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DYNAMICS OF THE LOGISTIC HARVESTING MODEL WITH INFINITE DELAY ON PERIODICALLY EVOLVING DOMAINS

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Abstract. In order to understand the impact of periodic evolution in habitats on the survival of species, a logistic reaction diffusion harvesting model with infinite delay in a periodically evolving domain is studied. By assuming that the evolving domain is uniform and isotropic, the model is converted into a reaction diffusion problem in a fixed domain. The asymptotic behavior of the model is obtained by using principal eigenvalue and the upper and lower solutions method, and a biological explanation of the impact of regional evolution on species is given. Our theoretical results and numerical simulations show that big evolution rate benefits the survival of species.

Keywords: reaction diffusion problem; logistic equation; evolving domain; persistence and extinction.

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1. Introduction

Ecology is the scientific analysis of interactions among organisms and their environment [1]. Mathematical models are usually used to describe an ecological system ranging in scale from an individual population to an ecological community and to gain understanding of the real system

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[2], such as the classic logistic equation [3]

$$\frac{dN}{dt} = rN(1 - \frac{N}{K}). \tag{1.1}$$

It depicts the changes in the number of species in its habitat, where N denotes total number of a population, r represents intrinsic growth factor, K is carrying capacity, that is, the maximum number of species that can be accommodated in the habitat. Taking the random movement of individuals in space into account, we arrive at a logistic reaction diffusion equation

$$\frac{\partial u}{\partial t} = d\Delta u + u(a - bu), \tag{1.2}$$

where $u = u(x,t)$ indicates population density, a represents the birth rate, b is the internal competition factor, $d > 0$ is the diffusion rate. Equation (1.2) was proposed by Fisher [4] in 1937 to discuss a steadily progressive wave of gene increase due to the local establishment of a favourable mutation. Kolmogoroff, Petrovsky and Piscoun-off [5] proved that the reaction diffusion equation (1.2) admits a travelling wave solution under certain conditions. Considering sustainable development and utilization of renewable resources, many scholars have been studying the corresponding harvesting model [6]

$$\frac{\partial u}{\partial t} = d\Delta u + u\big(a(t) - b(t)u\big) - E(t)u, \tag{1.3}$$

where $E(t)$ is the harvesting effort and its magnitude depends upon the resources devoted to harvest, Eu represents the amount of harvesting. Considering the time delay, Chen [7] studied the logistic harvesting model with infinite delay

$$\frac{\partial u}{\partial t} = d\Delta u + u\big(a - bu - c\int_{-\infty}^{t} K(t-s)u(x,s)ds\big) - Eu, \tag{1.4}$$

where K is a non-negative piecewise continuous kernel and satisfies

$$\int_0^{\infty} K(s)ds = 1, \quad \int_0^{\infty} sK(s)ds < +\infty.$$

In this paper, model (1.4) with Dirichlet boundary condition on periodically evolving domains will be discussed to understand the dynamics of the solution.

2. The model on periodically evolving domains

When ecological phenomena are characterized by reaction diffusion models, the domains involved are usually fixed. However, the changing of domain plays a significance role in the survival of species and attracts much attention. One of them is the problem with free boundary, which is caused by behaviors of species themselves. In [8], Du and Lin studied the logistic reaction diffusion model, and gave spreading-vanishing dichotomy, that is, the population either successfully expends to the entire new environment and tends to a positive equilibrium state, or eventually goes to extinction. See also some recent work [9-11] and references therein. Another problem with regional change is that with evolving domain [12], which is caused by climate and environment. For example, leaves where insects live grow with time, rivers and lakes where fishes habitat change seasonally, areas of which become larger in summer and smaller in winter.

As in [13], let $\Omega(t) \subset \mathbf{R}^n$ be a simply connected evolving domain at time $t \geq 0$ with its evolving boundary $\partial\Omega(t)$. For any point

$$x(t) = (x_1(t), x_2(t), \ldots, x_n(t)) \in \Omega(t),$$

we assume that $u(x(t),t)$ is the density of a species at position $x(t)$ and time $t \geq 0$. By Reynolds transport theorem [14], we have

$$\frac{\partial u}{\partial t} + \nabla u \cdot \mathbf{a} + u(\nabla \cdot \mathbf{a}) = d\Delta u + f(u,t) \text{ in } \Omega(t), \tag{2.1}$$

where $f(u,t) = u\big(a(t) - b(t)u - c(t)\int_{-\infty}^{t} K(t-s)u(y,s)ds\big) - Eu$, $\nabla u \cdot \mathbf{a}$ is called advection term while $(\nabla \cdot \mathbf{a})u$ is called dilution term. In order to circumvent the difficulty induced by the evolving domain, we modify equation (2.1) by using transformation of variables. Let $y_1,\ y_2,\ \ldots,\ y_n$ be fixed cartesian coordinates in fixed domain $\Omega(0)$ such that

$$x_1(t) = \hat{x}_1(y_1, y_2, \ldots, y_n, t),$$

$$x_2(t) = \hat{x}_2(y_1, y_2, \ldots, y_n, t),$$

$$\ldots,$$

$$x_n(t) = \hat{x}_n(y_1, y_2, \ldots, y_n, t).$$

Then u is mapped into the new function defined as

$$u(x_1(t),x_2(t),\dots,x_n(t),t) = v(y_1,y_2,\dots,y_n,t). \tag{2.2}$$

Thus equation (2.1) can be translated to another form which is defined on the fixed domain $\Omega(0)$ with respect to $y = (y_1,y_2,\dots,y_n)$. However, the new equation is still very complicated. To further simplify the model equation (2.1), we assume that domain evolution is uniform and isotropic. That is, the evolution of the domain takes place at the same proportion in all directions as time elapses. Mathematically, $x(t) = (x_1(t),x_2(t),\dots,x_n(t))$ can be described as follows:

$$(x_1(t),x_2(t),\dots,x_n(t)) = \rho(t)(y_1,y_2,\dots,y_n),\ y \in \Omega(0), \tag{2.3}$$

where the positive continuous function $\rho(t)$ is called evolution rate subject to $\rho(0) = 1$. Furthermore, if $\rho(t) = \rho(t+T)$ for some $T \geq 0$, the domain is periodically evolving, which has been discussed in [12]. If $\dot{\rho}(t) \geq 0$, the domain is then called growing one [13, 14], and if $\dot{\rho}(t) \leq 0$, the domain is shrinking, see [15] and references therein.

By (2.3), we have

$$\begin{aligned}
&v_t = u_t + \nabla u \cdot \mathbf{a},\\
&\mathbf{a} = \dot{x}(t) = \dot{\rho}(t)(y_1,y_2,\dots,y_n) = \frac{\dot{\rho}}{\rho}(x_1,x_2,\dots,x_n),\\
&\nabla \cdot \mathbf{a} = \frac{n\dot{\rho}}{\rho}, \quad \Delta u = \frac{1}{\rho^2(t)}\Delta v.
\end{aligned}$$

Then (2.1) becomes

$$v_t = \frac{d}{\rho^2(t)}\Delta v - \frac{n\dot{\rho}(t)}{\rho(t)}v + f(v,t),\ y \in \Omega(0),\ t > 0. \tag{2.4}$$

Considering the null Dirichlet boundary condition, which means that there is no species on the boundary, we then transform the logistic harvesting model with infinite delay on the periodically evolving domain $\Omega(t)$ into the following problem in a fixed domain $\Omega(0)$:

$$\begin{cases}
v_t = \frac{d(t)}{\rho^2(t)}\Delta v - \frac{n\dot{\rho}(t)}{\rho(t)}v - E(t)v & \\
\quad + v\big(a(t) - b(t)v - c(t)\int_{-\infty}^{t} K(t-s)v(y,s)ds\big), & y \in \Omega(0),\ t > 0,\\
v(y,t) = 0, & y \in \partial\Omega(0),\ t > 0,\\
v(y,t) = \eta(y,t)(:= u(x(t),t)), & y \in \Omega(0),\ -\infty < t \leq 0.
\end{cases} \tag{2.5}$$

3. Asymptotic behavior of the solution

To explore the impact of evolving domain on the asymptotic behavior of the solution to problem (2.5), we need to consider the eigenvalue problem of the corresponding linear periodic problem

$$\begin{cases} \varphi_t - \frac{d}{\rho^2(t)}\Delta\varphi = \frac{a(t)}{R_0}\varphi - (\frac{n\dot{\rho}(t)}{\rho(t)} + E(t))\varphi, & y \in \Omega(0),\ t > 0, \\ \varphi(y,t) = 0, & y \in \partial\Omega(0),\ t > 0, \\ \varphi(y,0) = \varphi(y,T), & y \in \Omega(0). \end{cases} \tag{3.1}$$

According to [16], we know that if $d > 0$ and $a(t),\ E(t),\ \rho(t)$ are all periodically continuous functions, then problem (3.1) has a unique principal eigenvalue $R_0 (> 0)$ with a positive T-periodic eigenfunction $\varphi(y,t)$. To reflect the dependence of R_0 on parameters d and $\rho(t)$, R_0 is written as $R_0 = R_0(d, \rho(t))$. From standard results on perturbation of simple eigenvalues [17], R_0 is a smooth function with respect to d and $\rho(t)$. Moreover, we have the following properties of $R_0(d, \rho(t))$.

Lemma 3.1. *Assuming that the environment is periodically evolving, then the principal eigenvalue of problem (3.1) can be explicitly expressed as*

$$R_0 = \frac{\int_0^T a(t)dt}{\int_0^T (E(t) + \frac{d\lambda_1}{\rho^2(t)})dt},$$

where $\lambda_1 > 0$ *is the principal eigenvalue of* $-\Delta$ *in* $\Omega(0)$ *subject to homogenous Dirichlet boundary condition, and* R_0 *is monotonically decreasing to* d *and increasing to* $\rho(t)$*.*

Proof. Since $a(t)$ is spatially independent, we rewrite the eigenfunction as

$$\varphi(y,t) = \alpha(t)\psi_1(y),$$

where $\alpha(t)$ is a function to be determined later, $\psi_1(y) > 0$ $(y \in \Omega(0))$ is the eigenfunction related with the principal eigenvalue $\lambda_1 > 0$ of the eigenvalue problem

$$\begin{cases} -\Delta\psi_1 = \lambda_1\psi_1, & y \in \Omega(0),\ t > 0, \\ \psi_1 = 0, & y \in \partial\Omega(0),\ t > 0. \end{cases} \tag{3.2}$$

From (3.1), we have

$$\dot{\alpha}(t)\psi_1(y) + \frac{d\lambda_1}{\rho^2}\alpha(t)\psi_1(y) + (\frac{n\dot{\rho}}{\rho} + E(t))\alpha(t)\psi_1(y) = \frac{a(t)}{R_0}\alpha(t)\psi_1(y),$$

which derives

$$\alpha(t) = \alpha(0)e^{\int_0^t (\frac{a(\tau)}{R_0} - \frac{n\dot{\rho}(\tau)}{\rho(\tau)} - E(\tau) - \frac{d\lambda_1}{\rho^2(\tau)})d\tau}.$$

Owing to $\varphi(y,t+T) = \varphi(y,t)$, we have $\alpha(t+T) = \alpha(t)$ and

$$\int_0^T (\frac{a(t)}{R_0} - E(t) - \frac{d\lambda_1}{\rho^2(t)})dt = 0.$$

Therefore,

$$R_0 = \frac{\int_0^T a(t)dt}{\int_0^T (E(t) + \frac{d\lambda_1}{\rho^2(t)})dt}. \tag{3.3}$$

The corresponding monotonicities are directly from expression (3.3). This completes the proof.

We now consider the steady state to problem (2.5).

Definition 3.1. We call the solution to the problem

$$\begin{cases} V_t = \frac{d}{\rho^2(t)}\Delta V - \frac{n\dot{\rho}(t)}{\rho(t)}V & \\ \qquad +V(a - E - bV - c\int_{-\infty}^t K(t-s)V(y,s)ds), & y \in \Omega(0),\ t > 0, \\ V(y,t) = 0, & y \in \partial\Omega(0),\ t > 0, \\ V(y,t+T) = V(y,t), & y \in \Omega(0),\ -\infty < t \leq 0 \end{cases} \tag{3.4}$$

as the steady state of problem (2.5).

Next we first present the existence and uniqueness of the steady state and then show the convergence result for the solution to the initial boundary problem (2.5) in relation to the periodic solution to problem (3.4).

Theorem 3.1. *Assume that $a(t)$, $b(t)$, $c(t)$, $\rho(t)$ and $E(t)$ are all continuous T-periodic functions.*

(i) If $R_0 > 1$, then problem (3.4) admits a unique positive periodic solution $V^*(y,t)$. Moreover, $V^*(y,t)$ is global attractor of problem (2.5), that is, for any solution $v(y,t)$ to problem (2.5) with positive bounded initial data, we have

$$\lim_{m\to\infty} v(y,t+mT) = V^*(y,t) \text{ on } \overline{\Omega}(0) \times [0,+\infty);$$

(ii) if $R_0 < 1$, then problem (3.4) has only trivial solution $V^\star(y,t) \equiv 0$. Moreover, $V^\star(y,t)$ is global attractor of problem (2.5), which means that

$$\lim_{t\to\infty} v(y,t) = 0 \text{ on } \overline{\Omega}(0).$$

Proof. We first present the existence and uniqueness of the solution to problem (3.4). When $R_0 > 1$, let φ be positive eigenfunction to problem (3.1), we choose

$$\hat{V} = \varepsilon\varphi,$$

where ε is sufficient small so that

$$\varepsilon b(t)\varphi(y,t)R_0 < a(t)(R_0 - 1),\ (y,t) \in \overline{\Omega}(0) \times [0,T],$$

then $\hat{V}$ is a lower solution of problem (3.4). It is easy to see that if we choose

$$\widetilde{V} \geq M := \max\{\max_{(y,t)\in\overline{\Omega}(0)\times[0,T]} \frac{a}{b}(t),\ \sup_{(y,t)\in\overline{\Omega}(0)\times(-\infty,0]} \eta(y,t)\},$$

then $\widetilde{V}$ is a upper solution of problem (3.4). We now select K^* so that $F(v,t) = K^*v + f(v,t)$ is nondecreasing with respect to v, and consider the following iteration process with initial values $\overline{V}^{(0)} = \widetilde{V}$ and $\underline{V}^{(0)} = \hat{V}$

$$\begin{cases} \overline{V}_t^{(m)} - \frac{d}{\rho^2(t)}\Delta\overline{V}^{(m)} + K^*\overline{V}^{(m)} = K^*\overline{V}^{(m-1)} - \frac{n\dot{\rho}(t)}{\rho(t)}\overline{V}^{(m-1)} + \overline{V}^{(m-1)}(a - E \\ \qquad -b\overline{V}^{(m-1)} - c\int_{-\infty}^{t} K(t-s)\overline{V}^{(m-1)}(y,s)ds), \quad y \in \Omega(0),\ t > 0, \\ \underline{V}_t^{(m)} - \frac{d}{\rho^2(t)}\Delta\underline{V}^{(m)} + K^*\underline{V}^{(m)} = K^*\underline{V}^{(m-1)} - \frac{n\dot{\rho}(t)}{\rho(t)}\underline{V}^{(m-1)} + \underline{V}^{(m-1)}(a - E \\ \qquad -b\underline{V}^{(m-1)} - c\int_{-\infty}^{t} K(t-s)\underline{V}^{(m-1)}(y,s)ds), \quad y \in \Omega(0),\ t > 0, \\ \overline{V}^{(m)}(y,t) = \underline{V}^{(m)}(y,t), \qquad y \in \partial\Omega(0),\ t > 0, \\ \overline{V}^{(m)}(y,t) = \overline{V}^{(m-1)}(y,t+T), \qquad y \in \Omega(0),\ -\infty < t \leq 0, \\ \underline{V}^{(m)}(y,t) = \underline{V}^{(m-1)}(y,t+T), \qquad y \in \Omega(0),\ -\infty < t \leq 0, \end{cases} \tag{3.5}$$

we then get the iterations $\{\overline{V}^{(m)}\}$ and $\{\underline{V}^{(m)}\}$, where $m = 1,2,3,\ldots$. Recalling positivity lemma [18] yields

$$\hat{V} \leq \underline{V}^{(m)} \leq \underline{V}^{(m+1)} \leq \overline{V}^{(m+1)} \leq \overline{V}^{(m)} \leq \widetilde{V},$$

and

$$\lim_{m\to\infty} \overline{V}^{(m)} = \overline{V}^*,\ \lim_{m\to\infty} \underline{V}^{(m)} = \underline{V}^*.$$

Then we have

$$\hat{V} \leq \underline{V}^{(m)} \leq \underline{V}^{(m+1)} \leq \underline{V}^* \leq \overline{V}^* \leq \overline{V}^{(m+1)} \leq \overline{V}^{(m)} \leq \widetilde{V}.$$

It follows from the regularity of partial differential equation that $\underline{V}^*$ and $\overline{V}^*$ are the solutions to problem (3.4). To illustrate the uniqueness of the solution, let V_1 and V_2 be two solutions. Define

$$\Lambda = \{s \in [0,1],\ sV_1 \leq V_2 \text{ on } \overline{\Omega}(0) \times [0,T]\}.$$

Clearly Λ contains a neighbourhood of 0. We claim that $1 \in \Lambda$. Suppose not, then

$$s_0 = \sup \Lambda < 1$$

and

$$(V_2 - s_0V_1)_t - \Delta(V_2 - s_0V_1) = f(V_2,t) - s_0 f(V_1,t).$$

Recalling that $f(V,\ t) + K^*V$ is increasing on $[0,\ \max V_2]$ gives

$$(V_2 - s_0V_1)_t - \Delta(V_2 - s_0V_1) + K^*(V_2 - s_0V_1)$$

$$= f(V_2,t) - s_0 f(V_1,t) + K^*V_2 - s_0K^*V_1$$

$$\geq f(s_0V_1,t) - s_0 f(V_1,t) \geq 0$$

for $y \in \Omega(0),\ t > 0$. On the other hand, for $y \in \partial\Omega(0),\ t > 0$, $V_2 - s_0V_1 = 0$. Using the strong maximum principle we have assertions as follows.

(i) $V_2 - s_0V_1 > 0$ on $\Omega(0) \times [0,T]$ with $\frac{\partial}{\partial \nu}(V_2 - s_0V_1) < 0$ on $\partial\Omega(0) \times [0,T]$. Then, clearly there is some $\varepsilon > 0$ such that $V_2 - s_0V_1 \geq \varepsilon V_1$. Thus $s_0 + \varepsilon \in \Lambda$, which contradicts the maximality of s_0.

(ii) $V_2 - s_0V_1 \equiv 0$ on $\Omega(0) \times [0,T]$. This case is also impossible since we would have the equation $f(V_2,t) = s_0 f(V_1,t)$, but $f(V_2,t) = f(s_0V_1,t) > s_0 f(V_1,t)$. Therefore, problem (3.4) has only a positive periodic solution $V^*(y,t)$.

Next, we will use the principle of induction and the uniqueness of the solution to problem (3.4) to prove the convergence result:

$$\lim_{m \to \infty} v(y,t+mT) = V^*(y,t) \text{ in } \overline{\Omega}(0) \times [0,+\infty).$$

Similarly as [18], let $v_m(y,t) = v(y,t+mT)$. If the initial data $\eta(y,t)$ is positive and bounded, then we can choose sufficiently small ε and big M such that

$$\varepsilon\phi = \hat{V} \leq \eta(y,t) \leq \widetilde{V} = M \text{ in } \overline{\Omega}(0) \times (-\infty,0],$$

then using the comparison principle yields

$$\hat{V}(y,t) \le v(y,t) \le \widetilde{V}(y,t) \text{ in } \overline{\Omega}(0) \times (-\infty,+\infty).$$

Then we can obtain

$$\underline{V}^{(1)}(y,t) = \underline{V}^{(0)}(y,t+T) = \hat{V}(y,t+T) \le v_1(y,t) \le \widetilde{V}(y,t+T) = \overline{V}^{(0)}(y,t+T) = \overline{V}^{(1)}(y,t)$$

in $\overline{\Omega}(0) \times (-\infty,0]$. By the comparison principle, we can see that

$$\underline{V}^{(1)}(y,t) \le v_1(y,t) \le \overline{V}^{(1)}(y,t) \text{ in } \overline{\Omega}(0) \times [0,+\infty).$$

Assume, by induction, that

$$\underline{V}^{(m-1)}(y,t) \le v_{m-1}(y,t) \le \overline{V}^{(m-1)}(y,t) \text{ in } \overline{\Omega}(0) \times [0,+\infty).$$

By the comparison principle, we can deduce that

$$\underline{V}^{(m)}(y,t) \le v_m(y,t) \le \overline{V}^{(m)}(y,t) \text{ in } \overline{\Omega}(0) \times [0,+\infty).$$

Due to the uniqueness of the solution to problem (3.4), we have

$$\lim_{m\to\infty} \overline{V}^{(m)} = \lim_{m\to\infty} \underline{V}^{(m)} = V^*(y,t),$$

and then

$$\lim_{m\to\infty} v(y,t+mT) = V^*(y,t) \text{ on } \overline{\Omega}(0) \times [0,+\infty).$$

When $R_0 < 1$, the proof is as above. This completes the proof.

Based on the above results, we further study the impact of diffusion rate d on the survival of species.

Theorem 3.2. *Assume that $a(t)$, $b(t)$, $c(t)$, $\rho(t)$ and $E(t)$ are all continuous T-periodic functions and satisfy $\int_0^T (a(t)-E(t))dt > 0$, $b(t) > 0$, then there exists a constant $D > 0$ such that when $d \in (0,D)$, problem (2.5) admits a unique steady state $V^*(y,t)$, which is globally asymptotically stable; when $d \in (D,+\infty)$, trivial solution $V^\star(y,t) \equiv 0$ is only the steady state of problem (2.5) and is globally asymptotically stable.*

Proof. By Lemma 3.1 and $\int_0^T(a(t)-E(t))dt>0,\ b(t)>0$, it is easy to know $R_0=1$ if and only if

$$d=D:=\frac{\int_0^T(a(t)-E(t))dt}{\lambda_1\int_0^T\frac{1}{\rho^2(t)}dt}. \tag{3.6}$$

Therefore, when $d\in(0,D)$, we have $R_0>1$, and then problem (2.5) has a positive steady state; when $d\in(D,+\infty)$, we have $R_0<1$, and problem (2.5) admits a unique trivial solution. This completes the proof.

Remark 3.1. *Theorem 3.2. implies that a small diffusion rate $d(<D)$ is benefit to the survival of species. Similar results have been found by many researches, for example, the authors showed in [19] that the slower diffusing species always wins the competition.*

4. Impact of domain evolution on survival of species

Assuming that $\rho(t)\equiv 1$ on initial domain $\Omega(t)$, that is $\Omega(t)=\Omega(0)$ is a fixed domain, we consider the following logistic harvesting model with infinite delay

$$\begin{cases}\frac{\partial\check{v}}{\partial t}=d\Delta\check{v}+\\ \qquad\check{v}(a-b\check{v}-c\int_{-\infty}^{t}K(t-s)\check{v}(x,s)ds)-E\check{v}, & y\in\Omega(0),\ t>0,\\ \check{v}(y,t)=0, & y\in\partial\Omega(0),\ t>0,\\ \check{v}(y,0)=\check{v}_0(y), & y\in\Omega(0).\end{cases} \tag{4.1}$$

The corresponding eigenvalue problem is

$$\begin{cases}\phi_t-\frac{d}{\rho^2(t)}\Delta\phi=\frac{a}{R_0^*}\phi-(\frac{n\dot{\rho}(t)}{\rho(t)}+E)\phi, & y\in\Omega(0),\ t>0,\\ \phi(y,t)=0, & y\in\partial\Omega(0),\ t>0,\\ \phi(y,0)=\phi(y,T), & y\in\Omega(0),\end{cases} \tag{4.2}$$

where R_0^* is the principal eigenvalue of problem (4.2) ([20]).

Definition 4.1. We call the solution to the problem

$$\begin{cases}\check{V}_t=\frac{d}{\rho^2(t)}\Delta\check{V}-\frac{n\dot{\rho}(t)}{\rho(t)}\check{V}\\ \qquad+\check{V}(a-E-b\check{V}-c\int_{-\infty}^{t}K(t-s)\check{V}(y,s)ds), & y\in\Omega(0),\ t>0,\\ \check{V}(y,t)=0, & y\in\partial\Omega(0),\ t>0,\\ \check{V}(y,t+T)=\check{V}(y,t), & y\in\Omega(0),\ t\le 0\end{cases} \tag{4.3}$$

as the steady state of problem (4.1).

Recalling Theorem 3.2 together with $\rho(t) \equiv 1$ yields

$$R_0^* = \frac{\int_0^T a(t)dt}{\int_0^T (E(t)+d\lambda_1)dt}, \quad D^* := \frac{1}{\lambda_1 T}\int_0^T (a(t)-E(t))dt. \tag{4.4}$$

We can give the corresponding global dynamics of problem (4.1).

Theorem 4.1. *Assume that $a(t)$, $b(t)$, $c(t)$, $\rho(t)$ and $E(t)$ are all continuous T-periodic functions and satisfy $\int_0^T (a(t)-E(t))dt > 0$, $b(t) > 0$, then there exists a constant $D^* > 0$ such that when $d \in (0, D^*)$, problem (4.3) admits a unique positive periobic solution $\check{V}^*(y,t)$. Moreover, $\check{V}^*(y,t)$ is globally asymptotically state to problem (4.1), that is, for any solution $\check{v}(y,t)$ to problem* (4.1)

$$\lim_{m\to\infty} \check{v}(y,t+mT) = \check{V}^*(y,t) \text{ on } \overline{\Omega}(0)\times[0,+\infty);$$

when $d \in (D^*, +\infty)$, problem (4.3) has only trivial solution $\check{V}^\star(y,t) \equiv 0$. Moreover, $\check{V}^\star(y,t)$ is global asymptotically state to problem (4.1), which means that

$$\lim_{t\to\infty} \check{v}(y,t) = 0 \text{ on } \overline{\Omega}(0).$$

To understand the impact of evolution rate on the global dynamics of the solution, we compare problem (4.1) ($\rho(t) \equiv 1$) with problem (2.5) for general $\rho(t)$, and have the following results which can be obtained by the explicitly expressions of R_0, R_0^*, D and D^*.

Theorem 4.2. *Assume that $a(t)$, $b(t)$, $c(t)$, $\rho(t)$ and $E(t)$ are all continuous T-periodic functions and satisfy $\int_0^T (a(t)-E(t))dt > 0$, $b(t) > 0$, then the following assertions are true:*
(i) if $\frac{1}{T}\int_0^T \frac{1}{\rho^2(t)}dt = 1$, then $R_0 = R_0^*$, that is, the survival and extinction of species on periodically evolving domains are the same as that on fixed domains;
(ii) if $\frac{1}{T}\int_0^T \frac{1}{\rho^2(t)}dt > 1$, then $R_0 > R_0^*$, which shows that the periodical domain evolution leads to fewer opportunities for the species to survive on evolving domains than that on fixed domains;
(iii) if $\frac{1}{T}\int_0^T \frac{1}{\rho^2(t)}dt < 1$, then $R_0 < R_0^*$, which shows that the periodical domain evolution results in more opportunities for the species to survive on evolving domains than that on fixed domains.

Theorem 4.3. *Assume that $a(t)$, $b(t)$, $c(t)$, $\rho(t)$ and $E(t)$ are all continuous T-periodic functions and satisfy $\int_0^T (a(t)-E(t))dt > 0$, $b(t) > 0$, then the following assertion is true:*

(i) if $\frac{1}{T}\int_0^T \frac{1}{\rho^2(t)}dt = 1$*, then* $D = D^*$*, and in consequence the range of diffusion rate which species can survive has no difference compared on periodically evolving domain with on fixed domain, that is, the periodic domain evolution has no impact on the survival of species;*
(ii) if $\frac{1}{T}\int_0^T \frac{1}{\rho^2(t)}dt > 1$*, then* $D < D^*$*, and hence the range of diffusion rate which species can survive is smaller on periodically evolving domain than on fixed domain, that is, the periodic domain evolution has a negative impact on the survival of species;*
(iii) if $\frac{1}{T}\int_0^T \frac{1}{\rho^2(t)}dt < 1$*, then* $D > D^*$*, and hence the range of diffusion rate which species can survive is larger on periodically evolving domain than on fixed domain, that is, the periodic domain evolution has a positive impact on the survival of species.*

Set $\overline{\rho^{-2}} = \frac{1}{T}\int_0^T \frac{1}{\rho^2(t)}dt$, where $\rho(t)$ is evolution rate. According to the above conclusions, one can find that $\overline{\rho^{-2}}$ is a threshold which can predict the impact of periodic domain evolution on the survival of species. In fact, by Remark 3.1, Theorems 4.2 and 4.3, we know that, if a species resides in a periodically evolving domain and its boundary is extremely infertile, then domain evolution will affect the survival and extinction of species which is closely related to the average value $\overline{\rho^{-2}}$. Specifically, if $\overline{\rho^{-2}} > 1$, then species must diminish its diffusion rate if it is to survive on its habitat. Otherwise, the species will become extinct. That is, the periodical domain evolution under this condition restrain the survival of species. If $\overline{\rho^{-2}} < 1$, then species on evolving domain can survive in a larger range of diffusion rates compared to species on a fixed domain. That is, as domain evolves, species can survive not only at the same rate as on fixed domains, but also at larger diffusion rates. Thus, the periodical domain evolution under this condition can promote the survival of species. And if $\overline{\rho^{-2}} = 1$, we can see that although the domain will show periodic growth and reduction, the survival or extinction of species is the same as that on fixed domains.

5. Numerical simulations

In this section, we present some numerical simulations in one dimensional space to illustrate our theoretical analysis.

Firstly, considering the domain evolution, we choose $\Omega(t)=(0,x(t))=(0,\rho(t)y)$, where $\rho(t)$ will be chosen later with $\rho(0)=1$ and $y\in(0,1)$.

Secondly, since time delay induces much difficulties in numerical simulations and our results are independent of time delay, we neglect time delay $(c=0)$ and only consider the following problem

$$\begin{cases} v_t=\frac{d}{\rho^2(t)}\Delta v-\frac{n\dot{\rho}(t)}{\rho(t)}v+v(a(t)-b(t)v)-E(t)v, & y\in\Omega(0),\ t>0,\\ v(y,t)=0, & y\in\partial\Omega(0),\ t>0,\\ v(y,0)=v_0(y)=(u_0(x(0))), & y\in\Omega(0). \end{cases} \tag{5.1}$$

At the same time we take the initial function $v_0(y)=\sin(\pi y)$ corresponding to $t=0$.

Next, we choose different values of parameters in problem (5.1) to verify our theoretical analysis.

Example 5.1. *To highlight the impacts of periodic evolution to domain on the survival of species, we assume that the parameters in problem* (5.1) *are constants* $a=7,\ b=0.5,\ E=1.5$ *and we choose a small diffusion rate* d*:*

$$d=d_1=0.5.$$

And then we choose different evolution rates $\rho(t)$*:*

$$\rho_1(t)\equiv 1 \textit{ and } \frac{1}{T}\int_0^T\frac{1}{\rho_1^2(t)}dt=1,$$

$$\rho_2(t)=e^{-0.09(1-\cos 4t)} \textit{ and } \frac{1}{T}\int_0^T\frac{1}{\rho_2^2(t)}dt=1.2069>1,$$

$$\rho_3(t)=e^{0.1(1-\cos 4t)} \textit{ and } \frac{1}{T}\int_0^T\frac{1}{\rho_3^2(t)}dt=0.8269<1.$$

Then we have conclusions as follows:

(i) $R_0(d_1,\rho_1)=\frac{\int_0^T a(t)dt}{\int_0^T(E(t)+\frac{d_1\lambda_1}{\rho_1^2(t)})dt.}=1.0878>1$. *By Theorem 3.1 we know that solution* $v(y,t)$ *of problem* (2.5) *asymptotically converges to the steady state* $V^*(y)$*. On the fixed domain* $(0,1)$*, the solution* $u(x,t)(=v(y,t))$ *asymptotically converges to the positive periodic steady state* $V^*(x)$*. This is shown in Fig. 1.*

(ii) $R_0(d_1,\rho_2)=\frac{\int_0^T a(t)dt}{\int_0^T(E(t)+\frac{d_1\lambda_1}{\rho_2^2(t)})dt.}=0.9389<1$. *By Theorem 3.1 we know that solution* $v(y,t)$ *of problem* (2.5) *asymptotically converges to the trivial solution* $V^{\star}(y)(\equiv 0)$*. On the evolving*

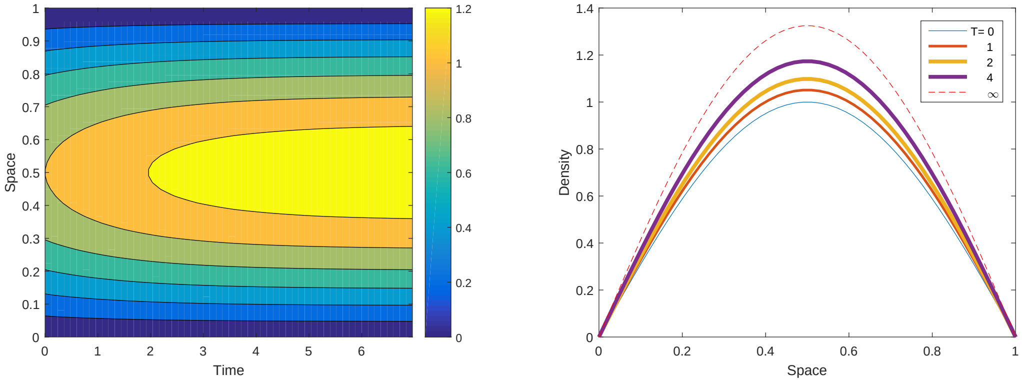

FIGURE 1. Left: The developing process of fixed domain corresponding to $\rho_1(t)\equiv 1$. Color bar on the right shows the density of species. Right: Convergence of temporal solutions to the positive periodic steady state (red dashed line).

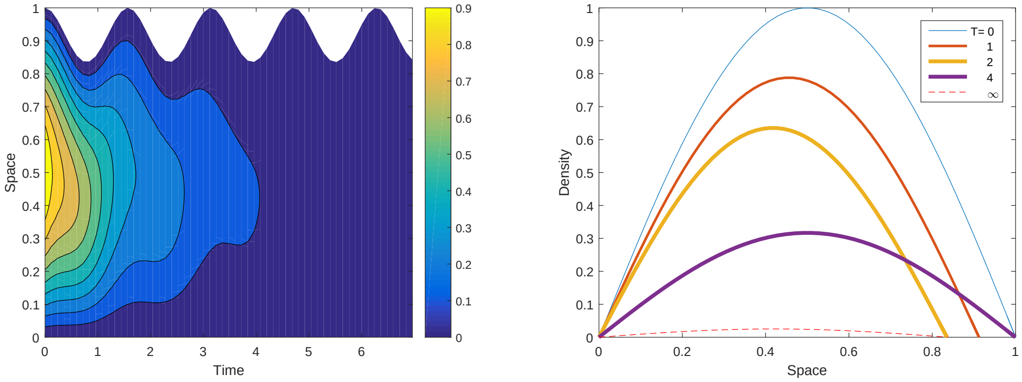

FIGURE 2. Left: The developing process of domain evolution corresponding to $\rho_2(t)=e^{-0.09(1-\cos 4t)}$. Color bar on the right shows the density of species. Right: Convergence of temporal solutions to the trivial solution (red dashed line).

domain $(0,x(t))$, *the solution* $u(x,t)(=u(\rho(t)y,t)=v(y,t))$ *asymptotically converges to the trivial solution, that is,* $\lim_{t\to\infty}u(\rho(t)y,t)=0$ *on* $\overline{\Omega}(0)$. *This is shown in Fig. 2.*

(*iii*) $R_0(d_1,\rho_3)=\frac{\int_0^T a(t)dt}{\int_0^T (E(t)+\frac{d_1\lambda_1}{\rho_3^2(t)})dt.}=1.2543>1$. *By Theorem 3.1 we know that solution* $v(y,t)$ *of problem* (2.5) *asymptotically converges to the positive periodic steady state* $V^*(y)$. *On the evolving domain* $(0,x(t))$, *the solution* $u(x,t)(=u(\rho(t)y,t)=v(y,t))$ *asymptotically converges*

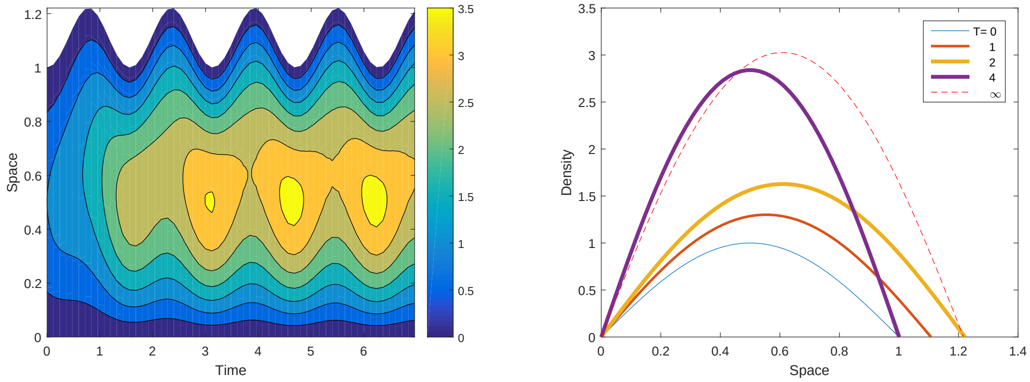

FIGURE 3. Left: The developing process of domain evolution corresponding to $\rho_3(t) = e^{0.1(1-\cos 4t)}$. Color bar on the right shows the density of species. Right: Convergence of temporal solutions to the positive periodic steady state (red dashed line).

to the positive periodic steady state, that is, $\lim_{m\to\infty} u(\rho(t)y, t+mT) = V^*(y,t)$ *on* $\overline{\Omega}(0)\times[0,+\infty)$. *This is shown in Fig.3.*

We can find when $\rho(t)\equiv 1$, *the species can survive on the fixed domain (see Fig. 1). However, when domain evolves with* $\overline{\rho^{-2}} > 1$, *the species will extinct in the future (see Fig. 2). It shows the periodical domain evolution with small* $\rho(t)$ $(\overline{\rho^{-2}} > 1)$ *has negative impact on the survival of species. By the way, from Fig. 3, when* $\overline{\rho^{-2}} < 1$, *the peak becomes larger, that is, the species on evolving domain with big* $\rho(t)$ *can live better. It shows the periodical domain evolution with big* $\rho(t)$ $(\overline{\rho^{-2}} < 1)$ *has positive impact on the survival of species.*

Example 5.2. *Assume that the parameters in problem* (5.1) *are constants* $a = 11$, $b = 0.5$, $E = 1.5$ *and we choose a big diffusion rate d:*

$$d = d_2 = 1.$$

We now choose different evolution rates $\rho(t)$:

$$\rho_4(t) \equiv 1 \text{ and } \frac{1}{T}\int_0^T \frac{1}{\rho_4^2(t)}dt = 1,$$

$$\rho_5(t) = e^{0.1(1-\cos 4t)} \text{ and } \frac{1}{T}\int_0^T \frac{1}{\rho_5^2(t)}dt = 0.8269 < 1,$$

$$\rho_6(t) = e^{-0.01(1-\cos 4t)} \text{ and } \frac{1}{T}\int_0^T \frac{1}{\rho_6^2(t)}dt = 1.0203 > 1.$$

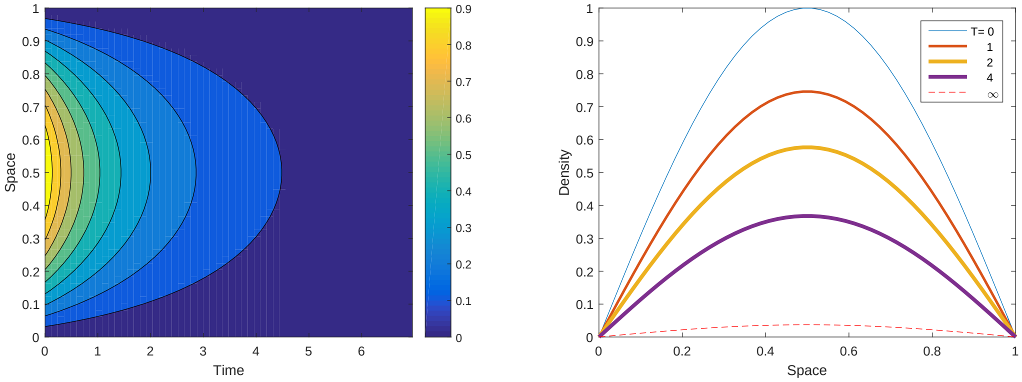

FIGURE 4. Left: The developing process of fixed domain corresponding to $\rho_4(t) \equiv 1$. Color bar on the right shows the density of species. Right: Convergence of temporal solutions to the trivial solution (red dashed line).

Then we have conclusions as follows:

(i) $R_0(d_2,\rho_4) = \frac{\int_0^T a(t)dt}{\int_0^T (E(t)+\frac{d_2\lambda_1}{\rho_4^2(t)})dt.} = 0.9675 < 1$. *By Theorem 3.1 we know that solution* $v(y,t)$ *of problem* (2.5) *asymptotically converges to the trivial solution* $V^\star(y)(\equiv 0)$. *On the fixed domain* $(0,1)$*, the solution* $u(x,t)(= v(y,t))$ *asymptotically converges to the trivial solution. This is shown in Fig. 4.*

(ii) $R_0(d_2,\rho_5) = \frac{\int_0^T a(t)dt}{\int_0^T (E(t)+\frac{d_2\lambda_1}{\rho_5^2(t)})dt.} = 1.1386 > 1$. *By Theorem 3.1 we know that solution* $v(y,t)$ *of problem* (2.5) *asymptotically converges to the steady state* $V^*(y)$. *On the evolving domain* $(0,x(t))$*, the solution* $u(x,t)(= u(\rho(t)y,t) = v(y,t))$ *asymptotically converges to the positive periodic steady state, that is,* $\lim_{m\to\infty} u(\rho(t)y,t+mT) = V^*(y,t)$ *on* $\overline{\Omega}(0)\times[0,+\infty)$. *This is shown in Fig. 5.*

(iii) $R_0(d_2,\rho_6) = \frac{\int_0^T a(t)dt}{\int_0^T (E(t)+\frac{d_2\lambda_1}{\rho_6^2(t)})dt.} = 0.9507 < 1$. *By Theorem 3.1 we know that solution* $v(y,t)$ *of problem* (2.5) *decays to zero. On the evolving domain* $(0,x(t))$*, the solution* $u(x,t)(= u(\rho(t)y,t) = v(y,t))$ *asymptotically converges to the trivial solution, that is,* $\lim_{t\to\infty} u(\rho(t)y,t) = V^\star(y,t) = 0$ *on* $\overline{\Omega}(0)$. *This is shown in Fig. 6.*

We can find if there is no evolution $(\rho(t)\equiv 1)$*, the species become extinct on the fixed domain (see Fig. 4). However, when domain evolves properly, that is,* $\overline{\rho^{-2}} < 1$*, the species will survive on evolving domain (see Fig. 5). It shows the periodical domain evolution with big* $\rho(t)$ $(\overline{\rho^{-2}} < 1)$ *has positive impact on the survival of species. Moreover, the species on evolving domain with*

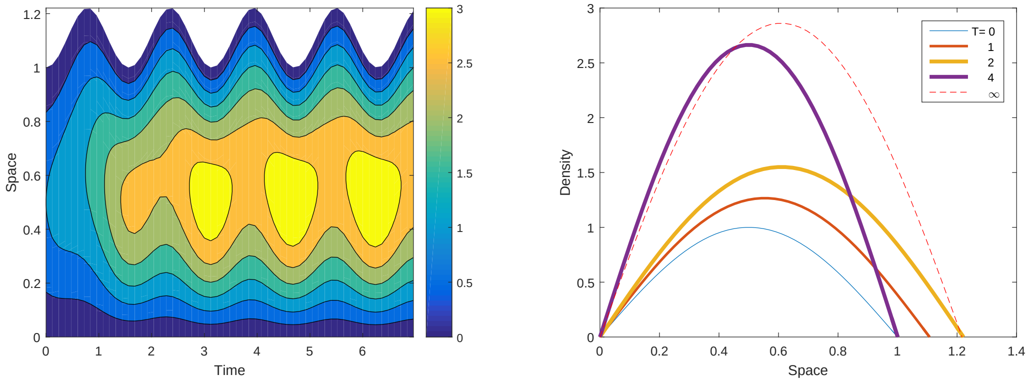

FIGURE 5. Left: The developing process of domain evolution corresponding to $\rho_5(t) = e^{0.1(1-\cos 4t)}$. Color bar on the right shows the density of species. Right: Convergence of temporal solutions to the positive periodic steady state (red dashed line).

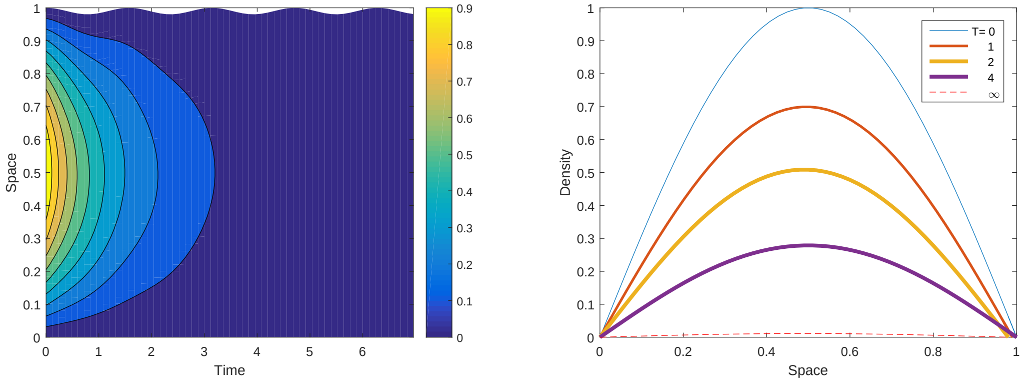

FIGURE 6. Left: The developing process of domain evolution corresponding to $\rho_6(t) = e^{-0.01(1-\cos 4t)}$. Color bar on the right shows the density of species. Right: Convergence of temporal solutions to the trivial solution (red dashed line).

small $\rho(t)$ $(\overline{\rho^{-2}} > 1)$ *will decay quickly to zero (see Fig. 6). It shows the periodical domain evolution with small* $\rho(t)$ $(\overline{\rho^{-2}} > 1)$ *has negative impact on the survival of species.*

Domain evolution is an interesting topic which has attracted a lot of attention [21-25]. However, most existing results on the long time behaviors of the solutions were investigated through numerical simulations. Our results shows that if $\overline{\rho^{-2}} > 1$, the periodical domain evolution is not conducive to species survival. If $\overline{\rho^{-2}} < 1$, the periodical domain evolution can promote species survival. And if $\overline{\rho^{-2}} = 1$, we can see that although the domain will show periodic growth and reduction, the survival or extinct of single species is the same as in fixed domains. We believe that our results can be extended to the reaction diffusion systems modelling two or more species models. We will continue to study for later.

Conflict of Interests

The authors declare that there is no conflict of interests.

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