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EPIDEMIOLOGICAL MODELS IN HIGH SCHOOL MATHEMATICS EDUCATION

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Abstract. In epidemiology, compartmental models also called epidemiological models are an effective tool for understanding and describing the dynamics of infectious diseases. They give a solid direction to policy makers and public health administration on how to effectively prevent and control these diseases. Further, epidemiological models are currently used to predict the epidemiological trend of coronavirus disease 2019 (COVID-19) in many countries. In this paper, we propose to teach and learn these models in high school mathematics education in order to introduce the basic concepts and notions of mathematical modeling of infectious diseases.

Keywords: epidemiology; mathematical modeling; simulation; mathematics education.

2010 AMS Subject Classification: 97M10, 97M60, 97N80.

1. Introduction

Currently, many infectious diseases threat human health and the economy of several countries. For instance, coronavirus disease 2019 (COVID-19) caused by a new type of virus belonging to coronaviruses family and recently named severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) [1]. On 11 March 2020, COVID-19 was reclassified as a pandemic by the World Health Organization (WHO). The disease spreads rapidly from country to country, causing enormous economic damage and lot of deaths worldwide. As of 21 April 2020, COVID-19

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has infected 2 397 217 worldwide and caused the death of 162 956 people [2]. In Morocco, the first case occurred on 2 March 2020 in Casablanca city. Since this event, the number of infected by COVID-19 has increased day after day. As of 20 April 2020, the confirmed cases reached 3046 and the number of recoveries reached 350 with a total number of 143 deaths [3].

Many epidemiological models have been proposed for understanding the transmission of infectious diseases and predicting the occurrence of their epidemics. The first mathematical model was introduced by Ross [4, 5] to study transmission of Malaria in early 1900. He explained the relationship between the number of mosquitoes and incidence of malaria in humans. Inspired by Ross' ideas, Kermack and Mckendrick [6] presented a Susceptible-Infected-Recovered (SIR) compartmental model in order to explain the evolution of the plague in island of Bombay over the period December 17, 1905 to July 21, 1906. This classical SIR model was extended by Hattaf and Yousfi [7] to predict the evolution of influenza A (H1N1) virus of 2009 in Morocco. On the other hand, Fanelli and Piazza [8] analyzed and forecasted of COVID-19 spreading in China, Italy and France by using a simple susceptible-infected-recovered-deaths (SIRD) model. The general version of the above models was proposed by Hattaf et al. [9] to consider different factors such as type of incidence rate and latent period of infection.

In education, mathematical modeling has attracted the attention of many researchers. Mousoulides et al. [10] proposed a theoretical model in order to examine students' modeling behavior. They analyzed the processes used by students when engaged in modeling activities and they examined how students' abilities to solve modeling problems changed over time. In [11], Blum presented empirical findings on the teaching and learning of mathematical modeling and he considered that modeling is an important and demanding activity for students and teachers. Based on meta-analytic techniques, Sokolowski [12] studied the effects of mathematical modeling on students' at the high school and college levels. Stohlmann [13] studied the modeling capacities of middle school students associated with the robot art model-eliciting activity. Ryanto et al. [14] used growth of population context to learn mathematics through modeling tasks in elementary school. Tong et al. [15] presented a study to develop students' mathematical modeling competency via teaching sine and cosine theorems.

For teaching and learning epidemiological models, Weisstein [16] described a strategy for teaching model-building in an introductory biology course, using the example of a model of an infectious disease outbreak. He implemented the obtained discrete model in Excel. Gaff et al [17] presented a game-based classroom lesson to teach many basic principles of epidemiology and mathematical modeling. Lofgren et al. [18] used zombie epidemics to make mathematical modeling of infectious diseases more accessible to public health professionals, students, and the general public.

According to an analysis that we carried out on the textbooks of the Moroccan high school, we noticed that there is no activity or exercise concerning the epidemiological models. Further, there are a few articles that focus on teaching and learning these models. For these reasons, the aim of this study is to give a simple approach to integrate mathematical modeling of infectious diseases in the teaching and learning of mathematics in Moroccan high schools. To do this, Section 2 devoted to the presentation of mathematics teaching in high school, and also skills and abilities expected in this phase of learning. Section 3 deals with four activities to built the visual, discrete and continuous version of a SIRD epidemiological model, and therefore to transmit current knowledge on mathematical modeling of COVID-19. Finally, the conclusions are summarized in the last section.

2. MATHEMATICS EDUCATION IN MOROCCO

According to the national education and training charter [19], the Moroccan education system consists of two years in preschool (ages 4-5), six years in primary school (ages 6-11), three years in middle school (ages 12-14) and three years in high school (ages 15-17). Students at the end of the third year of middle school education can choose one of the following branches: Science, Science and Technology, Letters and Human Sciences, or Original Education which is based on islamic disciplines and the arabic language. At the end of the first year of high school (common core), students are asked to choose a stream according to their preferences and skills, also taking into account the studies planned after the baccalaureate. The main streams of the first year of the baccalaureate are Experimental Sciences, Sciences and Technologies, Mathematical Sciences, Economic and Management Sciences, Letters and Human Sciences, and Original Education. Students are led at the end of the first year of the baccalaureate to choose a specialization stream in line with their desires and preferences and also according to the stream chosen previously. For example, the experimental sciences sector offers three branches: Earth Life Sciences (ELS), Physics Chemistry (PC) and Agronomic Sciences (AS). Also, the mathematical sciences stream gives the possibility of choosing between mathematical sciences A and mathematical sciences B. At the end of high school studies, students are required to take a national exam evaluating their studies in order to obtain the Baccalaureate certificate.

The teaching of mathematics in high school occupies a distinguished place. From the curriculum and educational guidelines (CEG) of 2007 [20], the general objectives of this teaching are:

(1) Make the student acquire positive values towards mathematics, arousing in him confidence in his ability to practice and making him valuing the role of mathematics in the development of the individual and society.
(2) Develop the student's ability to solve problems, by formulating them from mathematical or real situations and their representation by mathematical models, and make him acquire various strategies for solving them, and develop his ability to check the results, and their interpretation by returning to the original problem.
(3) Develop in the student the ability to communicate, model situations, explain reasoning and clarify a strategy by using oral or written expression, diagrams and graphs, or algebraic methods.
(4) Develop the student's ability to use different mathematical reasoning and their applications, to develop conjectures and construct examples or counterexamples.
(5) Develop the student's ability to develop connections, to look at mathematics as an integrated whole, and to describe results using mathematical representations or models.
(6) Provide the student with a solid foundation in mathematics, whether it be knowledge and skills that will enable him to study in the future or to integrate into practical life under relevant conditions, or skills essential for the use of new technologies.

In Morocco, the student turns to the mathematical sciences stream only in the first year of the baccalaureate. So the content program of this year is summarized in Table 1. Whereas, the content program of the second year is presented in Table 2.

Algebra and Geometry	Analysis
Logic concepts	Generalities on numerical functions
Sets and applications	Numerical sequences
Enumeration	Trigonometric calculus
Arithmetic in $\boldsymbol{Z}$	Limit of a numerical function
Barycenter	Derivability
Scalar product in plane	Study of numerical functions
Rotation	
Space vectors	
Analytic geometry of space	
Scalar product in space	
Vector product	

TABLE 1. Program for the first year of the stream of mathematical sciences.

Algebra and Geometry	Analysis
Complex numbers	Limit and continuity
Arithmetic in $\boldsymbol{Z}$	Numerical sequences
Internal composition laws	Derivation and study of functions
Groups	Primitive functions
Rings	Logarithmic and exponential functions
Fields	Mean value theorem
Real vector spaces	Integral calculus
Probability calculus	Differential equations

TABLE 2. Program for the second year of the stream of mathematical sciences.

3. Teaching of Epidemiological Models

Since the appearance of the first case on March 2, 2020 in Morocco, the most prominent information in the media such as radio, television and social networks are the new numbers of infected, recovered, and died cases of the pandemic. These media also talk about the basic reproduction number, mathematical modeling, the infection rate, etc. The purpose of the following activities is to popularize these concepts and to give an approach to integrate them in the teaching and learning of mathematics in high school.

Activity 1

The Moroccan government decided on April 18, 2020 to extend the state of health emergency until May 20, 2020 to the whole country in order to fight against the spread of COVID-19. You have seen in middle school in course of geology that the Richter scale measures the strength of a seism. Is there a scale to measure the strength of disease spread, and to say that the disease will disappear before or after the end of the state of emergency? To respond to this open problem, we first construct the disease transmission diagram. To do this, we ask the students to answer the following questions:

FIGURE 1. Schematic diagram of ....

(1) We assume that one day you will be susceptible to COVID-19. What could happen to you in the coming days? Express your answer with an arrow in the Figure 1.
(2) Now, we suppose that you are ill due to this disease. Add arrows to express possible cases of your state after illness.
(3) Write on each arrow added in the Figure 1 through the above questions, the appropriate word among the following words:

- recovery rate,
- death rate,
- transmission.

(4) What does the schematic diagram in Figure 1 represent? So, complete the title of this figure.

We want from the questions of activity 1 that the learner arrives to build the schematic diagram given in Figure 2. In the case of failure, the teacher can add a supplementary question concerning the comparison between Figure 2 and that obtained by the student by answering questions 3 and 4.

The arrows in Figure 2 indicate the course followed by each individual until he recovers or dies.

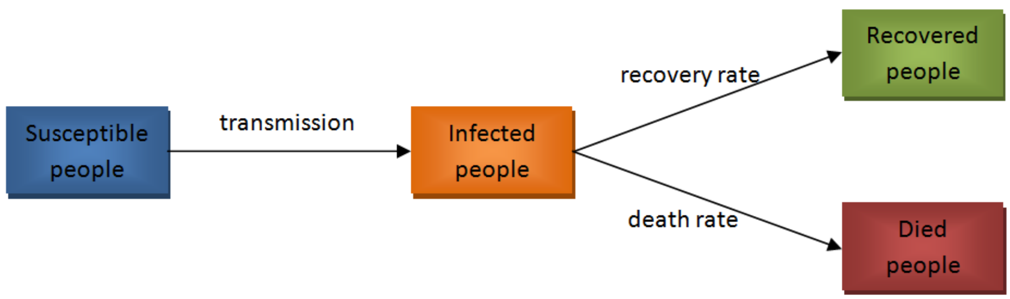

FIGURE 2. Schematic diagram of transmission disease.

Activity 2

191 new infected cases of coronavirus, 23 recovered people and 2 dead people were registered in Morocco during the day of 20 April 2020 [3]. Then we divide the population into three classes labeled by S, I, R and D, where $S(t)$, $I(t)$, $R(t)$ and $D(t)$ represent the number of susceptible, infected, recovered and died individuals at time t, respectively. In this activity, we ignore natural birth and death rates and assume that an individual can be infected only through contact with infectious individuals. Further, we assume that when an infected individual is cured, it acquires permanent immunity.

(1) Let $N(t)$ be the total population at time t. Write $N(t)$ as a function of $S(t)$, $I(t)$, $R(t)$ and $D(t)$.

(2) Write $S(t+1)$ by a sentence of words. This number depends on what?

(3) Write the relationships between $S(t+1)$, $S(t)$ and number of new infected cases during a short period Δt by "word equations".

(4) How many new infected cases would occur in each of the three populations P_1, P_2 and P_3, where (i) P_1 has a high transmission rate, many susceptible individuals and no infected individuals, (ii) P_2 has a high transmission rate, no susceptible individuals and many infected individuals, (iii) P_3 has a zero transmission rate, many susceptible and infected individuals.

(5) Let β be the transmission rate of COVID-19, choose the correct answer
- The number of new infected cases at time t is equal to $\beta S(t)I(t)$.
- The number of new infected cases at time t is equal to $\beta + S(t) + I(t)$.
- The number of new infected cases at time t is equal to $\beta S(t) - I(t)$.

(6) Express your answer in question 3 by a mathematical equation.

(7) We assume that 10% of the number of infected individuals at time t recover and 5% of them die. What will be the number of new recoveries and the number of new deaths at the same time t?

(8) Let r and d be the recovery and death rates, respectively. What does $rI(t)$ and $dI(t)$ represents?

(9) In the figure obtained in the activity 1, replace the words with the appropriate symbols from R, D, S, rI, βSI, dI and I.

(10) Prove that $I(t+1) = I(t) + \big(\beta S(t)I(t) - (r+d)I(t)\big)\Delta t$,
$R(t+1) = R(t) + rI(t)\Delta t$ and $D(t+1) = D(t) + dI(t)\Delta t$.

We want from the questions of activity 2 that the learner arrives to construct the following epidemiological discrete model:

$$
\begin{cases}
S(t+1) & = S(t) - \beta S(t)I(t)\Delta t, \\
I(t+1) & = I(t) + \big(\beta S(t)I(t) - (r+d)I(t)\big)\Delta t, \\
R(t+1) & = R(t) + rI(t)\Delta t, \\
D(t+1) & = D(t) + dI(t)\Delta t.
\end{cases}
\tag{1}
$$

Activity 3

(1) Calculate $\lim_{\Delta t\to 0} \frac{S(t+1)-S(t)}{\Delta t}$, $\lim_{\Delta t\to 0} \frac{I(t+1)-I(t)}{\Delta t}$, $\lim_{\Delta t\to 0} \frac{R(t+1)-R(t)}{\Delta t}$, $\lim_{\Delta t\to 0} \frac{D(t+1)-D(t)}{\Delta t}$ and deduce that

$$\begin{cases} S'(t) &= -\beta S(t)I(t), \\ I'(t) &= \beta S(t)I(t) - (r+d)I(t), \\ R'(t) &= rI(t), \\ D'(t) &= dI(t). \end{cases} \tag{2}$$

(2) Let $S(0)=S_0$, $I(0)=I_0$, $R(0)=R_0$, $D(0)=D_0$ be the initial conditions of system (2). Prove that

$$S(t) \le S_0, \text{ for all } t \ge 0.$$

(3) At the begun of the disease, the number of susceptible individuals is relatively constant and can be approximate by S_0. Check that

$$I'(t) = aI(t),$$

where $a = \beta S_0 - r - d$.

(4) Let $\mathscr{R}_0 = \frac{\beta S_0}{r+d}$. Study the variation of the function $I(t)$ according to the values of R_0.

(5) Interpret the results obtained in the last question.

The purpose of the above activity is to learn the student how to build the continuous version of the disease epidemic model from the discrete version and also the computation of the following threshold number:

$$\mathscr{R}_0 = \frac{\beta S_0}{r+d}, \tag{3}$$

which is called the basic reproduction number. Biologically, it represents the average number of secondary cases produced by one infected individual during its infectiousness period when all individuals in the population under study are uninfected. The learner can easily deduce that if $\mathscr{R}_0 > 1$, then the number of infected individuals will grow, and if $\mathscr{R}_0 < 1$ it will decay. Then the basic reproduction number plays the role of Richter scale for the infectious diseases and it also allows to say that the disease will disappear or not in the population.

Activity 4

The aim of this activity is to simulate system (2) by using Geogebra.

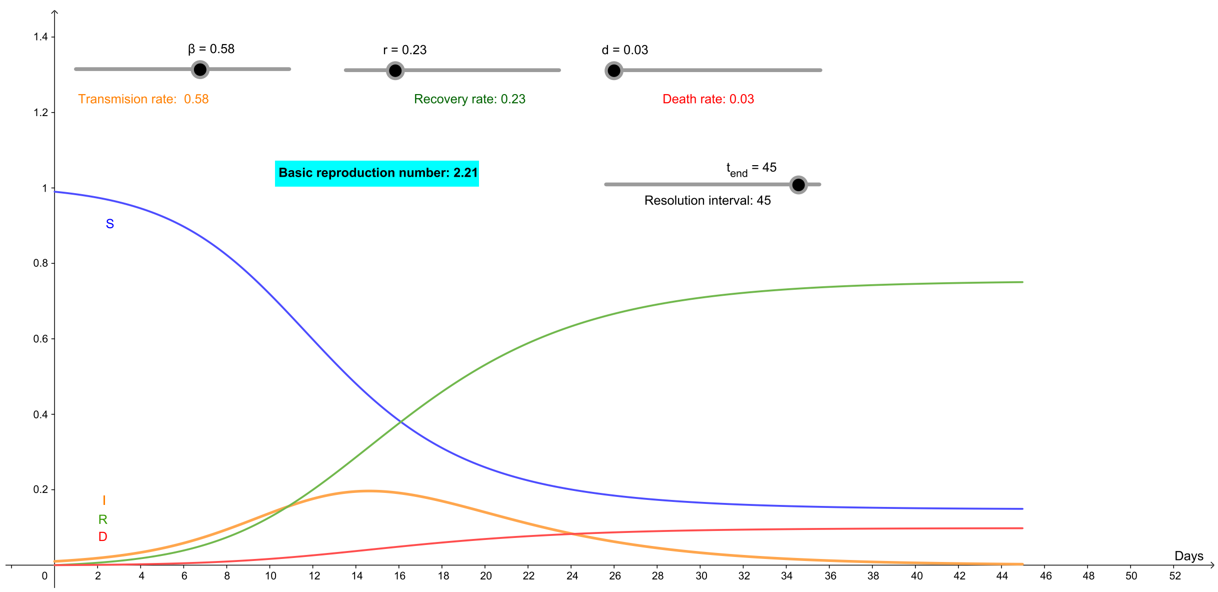

FIGURE 3. Simulation of the SIRD model presented by system (2) under different values of parameters.

From (2), we have $S(t)+I(t)+R(t)+D(t)=N$, where N is the number of the population in a city or a country. We can take $N=1$ to say 100% of the population.

(1) Enter the initial conditions S_0, I_0, R_0 and D_0 with $S_0=0.99$, $I_0=0.01$, $R_0=0$ and $D_0=0$.

(2) Using slider of Geogebra, enter the parameter values of the model β, r, d and also the resolution interval $[t_0,t_{end}]$. You can choose $\beta,r,d\in[0,1]$, $t_0=0$ and t_{end} varies from 0 to 50 days.

(3) Enter the following commands in the input field at the bottom of the screen, taking care to type "enter" at the end of each line.

$S'(t,S,R,D)=-\beta*S*I$

$I'(t,S,R,D)=\beta*S*I-r*I-d*I$

$R'(t,S,R,D)=r*I$

$D'(t,S,R,D)=d*I$

(4) To solve the system, enter in the input field the following command:

NSolve$\big(\{S', I', R', D'\}, t_0, \{S_0, I_0, R_0, D_0\}, t_{end}\big)$

(5) Calculate the basic reproduction number $\mathscr{R}_0 = \dfrac{\beta S_0}{r+d}$.

(6) Deduce when the disease will disappear or persist in the population.

We chose Geogebra to simulate the model because this software is familiar to students and downloadable free on the internet. Additionally, Geogebra is a dynamic geometry software (DGS) that promotes the interactions between different registers of semiotic representation in sense of Duval [21, 22].

4. Conclusions

In this work, we have developed four activities in order to introduce epidemiological models in the teaching of mathematics in high school and also familiarize the learner with the concepts of mathematical modeling of infectious diseases such as COVID-19. More precisely, the first activity helps the student to represent the evolution of the disease by a graphical scheme. This activity can be implemented in middle or primary school. The second activity aims to recognize the variables and parameters of the model as well as its discrete version, and it also allows the student to express the model by both word and mathematical equations. The third activity improves the ability of students to apply their knowledge such as the limit, derivative and variation of a function to new situations in order to deduce the continuous version of the epidemiological model and describe qualitatively its dynamical behavior. Also this activity can be used to introduce the notion of derivative in order to make sense to this notion. It can also be used as an activity to introduce the ordinary differential equation of the form $y^{'} = ay$. Finally, the fourth activity deals with the simulation of the model by Geogebra in order to see the effect of each parameter on the spread of the disease using the slider.

On the other hand, the four activities can develop in the learner several skills and abilities such as scientific reasoning, solving problems and communication skills. Additionally, they achieve the six general objectives of teaching mathematics at the Moroccan high school. So, it is more reasonable to experiment these activities in the classes of the Moroccan high school in order to validate our analytical results. We will study this in our future work.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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