Available online at http://scik.org
Commun. Math. Biol. Neurosci. 2020, 2020:24
https://doi.org/10.28919/cmbn/4599
ISSN: 2052-2541

A MATHEMATICAL MODELING WITH OPTIMAL CONTROL STRATEGY OF TRANSMISSION OF COVID-19 PANDEMIC VIRUS

ABDELFATAH KOUIDERE[1,*], BOUCHAIB KHAJJI[1], AMINE EL BHIH[1], OMAR BALATIF[2], MOSTAFA RACHIK[1]

[1]Laboratory of Analysis Modeling and Simulation, Department of Mathematics and Computer Science, Faculty of Sciences Ben M'Sik, Hassan II University, Sidi Othman, Casablanca, Morocco

[2]Laboratory of Dynamical Systems, Mathematical Engineering Team (INMA), Department of Mathematics, Faculty of Sciences El Jadida, Chouaib Doukkali University, El Jadida, Morocco

Abstract. In this paper, we propose a mathematical modeling that describe the dynamics of transmission of the novel Coronavirus 2019 (COVID-19), between potential people and infected people without symptoms, and those infected people with symptoms and then people with serious complications, as well as those under health surveillance and quarantine, in addition to people who recovered from the virus.

In addition, we propose an optimal strategy by carrying out awareness campaigns for citizens with practical measures to reduce the spread of the virus, and diagnosis and surveillance of airports and the quarantine of infected people.

Pontryagin's maximum principle is used to characterize the optimal controls, and the optimality system is solved by an iterative method. Finally, some numerical simulations are performed to verify the theoretical analysis using Matlab.

Keywords: SARS-COV-2; optimal control; mathematical model; COVID-19; quarantine.

2010 AMS Subject Classification: 93A30, 49J15.

---

*Corresponding author

E-mail address: kouidere89@gmail.com

Received March 30, 2020

1. INTRODUCTION

On December 31, 2019. In China, specifically in the city of Wuhan in the market for the sale of aquatic products, a dangerous, lethal and rapid spread appeared as the novel Coronavirus 2019 (COVID-19)$[1-2]$. which belongs to the Coronaviruses, which is a group of viruses that infects animals and mammals and then transmits to humans, the symptoms of the novel Coronavirus 2019 are similar to other types of Coronavirus, such as the severe acute syndrome virus SARS and others Coronavirus [3], the Middle East Mers syndrome [6], in a large number of characteristics and symptoms such as colds and high head temperature, and then lead to a pneumonia and collapse, leading to the failure of the respiratory system leading to death.

The novel Coronavirus 2019 (COVID-19) differs with the rest of the Coronavirus, which shows that the patient does not have symptoms of the virus for a period ranging between two to 14 days, and that is the seriousness of the disease, which facilitates the spread of the virus quickly, especially between the relatives and friends of the infected person, or through means of transportation, many cases have appeared in several neighboring countries and far from China.

Globally, the number of infected people in China has reached 81218 and about 3281 people have died. In Italy, 74386 people were infected and 7503 died. In USA, about the infected individuals reached 61081 and the deaths reached 819. In Spain, about 47610 people were infected and 3434 died. In Germany, about 37066 people were infected and 205 people died. In Iran, about 27017 people were infected and 2077 died. According to the last stats declared on Monday 25 March 2020. The epidemic has moved from China to Europe, which has become the main epidemic area according to the World Health Organization. and the number is able to increase due to the lack of a pre-vaccine virus for non-infected people and the absence of any treatment to treat patients, in addition to the emergence of other cases of neighboring countries as Japan, South Korea, Russia, and other countries as the United States of America, France and the Middle East countries such as the United Arab Emirates. The WHO declared a state of health emergency with an international dimension$[13,14]$.

The COVID-19 virus is considered more lethal and dangerous comparing to other viruses, whereas the number of people with severe acute respiratory syndrome (SARS) reached about

8098 people and the death of about 774 people, which also started from Asia and China specifically in 2002 and which researchers suggested transmission from bats to humans, as for the Middle East Syndrome (MERS) virus, 858 people died out of the total number of infected people, who numbered about 2494 people since its appearance in 2012$[4-6,7]$, which appeared initially in the Kingdom of Saudi Arabia.

Several countries, such as China, Italy, France and Spain have taken a number of practical measures, such as quarantine, establishing a state of health emergency throughout the country, stopping transport to and from other countries, due to the spread of the COVID-19 virus list, including declaring a high alert in all modes of transport, especially airports, diagnosing and monitoring all people arriving from these regions and sending them to quarantine. The World Health Organization has taken several steps to control the spread of the virus.

The COVID-19 pandemic has had serious repercussions on the global economy, estimated at billions of dollars, because after having closed its borders with other countries how many flights and navigation have been stopped, many companies have stopped many have lost their jobs.

A large number of mathematical models have been developed to simulate, analyse and understand the Coronavirus, in a related research work, Tahir et al [8] proposed a prevention strategies for mathematical model MERS-Coronavirus with stability analysis and optimal control and Zhi-Qiang Xia et al [9] modeling the transmission of middle east respirator syndrome Coronavirus in the republic of korea using a system of ordinary differential equations. Also, many researches have focused on this topic and other related topics ($[16-17,19-21]$).

Our aim in this paper is to highlight the seriousness of the spread and transmission of The COVID-19 virus between people, especially a person who is infected without symptoms (I_W) to an uninfected person or susceptible S. As well as transmission of a virus from an infected person with mild or moderate symptoms I to an uninfected person. We also highlight the complications of infection with the virus C. Then the quarantined people H on them and in the last recovered R.

In this paper, we propose a continous mathematical model that descibes the dynamic of a population infected by COVID-19 virus, and we give some basic properties of the model in Section 2. In Section 3, we present the optimal control problem for the proposed model where we give some results concerning the existence and characterization of the optimal controls using Pontryagin's maximum principle. Numerical simulations through MATLAB are given in section 4. Finally we conclude the paper in section 5.

2. Mathematical Model

We consider a mathematical model SI_WICHR that describe the dyanamic of transmission of COVID-19 virus in a given population. We divide the population denoted by N into six compartments.

2.1. Description of the Model.

The graphical representation of the proposed model is shown in Figure 1.

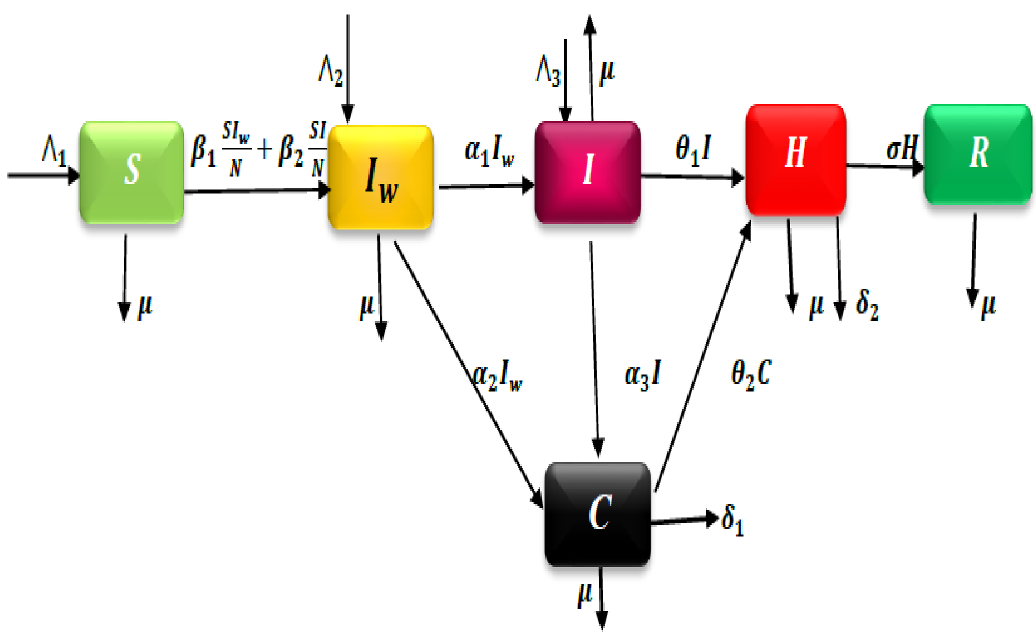

FIGURE 1. Figure compartments model

Compartment (S) is representing the number of people who may be infected with the Noval Coronavirus 2019. The compartment S is increasing by Λ_1 (denote the incidence of susceptible). This compartment is decreasing by the amount μ (natural mortality), $\beta_1 \frac{SI_W}{N}$ (The number of people who were infected with the virus by contact with the infected patients without symptoms) and $\beta_2 \frac{SI}{N}$ (The number of people who were infected with the virus by communicating with the infected patients with symptoms)

$$\frac{dS(t)}{dt} = \Lambda_1 - \mu\ S(t) - \beta_1 \frac{S(t)I_W(t)}{N} - \beta_2 \frac{S(t)I(t)}{N} \tag{1}$$

Compartment (I_W) is representing the number of Infected without symptoms. The copartment I_W is increasing by Λ_1 (denote the incidence of Immigrants and carriers of the virus without symptoms) and also by amounts $\beta_1 \frac{SI_W}{N}$ and $\beta_2 \frac{SI}{N}$. This compartment is decreasing by μ (natural mortality) and also decreased by $\alpha_1 I_W$ (the number of people become infected with symptoms) and also $\alpha_2 I_W$ (the number of people have developed a rapid and dangerous development of the virus due to immunodeficiency, old age or children).

$$\frac{dI_W(t)}{dt} = \Lambda_2 + \beta_1 \frac{S(t)I_W(t)}{N} + \beta_2 \frac{S(t)I(t)}{N} - (\ \mu\ + \alpha_1 + \alpha_2) I_W(t) \tag{2}$$

Compartment (I) is representing the number of individual infected with symptoms. The compartment I is increasing by Λ_2 (denote the incidence of Immigrants and carriers of the virus with symptoms) and also by increasing $\alpha_1 I_W$. This compartment is decreasing by $\alpha_3 I$ (People have severe complications such as pulmonary failure) and also decreasing by μ (natural mortality) and decreasing by $\theta_1 I$ (The number of people with symptoms of mild virus who have been quarantined).

$$\frac{dI(t)}{dt} = \Lambda_3 + \alpha_1 I_W(t) - (\alpha_3 + \theta_1 + \mu) I(t) \tag{3}$$

Compartment (C) is representing the number of infected with complications. The compartment C is increasing by $\alpha_2 I_W$ and increasing $\alpha_3 I$. This compartment is decreasing by $\theta_2 C$ (The number of people with serious complications who have been quarantined) and decreasing by $\delta_1 C$ (mortality rate due to complications) and also decreasing by μC (natural mortality).

$$\frac{dC(t)}{dt} = \alpha_2 I_W(t) + \alpha_3 I(t) - (\theta_2 + \mu + \delta_1) C(t) \tag{4}$$

Compartment (H) is representing the number of people who have been quarantined in hospitals with follow-up and health monitoring. The compartment H is increasing by $\theta_1 I$ and increasing $\theta_2 C$. This compartment is also decreasing by σH (The number of individual who recovered from the virus) and decreasing by $\delta_2 H$ (The number of people who died under quarantine in hospitals) and also decreasing by μH (natural mortality)

$$\frac{dH(t)}{dt} = \theta_1 I(t) + \theta_2 C(t) - (\mu + \sigma + \delta_2) H(t) \tag{5}$$

Compartment (R) is representing the number of recovered. The compartment H is increasing by σH. This compartment is decreasing by μR (natural mortality)

$$\frac{dR(t)}{dt} = \sigma H(t) - \mu R(t) \tag{6}$$

Hence, we present the COVID-19 mathematical model is governed by the following system of differential equation :

$$\begin{cases} \frac{dS(t)}{dt} = \Lambda_1 - \mu\ S(t) - \beta_1 \frac{S(t) I_W(t)}{N} - \beta_2 \frac{S(t) I(t)}{N} \\ \frac{dI_W(t)}{dt} = \Lambda_2 + \beta_1 \frac{S(t) I_W(t)}{N} + \beta_2 \frac{S(t) I(t)}{N} - (\ \mu\ + \alpha_1 + \alpha_2) I_W(t) \\ \frac{dI(t)}{dt} = \Lambda_3 + \alpha_1 I_W(t) - (\alpha_3 + \theta_1 + \mu) I(t) \\ \frac{dC(t)}{dt} = \alpha_2 I_W(t) + \alpha_3 I(t) - (\theta_2 + \mu + \delta_1) C(t) \\ \frac{dH(t)}{dt} = \theta_1 I(t) + \theta_2 C(t) - (\mu + \sigma + \delta_2) H(t) \\ \frac{dR(t)}{dt} = \sigma H(t) - \mu R(t) \end{cases} \tag{7}$$

where $S(0) \geq 0$, $I_W(0) \geq 0$, $I(0) \geq 0$, $C(0) \geq 0$, $H(0) \geq 0$ and $R(0) \geq 0$, are the initial state.

2.2. Model Basic Properties.

2.2.1. *Positivity of Solutions.*

Theorem 1. *If $S(0) \geq 0$, $I_W(0) \geq 0$, $I(0) \geq 0$, $C(0) \geq 0$, $H(0) \geq 0$ and $R(0) \geq 0$, the solutions $S(t)$, $I_W(t)$, $I(t)$, $C(t)$, $H(t)$ and $R(t)$ of system (7) are positive for all $t \geq 0$.*

Proof. It follows from the first equation of system (7) that

$$\begin{aligned}\frac{dS(t)}{dt} &= \Lambda_1 - \mu\, S(t) - \beta_1 \frac{S(t)I_W(t)}{N} - \beta_2 \frac{S(t)I(t)}{N} \\ &\geq -\mu\, S(t) - \beta_1 \frac{S(t)I_W(t)}{N} - \beta_2 \frac{S(t)I(t)}{N}\end{aligned}$$

$$\frac{dS(t)}{dt} + \left(\mu + \beta_1 \frac{I_W(t)}{N} + \beta_2 \frac{I(t)}{N}\right) S(t) \geq 0$$

where $F(t) = \mu + \beta_1 \frac{I_W(t)}{N} + \beta_2 \frac{I(t)}{N}$. The both sides in last inequality are multiplied by $\exp(\int\limits_0^t F(s)ds)$.

We obtain

$$\exp(\int\limits_0^t F(s)ds).\frac{dS(t)}{dt} + F(t)\exp(\int\limits_0^t F(s)ds).S(t) \geq 0$$

then $\frac{d}{dt}\left(S(t)\exp(\int\limits_0^t F(s)ds)\right) \geq 0$

Integrating this inequality from 0 to t gives:

$$\int\limits_0^t \frac{d}{ds}\left(S(s)\exp(\int\limits_0^t (\mu + \beta_1 \frac{I_W(s)}{N} + \beta_2 \frac{I(s)}{N})ds)\right) ds \geq 0$$

then

$$S(t) \geq S(0)\exp(\int\limits_0^t \left(\mu + \beta_1 \frac{I_W(s)}{N} + \beta_2 \frac{I(s)}{N}\right) ds)$$

$\Longrightarrow S(t) > 0.$

Similarly, we prove that $I_W(t) \geq 0$,$I(t) \geq 0, C(t) \geq 0, H(t) \geq 0$ and $R(t) > 0$. □

2.2.2. *Boudedness of the solutions.*

Theorem 2. *The set* $\Omega = \left\{(S, I_W, I, C, H, R) \in \mathbb{R}_+^6 /\ 0 \leq S + I_W + I + C + H + R \leq \frac{\Lambda}{\mu}\right\}$ *positively invariant under system* (7) *with initial conditions,* $S(0) \geq 0, I_W(0) \geq 0, I(0) \geq 0, C(0) \geq 0,$ $H(0) \geq 0$ *and* $R(0) \geq 0$.

Proof. Also, one assumes that:

$$\begin{aligned} \frac{dN}{dt} &= \Lambda - \mu N - \delta_1 C \leq \Lambda - \mu N \\ &\Longrightarrow \quad N(t) \leq \frac{\Lambda}{\mu} + N(0)e^{-\mu t} \end{aligned}$$

where $\Lambda = \Lambda_1 + \Lambda_2 + \Lambda_3$

If we take limit $t \to \infty$ we have $0 \leq N(t) \leq \frac{\Lambda}{\mu}$.

It implies that the region Ω is a postivily invariant set for the system (7). □

2.2.3. *Exictence of solutions.*

Theorem 3. *The system* (7) *that satisfies a given initial condition* $(S(0), I_W(0)\ , I(0).C(0), H(0), R(0))$ *has a unique solution.*

Proof. Let $X = \begin{pmatrix} S(t) \\ I_W(t) \\ I(t) \\ C(t) \\ H(t) \\ R(t) \end{pmatrix}$ and $\varphi(X) = \begin{pmatrix} \frac{dS(t)}{dt} \\ \frac{dI_W(t)}{dt} \\ \frac{dI(t)}{dt} \\ \frac{dC(t)}{dt} \\ \frac{dH(t)}{dt} \\ \frac{dR(t)}{dt} \end{pmatrix}$

so the system (7) can be rewritten in the following form:

$$\varphi(X) = AX + B(X) \tag{8}$$

where

$$A = \begin{pmatrix} -\mu & 0 & -0 & 0 & 0 & 0 \\ 0 & -(\mu + \alpha_1 + \alpha_2) & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & -(\alpha_3 + \theta_1 + \mu) & 0 & 0 & 0 \\ 0 & \alpha_2 & \alpha_3 & -(\theta_2 + \mu + \delta_1) & 0 & 0 \\ 0 & 0 & \theta_1 & \theta_2 & -(\mu + \sigma + \delta_2) & 0 \\ 0 & 0 & 0 & 0 & \sigma & -\mu \end{pmatrix}$$

and

$$B(X)=\begin{pmatrix} \Lambda_1-\beta_1\frac{S(t)I_W(t)}{N}-\beta_2\frac{S(t)I(t)}{N} \\ \Lambda_2+\beta_1\frac{S(t)I_W(t)}{N}+\beta_2\frac{S(t)I(t)}{N} \\ \Lambda_3 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The second term on the right-hand side of (8) satisfies

$$\begin{aligned}
|B(X_1)-B(X_2)| &= 2\left|\beta_1\frac{S_1(t)I_{W.1}(t)}{N}+\beta_2\frac{S_1(t)I_1(t)}{N}-\beta_1\frac{S_2(t)I_{W.2}(t)}{N}-\beta_2\frac{S_2(t)I_2(t)}{N}\right| \\
&= 2\left|\begin{array}{c} \beta_1\frac{S_1(t)I_{W.1}(t)}{N}+\beta_1\frac{S_1(t)I_{W.2}(t)}{N}-\beta_1\frac{S_1(t)I_{W.2}(t)}{N}-\beta_2\frac{S_1(t)I_1(t)}{N} \\ +\beta_2\frac{S_1(t)I_2(t)}{N}-\beta_2\frac{S_1(t)I_2(t)}{N}-\beta_1\frac{S_2(t)I_{W.2}(t)}{N}+\beta_2\frac{S_2(t)I_2(t)}{N} \end{array}\right| \\
&\leq 2\left(\begin{array}{c} \left|\frac{\beta_1 S_1(t)}{N}\right||I_{W.1}(t)-I_{W.2}(t)|+\left|\frac{\beta_1 I_{W.2}(t)}{N}\right||S_1(t)-S_2(t)| \\ +\left|\frac{\beta_2 S_1(t)}{N}\right||I_1(t)-I_2(t)|+\left|\frac{\beta_2 I_2(t)}{N}\right||S_1(t)-S_2(t)| \end{array}\right) \\
&\leq 2\frac{Z}{\mu}\left(\begin{array}{c} \left|\frac{\beta_1}{N}\right||I_{W.1}(t)-I_{W.2}(t)|+\left|\frac{\beta_1}{N}\right||S_1(t)-S_2(t)| \\ +\left|\frac{\beta_2}{N}\right||I_1(t)-I_2(t)|+\left|\frac{\beta_2}{N}\right||S_1(t)-S_2(t)| \end{array}\right) \\
&\leq M\left(|X_1(t)-X_2(t)|\right)
\end{aligned}$$

Where $M=2\frac{Z}{\mu}\left(\left|\frac{\beta_1}{N}\right|+\left|\frac{\beta_2}{N}\right|;\left|\frac{\beta_1}{N}\right|+\left|\frac{\beta_2}{N}\right|\right)$

then

$$\|\varphi(X_1)-\varphi(X_2)\|\leq V.\|X_1-X_2\|$$

Where $V=\max(M,\|A\|)<\infty$.

Thus, it follows that the function φ is uniformly Lipschitz continuous, and the restriction on $S(t)\geq 0$, $I_W(t)\geq 0$, $I(t)\geq 0$, $C(t)\geq 0$, $H(t)\geq 0$ and $R(t)\geq 0$, we see that a solution of the system exists[18]. □

3. THE OPTIMAL CONTROL PROBLEM

As of today 30 March 2020, there is no cure or vaccine for the virus, so we suggest the following strategies: there are three controls $u(t)$, $v(t)$ and $w(t)$ for $t\in[0,T]$. The first control can be interpreted as the proportion to be subjected to sensitation and prevention. So, we note that u is the awerness program to susceptible people at time t. The second control can be interpreted as quarantine and health monitoring, so, we note that $v(I(t)+C(t))$ is the proportion of quarantine and health monitoring of people with the virus and its complications. The third control can be interpreted as the proportion to be subjected to diagonosis and monitoring, so, we note that $wI_W(t)$ is the proportion diagnosis and monitoring of all persons arriving from areas spread the pandemic at time t.

(9)
$$\begin{cases} \frac{dS(t)}{dt} = \Lambda_1 - \mu\, S(t) - \beta_1(1-u(t))\frac{S(t)I_W(t)}{N} - \beta_2(1-u(t))\frac{S(t)I(t)}{N} \\ \frac{dI_W(t)}{dt} = \Lambda_2 + \beta_1(1-u(t))\frac{S(t)I_W(t)}{N} + \beta_2(1-u(t))\frac{S(t)I(t)}{N} - (\mu + \alpha_1 + \alpha_2)I_W(t) - w(t)I_W(t) \\ \frac{dI(t)}{dt} = \Lambda_3 + \alpha_1 I_W(t) - (\alpha_3 + \theta_1 + \mu)I(t) - v(t)I(t) \\ \frac{dC(t)}{dt} = \alpha_2 I_W(t) + \alpha_3 I(t) - (\theta_2 + \mu + \delta_1)C(t) - v(t)C(t) \\ \frac{dH(t)}{dt} = \theta_1 I(t) + \theta_2 C(t) - (\mu + \sigma + \delta_2)H(t) + w(t)I_W(t) + v(t)(I(t) + C(t)) \\ \frac{dR(t)}{dt} = \sigma H(t) - \mu R(t) \end{cases}$$

The problem is to minimize the objective functional

(10)
$$J(u,v,w) = I(T) + C(T) + I_W(T) + \int_0^T \left[I(t) + C(t) + I_W(t) + \frac{A}{2}u^2(t) + \frac{B}{2}v^2(t) + \frac{G}{2}w^2(t) \right] dt$$

Where $A \geq 0$, $B \geq 0$ and $G \geq 0$ are the cost coefficients. They are selected to weigh the relative importance of $u(t)$, $v(t)$ and $w(t)$ at time t, T is the final time.

In other words, we seek the optimal controls u^*, v^* and w^* such that

$$J(u^*, v^*, w^*) = \min_{u,v,w \in U} J(u,v,w) \tag{11}$$

Where U is the set of admissible controls defined by

$$U = \left\{ \begin{array}{c} u,v,w/0 \leq u_{\min} \leq u(t) \leq u_{\max} \leq 1\ , 0 \leq v_{\min} \leq v(t) \leq v_{\max} \leq 1 \\ \text{and } 0 \leq w_{\min} \leq w(t) \leq w_{\max} \leq 1, t \in [0,T] \end{array} \right\} \tag{12}$$

4. The Optimal Control: Existence and Characterization

We first show the existence of solutions of the system (9), thereafter we will prove the existence of optimal control([12]).

4.1. Existence of an Optimal Control.

Theorem 4. *Consider the control problem with system* (9) .

There exists an optimal control $(u^*,v^*,w^*) \in U^3$ *such that* $J(u^*,v^*,w^*) = \min_{u,v,w \in U} J(u,v,w)$

Proof. The existence of the optimal control can be obtained using a result by Fleming and Rishel [12], checking the following steps :

- It follows that the set of controls and corresponding state variables is nonempty. Using Boyce and DiPrima [15]

  To prove that the set of controls and the corresponding state variables is nonempty, we will use a simplified version of an existence result [15]. Let $X_i' = F_{X_i}(t; X_1, X_2, \cdots, X_6)$ with $i = 1;\cdots;6$ where $(X_1, X_2, \cdots, X_6) = (S, I_W, I, C, H, R)$ where $X_1, \cdots, X_5$ and X_6 form the right-hand side of the system of equations (9). Let u, v and w for some constants and since all parameters are constants and $X_1, \cdots, X_5$ and X_6 are continuous, then $F_S, F_{I_W}, F_I; F_C; F_H$ and F_R are also continuous. Additionally, the partial derivatives $\frac{\partial F_{X_i}}{\partial X_i}$ with $i = 1;\cdots;6$ are all continuous. therefore, there exists a unique solution (S, I_W, I, C, H, R) that satisfies the initial conditions. therefore, the set of controls and the corresponding state variables is nonempty and condition 1 is satisfied.
- $J(u,v,w) = I(T) + C(T) + I_W(T) + \int_0^T \left[I(t) + C(t) + I_W(t) + \frac{A}{2}u^2(t) + \frac{B}{2}v^2(t) + \frac{G}{2}w^2(t) \right]$ dt is convex in U.

  Take any controls $u, v \in U$ and $\lambda \in [0,1]$. then $0 \leq \lambda u + (1-\lambda)v$ Additionally, we observe that $\lambda u \leq \lambda$ and $(1-\lambda)v \leq (1-\lambda)$, then $\lambda u + (1-\lambda)v \leq \lambda + (1-\lambda) = 1$ Hence, $0 \leq \lambda u + (1-\lambda)v \leq 1$, for all $u, v \in U$ and $\lambda \in [0,1]$.
- The control space $U = \{(u,v,w)/(u,v,w)$ is measurable, $0 \leq u_{\min} \leq u(t) \leq u_{\max} \leq 1$,$0 \leq v_{\min} \leq v(t) \leq v_{\max} \leq 1$ and $0 \leq w_{\min} \leq w(t) \leq w_{\max} \leq 1, t \in [0,T]\}$. is convex and closed by definition.

- All the right hand sides of equations of system are continuous, bounded above by a sum of bounded control and state, and can be written as a linear function of u, v and w with coefficients depending on time and state.

From the system of differential equations (9),

$$\frac{dN}{dt} = \Lambda - \mu N$$

$$\lim_{t\to\infty} \sup N(t) \leq \frac{\Lambda}{\mu}$$

Therefore, all solutions of the model (9) are bounded. So, there exist positive constants Z_1, Z_2, Z_3, Z_4, Z_5 and Z_6 such that $\forall t \in [0,T]$:

$$S(t) \leq Z_1, I_W(t) \leq Z_2\ , I(t) \leq Z_3, C(t) \leq Z_4, H(t) \leq Z_5 \text{ and } R(t) \leq Z_6$$

We consider

$$\begin{cases} F_S = \dot{S}(t) \leq \Lambda_1 \\ F_{I_W} = \dot{I_W}(t) \leq \Lambda_2 \\ F_I = \dot{I}(t) \leq \Lambda_3 \\ F_C = \dot{C}(t) \leq \alpha_2 I_W(t) + \alpha_3 I(t) - v(t) Z_4 \\ F_H = \dot{H}(t) \leq \theta_1 I(t) + \theta_2 C(t) + w(t) I_W(t) + v(t)(Z_3 + Z_4) \\ F_R = \dot{R}(t) \leq \sigma H(t) \end{cases}$$

So, we can rewrite system (9) in matrix form as

$$F(t; S, I_W, I, C, H, R) \leq \Lambda + AX(t) - BU(t)$$

where

$$\begin{aligned} F(t;S,I_W,I,C,H,R) &= \begin{bmatrix} F_S & F_{I_W} & F_I & F_C & F_H & F_R \end{bmatrix}^T \\ \Lambda &= \begin{bmatrix} \Lambda_1 & \Lambda_2 & \Lambda_3 & 0 & 0 & 0 \end{bmatrix}^T \\ X(t) &= \begin{bmatrix} S & I_W & I & C & H & R \end{bmatrix}^T \\ U(t) &= \begin{bmatrix} u & v & w \end{bmatrix}^T \end{aligned}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_2 & \alpha_3 & 0 & 0 & 0 \\ 0 & 0 & \theta_1 & \theta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -\beta_1 \frac{SI_W}{N} - \beta_2 \frac{SI}{N} & 0 & 0 \\ \beta_1 \frac{SI_W}{N} + \beta_2 \frac{SI}{N} & 0 & I_W \\ 0 & I & 0 \\ 0 & C & 0 \\ 0 & I+C & I_W \\ 0 & 0 & 0 \end{bmatrix}$$

It gives a linear function of control vector and state variable vector. therefore, we can write

$$\begin{aligned} F(t;S,I_W,I,C,H,R) &\leq \|\Lambda\| + \|A\| \, \|X(t)\| + \|B\| \, \|U(t)\| \\ &\leq \varphi + \psi(\|X(t)\| + \|U(t)\|) \end{aligned}$$

where $\varphi \leq \|\Lambda\|$ and $\psi = \max(\|A\|, \|B\|)$.

Hence, we see the right-hand side is bounded by a sum of state and control vectors.

- The integrand in the objective functional $I(t) + C(t) + I_W(t) + \frac{A}{2}u^2(t) + \frac{B}{2}v^2(t) + \frac{G}{2}w^2(t)$ is clearly convex on U
- It rest to show that there exists constants $\zeta_1, \zeta_2, \zeta_3, \zeta_4 > 0$,

  and ζ such that $I(t) + C(t) + I_W(t) + \frac{A}{2}u^2(t) + \frac{B}{2}v^2(t) + \frac{G}{2}w^2(t)$

  satisfies $I(t) + C(t) + I_W(t) + \frac{A}{2}u^2(t) + \frac{B}{2}v^2(t) + \frac{G}{2}w^2(t) \geq \zeta_1 + \zeta_2 |u|^\zeta + \zeta_3 |v|^\zeta + \zeta_4 |w|^\zeta$.

The state variables being bounded, let $\zeta_1 = \inf_{t\in[0,T]} (I(t) + C(t) + I_W(t)), \zeta_2 = \frac{A}{2},\ \zeta_3 = \frac{B}{2},\ \zeta_5 = \frac{G}{2}$ and $\zeta = 2$ then it follows that: $I(t) + C(t) + S(t) + I_W(t) + \frac{A}{2}u^2(t) + \frac{B}{2}v^2(t) + \frac{G}{2}w^2(t) \geq \zeta_1 + \zeta_2 |u|^\zeta + \zeta_3 |v|^\zeta + \zeta_4 |w|^\zeta$.

Then from Fleming and Rishel [12] we conclude that there exists an optimal control.

□

4.2. Characterization of the optimal control. In order to derive the necessary conditions for the optimal control, we apply Pontryagin's maximum [11]

principle to the Hamiltonian $\hat{H}$ at time t defined by

$$\hat{H}(t) = I(t) + C(t) + I_W(t) + \frac{A}{2}u^2(t) + \frac{B}{2}v^2(t) + \frac{G}{2}w^2(t) + \sum_{i=1}^{6} \lambda_i(t) f_i(S, I_W, I, C, H, R) \tag{13}$$

where $f_{i,}$ is the right side of the difference equation of the i^{th} state variable.

Theorem 5. *Given the optimal controls* (u^*, v^*, w^*) *and the solutions* $S^*, I_W{}^*, I^*, C^*, H^*$ *and* R^* *of the corresponding state system* (9)*, there exists adjoint variables* $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ *and* λ_6 *satisfying:*

$$\lambda_1' = \lambda_1(\mu + \beta_1(1-u(t))\frac{I_W(t)}{N} + \beta_2(1-u(t))\frac{I(t)}{N}) - \lambda_2\left(\beta_1(1-u(t))\frac{I_W(t)}{N} + \beta_2(1-u(t))\frac{I(t)}{N}\right)$$

$$\lambda_2' = -1 + \lambda_1\beta_1(1-u(t))\frac{S(t)}{N} - \lambda_2\left(\beta_1(1-u(t))\frac{S(t)}{N} - \mu - \alpha_1 - \alpha_2 - w(t)\right) - \lambda_3\alpha_1 - \lambda_4\alpha_2 - \lambda_5 w(t)$$

$$\lambda_3' = -1 + \lambda_1\beta_2(1-u(t))\frac{S(t)}{N} - \lambda_2\beta_2(1-u(t))\frac{S(t)}{N} + \lambda_3(\alpha_3 + \theta_1 + \mu + v(t)) - \lambda_4\alpha_3 - \lambda_5(\theta_1 + v(t))$$

$$\lambda_4' = -1 - \lambda_4(\theta_2 + \mu + \delta_1 + v(t)) - \lambda_5(\theta_2 + v(t))$$

$$\lambda_5' = \lambda_5(\mu + \sigma + \delta_2) - \lambda_6\sigma$$

$$\lambda_6' = \lambda_6\mu$$

With the transversality conditions at time T_f : $\lambda_1(T_f) = 0$, $\lambda_2(T_f) = -1$, $\lambda_3(T_f) = -1$, $\lambda_4(T_f) = -1$, $\lambda_5(T_f) = 0$ *and* $\lambda_6(T_f) = 0$.

Furthermore, for $t \in [0, T]$*, the optimal controls* u^*, v^* *and* w^* *are given by*

$$u^* = \min\left(1, \max\left(0, \frac{(\lambda_2 - \lambda_1)}{A}\left(\beta_1\frac{S(t)I_W(t)}{N} + \beta_2\frac{S(t)I(t)}{N}\right)\right)\right) \tag{14}$$

$$v^* = \min\left(1, \max\left(0, \frac{(\lambda_3 I(t) + \lambda_4 C(t) - \lambda_5(I(t) + C(t))}{B}\right)\right) \tag{15}$$

$$w^* = \min\left(1, \max\left(0, \frac{(\lambda_2 - \lambda_5)}{G} \times I_W(t)\right)\right) \tag{16}$$

Proof. The Hamiltonian $\hat{H}$ is defined as follows:

$\hat{H}(t) = I(t)+C(t)+S(t)+I_W(t)+\frac{A}{2}u^2(t)+\frac{B}{2}v^2(t)+\frac{G}{2}w^2(t)+\sum_{i=1}^{6}\lambda_i(t)\,f_i(S,I_W,I,C,H,R)$

where :

$f_1(S,I_W,I,C,H,R) = \Lambda_1-\mu S(t)-\beta_1(1-u(t))\frac{S(t)I_W(t)}{N}-\beta_2(1-u(t))\frac{S(t)I(t)}{N}$

$f_2(S,I_W,I,C,H,R) = \Lambda_2+\beta_1(1-u(t))\frac{S(t)I_W(t)}{N}+\beta_2(1-u(t))\frac{S(t)I(t)}{N}-(\mu+\alpha_1+\alpha_2-w(t))I_W(t)$

$f_3(S,I_W,I,C,H,R) = \Lambda_3+\alpha_1 I_W(t)-(\alpha_3+\theta_1+\mu)I(t)-v(t)I(t)$

$f_4(S,I_W,I,C,H,R) = \alpha_2 I_W(t)+\alpha_3 I(t)-(\theta_2+\mu+\delta_1)C(t)-v(t)C(t)$

$f_5(S,I_W,I,C,H,R) = \theta_1 I(t)+\theta_2 C(t)-(\mu+\sigma+\delta_2)H(t)+w(t)I_W(t)+v(t)(I(t)+C(t))$

$f_6(S,I_W,I,C,H,R) = \sigma H(t)-\mu R(t)$

For $t\in[0,T]$, the adjoint equations and transversality conditions can be obtained by using Pontryagin's maximum principle[5, 10] such that

$$\lambda_1' = -\frac{\partial\hat{H}}{\partial S} = \lambda_1(\ \mu\ +\beta_1(1-u(t))\frac{I_W(t)}{N}+\beta_2(1-u(t))\frac{I(t)}{N})-\lambda_2\left(\beta_1(1-u(t))\frac{I_W(t)}{N}+\beta_2(1-u(t))\frac{I(t)}{N}\right)$$

$$\lambda_2' = -\frac{\partial\hat{H}}{\partial I_W} = -1+\lambda_1\beta_1\ (1-u(t))\frac{S(t)}{N}-\lambda_2\left(\beta_1(1-u(t))\frac{S(t)}{N}-\ \mu\ -\alpha_1-\alpha_2-w(t)\right)-\lambda_3\alpha_1-\lambda_4\alpha_2-\lambda_5 w(t)$$

$$\lambda_3' = -\frac{\partial\hat{H}}{\partial I} = -1+\lambda_1\beta_2(1-u(t))\frac{S(t)}{N}-\lambda_2\beta_2(1-u(t))\frac{S(t)}{N}+\lambda_3(\alpha_3+\theta_1+\mu+v(t))-\lambda_4\alpha_3-\lambda_5\left(\theta_1+v(t)\right)$$

$$\lambda_4' = -\frac{\partial\hat{H}}{\partial C} = -1-\lambda_4\left(\theta_2+\mu+\delta_1+v(t)\right)-\lambda_5\left(\theta_2+v(t)\right)$$

$$\lambda_5' = -\frac{\partial\hat{H}}{\partial H} = \lambda_5\left(\mu+\sigma+\delta_2\right)-\lambda_6\sigma$$

$$\lambda_6' = -\frac{\partial\hat{H}}{\partial R} = \lambda_6\mu$$

For $t\in[0,T]$, the optimal controls u^*, v^* and w^* can be solved from the optimality condition,

$$\frac{\partial\hat{H}}{\partial u} = 0$$

$$\frac{\partial\hat{H}}{\partial v} = 0$$

$$\frac{\partial\hat{H}}{\partial w} = 0$$

That are

$$
\begin{aligned}
-\frac{\partial \hat{H}}{\partial u} &= -Au(t)+(\lambda_2-\lambda_1)\left(\beta_1\frac{S(t)I_W(t)}{N}+\beta_2\frac{S(t)I(t)}{N}\right)=0\\
-\frac{\partial \hat{H}}{\partial v} &= -Bv(t)+\lambda_3 I(t)+\lambda_4 C(t)-\lambda_5(I(t)+C(t))=0\\
-\frac{\partial \hat{H}}{\partial w} &= -Gw(t)+(\lambda_2-\lambda_5)I_W(t)=0
\end{aligned}
$$

we have

$$u(t)=\frac{(\lambda_2-\lambda_1)}{A}\left(\beta_1\frac{S(t)I_W(t)}{N}+\beta_2\frac{S(t)I(t)}{N}\right)$$

$$v(t)=\frac{(\lambda_3 I(t)+\lambda_4 C(t)-\lambda_5(I(t)+C(t))}{B}$$

$$w(t)=\frac{(\lambda_2-\lambda_5)}{G}\times I_W(t)$$

By the bounds in U of the controls, it is easy to obtain u^*, v^* and w^* are given in (14,15, 16) the form of system(9). □

5. Numerical Simulation

In this section, we present the results obtained by numerically solving the optimality system. In our control problem, we have initial conditions for the state variables and terminal conditions for the adjoints. That is, the optimality system is a two-point boundary value problem with separated boundary conditions at times step $i=0$ and $i=T$. We solve the optimality system by an iterative method with forward solving of the state system followed by backward solving of the adjoint system. We start with an initial guess for the controls at the first iteration and then before the next iteration, we update the controls by using the characterization. We continue until convergence of successive iterates is achieved. A code is written and compiled in MATLAB using the following data.

Different simulations can be carried out using various values of parameters. In the present numerical approach, we use the following parameters values:

Paramter	Description	value in d^{-1}
μ	Natural mortality	0.02
β_1	The rate of people who were infected by contact with infected without sympt	0.2
β_2	The rate of people who were infected by contact with infected with sympt	0.1
α_1	The rate of people become normaly infected with symptoms	0.8
α_2	The rate of people have developed a rapid development of the virus	0.4
α_3	The rate of People have severe complications such as pulmonary failure.	0.2
θ_1	The rate of people with symptoms of mild virus who have been quarantined.	0.1
θ_2	The rate of people with serious complications who have been quarantined	0.2
δ_1	Mortality rate due to complications and also decreasing.	0.3
δ_2	The rate of people who died under quarantine in hospitals.	0.08
σ	The rate of people who recovered from the virus and decreasing	0.08
Λ_1	Denote the incidence of susceptible.	2000000
Λ_2	Denote the incidence of Immigrants and carriers of the virus without sympt	2000
Λ_3	Denote the incidence of Immigrants and carriers of the virus with sympt.	500

Table1: Parameter values used in numerical simulation

Since control and state functions are on different scales, the weight constant value is chosen as follows: $A = 10^5$, $B = 10^5$, and $G = 10^5$, with intial value $S(0) = 300000000$, $I_W(0) = 2500000$, $I(0) = 600000$, $C(0) = 5000$, $H(0) = 1000000$, $R(0) = 400000$,

From table 1 and figure 2, we note the number of infected with symptoms increased, as well as the increase in the number of infected with serious complications, as a result of the transmission of infection from one person to another through several methods including contact with an infected person or through respiratory secretions, such as sneezing and others, or by traveling, migrant workers, or families and relatives of the infected persons.

The proposed control strategy in this work helps to achieve several objectives.

5.1. Strategy A: Sensitization and prevention.

We use only the optimal control $u(t)$

This strategy aims to increase the number of people protected from virus, through figure (2) note that after applying different strategic as awarness program and protected, which reduces the high number of infected without symptoms.

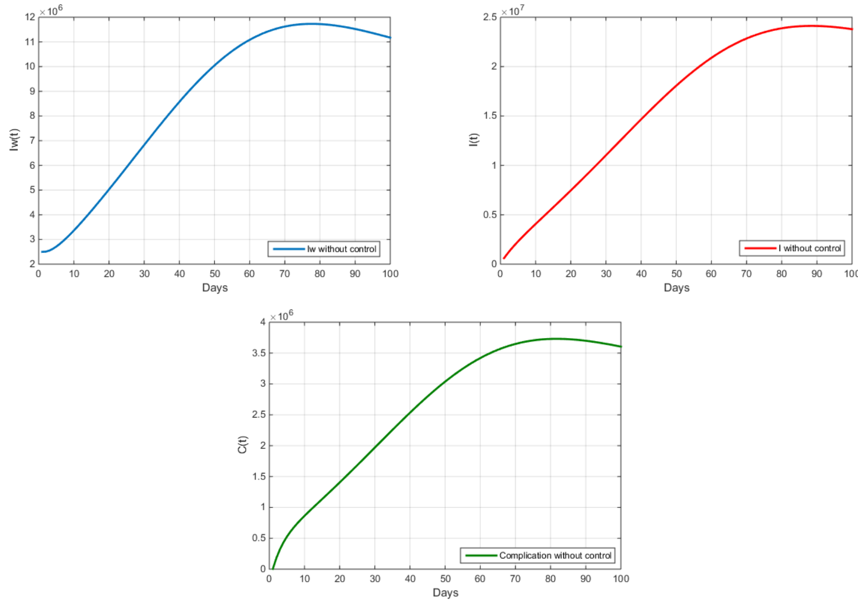

FIGURE 2. The evolution of the infected with and without complications without controls

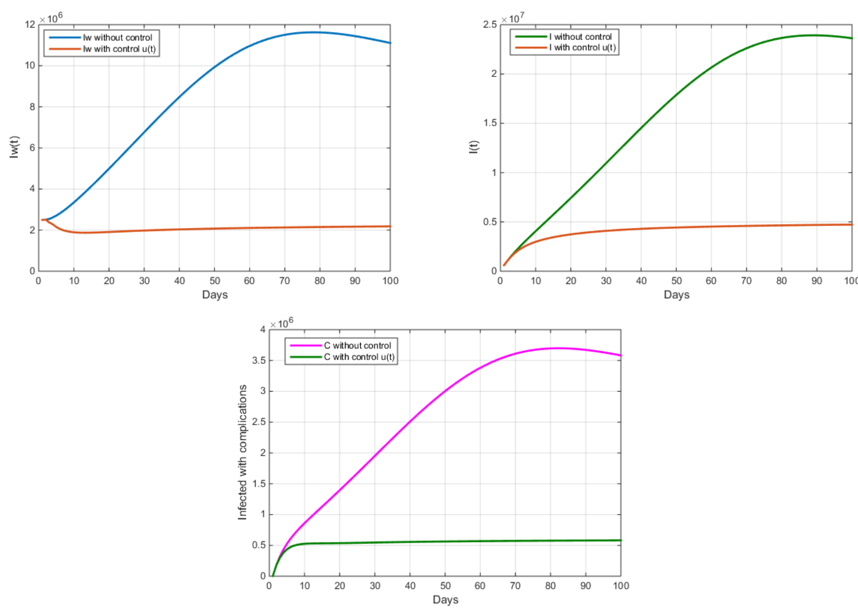

FIGURE 3. The evolution of the number of infected people with and without symptoms with control $u(t)$

Without and with control u(t) after 100 Days	Without control	With control	Percentage of decreasing
Infected without symptoms	1.11×10^7	2.18×10^6	80.35%
Infected with symptoms	2.36×10^7	4.72×10^6	80%
Infected with complication	3.58×10^6	6.95×10^5	80.6%

Table 2: Percentage of decreasing of number of infected with its types.

According to Figure 3 and table 2, the strategy of sensitization and prevention has been adopted, through awareness-raising campaigns for all citizens to inform them to the seriousness of the virus, through the mass media, and to take prevention measures by avoiding infected people, washing hands regularly, especially after sneezing, and wearing face masks, as well as reducing the spread of the virus.

Note: Up to the time limit, there is no vaccine or treatment for this virus.

5.2. Strategy B: Quarantine.

We use only the optimal control $v(t)$

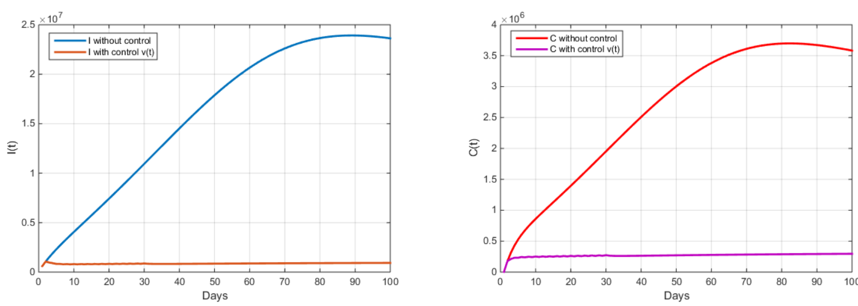

FIGURE 4. The evolution of the number of infected and infected with serious complications with control $v(t)$

Without and with control v(t) after 100 Days	Without control	With control	Percentage of decreasing
Infected with symptoms	2.36×10^7	9.3×10^5	96%
Infected with complication	3.58×10^6	2.96×10^5	92%

Table 3: Percentage of decreasing of number of infected with its types.

Based on figure 4 and table 3, the quarantine strategy was adopted for all people with symptoms, as well as those with serious complications inside hospitals, in addition to preventing entry and exit to and from the areas of the outbreak of the Coronavirus Emerging Virus 2019, this measure adopted by more countries were quarantined.

This is explained by figure 4, by the decrease in the number of people with symptoms and infected with complications after quarantine.

5.3. Strategy D: Diagnosis and monitoring.

We use only the optimal control $w(t)$

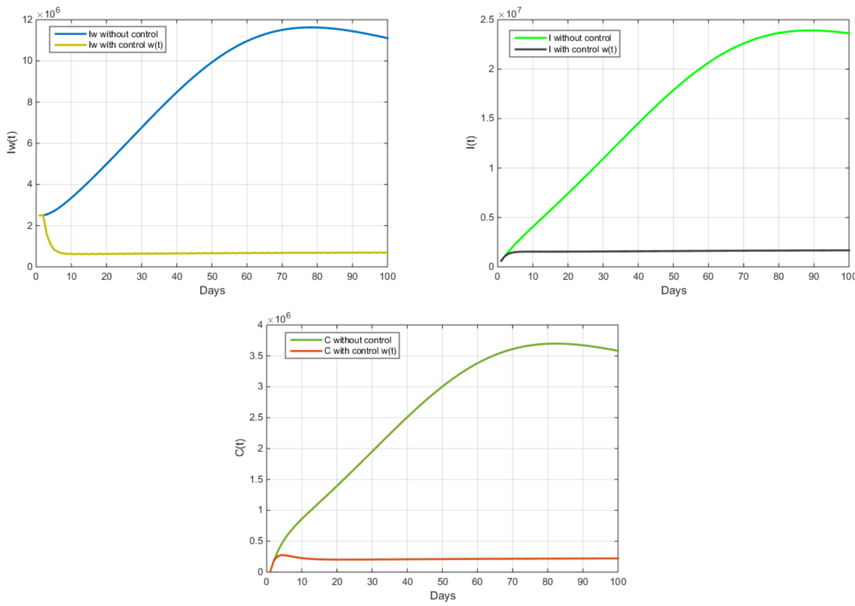

FIGURE 5. The evolution of the number of infected without symptoms and hospitalization with control $w(t)$

Without and with control u(t) after 100 Days	Without control	With control	Percentage of decreasing
Infected without symptoms	1.11×10^7	7.06×10^5	93.6%
Infected with symptoms	2.36×10^7	1.67×10^6	92.2%
Infected with complication	3.58×10^6	2.22×10^5	93.4%

Table 4: Percentage of decreasing of number of infected with its types.

Starting from figure 5 and table 4, the strategy of diagnosis and monitoring was adopted, this appears in the diagnosis of all cases of people likely to be infected with the virus, especially the family, relatives and neighbors of the person who was diagnosed, in addition to preventive precautions such as monitoring in all transport means such as metro, trains and airports and sending all arrivals from pandemic areas to quarantine. The main objective of this procedure is to limit the spread of virus to other countries.

Remark: We can also merge multiple assemblies as $(u(t), v(t))$ and $(u(t), v(t), w(t))$ thus get a variety of results.

6. Conclusion

In this paper, we introduced a matematical model of Novel Coronavirus (COVID-19), susceptible and infected by different step of developing of virus and guarantine and recovered, in order to minimize the number of infected and infected with several complications. We also introduced three controls which, respectively, represent sensitization and prevention, quarantine, diagnosis and monitoring. We applied the results of the control theory and we managed to obtain the characterizations of the optimal controls. The numerical simulation of the obtained results showed the effectiveness of the proposed control strategies.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

References

[1] World Health Organization.who.int/csr/don/12-january-2020-novel-Coronavirus-china.

[2] World Health Organization.who.int/emergencies/diseases/novel-Coronavirus-2019.

[3] World Health Organization.who.int/health-topics/Coronavirus/laboratory-diagnostics-for-novel-Coronavirus.

[4] Middle East respiratory syndrome Coronavirus (MERS-CoV) – The Kingdom of Saudi Arabia, 18 December 2019, who.int/csr/don/05-december-2019-mers-saudi-arabia/.

[5] A. Kouidere, O. Balatif, H. Ferjouchia, A. Boutayeb, M. Rachik, Optimal Control Strategy for a Discrete Time to the Dynamics of a Population of Diabetics with Highlighting the Impact of Living Environment, Discrete Dyn. Nat. Soc. 2019 (2019), 6342169.

[6] Middle East respiratory syndrome Coronavirus (MERS-CoV) Summary of Current Situation 21 July 2017.

[7] Middle East respiratory syndrome Coronavirus (MERS-CoV) – Qatar, 26 December 2019, www.who.int/csr/don/26-december-2019-mers-qatar.

[8] M. Tahir, I.A. Shah, G. Zaman, T. Khan, Prevention Strategies for Mathematical Model MERS-Coronavirus with Stability Analysis and Optimal Control. J. Nanosci. Nanotechnol. Appl. 3 (2018), 101.

[9] Z.-Q. Xia, J. Zhang, Y.-K. Xue, G.-Q. Sun, Z. Jin, Modeling the Transmission of Middle East Respirator Syndrome Corona Virus in the Republic of Korea, PLoS ONE. 10 (2015), e0144778.

[10] O. Balatif, B. Khajji, M. Rachik, Mathematical Modeling, Analysis, and Optimal Control of Abstinence Behavior of Registration on the Electoral Lists, Discrete Dyn. Nat. Soc. 2020 (2020), 9738934.

[11] L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze, and E. F. Mishchenko, The Mathematical Theory of Optimal Processes, Wiley, New York, 1962.

[12] W. H. Fleming and R. W. Rishel, Deterministic and Stochastic Optimal Control, Springer, New York, 1975.

[13] Novel Coronavirus(2019-nCoV) Situation Report - 14, who.int/docs/default-source/Coronaviruse/situation-reports/20200203-sitrep-14-ncov.

[14] Novel Coronavirus (2019-nCoV) technical guidance: Early investigations, who.int/emergencies/diseases/novel-Coronavirus-2019/technical-guidance/early-investigations.

[15] W. E. Boyce, R. C. DiPrima, Elementary Differential Equations and Boundary Value Problems, John Wiley & Sons, New York, 2000.

[16] A.B. Gumel et al. Modelling strategies for controlling SARS outbreaks, Proc. R. Soc. Lond. B, 271 (2004), 2223-2232.

[17] Nofe Al-Asuoad, Libin Rong, Sadoof Alaswad, Meir Shillo, Mathematical model and simulations of MERS outbreak: Predictions and implications for control measures, Biomath, 5 (2016), 1612141.

[18] G. Birkhoff and G.C. Rota, Ordinary Differential Equations, 4th edition, JohnWiley & Sons, New York, 1989.

[19] A. Kouidere, A. Labzai, B. Khajji, H. Ferjouchia, O. Balatif, A. Boutayeb, M. Rachik, Optimal control strategy with multi-delay in state and control variables of a discrete mathematical modeling for the dynamics of diabetic population, Commun. Math. Biol. Neurosci., 2020 (2020), Article ID 14.

[20] A. El Bhih, Y. Benfatah, S. Ben Rhila, M. Rachik, A. El Alami Laaroussi, A Spatiotemporal Prey-Predator Discrete Model and Optimal Controls for Environmental Sustainability in the Multifishing Areas of Morocco. Discrete Dyn. Nat. Soc. 2020 (2020), 2780651.

[21] A. El Bhih, R. Ghazzali, S. Ben Rhila, M. Rachik, A. El Alami Laaroussi, A Discrete Mathematical Modeling and Optimal Control of the Rumor Propagation in Online Social Network. Discrete Dyn. Nat. Soc. 2020 (2020), 4386476.