\int_0^1\int_0^1 w(\xi,\Psi,T)[\frac{\partial}{\partial\xi}\left((1-\xi)\frac{\partial C}{\partial\xi}\right)+\frac{1}{4\pi^2(1-\xi)}\frac{\partial^2 C}{\partial\Psi^2}+P_e\frac{\partial C}{\partial\xi}+\\ &\frac{P_e}{2\pi(1-\xi)}\frac{\partial C}{\partial\Psi}-\frac{1}{(1-\xi)^3}\frac{\partial C}{\partial T}+\frac{1}{(1-\xi)^3}\eta]d\xi d\Psi=0\end{aligned}\tag{23}$$

Using the condition of orthogonality between test function and trial function, Eq. (23) is expressed as

$$\begin{aligned}&\int_0^1\int_0^1[(1-\xi)\left(\frac{\partial C}{\partial\xi}\right)^2+\frac{1}{4\pi^2(1-\xi)}\left(\frac{\partial C}{\partial\Psi}\right)^2-P_eC\frac{\partial C}{\partial\xi}-\frac{P_e}{2\pi(1-\xi)}C\frac{\partial C}{\partial\Psi}\\ &+\frac{1}{(1-\xi)^3}\frac{\partial C^2}{\partial T}-\eta\frac{1}{(1-\xi)^3}C]d\xi d\Psi+\int_{\Gamma_1}C\frac{\partial C}{\partial n}d\Gamma+\int_{\Gamma_2}C(1-\xi)\frac{\partial C}{\partial n}d\Gamma=0\end{aligned}\tag{24}$$

The domain Ω is discretized into eighty elements and ninety six nodes (ξ is discretized as 0, 0.2, 0.4, 0.6, 0.8 and 1) which is shown in Fig.2

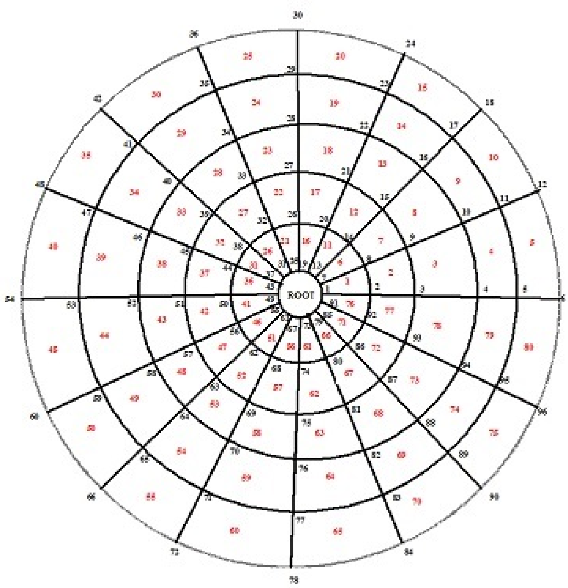


FIGURE 2. Finite Element discretization of the rhizosphere of a wheat root.

Discretized form of Eq. (24) is given by

$$I^{(e)}=\int_{\xi_i}^{\xi_j}\int_{\Psi_i}^{\Psi_k}[(1-\xi)\left(\frac{\partial C^{(e)}}{\partial\xi}\right)^2+\frac{1}{4\pi^2(1-\xi)}\left(\frac{\partial C^{(e)}}{\partial\Psi}\right)^2-P_eC^{(e)}\frac{\partial C^{(e)}}{\partial\xi}$$
$$-\frac{P_e}{2\pi(1-\xi)}C^{(e)}\frac{\partial C^{(e)}}{\partial\Psi}+\frac{1}{(1-\xi)^3}\frac{\partial(C^{(e)})^2}{\partial T}-\eta\frac{1}{(1-\xi)^3}C^{(e)}]d\xi d\Psi$$
$$+\int_{\Gamma_1}C^{(e)}\frac{\partial C^{(e)}}{\partial n}d\Gamma+\int_{\Gamma_2}C^{(e)}(1-\xi)\frac{\partial C^{(e)}}{\partial n}d\Gamma=0 \tag{25}$$

The following bilinear shape function for concentration variation of nutrient within each element is taken as

$$C^{(e)}=A_1^{(e)}+A_2^{(e)}\xi+A_3^{(e)}\Psi+A_4^{(e)}\xi\Psi \tag{26}$$

$$C^{(e)}=\bar{S}^TA^{(e)} \tag{27}$$

where

$$\bar{S}^T=\begin{bmatrix}1 & \xi & \Psi & \xi\Psi\end{bmatrix} \tag{28}$$

and

$$(A^{(e)})^T=\begin{bmatrix}A_1^{(e)} & A_2^{(e)} & A_3^{(e)} & A_4^{(e)}\end{bmatrix} \tag{29}$$

Substituting nodal conditions in Eq. (27),we get

$$\overline{C}^{(e)}=\bar{S}^{(e)}A^{(e)} \tag{30}$$

where

$$\overline{C}^{(e)}=\begin{bmatrix}C_i\\ C_j\\ C_k\\ C_l\end{bmatrix} \tag{31}$$

and

$$\bar{S}^{(e)} = \begin{bmatrix} 1 & \xi_i & \Psi_i & \xi_i\Psi_i \\ 1 & \xi_j & \Psi_j & \xi_j\Psi_j \\ 1 & \xi_k & \Psi_k & \xi_k\Psi_k \\ 1 & \xi_l & \Psi_l & \xi_l\Psi_l \end{bmatrix} \tag{32}$$

From the Eq.(30), we have

$$A^{(e)} = H^{(e)}\overline{C}^{(e)} \tag{33}$$

where

$$H^{(e)} = \left(\bar{S}^{(e)}\right)^{-1}$$

Substituting $A^{(e)}$ from Eq. (33) in Eq. (27), we get

$$C^{(e)} = \bar{S}^T H^{(e)}\overline{C}^{(e)} \tag{34}$$

The integrals $I^{(e)}$ given in expression Eq. (25) are evaluated using Eq. (34) and assembled to obtain

$$I^{(e)} = I_1^{(e)} + I_2^{(e)} + I_3^{(e)} + I_4^{(e)} + I_5^{(e)} + I_6^{(e)} + I_7^{(e)} + I_8^{(e)} \tag{35}$$

where

$$I_1^{(e)} = \int_{\xi_i}^{\xi_j}\int_{\Psi_i}^{\Psi_k} (1-\xi)\left(\frac{\partial C^{(e)}}{\partial \xi}\right)^2 d\xi d\Psi \tag{36}$$

$$I_2^{(e)} = \frac{1}{4\pi^2}\int_{\xi_i}^{\xi_j}\int_{\Psi_i}^{\Psi_k} \frac{1}{(1-\xi)}\left(\frac{\partial C^{(e)}}{\partial \Psi}\right)^2 d\xi d\Psi \tag{37}$$

$$I_3^{(e)} = -P_e\int_{\xi_i}^{\xi_j}\int_{\Psi_i}^{\Psi_k} C^{(e)}\frac{\partial C^{(e)}}{\partial \xi} d\xi d\Psi \tag{38}$$

$$I_4^{(e)} = -\frac{P_e}{2\pi}\int_{\xi_i}^{\xi_j}\int_{\Psi_i}^{\Psi_k}\frac{1}{1-\xi}C^{(e)}\frac{\partial C^{(e)}}{\partial \Psi}\,d\xi\, d\Psi \tag{39}$$

$$I_5^{(e)} = \int_{\xi_i}^{\xi_j}\int_{\Psi_i}^{\Psi_k}\frac{1}{(1-\xi)^3}\frac{\partial\left(C^{(e)}\right)^2}{\partial T}\,d\xi\, d\Psi \tag{40}$$

$$I_6^{(e)} = -\int_{\xi_i}^{\xi_j}\int_{\Psi_i}^{\Psi_k}\eta\frac{1}{(1-\xi)^3}C^{(e)},d\xi\, d\Psi \tag{41}$$

$$I_7^{(e)} = \int_{\Gamma_1} C^{(e)}\frac{\partial C^{(e)}}{\partial n}\,d\Gamma \tag{42}$$

$$I_8^{(e)} = \int_{\Gamma_2} C^{(e)}\,(1-\xi)\,\frac{\partial C^{(e)}}{\partial n}d\Gamma \tag{43}$$

The boundary conditions Eq. (20) and Eq. (22) are applied on the Eq. (43) and Eq. (42) respectively.

Extremizing the integral I w.r.t. each nodal potassium concentration as given below

$$\frac{dI}{d\overline{C}} = \sum_{e=1}^{80}\overline{M}^{(e)}\frac{dI^{(e)}}{d\overline{C}^{(e)}}\overline{M}^{(e)^T} = 0 \tag{44}$$

where

$$\overline{M}^{(e)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ - & - & - & - \\ - & - & - & - \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ - & - & - & - \\ - & - & - & - \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad and \quad \overline{C} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ - \\ - \\ - \\ C_{96} \end{bmatrix} \quad and \quad I = \sum_{e=1}^{80} I^{(e)} \tag{45}$$

$$\frac{dI^{(e)}}{d\overline{C}^{(e)}} = \frac{dI_1^{(e)}}{d\overline{C}^{(e)}} + \frac{dI_2^{(e)}}{d\overline{C}^{(e)}} + \frac{dI_3^{(e)}}{d\overline{C}^{(e)}} + \frac{dI_4^{(e)}}{d\overline{C}^{(e)}} + \frac{dI_5^{(e)}}{d\overline{C}^{(e)}} + \frac{dI_6^{(e)}}{d\overline{C}^{(e)}} + \frac{dI_7^{(e)}}{d\overline{C}^{(e)}} + \frac{dI_8^{(e)}}{d\overline{C}^{(e)}} \tag{46}$$

This leads to a following system of linear differential equations

$$[E]_{96\times 96}\left[\frac{d\overline{C}}{dT}\right]_{96\times 1} + [F]_{96\times 96}\left[\,\overline{C}\,\right]_{96\times 1} = [G]_{96\times 1} \tag{47}$$

Where E, F are the system matrices and G is the system vector.

Crank-Nicolson method is employed along the temporal scale to solve the system of Eq. (47). The time step is taken as 0.01. A program is developed in MATLAB 8.3.0.532 to find a numerical solution of the entire model.

4. Results and Discussion

The numerical results are obtained by using the following parameter values of potassium nutrient for wheat crop which are given in Table 1 and Table 2.

TABLE 1. Dimensional Root Parameters Value for Wheat Crop [28].

Parameter	Value	Unit
a	0.02	cm
V_{max}	3.73×10^{-6}	$\mu\ mol\ cm^{-2}\ s^{-1}$
K_m	1.0×10^{-2}	$\mu\ mol\ cm^{-3}$
V	38.05×10^{-7}	$cm\ s^{-1}$
c_{min}	1.05×10^{-3}	$\mu\ mol\ cm^{-3}$

The study is made for the following three cases

Case (i): Medium is only soil

Case (ii): Soil medium + Rock powder

TABLE 2. Dimensional Soil Parameters Value [28].

Parameter	Value	Unit
c_0	0.092	$\mu\ mol\ cm^{-3}$
ϕ	0.3	-
b	16.1	-
D_L	1.98×10^{-5}	$cm^2\ s^{-1}$
f	0.12	-

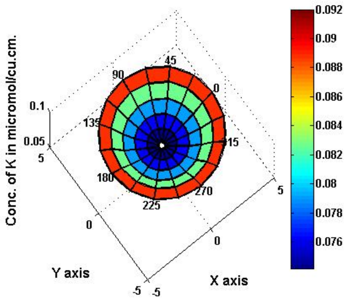
Case (iii): Soil medium + Rock powder + Yeast torulaspora globosa

It is assumed that the rock powder and the yeast is uniformly distributed in the whole rhizosphere of the root. Thus this implies that the source in the form of rock powder or bio-fertilizer compost of rock powder and yeast is constant throughout the whole rhizosphere. The data of rock powder and bio-fertilizer has been taken from the experimental work of Rosa-Magri et al. and is presented in Table 3.

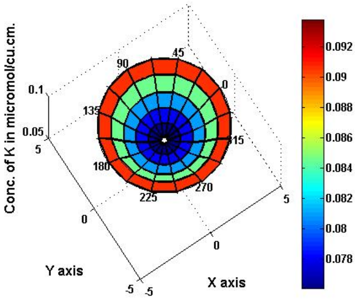
TABLE 3. Rate of release of K for (i) soil medium with rock powder (ii) soil medium with rock powder and yeast torulaspora globosa [25]

Time (days)	Case(ii)	Case(iii)
3	0.01	0.16667
6	0.00167	0.12167
9	0.00333	0.08889
12	0.00333	0.07167
15	0.00467	0.05867

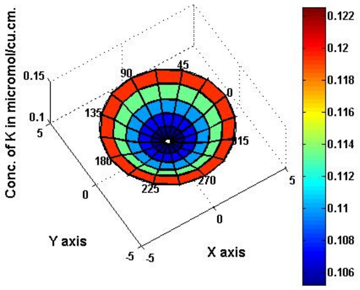
The concentration of potassium in micromole per cubic centimetre in the rhizosphere of a wheat root on the 3rd, 6th, 9th, 12th and 15th day is shown respectively in the Figures 3, 4, 5, 6 and 7. The dynamics of potassium in the rhizosphere of a wheat root in the absence of rock powder and yeast torulaspora globosa is shown respectively in the Fig. 3(a), 4(a), 5(a), 6(a) and 7(a). The dynamics of potassium in the rhizosphere of wheat root in the presence of rock



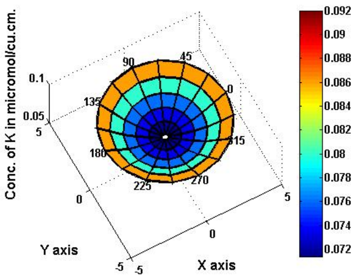
3(a) Conc. of K on 3rd day in absence of rock powder and yeast



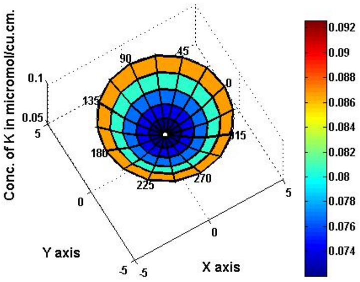
3(b) Conc. of K on 3rd day in presence of rock powder and absence of yeast



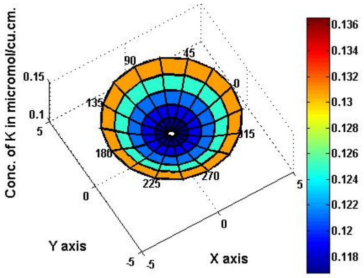
3(c) Conc. of K on 3rd day in presence of rock powder and yeast



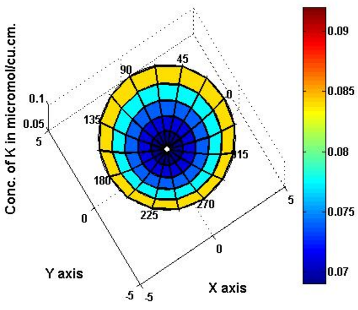
4(a) Conc. of K on 6th day in absence of rock powder and yeast



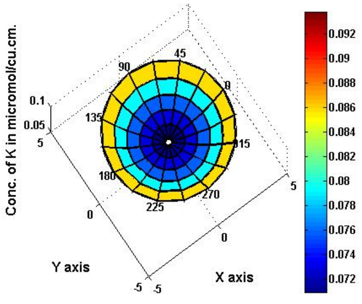
4(b) Conc. of K on 6th day in presence of rock powder and absence of yeast



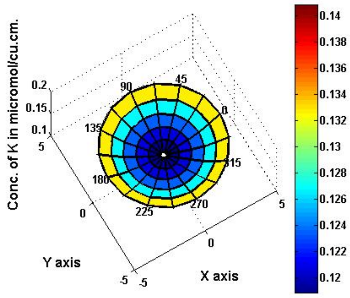
4(c) Conc. of K on 6th day in presence of rock powder and yeast



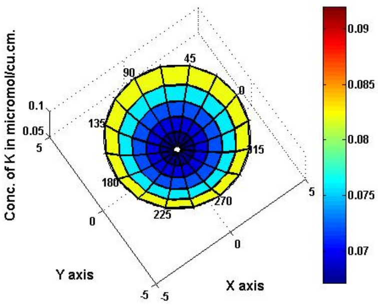
5(a) Conc. of K on 9th day in absence of rock powder and yeast



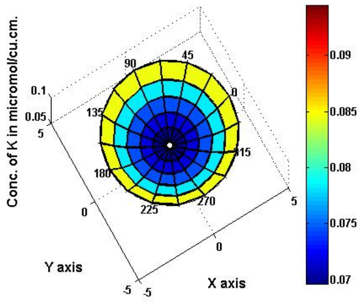
5(b) Conc. of K on 9th day in presence of rock powder and absence of yeast



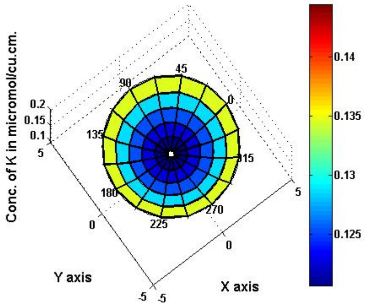
5(c) Conc. of K on 9th day in presence of rock powder and yeast



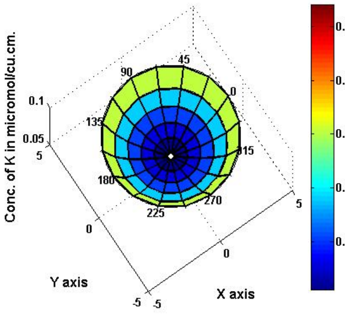
6(a) Conc. of K on 12th day in absence of rock powder and yeast



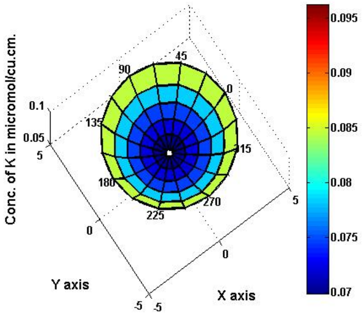
6(b) Conc. of K on 12th day in presence of rock powder and absence of yeast



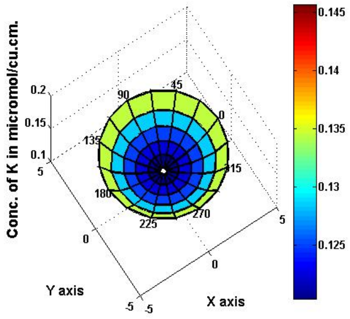
6(c) Conc. of K on 12th day in presence of rock powder and yeast



7(a) Conc. of K on 15th day in absence of rock powder and yeast

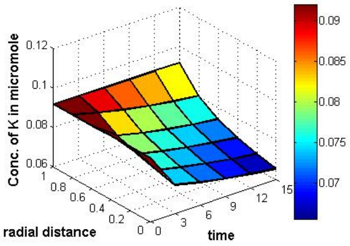


7(b) Conc. of K on 15th day in presence of rock powder and absence of yeast

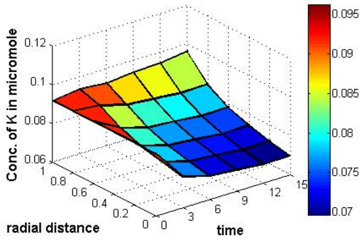


7(c) Conc. of K on 15th day in presence of rock powder and yeast

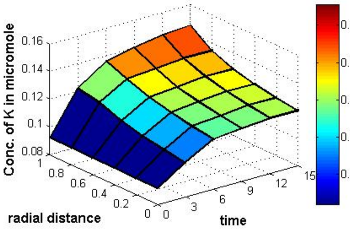
powder and in the absence of yeast torulaspora globosa is shown respectively in the Fig. 3(b), 4(b), 5(b), 6(b) and 7(b). The dynamics of potassium in the rhizosphere of a wheat root in the presence of rock powder and yeast torulaspora globosa is shown respectively in the Fig. 3(c), 4(c), 5(c), 6(c) and 7(c). It is observed in Fig. 3(a), 4(a), 5(a), 6(a) and 7(a) that in the absence of rock powder and yeast the concentration of potassium decreases in the rhizosphere with the passage of time. This is due to the fact that as there is no source present in the rhizosphere for release of potassium, the available quantity of potassium in the rhizosphere is being depleted due to the uptake of potassium by the root leading to the fall in the concentration of potassium in the rhizosphere of the root. Comparing Fig. 3(b), 4(b), 5(b), 6(b) and 7(b) with the Fig. 3(a), 4(a), 5(a), 6(a) and 7(a) it is observed that the concentration of potassium is higher in Fig. 3(b), 4(b), 5(b), 6(b) and 7(b) as compared to that in Fig. 3(a), 4(a), 5(a), 6(a) and 7(a). Also the fall in the concentration of potassium in the rhizosphere with the passage of time for case (i) [Fig. 3(a), 4(a), 5(a), 6(a) and 7(a)] is more as compared to that for case (ii) [Fig. 3(b), 4(b), 5(b), 6(b) and 7(b)]. This is because in case (ii) the presence of rock powder acts as a source and supplements the potassium concentration with the passage of time. Further on comparing figures of case (ii) [Fig. 3(b), 4(b), 5(b), 6(b) and 7(b)] with figures of case (iii) [Fig. 3(c), 4(c), 5(c), 6(c) and 7(c)] it is observed that the potassium concentration for case (iii) is higher than that to case (ii). Also the fall in the concentration of potassium in rhizosphere of root with the passage of time in case (iii) is lower than that in case (ii). This is due to the fact that the presence of yeast along with the rock powder accelerates the release of potassium from rock powder in the rhizosphere of the root thereby supplementing more potassium than that for case (ii) in which yeast was absent.



8(a)Conc. of K in absence of rock powder and yeast



8(b) Conc. of K in presence of rock powder and absence of yeast



8(c)Conc. of K in presence of rock powder and yeast

Figure 8 shows the concentration of potassium in micromole per cubic centimetre with respect to time in days and on the non-dimensional radial distance from the surface of the root for three cases (a) soil medium, (b) soil medium with rock powder and in absence of yeast (c) soil medium with rock powder and yeast torulaspora globosa. In Fig. 8(a) it is observed that the concentration is maximum at the point far away from the root and decreases in the region as we move towards the surface of the root where it is minimum. This is due to the fact that the root uptakes the potassium around it thereby reducing the potassium concentration in the region near the surface of the root. Further it is observed in Fig. 8(a) the potassium concentration in the rhizosphere of the root decreases with the passage of time. This is due to the fact that the potassium concentration is depleted with the passage of time due to its continuous uptake by the root. Similar behaviour of potassium concentration with respect to the position and time is also observed in Fig. 8(b). Comparing Fig. 8(b) with 8(a) it is observed that the fall in concentration with respect to position and time both in Fig. 8(a) is more as compared to that in Fig. 8(b). This is due to the fact that the presence of rock powder in the rhizosphere of root in case(ii) supplements with additional potassium concentration released from the rock powder thereby decreasing the fall in potassium concentration due to its depletion caused by its uptake by the root as compared to that in case (i) as there is no source present in case (i). In Fig. 8(c) it is observed that the potassium concentration increases sharply from 3rd day to 9th day and then gradually it increases further upto the 12th day and after that it falls slightly upto 15th day. This is due to the fact that between 3rd to 9th day sufficient rock powder is available in the rhizosphere from which the yeast is converting insoluble potassium into soluble potassium at a rate quite higher than the rate of uptake of potassium by the root. As the time passes the available rock powder depletes due to its conversion into soluble potassium by the yeast and therefore the rate of release of potassium by the yeast decreases due to decrease of quantity of rock powder in the soil. This implies that as the time passes further after 15 days a time will be reached when no rock powder will be left and the soil will be devoid of the source.

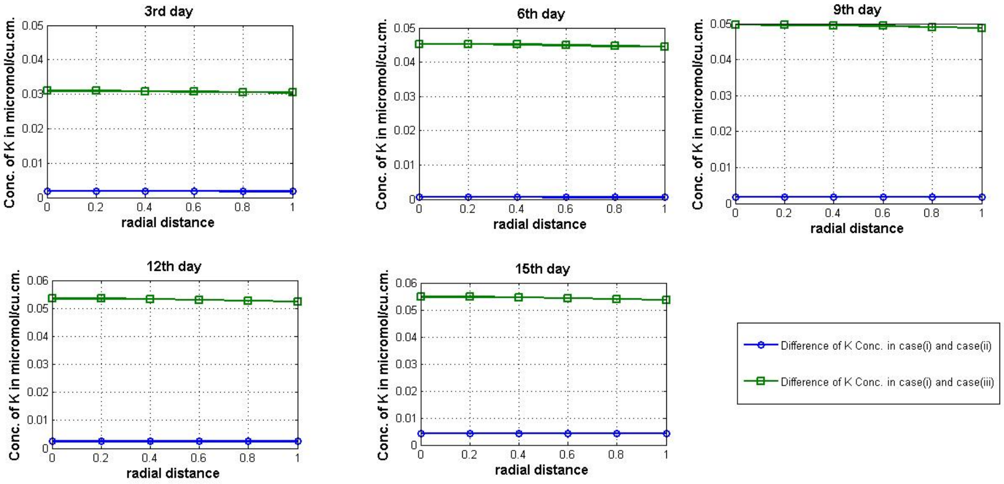


Figure 9 shows the difference in concentration of potassium in micromole per cubic centimetre between case (i) and case (ii) and between case (i) and case (iii) with respect to non-dimensional radial distance in the rhizosphere of the wheat root. Initially the concentration of potassium is same in all the three cases. The difference in concentration of potassium is large when yeast is added in the soil medium with rock powder. When the soil is having only rock powder the difference in concentration of potassium with the soil medium is very small. The reason for large difference in concentration difference of K, when yeast is added with the rock powder in soil is that yeast helps in releasing potassium from insoluble potassium present in rock powder into soluble potassium in the soil due to which concentration of soluble potassium increases in the rhizosphere. As time elapses the difference in concentration between absence of any source and in presence of bio fertilizer increases.

TABLE 4. Percentage change in potassium concentration between the case (i) and case (ii).

Radial Distance	Day 3	Day 6	Day 9	Day 12	Day 15
0	2.50511	0.869266	2.699112	3.70648	6.651197
0.2	2.431438	0.842727	2.614021	3.586387	6.430616
0.4	2.342593	0.810791	2.511832	3.442422	6.16661
0.6	2.22834	0.769816	2.381016	3.258509	5.829961
0.8	2.061619	0.710123	2.190826	2.991637	5.342322
0.9999	1.988269	0.662789	1.988333	2.651154	4.639474

In Table 4 a small percentage of change in potassium concentration is observed with the passage of time in the rhizosphere between case (i) and case (ii). In Table 4 the percentage change decreases on 6th day and again increases on 9th day onwards upto 15th day. This is due to the fact that initially there is some amount of soluble potassium in the rhizosphere and the potassium in rock powder is in insoluble form and it takes some time to convert this rock powder into soluble form of potassium by the rhizosphere of root and then continuously soluble form of potassium is being released from 9th day onwards. Also the percentage change in potassium after 9th day increases due to increase in difference of concentration of potassium

TABLE 5. Percentage change in potassium concentration between the case (i) and case (iii).

Radial Distance	Day 3	Day 6	Day 9	Day 12	Day 15
0	41.75185	63.45652	71.9764	79.68939	83.61502
0.2	40.52397	61.51918	69.7073	77.10738	80.84199
0.4	39.04323	59.18783	66.98225	74.01214	77.52307
0.6	37.13901	56.19663	63.49383	70.058	73.29091
0.8	34.36032	51.83906	58.42209	64.32025	67.16059
0.9999	33.13782	48.38357	53.02222	56.99981	58.32481

in case (i) and case(ii) as the potassium depletion due to uptake by root in case (i) increase the difference gap of potassium concentration between case (i) and case (ii). The percentage change in potassium concentration as shown in Table 5 is in between 33-83%. In table 5 the percentage change in potassium concentration is 13-20 times of that in Table 4. This implies that the presence of yeast accelerates the release of soluble potassium from the rock powder thereby causing a significant change in potassium concentration level in the rhizosphere of the root. Thus the bio-fertilizer enhances the availability of potassium in the soil quite significantly which is useful for the health anf growth of the plant.

5. Conclusion

A two dimensional finite element model is proposed and employed to study potassium dynamics in the rhizosphere of a wheat root for unsteady state case in the presence and absence of physical and bio-physical sources of potassium in the rhizosphere. From the results it is concluded that the presence of rock powder in absence of yeast slows down the depletion of potassium in the rhizosphere due to uptake by the root and helps in maintenance of potassium concentration level in the rhizosphere of the root. It also can be concluded that bio-fertilizer as a bio-physical source present in the soil not only retards the depletion of potassium in the rhizosphere due to uptake by the root but also provides high availability of potassium concentration in the rhizosphere of a wheat root. This good amount of potassium availability is very useful for maintenance of health and growth of a plant. Further this bio-fertilizer do not have any harmful effect on the crop and animals and humans who consume this crop. The finite element approach has proved to be quite versatile in the present study as it was possible to incorporate the variations in the parameters in the model. Such models can be developed further to explore the impact of other bio-fertilizers or bio-physical sources on concentration of potassium and other nutrients in the soil. On the basis of the present study the presence of bio-physical source consisting of rock powder and yeast is recommended as bio-fertilizer for wheat crop. In all it is a new research progress direction in the field of agriculture sciences and mathematical biology.

Acknowledgements

Authors are highly thankful to the Department of Biotechnology, New Delhi, India for providing the Bioinformatics Infrastructure facility at the Department of Mathematics, Maulana Azad National Institute of Technology, Bhopal for carrying out this work.

Conflict of Interests

The authors declare that there is no conflict of interests.

REFERENCES

[1] I.T. Abou-el-Seoud, A. Abdel-Megeed: Impact of rock materials and biofertilizations on P and K availability for wheat (Zea wheat) under calcareous soil conditions, Saudi J. Biol. Sci. 19(1) (2012), 55-63.

[2] R. Badge, N. Adlakha: Two dimensional finite element model to study the effect of water flux on the nitrate dynamics in the rhizosphere of a maize root, Int. J. Biomath. 10(4) (2017), Article ID 1750058.

[3] I. Bahadur, V.S. Meena, S. Kumar: Importance and application of potassic biofertilizer in Indian agriculture, Res. J. Chem. Sci. 3(12) (2014), 80–85.

[4] B.B. Basak, D.R. Biswas: Influence of potassium solubilizing microorganism (Bacillus mucilaginosus) and waste mica on potassium uptake dynamics by sudan grass (Sorghum vulgare Pers.) grown under two Alfisols, Plant Soil, 317(1-2) (2009), 235-255.

[5] M. Billah, A. Bano: Role of plant growth promoting rhizobacteria in modulating the efficiency of poultry litter composting with rock phosphate and its effect on growth and yield of wheat, Waste Manage. Res. 33(1) (2015), 63–72.

[6] N. Claassen, K.M. Syring, A. Jungk: Calculati on of K Uptake by Two Crops Grown under Field Conditions U sing a Mechanistic Model, Plant Soil, 95(2) (1986),209-220.

[7] N.B. Comerford, Cropper, W.P. Jr, H. Li, P.J. Smethurst, K.C.J. Van Rees, E.J. Jokela, H. Adgbidi, N.F. Barros: Soil Supply and Nutrient Demand (SSAND): a general nutrient uptake model and an example of its application to forest management, Can J. Soil Sci. 86(4) (2006), 655–673.

[8] P.R. Darrah, D.L. Jones, G.J.D. Kirk, T. Roose: Modelling the rhizosphere: a review of methods for upscalingto the wholeplant scale, J. Soil Sci. 57(1) (2006), 13-25.

[9] M.L. Dotaniya, S.K. Kushwah:Nutrients uptake ability of various rainy season crops grown in a vertisol of central India, African J. Agric. Res. 8(44) (2013), 5592–5598.

[10] N. Claassen, K. M. Syring, and A. Jungk: Verification of a mathematical model by simulating potassium uptake from soil, Plant Soil 95 (2) (1986), 209-220.

[11] M.G.Z. Girgis, H.M. Khalil, M.S. Sharaf MS: In vitro evaluation of rock phosphate and potassium solubilizing potential of some Bacillus strains, Aust. J. Basic Appl. Sci. 2(1) (2008), 68–81.

[12] A. Jungk, N. Claassen: Availability of phosphate and potassium as the result of interactions between root and soil in the rhizosphere, J. Plant Nutrition Soil Sci. 149(4) (1986), 411-427.

[13] J. Larsen, P. Jaramillo-Lpez, M. Njera-Rincon, C.E. Gonzlez-Esquivel: Biotic interactions in the rhizosphere in relation to plant and soil nutrient dynamics , J. Soil Sci. Plant Nutrition, 15(2) (2015), 449–463.

[14] D. Leitner, S. Klepsch, M. Ptashnyk, A. Marchant, G.J.D. Kirk, A. Schnepf, T. Roose: A dynamic model of nutrient uptake by root hairs, New Phytologist, 185(3) (2010), 792–802.

[15] M. Lija, A.O. Haruna, S. Kasim: Wheat (Zea mays L.) nutrient use efficiency as affected by formulated fertilizer with Clinoptilolite Zeolite, Emirates J. Food Agric. 26(3) (2014), 284.

[16] L. Louison, A. Omrane, H. Ozier-Lafontaine, D. Picart: Modeling plant nutrient uptake: mathematical analysis and optimal control, Evol. Equ. Control Theory, 4(2) (2015) 193-203.

[17] O. Martnez-Viveros, M.A. Jorquera, D.E. Crowley, G.M.L.M. Gajardo, M.L. Mora: Mechanisms and practical considerations involved in plant growth promotion by rhizobacteria, J. Soil Sci. Plant Nutrition, 10(3) (2010), 293–319.

[18] V.S. Meena, B.R. Maurya, J.P. Verma: Does a rhizospheric microorganism enhance K+ availability in agricultural soils?., Microbiol. Res. 169(5) (2014), 337–347.

[19] V.S. Meena, B.R. Maurya, J.P. Verma, A. Aeron, A. Kumar, K. Kim, V.K. Bajpai: Potassium solubilizing rhizobacteria (KSR): isolation, identification and K-release dynamics from waste mica, Ecol. Eng. 81 (2015), 340–347.

[20] E. Mursyida, N.R. Mubarik, A. Tjahjoleksono: Selection and Identification of Phosphate-Potassium Solubilizing Bacteria from the Area Around the Limestone Mining in Cirebon Quarry, Res. J. Microbiol. 10(6) (2015), 270.

[21] P. Parmar, S.S. Sindhu: Potassium solubilization by rhizosphere bacteria: influence of nutritional and environmental conditions, J. Microbiol. Res. 3(1) (2013), 25–31.

[22] W.T. Pettigrew: Potassium influences on yield and quality production for wheat, wheat, soybean and cotton, Physiologia plantarum, 133(4) (2008), 670-681.

[23] K.B. Prajapati, H.A. Modi: Isolation and characterization of potassium solubilizing bacteria from ceramic industry soil, CIBTech J. Microbiol. 1(2-3) (2012), 8–14.

[24] H. Rodrguez, R. Fraga: Phosphate solubilizing bacteria and their role in plant growth promotion, Ecol. Eng. 17(4) (1999), 319–339.

[25] M.M. Rosa-Magri, S.H. Avansini, M.L. Lopes-Assad, S.M. Tauk-Tornisielo, S.R. Ceccato-Antonini: Release of potassium from rock powder by the yeast Torulaspora globosa, Brazil. Arch. Biol. Techn. 55(4) (2012), 577–582.

[26] T. Roose, A.C. Fowler, P.R. Darrah: A mathematical model of plant nutrient uptake, J. Math. Biol. 42(4) (2001), 347–360.

[27] T. Roose, A.C. Fowler: A mathematical model for water and nutrient uptake by plant root systems, J. Theor. Biol. 228(2) (2004), 173–184.

[28] D. Samal, J.L. Kovar, B. Steingrobe, U.S. Sadana, P.S. Bhadoria, N. Claassen: Potassium uptake efficiency and dynamics in the rhizosphere of wheat (Zea mays L.), wheat (Triticum aestivum L.), and sugar beet (Beta vulgaris L.) evaluated with a mechanistic model., Plant Soil, 332(1-2) (2010), 105–121.

[29] T.C. Setiawati, L. Mutmainnah: Solubilization of Potassium Containing Mineral by Microorganisms From Sugarcane Rhizosphere, Agric. Agric. Sci. Proc. 9 (2016), 108–117.

[30] A.S. Shanware, S.A. Kalkar,M.M. Trivedi: Potassium solublisers: occurrence, mechanism and their role as competent biofertilizers, Int. J. Curr. Microbiol. App. Sci. 3 (2014), 622–629.

[31] S.B. Sharma, R.Z. Sayyed, M.H. Trivedi, T.A. Gobi: Phosphate solubilizing microbes: sustainable approach for managing phosphorus deficiency in agricultural soils, SpringerPlus, 2(1) (2013), Article ID 587.

[32] R. Shukla, Y.K. Sharma, A.K. Shukla: Molecular mechanism of nutrient uptake in plants, Int. J. Curr. Res. Aca. Rev. 2(12) (2014),142-154.

[33] J. Simunek, J.W. Hopmans: Modeling compensated root water and nutrient uptake, Ecol. Model. 220(4) (2009), 505–521.

[34] H.H. Supanjani, J.S. Jung, K.D. Lee: Wheat (Zea mays L.) Rock phosphate potassium and rock solubilizing bacteria as alternative sustainable fertilizers, Agron. Sustain. Dev. 26 (2006), 233-240.

[35] R. Uchida: Essential nutrients for plant growth: nutrient functions and deficiency symptoms, Plant Nutrient Manage. Hawaiis Soils, (2000), 31-55.

[36] Z.Y. Wang, J.M. Kelly, J.L. Kovar, Depletion of macro-nutrients from rhizosphere soil solution by juvenile corn, cottonwood, and switchgrass plants, Plant Soil, 270(1) (2007), 213-221.

[37] M. Wang, Q. Zheng, Q. Shen, S. Guo: The critical role of potassium in plant stress response., Int. J. Molecular Sci. 14(4) (2013), 7370-7390.

[38] D.O. Yawson, P.K. Kwakye, F.A. Armah, K.A. Frimpong: The dynamics of potassium (K) in representative soil series of Ghana, ARPN J. Agric. Biol. Sci. 6(1) (2011), 48-55.

[39] M. Zare, M. Zadehbagheri, A. Azarpanah: Influence of Potassium and Boron on Some Traits in Wheat (Triticum Aestivum Cv. Darab2), Int. J. Biotechn. 2(8) (2013), 141-153.

[40] A.A.B. Zakaria: Growth optimization of potassium solubilizing bacteria isolated from biofertilizer, Ph.D. Thesis, Malaysia Pahang Universitiy, (2009).

[41] Q.C. Zhang, G.H. Wang: Studies on nutrient uptake of rice and characteristics of soil microorganisms in a long-term fertilization experiments for irrigated rice, J. Zhejiang Univ. Sci. B, 6(2) (2005) 147154.

[42] K.C. Zygalakis, T. Roose: Derivation of a dual porosity model for the uptake of nutrients by root hairs, Oxford Centre Collab. Appl. Math. (2010), Report Number 10/06.