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MODELING THE DYNAMICS AND TRANSMISSION OF CASSAVA MOSAIC DISEASE IN TANZANIA

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Abstract. Cassava mosaic disease (CMD) is caused by cassava mosaic virus (CMV) and is transmitted by the whitefly vector called Bemisia tabaci. In this paper, the deterministic model for transmission dynamics of CMD is formulated by considering the whitefly vector, cassava resistant and susceptible breeds, and infected cassava. The basic reproduction number R_0 and sensitivity index for each parameter with respect to basic reproduction number R_0 are computed to determine which parameters are sensitive to the dynamics of cassava mosaic disease. Analysis shows that the death rate of whitefly vectors, the infection rate for susceptible vectors, the number of vectors that can be supported and the rate of loss of infected cassava due to disease are the most sensitive parameters to the dynamics of cassava mosaic disease. Numerical simulation indicates that, cassava new infections increase as the number of vectors that can be supported increase and acquire cassava mosaic disease. It shows that if control measures are not considered, then the susceptible breed and cassava resistant breed will be wiped out after five and ten months respectively. To control the disease, farmers are encouraged to apply control strategies such as spraying of insecticide, using of vector-resistant varieties, phytosanitation which involve the removal of infected cassava plants from the farm, crop hygiene and the use of free stem cutting method.

Keywords: cassava mosaic; dynamics; transmission; basic reproduction number; Tanzania; whitefly vector.

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1. Introduction

Cassava (Manihot esculenta) is one of the crops which was firstly introduced in West Africa from Brazil at the end of 16^{th} Century by Portuguese and spread to other African countries [3, 19]. Cassava is grown in tropical and subtropical areas which experience low rainfall as the crop survives in drought climate [10], and this makes cassava a major staple food in the world. According to FAO, about 700 million people depend on cassava as their main food in Africa [20]. Production of cassava in Africa is becoming low due to a number of causes, notably pests and diseases [6]. Cassava Brown Streak Disease (CBSD) and Cassava Mosaic Disease (CMD) are the most important biotic constraints which have led to decrease in yields [13, 1]. Cassava mosaic virus (CMV) contaminates the cassava leaves and is transmitted by the whitefly vector called Bemisia tabaci [3]. There are other 500 different plants including weeds and crops which are host to whitefly vector [15, 17]. Different causes for transmission of cassava mosaic disease have been reported, this includes the use of infected cassava stem, the use of infected plant materials by the farmers [12] as well as the use of CBSD resistant breed which later becomes vulnerable to cassava mosaic disease [22, 25].

The infected cassava plant is characterized by leaf mosaic patterns and it can persist during the premature stage of cassava leaf development. The cassava leaves which are infected by the disease are warped, reduced in size and distorted with yellow color separating the ordinary green color which is the health part of the leaves. They then deteriorate and the new leaves bend [7]. Tanzania is among the countries that face this problem and the disease has been spreading at a fast rate leading to food shortages [24]. According to Tanzania Commissions for Science and Technology (COSTECH) production of cassava in Tanzania is only 8t/ha which is lower compared to 20t/ha that can be produced, the main causes of lower production are pests and diseases.

Studies have been conducted to analyze the transmission dynamics of cassava mosaic disease and the impact of different control strategies. Holt et al: [8] studied the model with susceptible and infected cassava, and susceptible and infectious vectors. The study show that using infected cutting tools and elimination of infectious cassava have a little effect on the occurrence of the disease. Hebert M.P [4], use the Markov chain models to find the probability of eliminating the

disease by using the stochastic process models.The model was applied to CMV, the numerical and analytical results show that the vector aggregation is growing in intricacy as a well as the possibility of a disease to be recognized in host plant. Lawrence et al: [14] use the system of differential equation to find the equilibrium value of the whitefly vector and the cassava plants. The result was analyzed using the finite difference method to assess the spatiotemporal spread of the disease. Results obtained were compared to the field data and the implication of controlling the CMV through the practical were explored. The study concluded that using of ACMD resistant strains of cassava and windbreaks will have positive results on cassava yields. This paper studies the dynamics of cassava mosaic disease by considering cassava resistant breed which only catch cassava mosaic disease through unhealthy cutting and susceptible breed which catch mosaic disease through unhealthy cutting and contact with whitefly vectors before implementing controls.

2. Materials and Methods

2.1. Model Development

The model is formulated by modifying the model which was developed by Holt *et al*. [8] to include breed which catches cassava mosaic disease through unhealthy cutting and susceptible breed that catches mosaic disease through unhealthy cutting and through contact with whitefly vector. The model consists of two groups of population. The first group includes the cassava population (N_C) which is divided into resistant (S_r) and Susceptible (S_C) breeds, and infected cassava (I_C). Second group includes the whitefly vector population (N_V) which consists of susceptible vector (S_V) and infectious vector (I_V).

Cassava resistant breed is replanted at a rate r_1 and is infected by cassava mosaic disease through unhealthy cutting at a rate β_1 and they are harvested at a rate ρ_1. The term k_1, represents the maximum plants for cassava resistant breed which can be planted. Cassava susceptible breed is replanted at a rate r_2, and is infected by cassava mosaic disease following contact with infected whitefly vector and unhealthy cutting at a rate β_2 while it is harvested at a rate ρ_2. The maximum plants of cassava susceptible breed that can be planted is k_2. Infected cassava

flourish following infection of cassava resistant breed through unhealthy cutting at a rate β_1, and infection of cassava susceptible breed through unhealthy cutting and contact with infected whitefly vector at a rate β_2 and they decrease due to the effect of cassava mosaic disease at a rate a and harvested at a rate ρ_3. Susceptible vector is recruited by birth at a rate b and catch infection following contact with infected cassava at a rate β_2. Also, k_3 is the maximum number of vectors that can be supported. Infected vector is recruited when susceptible vector catch infection following contact with infected cassava at a rate β_3 and γ is the death rate of whitefly vector.

2.2. Assumptions of the Model

The model assumes that, all whitefly vectors are born susceptible to cassava mosaic disease. The replanted cassava for both breeds are susceptible to CMD. The whitefly vector cannot transmit cassava mosaic disease to cassava resistant breed except through unhealthy cutting. Cassava susceptible breed gets cassava mosaic disease through contact with infected whitefly and through unhealthy cutting. Susceptible vectors can be infected when they come into contact with the infected cassava. The interaction between cassava and vector population is shown in Figure 1. Variables and parameters are described in Table 1 and 2 respectively.

TABLE 1. Variables' Descriptions

Variables	**Description**
S_r	Cassava resistant breed at time t.
S_C	Cassava susceptible breed at time t.
I_C	Infected cassava at time t.
S_V	Susceptible vectors at time t.
I_V	Infectious vectors at time t.

TABLE 2. Parameters' Descriptions

Parameters	Description
r_1	The rate of planting cassava resistant breed
ρ_1	The rate of harvesting cassava resistant breed
β_1	The rate of infection for cassava resistant breed.
r_2	The rate at which cassava susceptible breed is replanted.
ρ_2	The rate at which cassava susceptible breed is harvested
β_2	The rate at which cassava susceptible breed is infected
ρ_3	The rate at which infected cassava is harvested
a	The rate of loss of infected cassava due to disease
b	Recruitment rate for whitefly.
β_3	Vector infection rate
γ	The death rate of whitefly vectors
k_l	The maximum number of resistant breed that can be planted.
k_2	The maximum number of susceptible breed that can be planted
k_3	Maximum number of vectors that can be supported

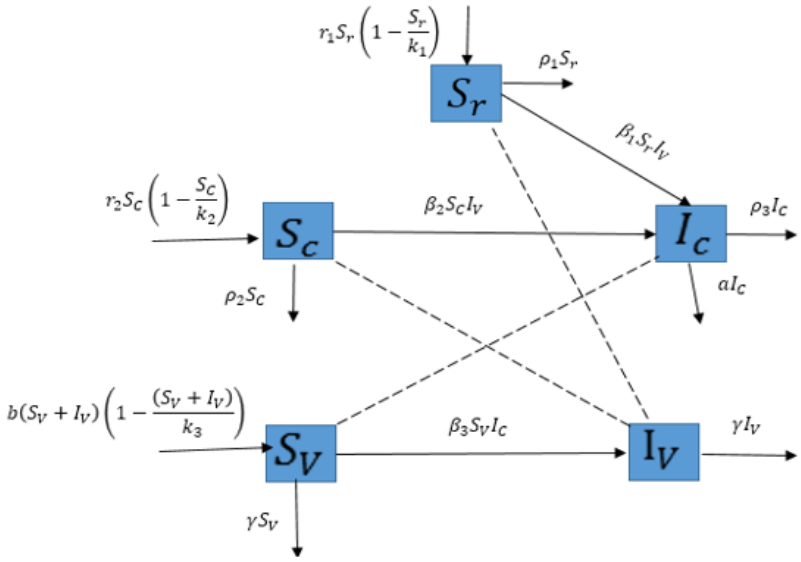


FIGURE 1. Compartmental Model for the transmission dynamics of Cassava Mosaic Disease

2.3. Model equations for the two groups

$$\frac{dS_r}{dt} = r_1 S_r \left(1 - \frac{S_r}{k_1}\right) - \beta_1 S_r I_V - \rho_1 S_r, \tag{1a}$$

$$\frac{dS_C}{dt} = r_2 S_C \left(1 - \frac{S_C}{k_2}\right) - \beta_2 S_C I_V - \rho_2 S_C, \tag{1b}$$

$$\frac{dI_C}{dt} = \beta_2 S_c I_v + \beta_1 S_r I_v - \rho_3 I_c - a I_c, \tag{1c}$$

$$\frac{dS_V}{dt} = b\,(S_V + I_V)\left(1 - \frac{S_V + I_V}{k_3}\right) - \beta_3 S_V I_C - \gamma S_V, \tag{1d}$$

$$\frac{dI_V}{dt} = \beta_3 S_V I_C - \gamma I_V, \tag{1e}$$

Subject to $S_r > 0, S_C > 0, I_C \geq 0, S_V \geq 0, I_V \geq 0$.

The total population of cassava is given as $S_r + S_C + I_C = N_C$ and the total population of vector is given as $N_V = S_V + I_V$.

2.4. Basic Properties of the Model

Invariant Region: Metzer matrix is used to show the feasible region, in which the variables are positive $\forall t \geq 0$. To deduce the feasible region; the model system (1a)-(1e) can be written as:

$$\frac{dx}{dt} = Ax + F, \tag{2}$$

where $x = (S_r, S_C, I_C, S_V, I_V)^T$ and a constant term $F = (0,0,0,0,0)^T$ such that:

$$Ax = \begin{pmatrix} -q_1 & 0 & 0 & 0 & 0 \\ 0 & -q_2 & 0 & 0 & 0 \\ \beta_1 I_V & \beta_2 I_V & -q_3 & 0 & (\beta_2 S_C + \beta_1 S_r) \\ 0 & 0 & 0 & -q_4 & (b - 2\frac{(S_V + I_V)}{k_3}) \\ 0 & 0 & 0 & \beta_3 I_V & -\gamma \end{pmatrix}, \tag{3}$$

for;

$q_1 = \beta_1 I_V + \rho_1 - r_1\left(1 - 2\frac{S_r}{k_1}\right)$, $q_2 = \beta_2 I_V + \rho_2 - r_2\left(1 - 2\frac{S_C}{k_2}\right)$,

$q_3 = \rho_3 + a$, $q_4 = \gamma + \beta_3 I_C - b - 2\frac{(S_V+I_V)}{k_3}$.

In equation (3), A is a Metzler matrix $\forall x \in \mathbb{R}^5$ and due to the fact that $F \geq 0$, the model system (1a) - (1e) is positive invariant in $\mathbb{R}^5$ and F is Lipschitz continuous. Therefore the feasible region Ω is a set of $\Omega = \{S_r, S_C, I_C, S_V, I_V \in \mathbb{R}^5\}$ with initial condition $S_r > 0$, $S_C > 0$, $I_C \geq 0$, $S_V > 0$, $I_V \geq 0$.

Positivity of the solutions:

Let the initial condition be $S_r(0), S_C(0), I_C(0), S_V(0), I_V(0)$, the solutions S_r, S_c, I_c, S_v, I_v of the model system (1a) - (1e) are positive $\forall t > 0$. We show that, the solution of the model system (1a) - (1e) are positive by starting with equation (1a) that:

$$\frac{dS_r}{dt} \geq -(\beta_1 S_r I_V + \rho_1 S_r). \tag{4}$$

Separate the variables and integrate both sides of the equation,

$$\int \frac{1}{S_r} \mathrm{d}S_r \geq \int -(\beta_1 I_v + \rho_1)\, \mathrm{d}t, \tag{5}$$

$$\ln(S_r) \geq -(\beta_1 I_v + \rho_1)\, t + C. \tag{6}$$

This give the values of S_r as:

$$S_r(t) \geq A e^{-(\beta_1 I_v + \rho_1)t}. \tag{7}$$

At initial condition time, $t = 0$, equation (7) above becomes

$$S_r(0) \geq A, \tag{8}$$

Therefore

$$S_r(t) \geq S_r(0)\, e^{-(\beta_1 I_v + \rho_1)t}. \tag{9}$$

Thus, $S_r(0) \geq 0$, $\forall t > 0$.

Apply the same procedure to the remaining equations (1b), (1c), (1d) and (1e):
We get

$$S_C(t) \geq S_C(0)\, e^{-(\beta_2 I_v + \rho_2)t}. \tag{10}$$

$$I_C(t) \geq I_C(0)\, e^{-(\rho_3 + a)t}. \tag{11}$$

$$S_V(t) \geq S_V(0)\, e^{-(\beta_3 I_c + \gamma)t}. \tag{12}$$

$$I_v(0) \geq I_v(0)\, e^{-\gamma t}. \tag{13}$$

Here we conclude that, the requirement to study the dynamics of CMD is satisfied considering that, all the solutions of the model (1a) - (1e) are positive and bounded in the region:

$$\Omega = \{S_r(t), S_C(t), I_C(t), S_V(t), I_V(t)\}. \tag{14}$$

2.5. Cassava Mosaic Free Equilibrium

The steady state when there is no cassava mosaic disease is called cassava mosaic free equilibrium. We compute cassava mosaic free equilibrium when $I_c = I_v = 0$. At this state, the total cassava plants is the sum of susceptible and resistant breeds. However, the population of the vector at this state consists of susceptible whitefly vector. Cassava mosaic free equilibrium is given by:

$$F^0 = (S_r, S_C, I_C, S_V, I_V) = \left(\frac{(r_1 - \rho_1)\, k_1}{r_1}, \frac{(r_2 - \rho_2)\, k_2}{r_2}, 0, \frac{(b - \gamma)\, k_3}{b}, 0 \right). \tag{15}$$

2.6. Basic Reproduction Number R_0

The basic reproduction number is denoted by R_0. It refers to an expected number of secondary infections from an infected whitefly when introduced into a susceptible population of cassava plants [5]. If $R_0 > 1$, the infectious whitefly can transmit the cassava mosaic disease to more

than one cassava plants, and if $R_0 < 1$, an infectious whitefly transmits the cassava mosaic disease to less than one cassava plants, hence the disease is dying out. The basic reproductive number will be determined by next generation matrix [11] as follows:

Assume that, $f_i(\bar{x})$ is the rate of cassava and whitefly new infections and $V_i(\bar{x}) = V_i^-(\bar{x}) - V_i^+(\bar{x})$ where $V_i^+(\bar{x})$ are the terms that are transferred into the compartment and $V_i^-(\bar{x})$ are the terms that are transferred out of the compartment such that: [5].

$$F = \frac{\partial f_i(x_0)}{\partial(x_j)} \quad and \quad V = \frac{\partial V_i(x_0)}{\partial(x_j)}, \tag{16}$$

where $i, j = 1, 2, ..., m$ and x_0 indicates the cassava mosaic free equilibrium. From the model system (1a)- (1e), f_i and V_i are defined by:

$$f_i = \begin{pmatrix} \beta S_r I_V + \beta_2 S_C I_V \\ \beta_3 S_V I_C \\ \end{pmatrix} \tag{17}$$

and

$$V_i = \begin{pmatrix} \rho_3 + aI_C \\ \gamma I_V \\ \end{pmatrix}. \tag{18}$$

Matrices F and V are obtained by differentiating equation (17) and (18) respectively, with respect to I_c and I_v so that:

$$F = \begin{pmatrix} 0 & \beta S_r + \beta_2 S_C \\ \beta_3 S_V & 0 \\ \end{pmatrix} \tag{19}$$

and

$$V = \begin{pmatrix} \rho_3 + aI_C & 0 \\ 0 & \gamma \\ \end{pmatrix}. \tag{20}$$

The next generation matrix is given by:

$$FV^{-1} = \begin{pmatrix} 0 & \frac{\beta_2 S_C + \beta_1 S_r}{\gamma} \\ \frac{\beta_3 S_V}{\rho_3 + a} & 0 \end{pmatrix}. \tag{21}$$

The basic reproduction number R_0 for cassava plants and vector is a dominant eigenvalue of the next generation matrix FV^{-1} [18]. The basic reproduction number R_0 is therefore given by:

$$R_0 = \sqrt{\frac{\beta_3 (b-\gamma) k_3}{b(\rho_3 + a)\gamma} \left(\frac{(r_1 - \rho_1) k_1 \beta_1}{r_1} + \frac{(r_2 - \rho_2) k_2 \beta_2}{r_2} \right)}. \tag{22}$$

From equation (22), basic reproduction number R_0 is determined by all parameters from the model. The basic reproduction number R_0 increases in proportion to $\beta_3, b, k_3, \beta_1, \beta_2, k_1, r_1, k_2$ and r_2, and decreases as $\gamma, \rho_3, \rho_2, a$ and ρ_1 increase.

3 . Sensitivity analysis

Sensitivity index of a parameter tells how a parameter is sensitive to the disease. In this section, sensitivity index of each parameter with respect to basic reproduction number R_0 is derived to determine how each parameter influences the disease. If f is a parameter in reproduction number R_0 then, sensitivity index of f with respect to R_0 is given by:

$$\Upsilon_f^{R_0} = \frac{dR_0}{df} \times \frac{f}{R_0}. \tag{23}$$

3.1. Parameters Adoption

Parameter values from the literature and assumed ones are used. Table 3 summarizes the parameter values, range and their sources.

TABLE 3. Parameter Values.

Parameters	**Value**	**Range**	**Source**
r_1	$0.025 day^{-1}$		Assumed
ρ_1	$0.005 day^{-1}$		Assumed
β_1	$0.0012 vector^{-1} day^{-1}$		Assumed
r_2	$0.2 day^{-1}$	$0.025-0.2$	[14]
ρ_2	$0.003 day^{-1}$	$0.002-0.004$	[8]
β_2	$0.003 vector^{-1} day^{-1}$	$0.002-0.032$	[8]
ρ_3	$0.003 day^{-1}$	$0.002-0.004$	[8]
a	$0.033 day^{-1}$	$0-0.033$	[8]
b	$0.5 vector^{-1} day^{-1}$	0.1-1.0	[26]
β_3	$0.002 plant^{-1} day^{-1}$	$0.002-0.032$	[8]
γ	$0.0782 day^{-1}$	$0.06-0.18$	[8]
k_1	3000		[21]
k_2	2000		Assumed
k_3	350	0-2500	[8]

Using forward normalized sensitivity index for each parameter with respect to basic reproduction number R_0, sensitivity index for β_2 is derived as follows:

$$\Upsilon_{\beta_2}^{R_0} = \frac{dR_0}{d\beta_2} \times \frac{\beta_2}{R_0}, \tag{24}$$

$$\frac{dR_0}{d\beta_2} = 1/2 \frac{\beta_3 (b-\gamma) k_3 (r_2-\rho_2) k_2}{r_2 b (\rho_3+a) \gamma} \frac{1}{\sqrt{\frac{\beta_3(b-\gamma)k_3}{b(\rho_3+a)\gamma}\left(\frac{(r_1-\rho_1)k_1\beta_1}{r_1}+\frac{(r_2-\rho_2)k_2\beta_2}{r_2}\right)}}. \tag{25}$$

Full computation gives:

$$\Upsilon_{\beta_2}^{R_0} = +0.3362. \tag{26}$$

We apply the same method to obtain sensitivity indices for other parameters. Table 4 summarizes sensitivity indices for all parameters with respect to basic reproduction number R_0.

TABLE 4. Sensitivity Indices.

Parameters	**Sensitivity index**	**Parameters**	**Sensitivity index**
β_3	+0.5000	r_2	+0.0051
β_1	+0.1638	γ	−0.5927
β_2	+0.3362	ρ_3	−0.0417
k_3	+0.5000	ρ_2	−0.0051
b	+0.0927	ρ_1	−0.0410
k_1	+0.1638	a	−0.4583
r_1	+0.0410	k_2	+0.3362

From the Table 4 parameters $\beta_2, \beta_3, \beta_1, k_1, k_2, k_3, b, r_1, r_2$ have positive indices, this means that the basic reproduction number R_0 increase in proportion to these parameters. Parameters a, ρ_1, ρ_2, ρ_3 and γ have negative indices. This means that the basic reproduction number R_0 decrease when a, ρ_1, ρ_2, ρ_3 and γ increase. The most sensitive parameter is the death rate of whitefly vectors γ, the increase of this parameter decrease the basic reproduction number R_0.

4 . Global Stability of Cassava Mosaic Free Equilibrium

The global stability of cassava mosaic free equilibrium is established by approach used by Castillo-Chavez [2]. When this approach is used, system (1a) - (1e) is written as follows:

$$\frac{dX_1}{dt} = H(X_1 - X_{F_0}) + H_1 X_2, \tag{27}$$

$$\frac{dX_2}{dt} = GX_2, \tag{28}$$

where X_1 presents the noninfectious classes and X_2 infectious classes. $X_{(F_0)}$ present mosaic free equilibrium. Mosaic free equilibrium is said to be globally asymptotically stable if eigenvalues of matrix H are negative and matrix G is a Metzler matrix [9]. We thus define X_1, X_2 and

X_{F_0} by:

$$X_1 = \begin{pmatrix} S_r \\ S_C \\ S_V \end{pmatrix}. \tag{29}$$

$$X_2 = \begin{pmatrix} I_C \\ I_V \end{pmatrix}. \tag{30}$$

$$X_{F_0} = \begin{pmatrix} \frac{(r_1-\rho_1)k_1}{r_1} \\ \frac{(r_2-\rho_2)k_2}{r_1} \\ 0 \\ \frac{(b-\gamma)k_3}{b} \\ 0 \end{pmatrix}. \tag{31}$$

Matrices H_1 and H are defined by:

$$H_1 = \begin{pmatrix} 0 & -\beta_1 S_r \\ 0 & -\beta_2 S_C \\ -\beta_3 S_V & b - \frac{2b(S_V+I_V)}{k_3} \end{pmatrix} \tag{32}$$

and

$$H = \begin{pmatrix} -q_1 & 0 & 0 \\ 0 & -q_2 & 0 \\ 0 & 0 & -q_3 \end{pmatrix}, \tag{33}$$

where

$q_1 = (r_1 + 2\frac{r_1 S_r}{k_1} + \beta_1 I_V + \rho_1)$, $q_2 = (r_2 + 2\frac{r_2 S_C}{k_2} + \beta_2 I_V + \rho_2)$, $q_3 = (b + \frac{2b(S_V+I_V)}{k_3} + \beta_3 I_C + \gamma)$.

Matrix H has negative eigenvalues and matrix G is Metlzer matrix since elements in the main diagonal are negative and the off diagonal elements are positive provided the rate of planting cassava is greater than the rate at which they are harvested and the recruitment rate of whitefly vectors is greater than their death rate. Therefore, when the basic reproduction number R_0,

is less than one $(R_0 < 1)$ and greater than one $(R_0 > 1)$, then the disease free equilibrium is globally asymptotically stable and unstable respectively.

5 . Global Stability of Cassava Mosaic Free Equilibrium

Due to non-linear nature of the model, it is not possible to obtain cassava mosaic equilibrium explicitly. To prove existence of cassava mosaic equilibrium, we state and prove the following theorem:

Theorem: Cassava mosaic equilibrium exists if $S_r^* > 0, S_C^* > 0, I_C^* > 0, S_V^* > 0, I_V^* > 0$.

Proof: Approach in Tumwine et al. [23] and Massawe et al: [16] is adopted in proving existence of cassava mosaic equilibrium. We use the sum of cassava plants and whitefly vectors when their rate of change is zero. When we consider total cassava plants at cassava mosaic equilibrium, we have:

$$r_1 S_r^* \left(1 - \frac{S_r^*}{k_1}\right) + r_2 S_C^* \left(1 - \frac{S_C^*}{k_2}\right) - \rho_1 S_r^* - \rho_2 S_C^* - (\rho_3 + a) I_C^*. \tag{34}$$

This lead to :

$$\rho_1 S_r^* + \rho_2 S_C^* + (\rho_3 + a) I_C^* = r_1 S_r^* \left(1 - \frac{S_r^*}{k_1}\right) + r_2 S_C^* \left(1 - \frac{S_C^*}{k_2}\right). \tag{35}$$

Since $S_r^* < k_1, S_C^* < k_2$ and all the parameters are positive.

Then:

$$r_1 S_r^* \left(1 - \frac{S_r^*}{k_1}\right) + r_2 S_C^* \left(1 - \frac{S_C^*}{k_2}\right) > 0, \tag{36}$$

showing that: $S_r^* > 0, S_C^* > 0$ and $I_C^* > 0$. Using the same approach for whitefly vector we have $S_V^* > 0$ and $I_V^* > 0$. This shows that cassava mosaic equilibrium exists.

5.1. Global Stability of Cassava Mosaic Equilibrium

The global stability of cassava mosaic equilibrium is investigated by logarithmic Lyapunov function which is given by:

$$L = \sum G_i \left(P_i - P_i^* ln P_i\right), \tag{37}$$

where G_1, is a positive constant which is to be chosen carefully, P_i is a variable in a compartment i and P^* present a compartment variable at equilibrium point. Using system (37) the Lyapunov function is defined by;

$$\begin{aligned} L(S_rS_C,I_C,S_V,I_V) = G_1\left(S_r - {S_r}^* lnS_r\right) \\ +G_2\left(S_C - {S_C}^* lnS_C\right) \\ +G_3\left(I_C - {I_C}^* lnI_C\right) \\ +G_4\left(S_V - {S_V}^* lnS_V\right) \\ +G_5\left(I_V - {I_V}^* lnI_V\right). \end{aligned} \tag{38}$$

Differentiate the Lyapunov function (38) above with respect to time, we get

$$\begin{aligned} \frac{dL}{dt} =& G_1\left(1-\frac{S_r^*}{S_r}\right)\frac{dS_r}{dt} + G_2\left(1-\frac{S_C^*}{S_C}\right)\frac{dS_C}{dt} + G_3\left(1-\frac{I_C^*}{I_C}\right)\frac{dI_C}{dt} \\ &+ G_4\left(1-\frac{S_V^*}{S_V}\right)\frac{dS_V}{dt} + G_5\left(1-\frac{I_V^*}{I_V}\right)\frac{dI_V}{dt} \end{aligned} \tag{39}$$

From equations (39), we have:

$$\begin{aligned} \frac{dL}{dt} =& G_1\left(1-\frac{S_r^*}{S_r}\right)\left(r_1S_r\left(1-\frac{S_r}{k_1}\right) - \beta_1S_rI_V - \rho_1S_r\right) \\ &+ G_2\left(1-\frac{S_C^*}{S_C}\right)\left(r_2S_C\left(1-\frac{S_C}{k_2}\right) - \beta_2S_CI_V - \rho_2S_C\right) \\ &+ G_3\left(1-\frac{I_C^*}{I_C}\right)\left(\beta_2S_CI_V + \beta_1S_rI_V - \rho_3I_C - aI_C\right) \\ &+ G_4\left(1-\frac{S_V^*}{S_V}\right)\left(b\left(S_V+I_V\right)\left(1-\frac{S_V+I_V}{k_3}\right) - \beta_3S_VI_C - \gamma S_V\right) \\ &+ G_5\left(1-\frac{I_V^*}{I_V}\right)\left(\beta_3\ S_VI_C - \gamma I_V\right). \end{aligned} \tag{40}$$

At cassava mosaic equilibrium, equation (40) becomes:

$$\begin{aligned}\frac{dL}{dt} =& -G_1\rho_1\frac{(S_r-S_r^*)^2}{S_r} - G_2\rho_2\frac{\left(S_C-S_C^*\right)^2}{S_C} - G_3(\rho_3+a)\frac{\left(I_C-I_C^*\right)^2}{I_C} - G_4\gamma\frac{(S_V-S_V^*)^2}{S_V}\\ &- G_5\gamma\frac{(I_V-I_V^*)^2}{I_V} - G_1\beta_1\frac{(S_r-S_r^*)\left(S_rI_V-S_r^*I_V^*\right)}{S_r} - G_2\beta_2\frac{(S_C-S_{C^*})\left(S_CI_V-S_C^*I_V^*\right)}{S_r}\\ &- G_4\beta_3\frac{(S_V-S_V^*)\left(S_VI_C-S_V^*I_V^*\right)}{S_V},\end{aligned} \tag{41}$$

this simplifies to:

$$\begin{aligned}\frac{dL}{dt} =& -G_1\rho_1\frac{(S_r-S_r^*)^2}{S_r} - G_2\rho_2\frac{\left(S_C-S_C^*\right)^2}{S_C} - G_3(\rho_3+a)\frac{\left(I_C-I_C^*\right)^2}{I_C} - G_4\gamma\frac{(S_V-S_V^*)^2}{S_V}\\ &- G_5\gamma\frac{(I_V-I_V^*)^2}{I_V} + F(\Omega),\end{aligned} \tag{42}$$

where:

$$\begin{aligned}F(\Omega) =& -G_1\beta_1\frac{(S_r-S_r^*)\left(S_rI_V-S_r^*I_V^*\right)}{S_r} - G_2\beta_2\frac{\left(S_C-S_C^*\right)\left(S_CI_V-S_C^*I_V^*\right)}{S_r}\\ &- G_4\beta_3\frac{(S_V-S_V^*)\left(S_VI_C-S_V^*I_C^*\right)}{S_V}.\end{aligned} \tag{43}$$

The function $F(\Omega)$ is negative or zero in Ω, therefore $\frac{dL}{dt} \leq 0$ in Ω and it is zero for $\Omega = \Omega^*$. Since $\frac{dL}{dt} = 0$ when $\Omega = \Omega^*$ and $\frac{dL}{dt} \leq 0$ in Ω then the largest invariant set in Ω when $\frac{dL}{dt} = 0$ is a singleton Ω^* which is cassava mosaic equilibrium point. By LaSalles invariant principle, the casssava mosaic equilibrium Ω^* is globally asymptotically stable when $R_0 > 1$.

6. Numerical Simulation of the Basic Model

In this section, we simulate model system (1) to determine the long term impact of cassava mosaic disease. We simulate the dynamics of cassava mosaic disease by considering sensitive parameters.

The dynamics of cassava mosaic disease is demonstrated in Figure 2. All susceptible vectors contract the disease before five months, this is reflected by susceptible cassava which also decrease due to the disease. Cassava resistant breed takes longer to get cassava mosaic disease as demonstrated in Figure 2. Figure 3 illustrates cassava and vector populations.

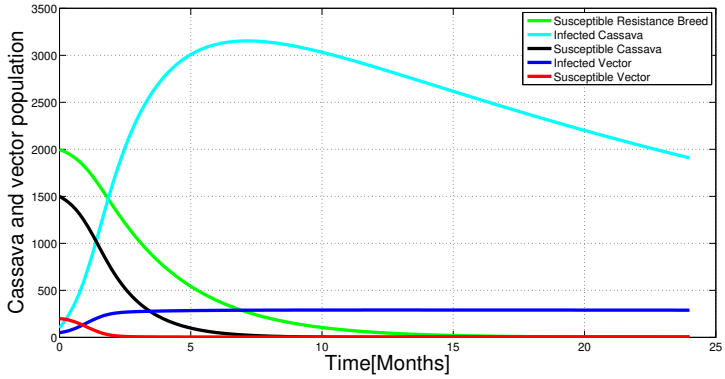
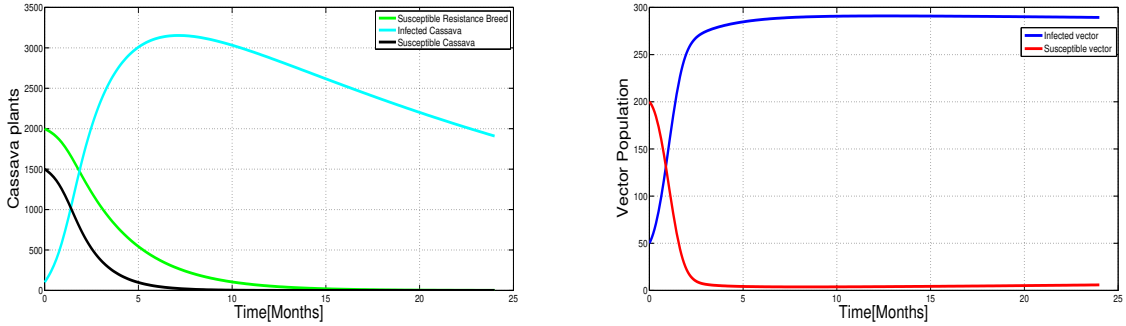


FIGURE 2. Total Population



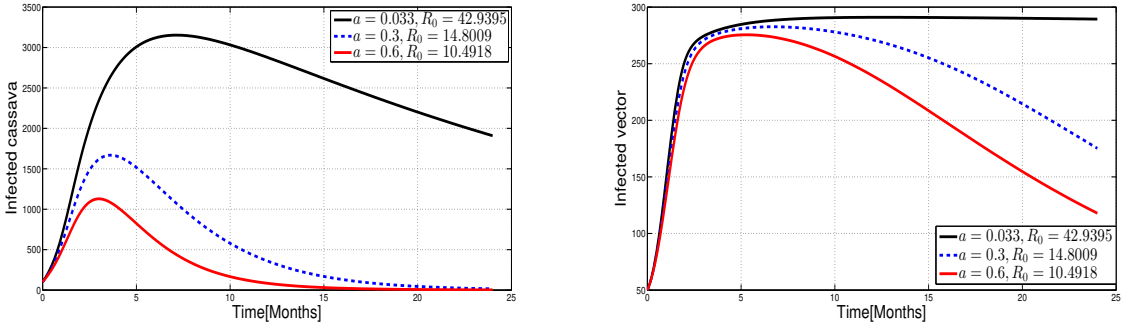
(A) (B)

FIGURE 3. Vector and Cassava Population

The variation of sensitive parameters shows that cassava mosaic disease increase proportionally to recruitment rate of whitefly vectors and decreases as the rate of loosing infected cassava increases. All classes are demonstrated in Figures below as follows.

Figure 4 demonstrates the variation of the rate of loss of infected cassava to the infected classes. It shows the behavior of infected cassava and infected vectors when the parameter a vary, the increase of a lead to the decrease of infected cassava and the decrease of infected

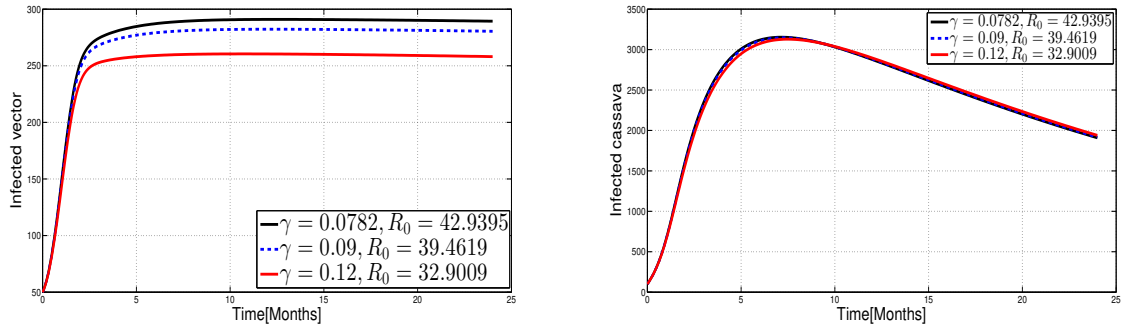
vector.



(A) Infected cassava. (B) Infected vector.

FIGURE 4. Variation of loss of infected cassava rate in infected class.

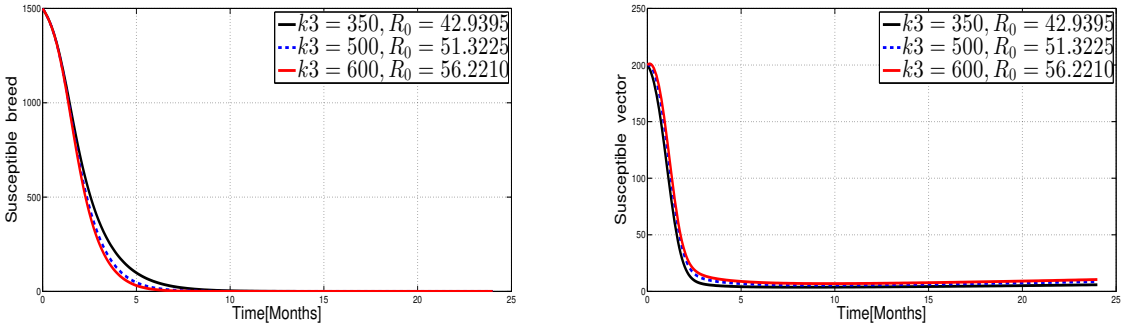
Figure 5, shows the variation of vector mortality rate γ to the infectious vector and infected cassava class, if the rate of vector mortality increase the number of infected vector and infected cassava decreases.



(A) Infected vector. (B) Infected cassava.

FIGURE 5. Variation of vector mortality rate in infectious class.

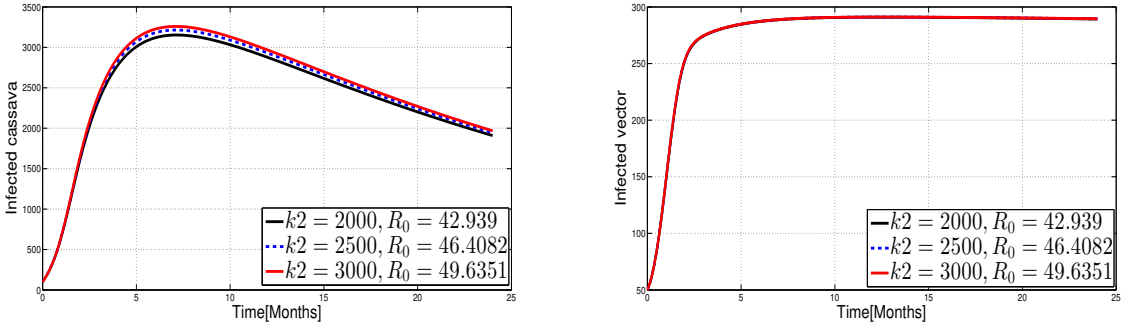
From Figure 6 the graphs demonstrate the variation of vector carrying capacity k_3 to the susceptible class of cassava and susceptible class of vector. The graphs show as the carrying capacity of whitefly vectors increase the number of susceptible cassava breed decrease, the number of susceptible vector increase.



(A) Susceptible breed. (B) Susceptible vector.

FIGURE 6. Variation of vector carrying capacity to the susceptible class.

Figure 7 shows the impact of varying the carrying capacity of susceptible breed of cassava to the infected cassava class and infected vector . It shows that as the carrying capacity of susceptible breed increases the number of infected cassava and infected vector increases.



(A) Infected cassava. (B) Infected vector.

FIGURE 7. Variation of cassava susceptible breed carrying capacity to the infected class.

7. Conclusion

In this paper, the deterministic model for transmission dynamics of CMD which includes population of cassava and whitefly vector is presented and analyzed. The sensitivity analysis was performed to identify sensitive parameters. Analysis shows that the number of vectors that can be supported, the rates at which vectors acquire disease and the carrying capacity of susceptible cassava breed, play important role in the transmission dynamics of cassava mosaic

disease. New infections will increase as the carrying capacity of susceptible cassava and the rate of infection of vectors increases. To improve cassava productivity, campaigns to eradicate cassava mosaic disease should focus on strategies which reduce vectors' population. These strategies include spraying insecticide, use of vector-resistant varieties, phytosanitation which involve the removal of infected cassava plants from the place that will be used for the new plantings, crop hygiene and the use of free stem cutting method.

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Conflict of Interests

The authors declare that there is no conflict of interests.

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