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AN APPLICATION OF OPTIMAL CONTROL TO THE MARINE ARTISANAL FISHERY IN GHANA

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Abstract. This study focuses on the determination of an optimal fishing effort for the marine artisanal fishery in Ghana. That is, the rate of harvest maximizing the net economic benefits while also maximizing the stock size. Employing the Gordon-Schaefer bioeconomic model and using empirical data on the round sardinella (*Sardinella aurita*), the static reference points of the model comprising the maximum sustainable yield (MSY), maximum economic yield (MEY) and open access yield (OAY) are determined and discussed. Further, the dynamic reference point of the model, the optimum sustainable yield (OSY), is also explored. Bifurcation analysis of the model shows that it undergoes transcritical bifurcation, and it is structurally stable for rate of effort not exceeding the bifurcation point. The characterization of the optimal control indicates that the resource should only be harvested by exerting up to the maximum available effort if the net revenue per unit harvest exceeds (or equals) the shadow price. Numerical simulations carried out on the dynamic model indicate that the optimal fishing effort should be set at $351,328$ trips per year, provided the initial stock size is at least $554,654$ tonnes. However, given the current high rate of fishing effort, due to the open access nature of the fishery, the recommended optimal effort strategy is bang-bang, which translates into implementation of closed fishing seasons.

Keywords: optimal control; Ghana marine artisanal fishery; round sardinella; bifurcation point; shadow price; equilibrium reference points.

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1. INTRODUCTION

Renewable resources, of which fisheries form an important component, are vital to the socio-economic well-being of mankind. A renewable resource like fish is sometimes the only source of protein for most marginalized communities in developing nations across the globe. According to a Food and Agriculture Organization of the United Nations (FAO) report regarding the state of fisheries of the world, as high as 75% of the fisheries are fully exploited or partially exploited [21, 30]. Therefore, there is the the need for effective and prudent management for the sustainability of such a valuable and important resource.

The Ministry of Fisheries and Aquaculture Development (MOFAD) reports that the Ghana fisheries sector comprises three areas – marine, inland and aquaculture. While aquaculture and inland fisheries are mainly small scale, the marine consists of a combination of small and large scale industrial fisheries.

The sector plays an enormous role in the socioeconomic development of the country. It serves as a major source of employment, wealth creation and livelihood, among others, especially for coastal and inland communities. The Ghana Statistical Service (GSS) reports that employment generated by the sector is over 2.7 million out of the country's population of around 28 million [12]. This comprises fishers as well as boat owners, boat builders, processors of landed fish and other ancillary jobs. The sector essentially provides over 70% of the total fish requirements and, by extension, the bulk of the country's protein requirements [23].

Furthermore, the sector contributes positively to the nation's objective of promoting food security, improved foreign exchange earnings and sizable individual incomes. In 2015, production of fish domestically totaled $451,099.4$ tonnes, which include $320,221.4$ tonnes of marine capture (71.0%), nearly $86,268.3$ tonnes of inland capture (19.1%) and $45,610$ tonnes of aquaculture (9.9%). To supplement domestic production, $145,910.3$ tonnes of fish valued at US\$$120,443,785$ were imported into the country. In terms of foreign exchange earnings accruing to the country, US\$$309,790,723.90$ was generated from exporting an estimated $53,750$ tonnes of fish and seafood [23].

The marine fishery sector in Ghana comprises three main sub-sectors; namely, small scale (for artisanal or canoe), semi-industrial (for inshore) and industrial [25]. As already alluded to, this sector is the most important of all the fisheries sectors, accounting for more than two-thirds of domestic fish production. Also, among the sub-sectors in the marine sector, the artisanal fisheries are predominant. According to a report in the Republic of Ghana Fisheries and Aquaculture Sector Development Plan (FASDP), out of the estimated $135,000$ fishers in the marine sector, up to $124,000$, constituting 92%, are artisanal fishers [9].

The artisanal fisheries are characterized by the use of several fishing gears operating from dugout canoes carved out of a single log of wood species called 'wawa' (*Triplochiton scleroxylon*) and 'Onyina' (*Ceiba petendra*). The preferred gear of the fishermen are purse seine, beach seine, gill net, lobster net, among others. There are more than $12,700$ canoes operating in 315 landing beaches and 190 fishing villages producing between 75 and 80% of the total marine production [1, 3, 25].

The artisanal fisheries are dominated by the small pelagic species, which include round sardinella (*Sardinella aurita*), flat sardinella (*Sardinella maderensis*), chub mackerel (*Scomber japonicus*) and anchovy (*Engraulis encrasicolus*). The round sardinella forms the bulk of the small pelagics and does well when the temperatures are below 26 °C [23]. The sardinella fishery has a long history in Ghana and it is very important both socially and economically. Investigations have shown that sardinella is influenced by climatic and oceanographic conditions; and it is seasonal and most abundant during the period of the major coastal upwelling (July to September), although juveniles are fished throughout the year. The minor upwelling season, which usually lasts for only three weeks, normally begins in December or January and end, by the latest, in February. Therefore, there are two fishing seasons for the sardinella in Ghana [19].

Presently, the sardinella fishery teeters on the verge of collapse. The annual catch has plummeted year after year while the number of artisanal fishing canoes has risen year after year due to the open access nature of the fishery. Annual catch fell to just over $17,000$ tonnes in 2012 from a high of $120,000$ tonnes just a dozen years earlier. Therefore, from all indications, the sardinella fishery is in a state of crisis since this the longest period of decline after the start of heavy exploitation of the resource [33].

The original bioeconomic model on fishery formulated by Gordon [11] and Schaefer [28] was static in nature, and Clark and Munro [6] were among the first researchers to propose a dynamic version of it. The latter extended the model, first by making it non-autonomous and second, nonlinear. The Gordon-Schaefer model is a bioeconomic model in the sense that it contains both biological and economic parameters; and it describes the growth of a renewable resource undergoing harvesting. The addition of the economic parameters allows for the economic benefits, or lack thereof, of the fishing activity to be assessed. Invoking steady-state or equilibrium conditions allows for the identification of the three widely acclaimed static reference points: maximum sustainable yield (MSY) and maximum economic yield (MEY) as desirable management targets, and open access yield (OAY) as an avoidable target [21, 30]. In addition, the optimum sustainable yield (OSY), which is a dynamic equilibrium condition corresponding to the singular path of the optimal control, is discussed. Under this dynamic reference point, the discount rate δ is taken into consideration, unlike in the case of the static reference points. It is also of interest to note that when δ is zero OSY equals MEY, whereas OSY equals OAY as δ approaches infinity.

Lleonart and Merino [21] in their extension to the Gordon-Schaefer model, proposed an alternative approach to poorly managed or unregulated fisheries by modeling the biomass yield through time. They concluded that unregulated or poorly managed fisheries tend to be overexploited, but not at levels sufficient to dissipate the economic rent (or net revenue) in its entirety. In another study, Udumyan et al. [32] addressed marine habitat concerns by incorporating the dynamics of the carrying capacity, as an indicator of the state of marine habitats, into the Gordon-Schaefer model. Their results collaborated the claims by marine biologists and fisheries managers that habitat degradation is one of the main factors accounting for the decline of fisheries in many parts of the world.

In light of the numerous problems confronting the sardinella fishery in Ghana, it is imperative to develop a bioeconomic model to help address these challenges and also ensure the optimal management of the resource. There are hardly any papers using dynamic bioeconomic models to explore the sardinella fishery in Ghana. The only publication that has been encountered in the course of reviewing the literature makes use of discrete-time, difference equations, and it is authored by Bailey et al. [2]. However, with this present study, a continuous-time, differential equations model is proposed for the optimal exploitation of the sardinella under the proportional (or constant effort) harvesting scheme. It is assumed that the harvest (also known as catch or yield) is proportional to the effort expended in the fishing activity. Using real-world sardinella data, this paper explores the state dynamics of the model, as well as how the model performs under dynamic conditions, unlike many papers on the subject which limit themselves to the static conditions of the model. In addition, the dynamic model is investigated under two scenarios: the short-run dynamics where the time horizon is finite and the long-run dynamics where the horizon is infinite. Another novelty of this study is the exploration of the relationship between the shadow price and the net revenue as it relates to the optimal fishing effort.

In Section 2, the optimal control model is formulated comprising first, the biological model and then the complete bioeconomic model. Bifurcation analysis of the model is performed in Section 3. Most studies on the subject do not undertake stability analysis of the model. However, in this study, the Schaefer model is subjected to bifurcation analysis and the equilibrium points and their stability properties determined. Optimality of the model, which consists of the characterization of the optimal control as well as the singularity analysis of the model, is discussed in Section 4. Numerical and graphical illustrations of the model are portrayed in Section 5 while the last section deals with the discussion and conclusions.

2. Model formulation

The model is formulated taking cognizance of the fact that fishing is as much of a biological activity as an economic one. Therefore, fishery managers should be guided by these considerations when taking important decisions concerning the fishery.

2.1. The biomass dynamics.

The biological dynamics of the model, also referred to as the Schaefer model, can be formulated as

$$\frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right) - qE(t)x(t)\,, \qquad x(0) = x_0, \tag{1}$$

where $x(t)$ is the biomass of the fish population (or stock size) at time t, x_0 is the initial biomass level, r is the intrinsic growth rate of fish biomass, q is the catchability coefficient, which represents the proportion of the current stock caught by one standard vessel in one time unit, $E(t)$ is the rate of fishing effort at time t, and K denotes the carrying capacity for the given population; that is, the maximum sustainable fish biomass [5, 13].

Note that the harvest or yield is given by

$$h(t) = qE(t)x(t). \tag{2}$$

There are two equilibrium points associated with (1); namely, 0 and a positive equilibrium point

$$x_{eqm} = K\left(1 - \frac{qE}{r}\right), \tag{3}$$

provided $E < \frac{r}{q}$.

When $E \geq \frac{r}{q}$, $x_{eqm} \leq 0$ and the population goes into extinction. Therefore, $E = \frac{r}{q}$ is a transcritical bifurcation point for the model.

Maximum sustainable yield (MSY)

This corresponds to the level of harvesting that maximizes the sustainable yield. That is, the maximum harvest which can be maintained indefinitely.

Substituting (3) into (2), with $x = x_{eqm}$, gives the sustainable yield

$$h_S = qEK\left(1 - \frac{qE}{r}\right). \tag{4}$$

The effort that maximizes the sustainable yield h_S is found as

$$E_{MSY} = \frac{r}{2q}. \tag{5}$$

The value of MSY, denoted h_{MSY}, is found by plugging (5), with $E = E_{MSY}$, into (4). Hence,

$$h_{MSY} = \frac{rK}{4} \tag{6}$$

and the biomass level at the MSY is

$$x_{MSY} = \frac{K}{2}. \tag{7}$$

2.2. The bioeconomic model. Incorporating economic parameters into the afore-mentioned biological model gives the static bioeconomic model.

Open access yield (OAY)

Operating under an open access regime where there is little or no regulation of the resource, effort E tends to a level where the sustainable economic rent (or net revenue) is zero. This gives rise to what is known as economic overfishing.

The net revenue is the difference of total sustainable revenue TR_S and total cost TC, and can be expressed as

$$\begin{aligned} Sustainable\ net\ \ revenue &= ph_S - cE \\ &= pqEK\left(1 - \frac{qE}{r}\right) - cE, \end{aligned} \tag{8}$$

where p is the price per unit harvest and c is the cost per unit effort (the costs may include cost of fuel powering the fishing vessels).

Letting (8) go to zero gives

$$E_{OAY} = \frac{r}{q}\left(1 - \frac{c}{pqK}\right), \tag{9}$$

provided $pqK > c$.

To get the biomass level x_{OAY} associated with E_{OAY}, substitute (9), with $E = E_{OAY}$, into (3) to get

$$x_{OAY} = \frac{c}{pq}. \tag{10}$$

The associated harvest level is

$$h_{OAY} = \frac{rc}{pq}\left(1 - \frac{c}{pqK}\right). \tag{11}$$

It is worth noting that when $E_{OAY} > E_{MSY}$, the rate of harvest (at least in the short term) exceeds h_{MSY} resulting in $x_{OAY} < x_{MSY}$. This situation is known as biological overfishing and eventually leads to the depletion of the resource. Thus OAY can lead to both biological and economic overfishing.

The OAY is also known as bionomic equilibrium (BE) because it simultaneously gives equilibrium in both the biological and economic sense [31].

Maximum economic yield (MEY)
This is the rate of harvesting that maximizes the sustainable net revenues. That is, the maximum net revenues which can be maintained indefinitely implementing this rate of harvest.

The effort level that maximizes the the net revenue is found from (8) as

$$E_{MEY} = \frac{r}{2q}\left(1 - \frac{c}{pqK}\right). \tag{12}$$

Using (3), the associated biomass level is

$$x_{MEY} = \frac{K}{2}\left(1 + \frac{c}{pqK}\right). \tag{13}$$

The corresponding harvest level is

$$h_{MEY} = \frac{rK}{4}\left(1 - \left(\frac{c}{pqK}\right)^2\right). \tag{14}$$

It is instructive to note that when effort level $E < E_{OAY}$, the net revenue is positive (revenue exceeds cost), thereby attracting more fishers into the industry. On the other hand, if the effort level $E > E_{OAY}$, the net revenue is negative (cost exceeds revenue), making fishers to abandon the industry. Therefore, there is always a tendency towards E_{OAY} under open access fishery.

The Gordon-Schaefer model, as already mentioned, contains both biological and economic parameters. Therefore any numerical study of the model needs to obtain the values for these parameters.

The biological data is sourced from a published work on a *Sardinella aurita* study conducted by the FAO in collaboration with the Nansen Program [24]. As regards the economic data, the values are based on a field study conducted by the Marine Fisheries Research Division (MFRD) of the Fisheries Commission of Ghana [22]. To reflect current values, the original price and cost figures have been adjusted for inflation [2, 4].

Table 1 presents the biological and economic parameter values used for the study.

TABLE 1. Parameter values for model

Parameter	Description	Value	Units
δ	Discount rate	0.15	year^{-1}
q	Catchability coefficient	1.8×10^{-6}	trip^{-1}year^{-1}
r	Intrinsic growth rate	1.42	year^{-1}
K	Carrying capacity	$1,000,000$	tonnes
p	Ex-vessel price of fish	600	\$ tonne^{-1}
c	Cost per trip	195	\$ trip^{-1}year^{-1}

Recall that the sustainable net revenue for the static model is given by (8) Therefore, substituting the values of the effort and harvest levels at the three equilibrium reference points; MEY, MSY and OAY into the net revenue equation, the annual sustainable net revenue can be computed, as shown in Table 2.

TABLE 2. Annual net revenue at MEY, MSY and OAY levels of harvesting

Reference point	MEY	MSY	OAY
x	$590,278$	$500,000$	$180,556$
E	$323,225$	$394,444$	$646,450$
h	$343,427$	$355,000$	$210,096$
Sustainable net revenue ($ year^{-1})	$143,027,325$	$136,083,420$	0

Table 2 validates the theoretical results concerning the Gordon-Schaefer model. At MEY levels, the table gives the maximum revenue as well the highest fish stock size. Thus, apart from providing the most revenue, it is also the most conservationist among the three reference points. Of course, the level at MSY provides the highest yield; and at the OAY level, it provides the most effort (twice the effort at MEY), lowest fish stock size, lowest yield, and of course, zero revenue. Note that the sustainable net revenue for OAY is exactly zero when all the significant digits of h and E are used in the computation.

Palm [26] asserted that much of the analysis of fisheries is based on the concept of equilibrium, as shown from the preceding analysis on the static model. However, equilibrium is an idealization and is never actually encountered in reality because continually changing environmental influences act as disturbances which displace the system from its equilibrium condition. Thus for systems that are unstable, this is disastrous as equilibrium is never regained; and for other systems that are stable with large time constants, the return to equilibrium might take so long as to negate the assumptions and usefulness of the equilibrium analysis. Therefore, there is the need for static or equilibrium-based analysis to be supplemented with dynamic methods which take into account the complex nature of the fishery.

Hence the dynamic optimization version of the Gordon-Schaefer model can be presented in the following form:

$$
\begin{aligned}
&\max_{E} J(E) = \int_0^\infty e^{-\delta t}(pqx(t)-c)E(t)\,dt \\
&\text{subject to} \quad \frac{dx(t)}{dt} = rx(t)\left(1-\frac{x(t)}{K}\right) - qE(t)x(t) \\
&\qquad\qquad x(0) = x_0 \\
&\qquad\qquad 0 \le E(t) \le E_{max}\,,
\end{aligned} \tag{15}
$$

where E_{max} is the maximum effort capacity. The model aims to determine the effort strategy $E(t)$ that results in the largest possible net economic benefit as expressed by the present value integral J of (15).

It must be noted that (15) is cast a an infinite horizon problem where the terminal time $T \to \infty$. This reflects the fact that the fishery is expected to persist indefinitely. However, if transient analysis of the fishery is called for, T is finite. See Ibrahim and Benyah [14] for further details on transient analysis.

3. Bifurcation analysis

A bifurcation can be described as the change in the number of equilibrium points or periodic orbits, or in the stability properties of a dynamical system if a parameter is varied. The value of the parameter where the stability dynamics change is called a bifurcation point [7, 14].

Recall that there are two equilibrium points associated with the state dynamics of the model (1) when the effort is less than the bifurcation point. These are 0 and $x_{eqm} = K\left(1-\frac{qE}{r}\right)$, and the bifurcation point is given by $E = \frac{r}{q}$.

From the parameter values given in Table 1, the bifurcation point for the model is computed as $E = 788,888.89$. That is, approximately $788,889$ trips annually for the fishery.

The following two definitions by King et al. [18] are relevant to the discussion of the stability properties of dynamical systems.

Definition 3.1: Consider the following first order autonomous differential equation $\frac{dx}{dt} = g(x,E)$.

(1) If $g(x_1, E) = 0$, then $x = x_1$ is an equilibrium solution to the differential equation. Therefore, x_1 is an equilibrium point.
(2) If $g_x(x_1, E) \neq 0$, then $x = x_1$ is a hyperbolic equilibrium point.
(3) If $g_x(x_1, E_1) = 0$, where $E = E_1$ is the bifurcation point, then $x = x_1$ is a nonhyperbolic equilibrium point.

Definition 3.2: A dynamical system that contains one or more nonhyperbolic equilibrium points is said to be structurally unstable.

This means that a small perturbation, not to the solution but to the model itself in the form an addition of a small extra term to $g(x, E)$, can lead to a qualitative difference in the structure of the set of solutions; for instance, a change in the number of equilibrium points or their stability.

The slope fields and solution curves of the state equation are plotted to highlight the equilibrium and stability properties of the model. For ease of analysis, q, $E(t)$, K and $x(t)$ are scaled to a thousand units.

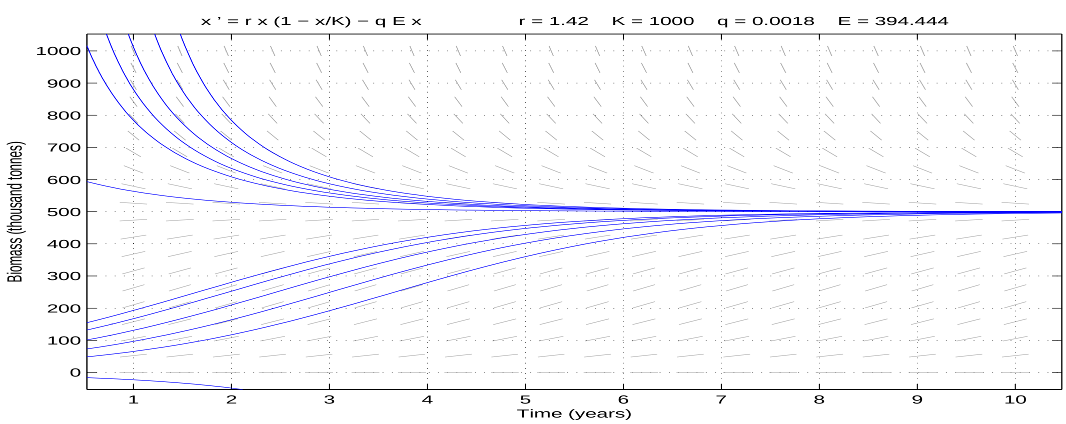

FIGURE 1. Solution curves for $E = 394,444$

Solution curves corresponding to the situation where $E = E_{MSY} = 394,444$ trips are presented in Figure 1. It can be observed that there are two hyperbolic equilibrium points: 0 and $x_{eqm} = x_{MSY} = 500,000$ tonnes. For any initial biomass level $x_0 > x_{MSY}$, the population approaches the equilibrium population, x_{MSY} in the long run. Similarly, for $0 < x_0 < x_{MSY}$, the population asymptotically approaches x_{MSY}. Thus the biomass level 0 is unstable while x_{MSY} is stable (making the system structurally stable). Of course, biomass levels starting from the equilibrium levels, 0 and x_{MSY} remain there indefinitely. Hence, an effort level corresponding to E_{MSY} induces a long-term biomass level of exactly half the carrying capacity.

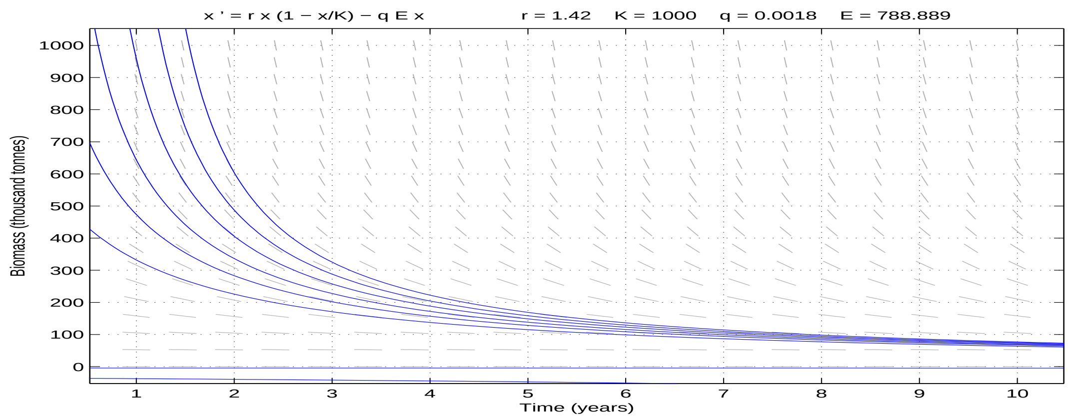

FIGURE 2. Solution curves for $E = 788,889$

Solution curves corresponding to the case where $E = 788,889$ trips, the bifurcation point, are presented in Figure 2. For any initial biomass level $x_0 > 0$, the population approaches the equilibrium population, 0 in the long run. Thus, at the transcritical bifurcation point, the single nonhyperbolic equilibrium biomass level, 0 is semi-stable (making the system structurally unstable). Of course, biomass levels starting from the equilibrium level, 0 remain there indefinitely. Hence, for any initial biomass level, the long-term population of fish stock is towards extinction.

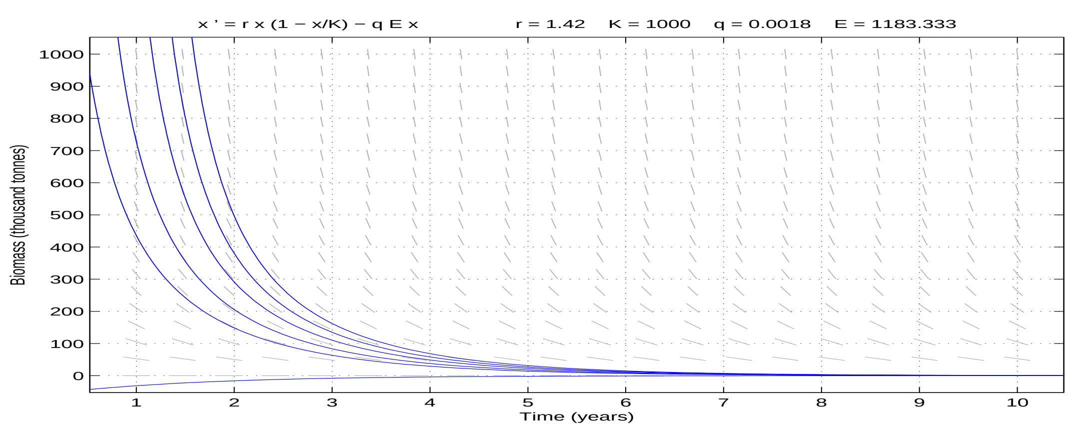

FIGURE 3. Solution curves for $E = 1,183,333$

The case where effort level, $E = 1,183,333$ trips, is greater than the bifurcation point is shown in Figure 3. For any initial biomass level $x_0 > 0$, the population approaches the hyperbolic equilibrium population, 0 in finite time. Thus, corresponding to this effort level exists a nonnegative equilibrium biomass level, 0 which is stable (making the system structurally stable). Hence, in this situation, whatever the initial fish population, the fish will die out as a result of overfishing or excessive harvesting in finite time.

In summary, fish stocks exploited below $788,889$ trips (the bifurcation point) are more likely to persist. Specifically, exploited stocks at MEY and MSY effort levels, $323,255$ and $394,444$ trips, respectively, are almost guaranteed to persist. Moreover, at the OAY level of fishing effort ($646,450$ trips), the model predicts that the fish population will persist indefinitely at any level x_{OAY} no matter how low; and also the population will always recover if fishing ceases. However, as a matter of fact, many fish populations never recover from severe overfishing. This is as a result of, perhaps, the level of reproduction not high enough to sustain a population or the species is displaced by a competing species [5]. Stocks exploited at or above the bifurcation point ($1,183,333$ trips) are more than likely to go into extinction, if not in finite time, then probably in the long run.

Regarding the artisanal fishery in Ghana, as stated earlier, there are currently a little over $12,700$ canoes operating within the sector targeting the four main fish species: round sardinella, flat sardinella, anchovies and mackerels. Assuming 40% of the boats are dedicated to the round sardinella, and also assuming 20 fishing days in a month (since most of the fishers along the coast do not fish on Tuesdays, and also excluding days fishers use to mend their broken nets as well as days of adverse weather conditions), then we have a total of $1,219,200$ fishing days in a year. Furthermore, equating one trip to one fishing day, then there are a total of $1,219,200$ trips undertaken annually by the fishermen exploiting, in particular, the round sardinella. Juxtaposing these current effort levels by the artisanal fishermen with the preceding reference points, they seem way above the bifurcation point of the fishery. In fact, the current effort levels are more than three times the effort needed at MSY. This can only mean one thing – if current effort levels are not checked, the long term sustainability of the resource will be in serious jeopardy, as attested to by current very low yields.

4. Optimality of the model

The characterization of the optimal control is sought for in this section. Also, the model is analyzed to determine whether or not the singular path is attainable by the control.

4.1. Characterization of the optimal control.

The goal, as stated earlier, is to maximize the present-value of the net revenue. Thus, we seek an optimal control E^* such that

$$J(E^*) = \max\{J(E) \,\big|\, E \in U\},$$

where the control set is Lebesgue measurable for an infinite time horizon, and defined by

$$U = \{E \,|\, 0 \le E(t) \le E_{max},\ t \in [0,\infty)\}.$$

To derive the necessary conditions for the optimal control, Pontryagin's Maximum Principle is employed [15, 27]. The current-value Hamiltonian for the optimal control problem (15) is

$$H(x,\lambda,E,t) = (pqx-c)E + \lambda\left[rx\left(1-\frac{x}{K}\right) - qEx\right]. \tag{16}$$

The adjoint variable λ is governed by

$$\begin{aligned}\lambda' &= \delta\lambda - \frac{\partial H}{\partial x}\\ &= \delta\lambda - pqE - \lambda\left(r - \frac{2rx}{K} - qE\right).\end{aligned} \tag{17}$$

The switching function is defined by

$$\begin{aligned}\psi(t) &= \frac{\partial H}{\partial E}\\ &= (pqx-c) - \lambda qx.\end{aligned} \tag{18}$$

The characterization of the optimal control is

$$\begin{cases} E^* = 0 & \text{if} \quad \psi(t) < 0\\ 0 < E^* < E_{max} & \text{if} \quad \psi(t) = 0\\ E^* = E_{max} & \text{if} \quad \psi(t) > 0.\end{cases} \tag{19}$$

4.2. Singularity analysis of the model. This analysis determines whether the optimal control would be bang-bang or follow a singular path. For a singular control, it is assumed that there is an interval I for all $t \in I \subset [0,\infty)$ such that

$$\psi(t) = 0\,. \tag{20}$$

Thus, from (18) and (20),

$$(pqx - c) - \lambda qx = 0\,. \tag{21}$$

So, solving for λ we find

$$\lambda = p - \frac{c}{qx}. \tag{22}$$

Differentiating (22) with respect to t, substituting for the state equation in (15) and simplifying, it follows that

$$\lambda' = \frac{crK - cqEK - crx}{qxK}. \tag{23}$$

By plugging the λ expression (22) into the adjoint equation (17), we get

$$\lambda' = \frac{\delta pqxK + 2pqrx^2 + crK - pqrxK - cqEK - \delta cK - 2crx}{qxK}. \tag{24}$$

Setting the expressions (23) and (24) equal to each other and simplifying, we obtain the positive optimal biomass level as

$$x^* = \frac{K}{4}\left[\left(1 + \frac{c}{pqK} - \frac{\delta}{r}\right) + \sqrt{\left(1 + \frac{c}{pqK} - \frac{\delta}{r}\right)^2 + \frac{8\delta c}{pqrK}}\right]. \tag{25}$$

The effort level corresponding to x^* is found from the state equation in (15), after noting that $x' = 0$, since x^* is a constant. Thus

$$E^* = \frac{r}{q}\left(1 - \frac{x^*}{K}\right). \tag{26}$$

The corresponding harvest level is

$$h^* = qE^*x^*\,. \tag{27}$$

The optimal annual sustainable net revenue is given by

$$\pi = ph^* - cE^*\,. \tag{28}$$

Hence the characterization of the optimal fishing effort is

$$E^* = \begin{cases} 0 & \text{if} \quad \lambda > p - \dfrac{c}{qx} \\ \dfrac{r}{q}\left(1 - \dfrac{x^*}{K}\right) & \text{if} \quad \lambda = p - \dfrac{c}{qx} \\ E_{max} & \text{if} \quad \lambda < p - \dfrac{c}{qx}. \end{cases} \tag{29}$$

This implies that the optimal control comprises both the extreme controls (bang-bang) and the singular control. The extreme controls indicate that the resource should be harvested (by exerting up to the maximum available effort rate) if and only if the net revenue per unit harvest (or the marginal net revenue of harvest) exceeds the current-value shadow price of the resource (or the marginal net revenue of stock) [8, 14].

In addition, the harvested resource could follow the OSY path (or singular path) if the current-value shadow price exactly equals the net revenue per unit harvest. Thus the OSY parameters are E^*, x^* and h^*.

Sometimes, the optimal control alternates between the singular and bang-bang controls known as a bang-singular control (also referred to as singular control) [17, 20].

It is normal in optimal control problems to ensure the existence of the optimality system. In this vein, it should be noted that the state equation, which is logistic with harvesting is a priori bounded. Also, the state equation and the objective functional are both linear in the control E. Therefore, by standard arguments, an optimal control as well as the optimal state exists [10, 15].

Table 3 features the results of harvesting at OSY levels.

TABLE 3. Annual net revenue at OSY levels of harvesting

δ	x^*	E^*	h^*	Sustainable net revenue ($ year^{-1})
0	590,278	323,225	343,427	143,027,325
0.05	578,170	332,777	346,323	142,902,285
0.10	566,293	342,146	348,759	142,536,930
0.15	554,654	351,328	350,758	141,945,840
0.20	543,261	360,317	352,343	141,143,985
0.25	532,119	369,106	353,535	140,145,330
0.50	480,392	409,913	354,454	132,739,365
0.75	435,645	445,214	349,119	122,654,670
0.95	404,926	469,447	342,165	113,756,835
∞	180,556	646,450	210,096	0

In Table 3, it is shown that when the discount rate is zero the OSY levels are identical to the MEY levels of the model. As the discount rate increases, effort rate increases while the biomass and the net revenue decrease. However, the harvest rate increases up to point (attaining its maximum where the biomass level is near x_{MSY}), then starts to decrease. When the discount rate approaches infinity, the levels of OSY mirror those of OAY. In order to avoid the undesirable situation of OAY, a strong regulatory scheme must be strictly enforced to curb the excessive harvesting of the resource [16].

Note that the optimal sustainable population size, x^* (represented by (25)) contains x_{MSY}, x_{MEY} and x_{OAY} as special cases. That is, x_{MSY} is optimal under zero costs and zero discounting, x_{MEY} is optimal under zero discounting and x_{OAY} is optimal at infinite discounting. In other words, with zero future discounting, the optimal harvest strategy would be to maximize long-term economic benefits (or net revenue), which corresponds to MEY.

It can also be deduced that

$$x^* < x_{MEY} \quad \text{if } \delta > 0,$$

and also, as attested to by Table 3,

$$x^* \text{ is a decreasing function of } \delta.$$

This means that higher discount rates imply lower levels of conservation. In fact, as already observed from Table 3,

$$\lim_{\delta \to \infty} x^* = x_{OAY}.$$

Scott [29] was the first to argue that infinite discounting leads to the same outcome as OAY. In other words, under high discounting, future revenues count for very little, so present revenues are maximized, and this implies rapidly harvesting down to x_{OAY}. As another way of looking at it, under open access competition, resource users must completely discount the future because they cannot expect positive returns in the long run [5].

The sensitivity of x^* to various economic parameters is listed in Table 4.

TABLE 4. Sensitivity of x^* to economic parameters

Parameter	Sensitivity to x^*
price, p	–
effort cost, c	+
discount rate, δ	–

From Table 4 , it can be seen that the optimal population size, x^* increases if the cost of fishing effort increases. On the other hand, x^* decreases if the price per unit harvest, or the discount rate increases.

5. NUMERICAL SIMULATIONS

Since MEY, MSY and OAY are the static equilibrium reference points, the dynamic reference point, OSY, is explored further employing simulations.

To this end, the optimality system is implemented using an iterative method involving the Runge-Kutta Fourth order scheme. This involves the Forward-Backward Sweep approach from Lenhart and Workman [20], implementable in Matlab. Simulations results are presented corresponding to OSY (which is optimal under dynamic conditions). First to be considered is the equilibrium (or long-run) scenario where the time horizon $T \to \infty$; and second, take a look at the transient (or short-run) case where T is finite.

5.1. Long-run dynamics of the model. The long-run scenario, as shown in Figure 4, depicts fishing at a maximum effort rate of $351,328$ trips (OSY effort rate), $x_0 = 800,00$ tonnes and $T = 20$ years, where the shadow price initially increases, appears to remain constant, then decreases. Meanwhile the net revenue per unit harvest initially decreases up a point, then appears to remain constant. At the start of the harvesting period, the shadow price, US \$288.94 is significantly lower than the net revenue, US \$464.58. This signifies that the revenue due an additional tonne of fish being added to the biomass is less than the expected revenue from harvesting the fish. So at this instance it is prudent to harvest. As time progresses the shadow price appears to stabilize and intersect with the net revenue for the majority of the time horizon. Thereafter, the shadow price experiences a sharp fall down to zero at the final time horizon, $T = 20$ years. On the other hand, the net revenue appears to remain constant, and finally ends at US \$404.76. The optimal control alternates between the singular and bang-bang controls. This shows that for a long-term time horizon (or under equilibrium conditions), it is optimal to exert the maximum effort, or the effort at the OSY level ($E_{max} = E^* = 351,328$ trips; see Figure 5).

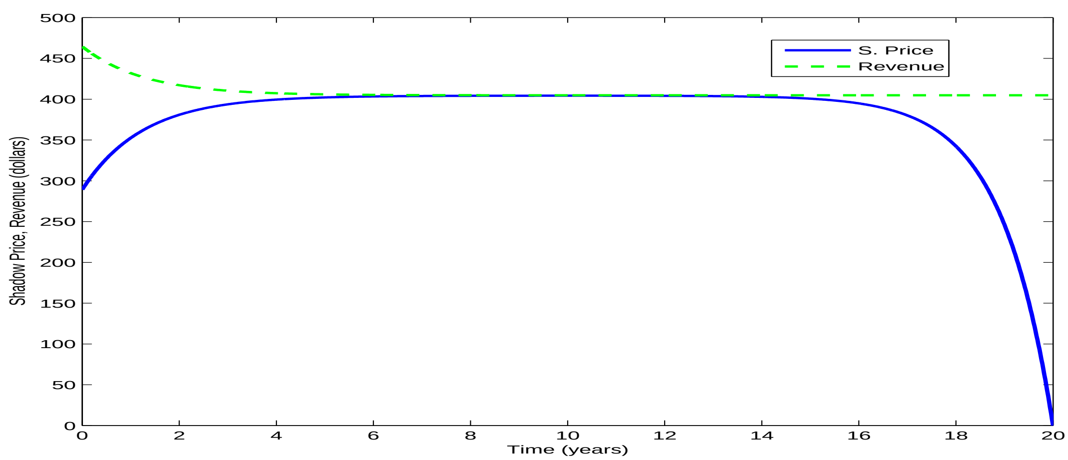

FIGURE 4. Shadow price and net revenue for $E_{max} = 351,328$, $x_0 = 800,000$

Simulation results for the fishing effort and biomass level relating to the case where $E_{max} = 351,328$ trips and $T = 20$ years are presented in Figure 5. In the effort plot, it is observed that when the maximum effort rate E_{max} is set at the OSY level, the optimal effort rate appears to follow the same path very close to the OSY level throughout the twenty-year horizon for the different initial biomass levels. However, the fish biomass levels initially follow different trajectories. The biomass decreases for the higher initial value of $800,000$ tonnes and also decreases for the lower initial value of $600,000$ tonnes, and finally ends at the equilibrium value of around $554,654$ tonnes.

Assuming an initial population size of $600,000$ tonnes, the total net revenue over the twenty-year horizon corresponding to the given rate of fishing effort is computed as US\$$917,100,000$; a decrease of 7% of the net revenue, US\$$982,870,000$, for $x_0 = 800,000$ tonnes.

It is worth noting that, if $E_{max} > E^*$ the iterates of the simulation process fail to converge as $T \to \infty$. In other words, the model does not attain dynamic equilibrium status for rates of fishing effort exceeding the effort rate corresponding to OSY. In particular, for any initial biomass level and effort levels at $E_{max} = E_{MSY} = 394,444$ trips and $E_{max} = E_{OAY} = 646,450$ trips, the iterates failed to converge for a time horizon beyond four years and one year, respectively. This validates the assertion that OSY is optimal under dynamic conditions.

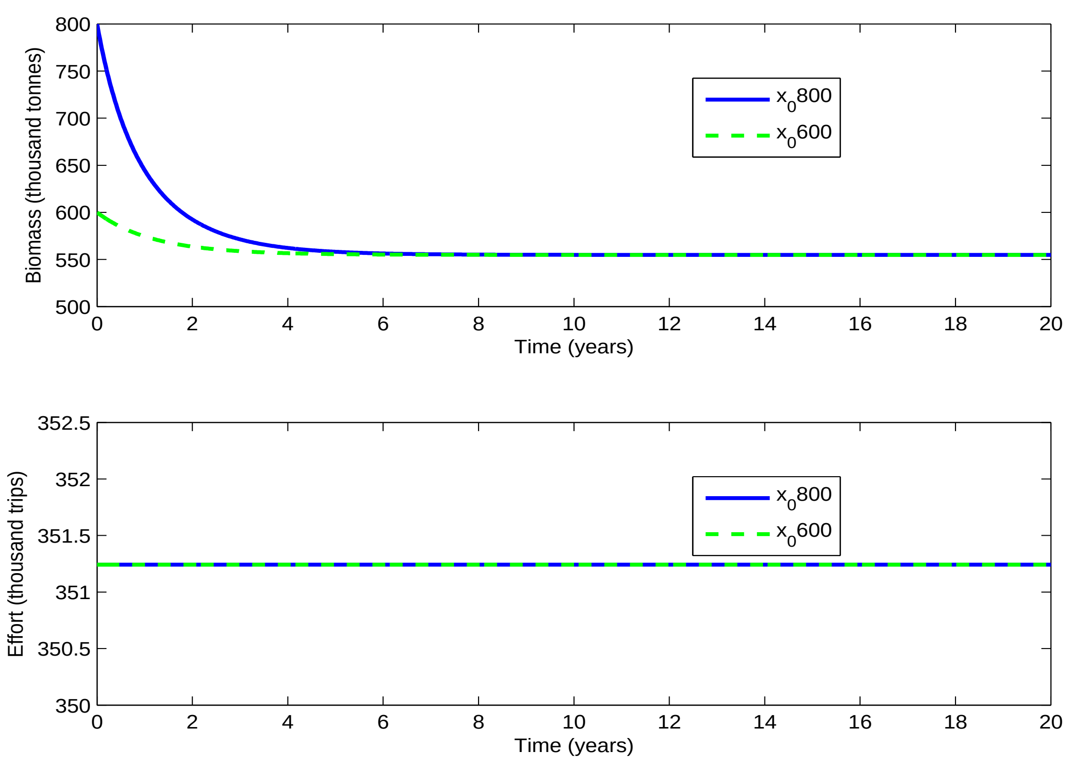

FIGURE 5. Effort strategy and biomass level for $E_{max} = 351,328$

Simulation results for the effort rate and biomass level relating to the case where $x_0 = 750,000$ tonnes, $T = 20$ years, $E_{max} = 351,328$ trips and $E_{max} = 323,225$ trips are presented in Figure 6. In the effort plot, it is observed that when the maximum effort level E_{max} is set at the OSY and MEY levels, the optimal effort level appears to follow different paths corresponding respectively to the equilibrium OSY and MEY levels throughout the twenty-year horizon. Furthermore, the fish biomass levels follow different trajectories. The biomass decreases for the effort level at OSY to its equilibrium value of around $554,654$ tonnes, and also decreases for the effort level at MEY to its equilibrium value of around $590,278$ tonnes.

Assuming an initial population size of $750,000$ tonnes, the total net revenue over the twenty-year horizon corresponding to the effort levels at OSY and MEY are computed as US\$$967,990,000$ and US\$$956,600,000$, respectively. The net revenue for MEY is lower by only 1%.

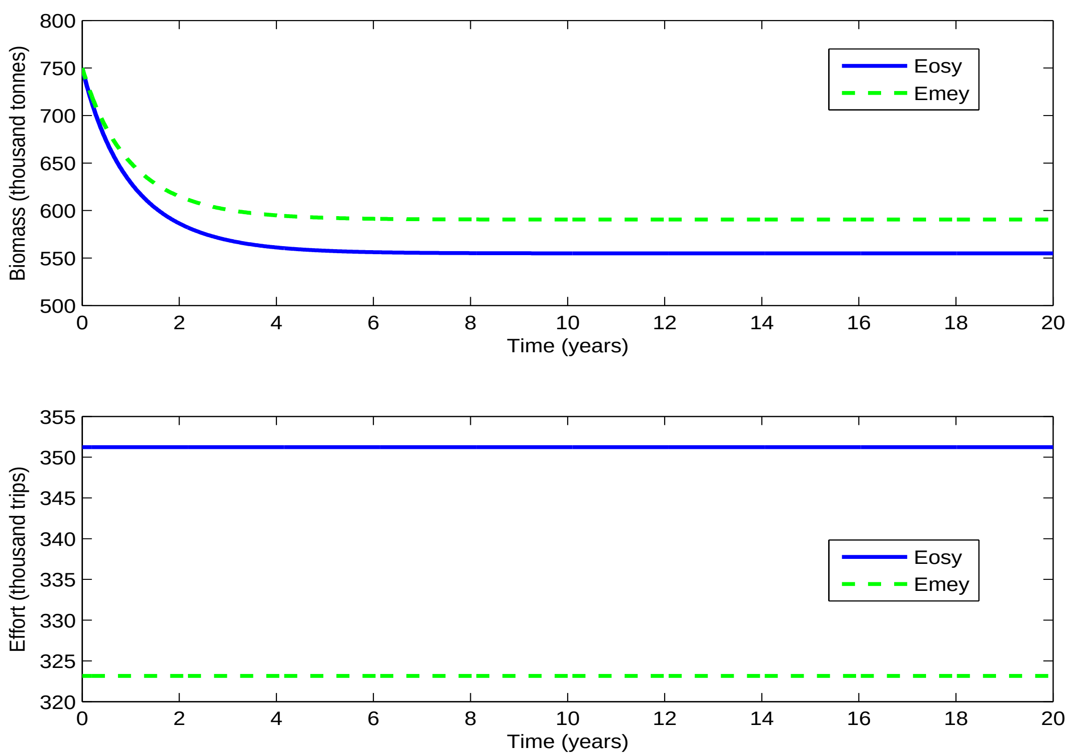

FIGURE 6. Effort strategy and biomass level for $E_{max} = 351,328;\ 323,225$

5.2. Short-run dynamics of the model. Regarding the short-run scenario, the time horizon considered for this study is one year. As shown in Figure 7, fishing is at a maximum effort rate of $800,000$ trips, $x_0 = 200,00$ tonnes and $T = 1$ year, where the shadow price appears to be decreasing and the net revenue per unit harvest initially increases up a point, then decreases. At the start of the harvesting period, the shadow price, US \$392.56 is significantly higher than the net revenue, US \$58.33. This signifies that the revenue due an additional tonne of fish being added to the biomass is greater than the expected revenue from harvesting the fish. So at this instance it is prudent not to harvest. As time progresses the shadow price experiences a gradual decline in value while the net revenue sharply increases until the two intersect at US \$297.46 with switching time $t^* = 0.56$ year. Thereafter, the shadow prices experiences a sharp fall down to zero at the final time horizon, $T = 1$ year, while the net revenue gradually declines to US \$227.65. Thus, after 6.7 months (the switching time) it is now optimal to harvest the fish stocks as the net revenue would be greater than the shadow price.

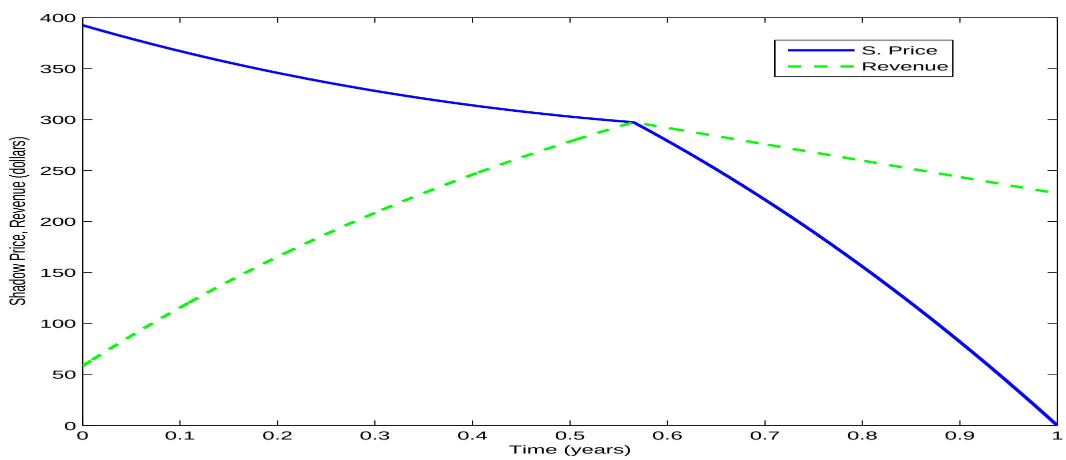

FIGURE 7. Shadow price and net revenue for $E_{max} = 800,000$, $x_0 = 200,000$ and $T = 1$

Simulation results for the fishing effort strategy and biomass level relating to the special case where $E_{max} = 800,000$ trips (above the bifurcation point), $x_0 = 200,000$ tonnes (far below x_{MSY}) and $T = 1$ year are presented in Figure 8. In the effort plot, it is observed that the switching time occurs at $t^* = 0.56$ year (see Figure 7) indicating that for the initial 6.7 months of the year, no fishing effort should be applied (or harvesting occurring). Thereafter, the maximum rate of fishing effort should be applied. A significant change in the growth of the fish stock can be observed once the maximum rate of fishing effort is exerted, as seen in the biomass plot. The biomass level ends at around 290,000 tonnes. Thus, the optimal effort strategy recommends the bang-bang approach.

Assuming an initial population size of one-fifth the carrying capacity, 200,000 tonnes, the total net revenue over the one year horizon corresponding to a high rate of fishing effort of more than twice the MSY level (800,000 trips) is computed as US \$623,710,000.

This extreme scenario vividly depicts the current sardinella fishery situation in Ghana. The fishery is open access (very high effort rates) and the present low yields presume a low level of stock size (biomass below sustainable levels). The model strongly recommends that in a crisis situation, as the sardinella fishery in Ghana evidently is, closed fishing seasons must be implemented. At least six months of no fishing activity every year would go a long way to restoring the fish stock to an appreciable level.

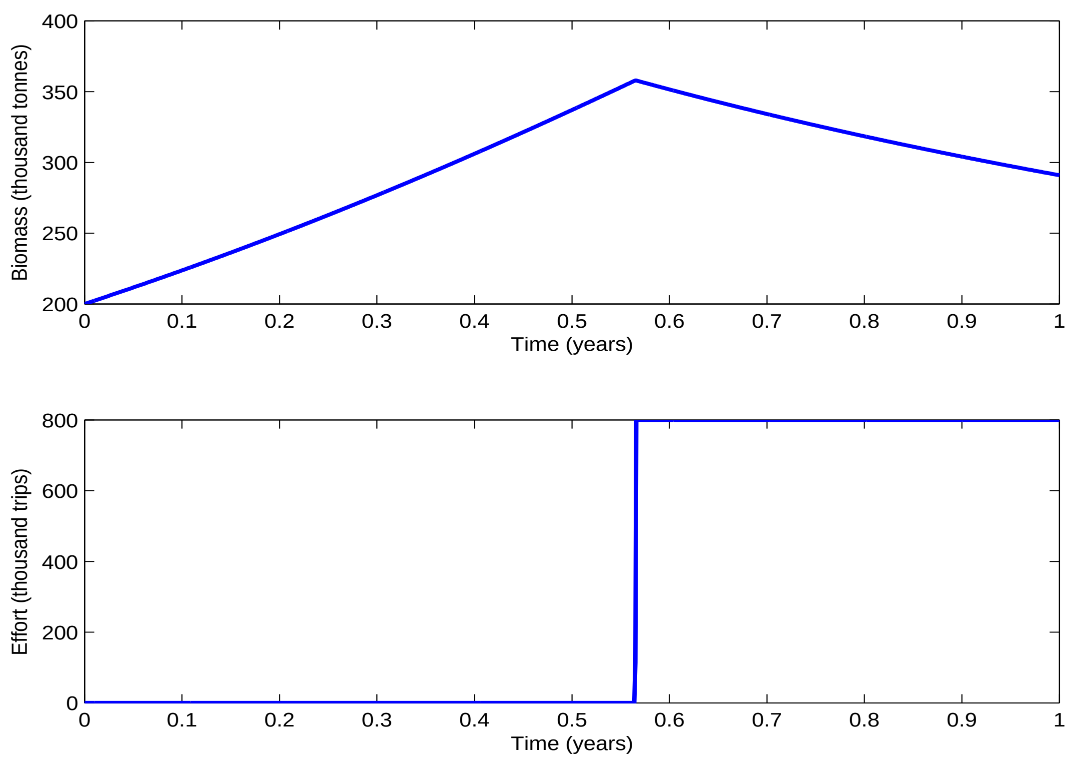

FIGURE 8. *Effort strategy for* $E_{max} = 800,000$, $x_0 = 200,000$ *and* $T = 1$

6. CONCLUSIONS

This study has looked into the fishing effort strategies of the sardinella fishery in Ghana under the canonical Gordon-Schaefer model so as to determine the optimal strategy. The biological model (or Schaefer model) has been subjected to bifurcation analysis and the results show that harvesting of the resource should only be considered when the rate of fishing effort is far below the bifurcation point. The static reference points have been computed and the results confirm that the effort strategy at the MEY level is the most conservationist among the three as well as providing the greatest net revenue.

In order to determine the optimality of the model, characterization of the optimal control, which is the optimal rate of fishing effort, has been carried out. Further, singularity analysis of the model showed that both the bang-bang and singular paths are applicable. Analysis of the dynamic reference point, OSY, validates the assertion that the MEY and OAY levels are its special cases corresponding to the discount rate being zero and infinity, respectively. Furthermore, the MSY, another special case, is optimal at zero discounting and zero costs.

From the simulation results, the recommended rate of fishing effort is at the OSY level, with an initial biomass level of at least $x^* = 554,654$ tonnes. When the initial biomass level is below the MSY level, indicating an overexploited fishery, coupled with a very high rate of fishing effort (above the bifurcation point), the model indicates that fishing cannot be continuous all year round. There must be a break in fishing of up to six months. This calls for the implementation of closed fishing seasons.

Additionally, the current situation in the marine artisanal sector where all the fishers are at liberty to target any of the fish species should be reconsidered. A licensing regime should be implemented to enable particular fishers to exclusively target certain species so that differentiation of fishing effort can be established. This would ensure proper monitoring and long term sustainability of the resource.

Conflict of Interests

The authors declare that there is no conflict of interests.

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