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SMOKING HABIT: A BIO MATHEMATICAL STUDY

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Abstract. Smoking habit is an addiction to both physical and psychological. Nicotine from cigarettes temporarily experiences physical withdrawal symptoms and cravings. Because of nicotine effects on the brain, as a quick and reliable way to boost our outlook, relieve stress, and unwind. To stop the smoking habit, you'll need to address both the addiction and habits. With the proper support and right planning, any smoker can kick the addiction. In this research work, we formulate a mathematical model which analyse the smoking habit in the human population. Here we divide the total population into three classes: potential smokers, smokers and media aware population. We discuss the dynamical behaviour of the model. Finally, we justify our finding through numerical simulation. Our mathematical study reflects that anti smoking campaign plays a pivotal role for reducing number of smoker as well as smoking habit.

Keywords: potential smokers; smoking habit; stability analysis; smoking reproduction ratio.

2010 AMS Subject Classification: 93A30.

1. Introduction

Tobacco epidemic is one of the biggest health threats in the world. Worldwide around 8 million people die due to tobacco epidemic, among which approx. 7 millions of this death are the result

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of the direct habit of tobacco use whereas approx. 1.2 million people are the result of passive smoking. There are more than 7000 chemicals in tobacco smoke of which 250 chemicals are known as harmful and at least 69 chemicals are the major causes of cancer. Thus the second hand or nonsmokers are affected by these harmful chemicals in enclosed spaces like restaurants, offices. Approx. 65000 childrens die each year from illness attributes to second-hand smoking. In adults, second-hand smoke causes serious cardiovascular respiratory disease, including heart disease, lung disease. It causes pregnancy complications for pregnant women.

Good monitoring tracks the extent and character of the tobacco epidemic. Only 38% countries of the world population monitors tobacco use. In 2015 Global Adult Tobacco Survey (GATS) in China revealed that 26.6% of the Chinese people believes smoking causes lung cancer, heart disease. This change has been occurred due to awareness programme by which few people understand the specific health risk of tobacco use. Anti-tobacco media campaigns protect children and other potential smoker groups and reduces in the population of new tobacco users.

Anti Tobacco campaigning carried out in Brazil, Canada, Singapore and Thailand. Which consistently gives a significant results to control the smoking habit. Mass media campaigns can also play a pivotal role in smokers by convincing people to stop using tobacco. Within the last 2 years, around 1.7 billion people in the 39 countries are covered under the anti-tobacco mass media campaign. Medical doctors, scientists, social workers are trying to reduce the ratio of smokers in the total population.

Mathematicians are also working to formulate the mathematical models to find out how to control of smoking habit. For this purpose, the first smoking model was presented by Castello et al. [1] in which he proposed three compartments like potential smokers, chain smokers and quit smokers. Then Sharomi and Gumel [2] improved the model by using the chain smoker class. Zaman [3] presented the optimal campaigns in the smoking dynamics. Adhana and Mekonnen [4] studied by adding smoking cause death rate and one more compartments exposed class in the model proposed by Sharomi and Gumel [2]. Huo and Zhu [5] analysed a model taking into account light smokers compartment, recovery compartment, and two relapses in the giving up smoking model. Zeb et. al. [6] considered a delayed smoking model in which the potential smokers are assumed to satisfy the logistic equation. Alkhudhari et al. [7] studied the effect of

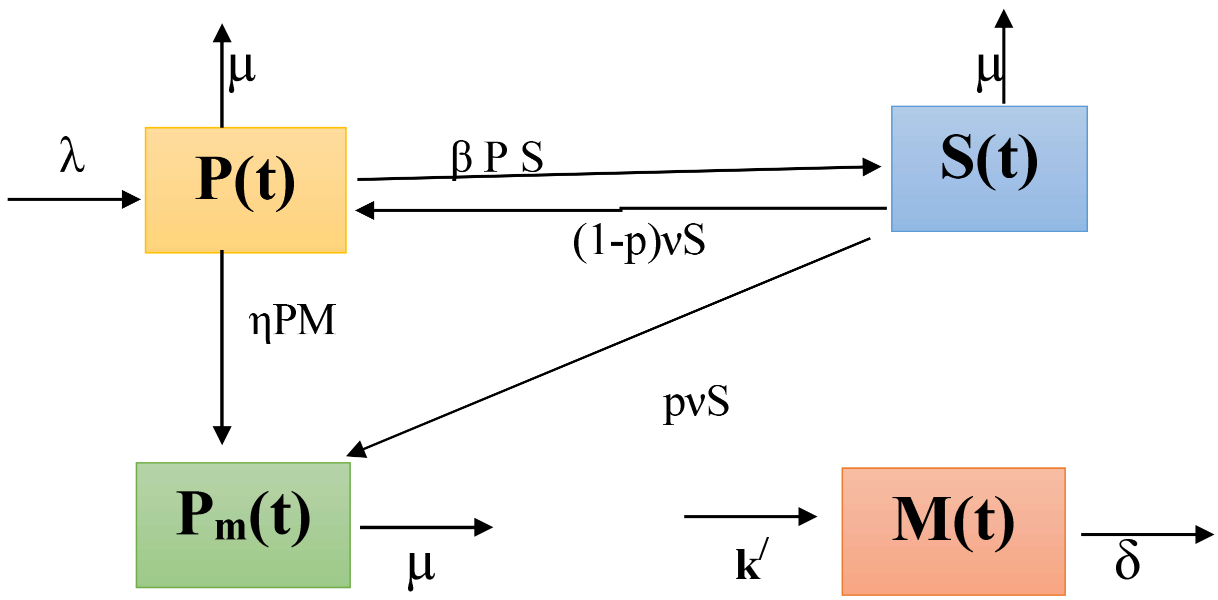

FIGURE 1. Schematic explanation of the model (1).

smokers on temporary quitters. Verma and Bhadaurian [8] studied the dynamics of progression of potential smoker class to the class of smokers followed by the movement of smokers to the state in which they permanently quit smoking due to anti-smoking campaigns. Sikander et al. [9] used Variation of Parameter Method (VPM) with an auxiliary parameter to obtain the approximate solutions for the epidemic model for the evolution of smoking habit in a constant population. It has been observed that media awareness model plays an important role to control infection [10],[11],[12],[13]. Yadav et al. [14] studied the dynamics of smoking behavior under the influence of educational programs and also the individual's determination to quit smoking. Veeresha et al.[15] studied fractional smoking epidemic model with the aid of a novel technique called q-homotopy analysis transform method (q-HATM).

This article has been arranged in the following manner. Firstly we have formulated the basic mathematical model on the basis of Smoking infection in presence of media awareness (Section 2). In Section 3, we have studied the model analytically. In section 4, we have numerically studied the model. In the last section (Section 5) we have discussed about implication of the results which we have found out in different sections.

2. The Model Formulation

In this article, we establish the smoking model in presence of awareness as in Figure 1. From the schematic figure, the total human population $N(t)$ is divided into three components, namely the potential smokers compartment ($P(t)$), persistent smokers ($S(t)$), and the aware population is denoted as $P_m(t)$. Also $M(t)$ denotes the commutative density of awareness programme driven by the media in the region at time t. It is assumed that the habit or infection spread through direct contact between persistent smokers and potential smokers at a rate of β.

We also assume that infection is removed at a rate of γ. After recovery, a fraction p of persistent people will become aware and join the potential smoker class, whereas the remaining fraction $(1-p)$ will become persistent smoker. We also assume that the potential smokers move to aware class at a rate of ηPM due to media campaigns. It is also assumed that some media campaign fade or less their impact on people. We assume $k' = k\mu$ denotes the rate of implementation of a media campaign which is proportional to the death rate of smokers and the δ denotes the fading rate of media campaigns are maintained at a level M_0 which should be maintained in the system.

$$\begin{aligned}
\frac{dP}{dt} &= \Lambda - \beta PS - \mu P + (1-p)\nu S - \eta PM, \\
\frac{dS}{dt} &= \beta PS - \mu S - \nu S, \\
\frac{dP_m}{dt} &= \eta PM - \mu P_m + p\nu S, \\
\frac{dM}{dt} &= k'S - \delta(M - M_0).
\end{aligned} \tag{1}$$

Here we assume that $P(0) \geq 0,\ S(0) \geq 0,\ P_m(0) \geq 0$ and $M(0) \geq 0$. We also note that

$$N(t) = P(t) + S(t) + P_m(t). \tag{2}$$

By combining

$$\begin{aligned}
\frac{dN}{dt} &= \frac{dP}{dt} + \frac{dS}{dt} + \frac{dP_m}{dt}, \\
&\leq \Lambda - \mu N,
\end{aligned} \tag{3}$$

which implies that $\lim_{t\to\infty} \sup N(t) \leq \frac{\Lambda}{\mu}$.

Here we assume that all variables and parameters to be non negative for all $t \geq 0$. We study the above model in the positive invariant set Ω defined as

$$\Omega = \{(P,S,P_m,M)\varepsilon R^4_+; 0 \leq P,S,P_m \leq \frac{\Lambda}{\mu}, 0 \leq M \leq \frac{k'}{\delta}\}, \tag{4}$$

the region of attraction of the model. Setting the right hand side of the equation to zero, we find two equilibrium of the model :

- (i) Smoking Free Equilibrium $\bar{E} = (\bar{P}, 0, \bar{P_m}, \bar{M})$,
- (ii) Smoking Present Equilibrium $E^* = (P^*, S^*, P^*_m, M^*)$.

(A) For the Smoking free equilibrium (SFE), $\bar{P} = \frac{\Lambda}{\mu+\eta M_0}$, $\bar{S} = 0$, $\bar{P_m} = \frac{\eta M_0 \bar{P}}{\mu}$ and $\bar{M} = M_0$.

(B) The Smoking Present Equilibrium (SPE),

$P^* = \frac{\mu+\nu}{\beta}$, $P^*_m = \frac{1}{\mu}[\eta(\frac{\mu+\nu}{\beta})(M_0 + \frac{k'S^*}{\delta}) + p\nu S^*]$, $M^* = M_0 + \frac{k'S^*}{\delta}$, and $S^* = \frac{\delta[\Lambda\beta-(\mu+\nu)(\mu+\eta M_0)]}{\beta\delta(\mu+p\delta)+\eta k'(\mu+\delta)}$.

If $\Lambda\beta - (\mu+\nu)(\mu+\eta M_0) > 0$, then $S^* > 0$.

Here the smoker generation number is defined as $S_0 = \frac{\Lambda\beta}{(\mu+\nu)(\mu+\eta M_0)}$.

The Smoking free equilibrium of the system is obtained by setting all the smokers class and recovered classes equal to zero.

The system become smoking free, when $M = M_0$, which means 100% programme execution in the ideal situation.

The model exhibits SPE, $E^*(P^*, S^*, P^*_m, M^*)$.

By setting $\frac{dP}{dt} = \frac{dS}{dt} = \frac{dP_m}{dt} = \frac{dM}{dt} = 0$,

we get P^*, S^*, P^*_m, M^*, which is mentioned in (B). By analysing the existence of SPE, we have following cases

Case 1: If $S_0 > 1$, then S^* and thus $M^* > 0$, $P^*_m > 0$. Which shows that for $S_0 > 0$, there exist unique positive equilibrium exist.

Case 2: If $S_0 = 1$ then $S^* = 0, M^* = M_0$ and E^* does not exist.

Case 3: If $S_0 < 1$ then $S^* < 0$, there exist a transcritical bifurcation at $S_0 = 1$.

3. Analysis of the Model

In this section we study the stability analysis of the system at $\bar{E}$ and E^*.

The Jacobian Matrix of the system is given by

$$J = \begin{pmatrix} -(\beta S+\mu+\eta M) & -\beta P+(1-p)\nu & 0 & -\eta P \\ \beta S & \beta P-\mu-\nu & 0 & 0 \\ \eta M & p\nu & -\mu & \eta P \\ 0 & k' & 0 & -\delta \end{pmatrix}.$$

The Jacobian matrix for Smoking free equilibrium ($\bar{E}$) is given by

$$\bar{J} = \begin{pmatrix} -(\mu+\eta M_0) & -\beta\bar{P}+(1-p)\nu & 0 & -\eta\bar{P} \\ 0 & \beta\bar{P}-\mu-\nu & 0 & 0 \\ \eta M_0 & p\nu & -\mu & \eta\bar{P} \\ 0 & k' & 0 & -\delta \end{pmatrix}.$$

It appears that all eigen values of $\bar{J}$ are $-\delta,\ \mu,\ -(\mu+\eta M_0)$ and $\beta\bar{P}-(\mu+\nu)$.

If $\beta\bar{P}-(\mu+\nu)<0$, then we can easily say that all eigen values are negative.

Now,

$$\begin{aligned} \beta\bar{P}-(\mu+\nu) &< 0 \\ \Rightarrow \frac{\Lambda\beta}{(\mu+\nu)(\mu+\eta M_0)} &< 1 \\ \Rightarrow S_0 < 1 & \end{aligned} \tag{5}$$

If $S_0<1$, then all eigen values of $\bar{J}$ are negative. Hence we can conclude that SFE is asymptotically stable if $S_0<1$. If $S_0>1$, then only one eigenvalue is positive. Hence the system at $\bar{E}$ becomes unstable. Therefore the SFE attains a transcritical bifurcation at $S_0=1$.

Remark: The smoker generation number S_0 plays a pivotal role as like basic reproduction number plays in epidemic model. This basic reproduction number represents the expected number of secondary infection arisen from a single individual during the entire infection period [16]. Here we rename as smoker generation number and denoted it as S_0 determined by

$$S_0 = \frac{\Lambda\beta}{(\mu+\nu)(\mu+\eta M_0)}, \tag{6}$$

which represents that one smoker creates $\Lambda\beta$ smokers in its whole life if $\frac{1}{(\mu+\nu)(\mu+\eta M_0)}$ spent in smoking class. From this it is clearly observed that as media awareness ηM_0 increases, S_0

reduces. If the media campaigning reduces S_0 increases which causes the life risk. Thus anti smoking campaign plays a pivotal role for reducing number of smoker as well as smoking habit. Now for the endemic equilibrium E^*, the Jacobian Matrix J^* is given by

$$J^* = \begin{pmatrix} -(\beta S^* + \mu + \eta M^*) & -\beta P^* + (1-p)\nu & 0 & -\eta P^* \\ \beta S^* & \beta P^* - \mu - \nu & 0 & 0 \\ \eta M^* & p\nu & -\mu & \eta P^* \\ 0 & k & 0 & -\delta \end{pmatrix}.$$

Hence the Characteristic equation is

$$(\xi + \mu) \begin{vmatrix} \xi + \beta S^* + \mu + \eta M^* & \beta P^* - (1-p)\nu & \eta P^* \\ -\beta S^* & \xi - \beta P^* + \mu + \nu & 0 \\ 0 & k' & \xi + \delta \end{vmatrix} = 0. \tag{7}$$

Clearly one eigen value $-\mu < 0$.

Other three eigen values are found by solving the characteristic polynomials for J^* which is given by

$$\begin{aligned} \xi^3 + a_1\xi^2 + a_2\xi + a_3 &= 0, \qquad (8)\\ a_1 &= (\mu + \beta S^* + \eta M^*) + (\mu + \nu\delta - \beta P^*) + \delta, \\ a_2 &= (\mu + \beta S^* + \eta M^*)(\mu + \nu\delta - \beta P^*) + \beta^2 S^{*2}((1-p)\nu, \\ &\quad + \eta k' P^* - \beta P^*) + (\mu + \beta S^* + \eta M^* + \mu + \nu\delta - \beta P^*)\delta, \\ a_3 &= \delta(\mu + \beta S^* + \eta M^*)(\mu + \nu\delta - \beta P^*) \\ &\quad - \beta S^*((1-p)\nu + \eta k' P^* - \beta P^*). \qquad (9) \end{aligned}$$

Here,

$$\begin{aligned} a_1 a_2 - a_3 &= (\mu + \beta S^* + \eta M^*)(\mu + \delta\nu - \beta P^*)(1-\delta) \\ &\quad + \delta P^* S^*[(1-p)\nu + P^*(\eta k' - \beta)] > 0. \qquad (10) \end{aligned}$$

By using Routh-Hurwitz Criterion [17], we can say the smoking present equilibrium E^* is locally asymptotically stable if $a_i > 0,\ i = 1,2,3$ and $a_1 a_2 - a_3 > 0$ are satisfied.

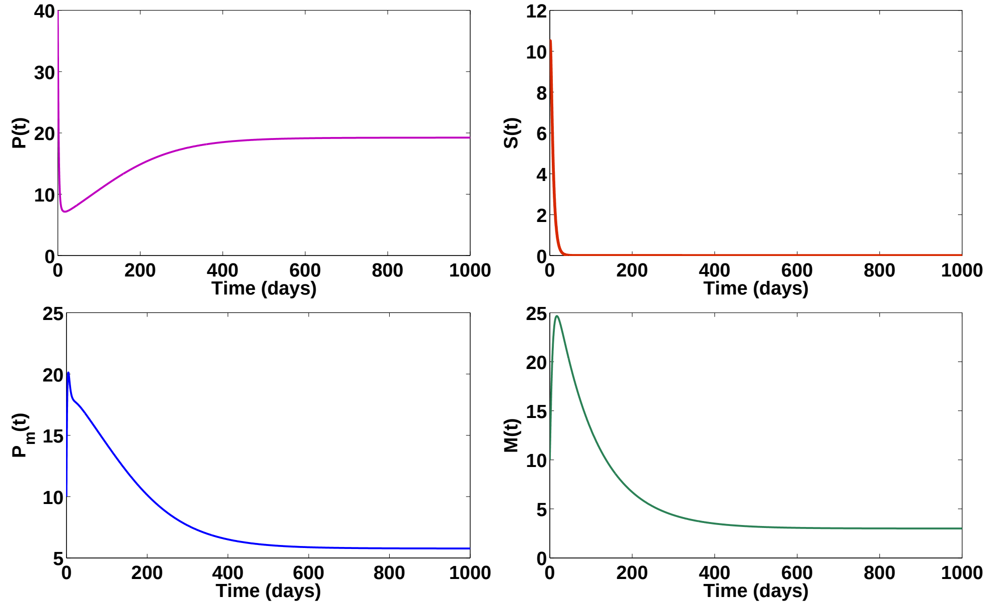

FIGURE 2. Trajectories showing the time dependent changes in population of the model variables when $S_0 < 1$.

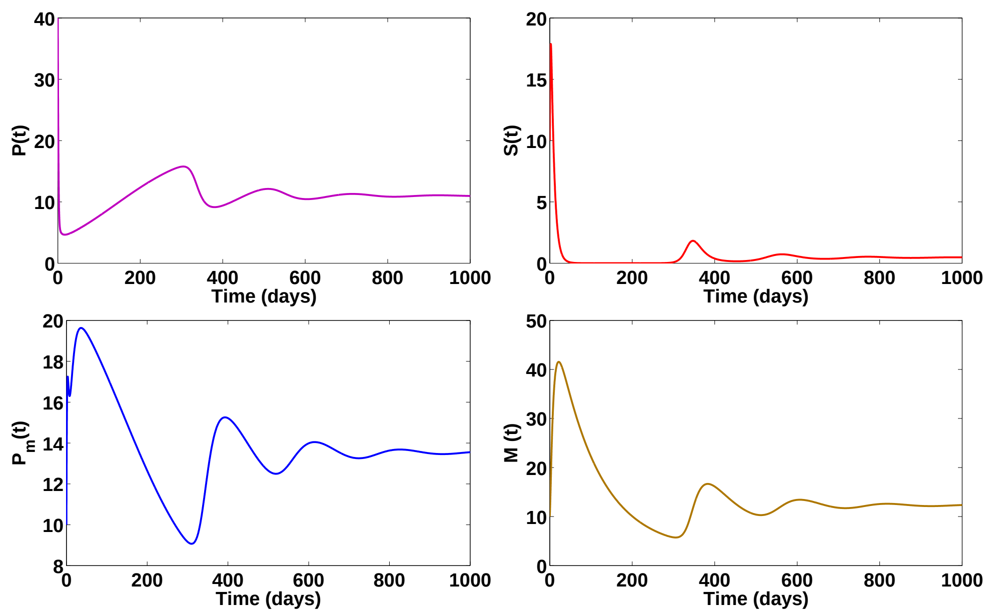

FIGURE 3. Trajectories showing the time dependent changes in population of the model variables when $S_0 > 1$.

Theorem 1: (i) If $S_0 < 1$, then SFE $(E^*)$is locally asymptotically stable. (ii) If $S_0 > 1$, then SFE is unstable and the endemic equilibrium E^* exist which is asymptotically stable if $a_i > 0,\ i = 1,2,3$ and $a_1 a_2 - a_3 > 0$.

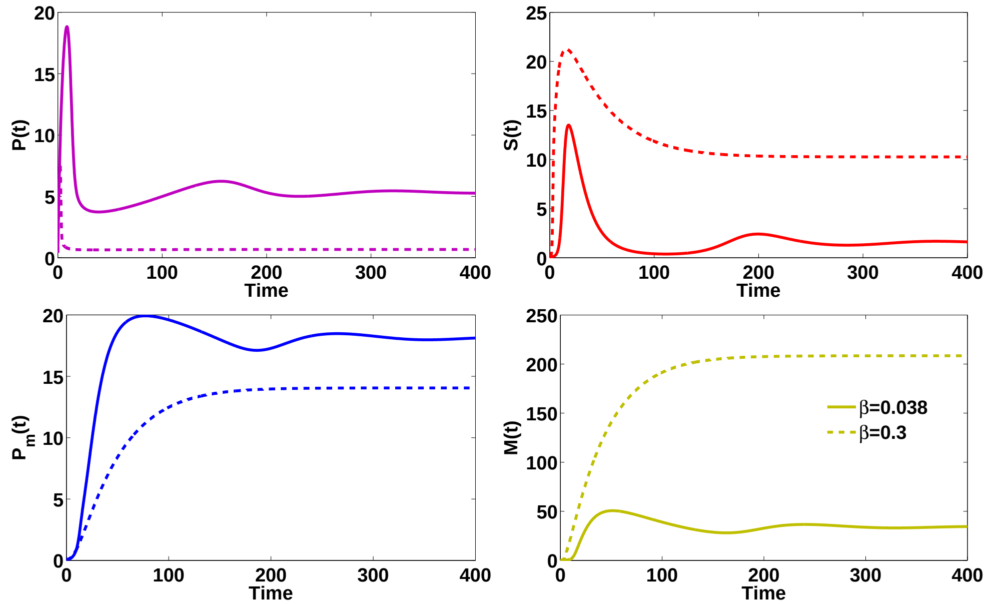

FIGURE 4. Trajectories of populations and awareness program changes for different values of β.

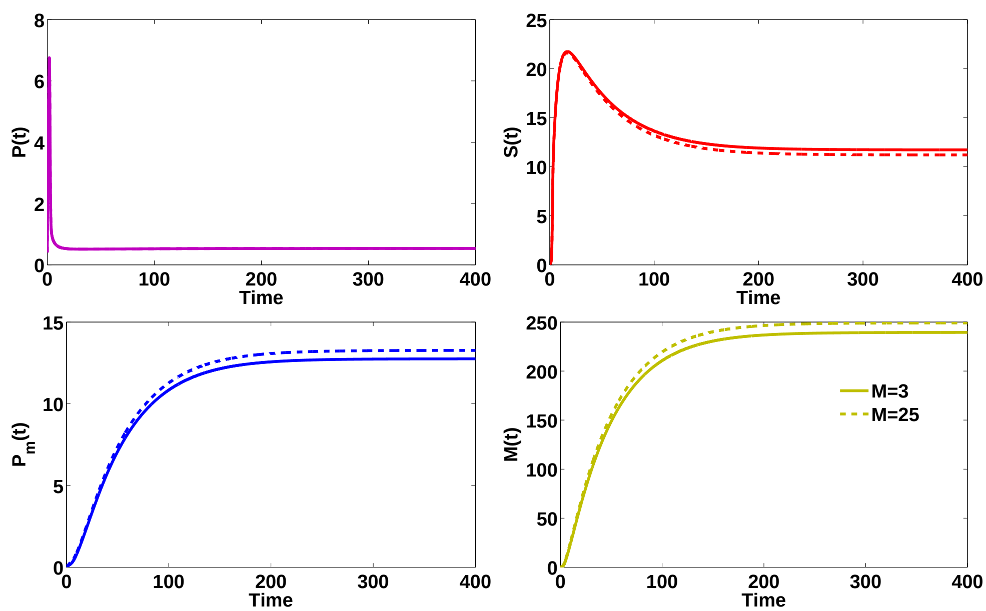

FIGURE 5. Trajectories of populations and awareness program changes for different values of M.

4. Global Stability

To study the global stability of the system we consider the Lyapunov's function as

$$L = \frac{1}{2}(P-P^*)^2 + \frac{\omega_1}{2}(S-S^*)^2 + \frac{\omega_2}{2}(P_m-P_m^*)^2 + \frac{\omega_3}{2}(M-M^*)^2. \tag{11}$$

Here $\omega_1,\ \omega_2,\omega_3$ are some positive constants to be chosen.

On differentiating L with respect to t along the solutions of given model we get

$$\begin{aligned}
\frac{dL}{dt} &= (P-P^*)\frac{dP}{dt} + \omega_1(S-S^*)\frac{dS}{dt} + \omega_2(P_m-P_m^*)\frac{dP_m}{dt} + \omega_3(M-M^*)\frac{dM}{dt} \\
&= -m_{11}(P-P^*)^2 - m_{22}(S-S^*)^2 - m_{33}(P_m-P_m^*)^2 - m_{44}(M-M^*)^2 + m_{12}(S-S^*)(P-P^*) \\
&\quad + m_{13}(P-P^*)(P_m-P_m^*) + m_{14}(P-P^*)(M-M^*) + m_{23}(S-S^*)(P_m-P_m^*) \\
&\quad + m_{24}(S-S^*)(M-M^*) + m_{34}(P_m-P_m^*)(M-M^*) \\
&= -\frac{m_{11}}{2}(P-P^*)^2 + m_{12}(S-S^*)(P-P^*) - \frac{m_{22}}{4}(S-S^*)^2 \\
&\quad -\frac{m_{11}}{2}(P-P^*)^2 + m_{13}(P-P^*)(P_m-P_m^*) - \frac{m_{33}}{3}(P_m-P_m^*)^2 \\
&\quad -\frac{m_{11}}{2}(P-P^*)^2 + m_{14}(P-P^*)(M-M^*) - \frac{m_{44}}{3}(M-M^*)^2 \\
&\quad -\frac{m_{22}}{4}(S-S^*)^2 + m_{23}(S-S^*)(P_m-P_m^*) - \frac{m_{33}}{3}(P_m-P_m^*)^2 \\
&\quad -\frac{m_{22}}{4}(S-S^*)^2 + m_{24}(S-S^*)(M-M^*) - \frac{m_{44}}{3}(M-M^*)^2 \\
&\quad -\frac{m_{33}}{3}(P_m-P_m^*)^2 + m_{34}(P_m-P_m^*)(M-M^*) - \frac{m_{44}}{3}(M-M^*)^2 \qquad (12)
\end{aligned}$$

Where $m_{11} = \beta S^* + \mu + \eta M^*,\ m_{22} = \omega_1\mu - \omega_2 P^* + \omega_1\nu,\ m_{33} = \omega_2\mu,\ m_{44} = \omega_3\delta,\ m_{12} = \beta P^* + (1-p)\nu + \omega_1\beta,\ m_{13} = \omega_2 M^*,\ m_{23} = \omega_2 p\nu,\ m_{24} = \omega_3 k',\ m_{34} = \omega_2 p^*$.

Now if, $m_{12}^2 < \frac{1}{2}m_{11}m_{22},\ m_{13}^2 < \frac{2}{3}m_{11}m_{33},\ m_{14}^2 < \frac{1}{3}m_{11}m_{44},\ m_{23}^2 < \frac{1}{3}m_{22}m_{33},\ m_{24}^2 < \frac{1}{3}m_{22}m_{44},\ m_{34}^2 < \frac{4}{9}m_{33}m_{44}$ then $\frac{dL}{dt} < 1$, which shows the condition of negative definite.

For this condition we can say that the model in Smoking present equilibrium, E^* is globally asymptotically stable. On the basis of above study we have found out the condition of global stability stated below

- (i) $\{-\beta P^* + (1-p)\nu + \omega_1\beta\}^2 < \frac{1}{2}\{\beta S^* + \mu + \eta M^*\}\{\omega_1(\mu+\nu) - \omega_2 P^*\}$.
- (ii) $(\omega_2 M^*)^2 < \frac{2}{3}\{\beta M^* + \mu + \eta M^*\}\omega_2\mu$.
- (iii) $(\eta P^*)^2 < \frac{1}{3}\{\omega_1(\mu+\nu) - \omega_2 P^*\}\omega_2\delta$.

- (iv)$(\omega_2 p\nu)^2 < \frac{1}{3}\{\omega_1(\mu+\nu)-\omega_2 P^*\}\omega_2\mu$.
- (v) $(\omega_3 k')^2 < \frac{1}{3}\{\omega_1(\mu+\nu)-\omega_2 P^*\}\omega_3\delta$.
- (vi) $(\omega_2 P^*)^2 < \frac{4}{9}\{\omega_1(\mu+\nu)-\omega_2 P^*\}\omega_3\delta$.

5. Sensitivity Analysis of S_0

Using the approach in Chitnis et al. [18], we calculate the normalized forward sensitivity indices of $S_0 = \frac{\Lambda\beta}{(\mu+\nu)(\mu+\eta M_0)}$.
Let $\Upsilon_p^{S_0} = \frac{\partial S_0}{\partial p} \times \frac{p}{S_0}$.
Where $\Upsilon_p^{S_0}$ denotes the sensitivity index of S_0 with respect to the parameters p.

Here we get

$$\begin{aligned}
\frac{\partial S_0}{\partial \Lambda} &= \frac{\beta}{(\mu+\nu)(\mu+\eta M_0)},\\
\frac{\partial S_0}{\partial \beta} &= \frac{\Lambda}{(\mu+\nu)(\mu+\eta M_0)},\\
\frac{\partial S_0}{\partial \mu} &= -\frac{\beta\Lambda}{(\mu+\nu)^2(\mu+\eta M_0)^2} \times [2\mu+(\nu+\eta M_0)],\\
\frac{\partial S_0}{\partial \nu} &= -\frac{\beta\Lambda}{(\mu+\nu)^2(\mu+\eta M_0)},\\
\frac{\partial S_0}{\partial \eta} &= -\frac{\beta\Lambda M_0}{(\mu+\nu)(\mu+\eta M_0)^2}.
\end{aligned} \tag{13}$$

Hence,

$$\begin{aligned}
\Upsilon_\beta^{S_0} &= \Upsilon_\Lambda^{S_0} = 1,\\
\Upsilon_\mu^{S_0} &= -\frac{\mu[2\mu+\nu+\eta M_0]}{(\mu+\nu)(\mu+\eta M_0)},\\
\Upsilon_\nu^{S_0} &= -\frac{\nu}{(\mu+\nu)},\\
\Upsilon_\eta^{S_0} &= -\frac{\eta M_0}{\mu+\eta M_0}.
\end{aligned} \tag{14}$$

Since $\Upsilon_\beta^{S_0} = \Upsilon_\Lambda^{S_0} = 1$, the largest sensitivity indexes. If β or Λ reduces 1% then S_0 reduces by 1%. If μ, ν or η increases then S_0 reduces. Hence media coverage has the negative effect on the S_0.

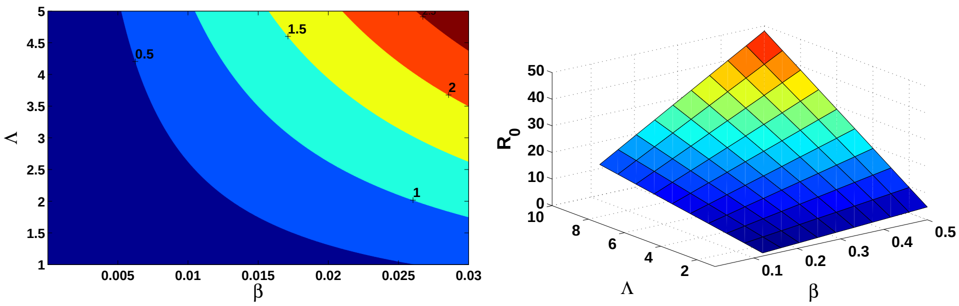

FIGURE 6. Left Panel: Contour Plot of S_0 as a function of β and Λ. Right Panel: Mesh diagram of S_0 as a function of β and Λ.

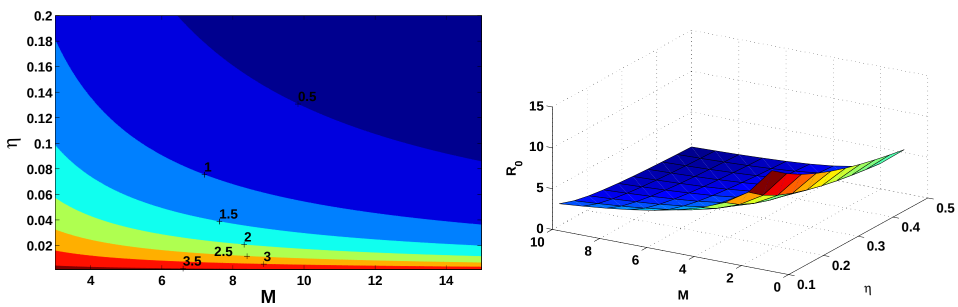

FIGURE 7. Left Panel: Contour Plot of S_0 as a function of M and η. Right Panel: Mesh diagram of S_0 as a function of M and η.

6. Numerical Simulation

In this section, we illustrate some numerical solutions of the model 1 for different values of the parameters as in Table 1.

For $S_0 < 1$, Figure 2 shows that for $\beta = 0.038$ the Smoking reproduction number $S_0 < 1$ and thus the solution trajectories tend to the equilibrium E_0. Hence the model 1 is locally asymptotically stable. In figure 2, we use the same parameters and the initial values as previously with $\beta = 0.03$. Figure 3 shows that the number of potential smoker population decreases and approaches at P^*. This figure also shows that the number of smokers increases initially and then it decreases and approaches to S^*. Hence for $S_0 > 1$ the system trajectories approaches to the smoking present equilibrium E^*. Hence the model is locally asymptotically stable about E^* for the above parameter set.

Figure 4 shows the model trajectories for $\beta = 0.038$ and for 0.3. We note from the figure that as β increases the potential smokers and aware population decreases with time and moves to its equilibrium point. Whereas the smoker population increases with increasing value of

TABLE 1. List of parameters for system.

Parameters	Explanation	Default Values	Units	Reference
Λ	Recruitment rate of potential smoker.	0.02	day^{-1}	[6]
μ	Naturally death rate.	0.02	day^{-1}	[8]
β	transmission rate	0.0038	day^{-1}	[8]
ν	Recovery rate of smoker.	0.2	day^{-1}	[8]
η	Dissemination rate by awareness programs	0.02	day^{-1}	[8]
k'	Implementation rate of awareness programs	0.002	day^{-1}	[8]
δ	Depletion rate of awareness program	0.05	day^{-1}	[8]

β. Figure 5 shows the trajectories when $\beta = 0.3$ and $M_0 = 3$ and 25. From this figure, it is clearly observed that if we increase the awareness maintenance level from 3 to 25, then the aware smoke population increases and the smoker population decreases with respect to time and attain its equilibrium stage. Figure 6 shows the graph of S_0 as a function of the rate of contact β and recruitment rate Λ. From this figure, it clearly observed that S_0 can be less than one if β and Λ are small. As β and Λ are both large, S_0 can blow up. Figure 7 shows the graph of S_0 as a function of the rate of contact M and the recruitment rate η. From this figure, it clearly observed that S_0 can be less than one if β and Λ are large. Whereas β and Λ are both attain low value, S_0 can blow up. Figure 8 shows the partial rank correlation coefficient sensitivity analysis. All relevant parameters are varied against S_0 throughout the range given in Table 1. Parameters with $PRCCs > 0$ will increase S_0 when they are increased, while the parameters with $PRCCs < 0$ will decrease S_0 when the corresponding parameters increases. From this figure, it shows that β, Λ and μ plays the most significant role to control over the epidemic. The variables β and Λ are the parameters that have the largest impact on the outcome.

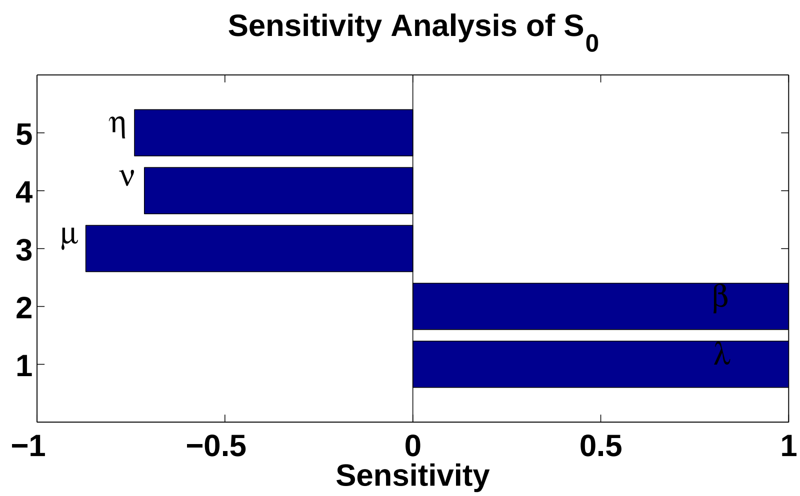

FIGURE 8. Tornado plot of sensitivity analysis of all six parameters that influence S_0.

7. DISCUSSION AND CONCLUSION

In this paper, we have presented a mathematical model to analyse the role of media awareness on the smoking dynamics. We have shown that there exist two equilibrium (i) Smoking free equilibrium $\bar{E}$ and (ii) Smoking present equilibrium E^*. We have found Smoking generation number $S_0 = \frac{\Lambda\beta}{(\mu+\nu)(\mu+\eta M_0)}$. When $S_0 < 1$, the smoking free equilibrium is stable whereas if $S_0 > 1$, the smoking present equilibrium exist. We have used a Lyapunov function to show the global stability of the smoking present equilibrium E^*. The Smoking present equilibrium E^* is globally stable under the condition stated in Theorem 1. We have also studied the sensitivity analysis of S_0. From this analysis, it is observed that for β and Λ, sensitivity index become the largest. Thus if we can reduce β and Λ then we can control the smoking habit. In contrast, if we can increase the value of the parameters μ, η and ν then S_0 decreases. Thus media coverage plays a positive role to increase the dissimilation rate of the awareness programme for which the smoking habit reduces. Numerical simulation supports our analytical findings.

From our analytical as well as numerical simulation we can conclude that the smoking habit can be controlled if we can reduce S_0. This is possible only by increasing education and hence by increasing the education programme and we can eradicate the smoking habit. Through this study, we can conclude that a high transmission rate can be controlled through the awareness programme. Hence, through this study, we can recommend that continuous health examination programme should be organised. Anti Tobacco campaign plays an important role to control or eradicate the smoking habit.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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