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RELIABLE ITERATIVE METHODS FOR MATHEMATICAL MODEL OF COVID-19 BASED ON DATA IN ANHUI, CHINA

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Abstract: In this paper, five reliable iterative methods: Daftardar-Jafari method (DJM), Tamimi-Ansari method (TAM), Banach contraction method (BCM), Adomian decomposition method (ADM) and Variational iteration method (VIM) to obtain approximate solutions for a mathematical model that represented the coronavirus pandemic (COVID -19 pandemic). The accuracy of the obtained results is numerically verified by evaluating the maximum error remainder. In addition, the approximate results are compared with the fourth-order Runge-Kutta method (RK4) and good agreement have achieved. The convergence of the proposed methods is successfully demonstrated and mathematically verified. All calculations were successfully performed with MATHEMATICA®10.

Keywords: Reliable iterative method; Daftardar-Jafari method; Temimi-Ansari method; Banach contraction method; Adomian decomposition method; variational iteration method; maximum error remainder.

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1. INTRODUCTION

One of the disaster events in 2020 was the spread of COVID -19 around the world. The term COVID -19 can be divided into three sections, the first CO one representing the first letters of the

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word corona, and the second syllable which is VI the abbreviation of the word virus. The third syllable consists of one letter, the letter D, which is an abbreviation of the word disease. The aim of the corona virus is to have a specific scientific name which distinguishes it from other viruses and other diseases. As for the number 19, it is an abbreviation of the year in which the corona virus first appeared, 2019, but before the name COVID -19 was published for the disease, scientists called Corona the new (nCoV-19 Corona Virus), which literally means corona virus 2019.

It was first recognized as a pathogen of microorganisms from human diseases in the mid-1960s [1]. Coronaviruses are viruses that cause diseases such as acute colds and develop into more serious diseases, such as respiratory diseases such as Middle East syndrome, and acute respiratory syndrome (SARS) [2].

The name "Corona virus" is derived from the Latin crown, which means "crown" or "wreath", which in itself is a borrowing of the Greek "wreath". The name refers to the unmistakable appearance of viruses (the infectious form of the virus) by means of an electron microscope, which contains a set of bulb-like light bulbs on the bulb or the image of the solar aura [3-4]. Corona viruses kill 30% or more of those infected (such as MERS-COV) [5].

Signs of the disease: fever, cough, malaise and breathing difficulties. Severe cases of illness lead to pneumonia, kidney failure and even death [6]. An unknown pneumonia was diagnosed in December 2019 in Wuhan, Hubei in Wuhan province, China. The virus responsible was later confirmed as a new coronavirus [7]. The disease has spread from Wuhan to all parts of the country and abroad. On March 12, 2020, at 5:44 pm, 80,981 people were injured in China, with a total death toll of 31.73. New cases of COVID -19 occurred in 109 foreign countries (including Japan and South Korea, etc.), and 38,620 cases and 1,446 deaths were confirmed in China [8]. The Wuhan Dynasty was identified as a new strain 70% genetically similar to n-COV [9]. The virus is similar to 96% of coronary bat viruses, so there is a widespread suspicion that it also originates from bats [10]. The epidemic has led to travel restrictions and closures in many countries. Typical recommendations for infection prevention include regular hand washing, covering the mouth and nose when sneezing, coughing, avoiding contact with people with respiratory symptoms such as coughing and sneezing

Iterative methods are used to obtain analytic-approximate solutions to various nonlinear problems in approximate forms. New iterative method introduced in 2011 by Temimi and Ansari (TAM) [11], for solving nonlinear problems. The TAM was inspired from the homotopy analysis method (HAM) [12] and used to solve several ODEs [13], PDEs and KdV equations [14], differential algebraic equations (DAEs) [15], Duffing equations [16], some chemical problems [17], thin film flow problem [18] and Fokker-Planck's equations [19]. Another iterative method suggested by Daftardar and Jafari, in 2006 (DJM) [20], this method has been used to solve different equations such as fractional differential equations [21], partial differential equations [22], Volterra integro-differential equations and some applications for the Lane-Emden equations [23], evolution equations [24].

Also, other iterative method called the Banach contraction method (BCM) based on the Banach contraction principle [25]. The BCM, was used to solve different types of differential and integral equations [26]. Moreover, the Adomian decomposition method (ADM), is analytic method that was introduced and developed by George Adomian 1976 [27-28]. ADM is a reliable method to solve many various kinds of problems in applied science. This method has been used by many researchers and has extensive applications of linear and nonlinear ordinary differential equations, partial differential equations and integral equations [29-30]. In addition, the variational iteration method (VIM), is an iterative analytic method that was established by He in (1999) [31,32] and used to solve a wide variety of linear and nonlinear, homogeneous and inhomogeneous equations. In this paper, the five iterative methods: DJM, TAM, BCM, ADM and VIM will be used to solve the COVID- 19 models to obtain approximate solutions. These solutions will be numerically compared with the fourth order Runge- Kutta method. The convergence and some error indicator will be introduced and discussed.

We have organized this paper as follows: The mathematical biological model will be introduced in section 2. In section 3 the basic ideas of the three iterative methods are given for the COVID-19 models. The convergence of the methods will be presented in section 4. Section 5, the solution of the COVID-19 models by the proposed methods will be given. Numerical results and convergence will be presented in section 6. Finally, the conclusion is presented in section 7.

2. MATHEMATICAL MODELS OF COVID-19

In this section, the epidemic in Anhui, China can be divided into three phases, from January 10 to February 11, 2020 as given in [1].

Phase I (prior to January 23, 2020): The population can be divided into four categories: susceptible (S), exposed and pre-symptomatic population (E), symptomatic population (I), and recovered population (R), which can be presented as follows:

$$(1) \quad \begin{cases} \frac{dS}{dt} = \frac{-\beta S(kE+I)}{M}, \\ \frac{dE}{dt} = \frac{\beta S(kE+I)}{M} - qE, \\ \frac{dI}{dt} = qE - \gamma I, \\ \frac{dR}{dt} = \gamma I. \end{cases}$$

The initial conditions are

$$(2) \quad S(0) = r_1, E(0) = r_2, I(0) = r_3 \quad \text{and} \quad R(0) = r_4.$$

where, β is the transmission rate of disease, k controls the infectiousness of exposed pre-symptomatic individuals relative to symptomatic individuals, q is the rate at which exposed individual showing symptom, γ is the cure rate at which symptomatic individuals moving to recovered class and the total population M .

Phase II (between January 23 and February 6, 2020): On the basis of Eq. (1), we assume all symptomatic cases were quarantined in quarantined class (Q) and change the exposure rate to β_1 , σ cure rate at which quarantined cases moving to recovered class.

which can be presented as follows, see [1].

$$(3) \quad \begin{cases} \frac{dS}{dt} = \frac{-\beta_1 SE}{M}, \\ \frac{dE}{dt} = \frac{\beta_1 SE}{M} - qE, \\ \frac{dQ}{dt} = qE - \sigma Q, \\ \frac{dR}{dt} = \sigma Q. \end{cases}$$

The initial conditions are

$$(4) \quad S(0) = r_1, E(0) = r_2, Q(0) = r_3 \quad \text{and} \quad R(0) = r_4.$$

Phase III (after February 6, 2020): Exposure rate changed to β_2 , model at this phase it is the same Eq.(3), except replace β_2 instead of β_1 .

$$(5) \quad \begin{cases} \frac{dS}{dt} = \frac{-\beta_2 SE}{M}, \\ \frac{dE}{dt} = \frac{\beta_2 SE}{M} - qE, \\ \frac{dQ}{dt} = qE - \sigma Q, \\ \frac{dR}{dt} = \sigma Q. \end{cases}$$

The initial conditions are

$$(6) \quad S(0) = r_1, E(0) = r_2, Q(0) = r_3 \text{ and } R(0) = r_4.$$

3.THE BASIC CONCEPTS OF THE PROPOSED ITERATIVE METHOD

In this section the five iterative methods will be introduced to solve a system of ordinary differential equations contains linear and nonlinear terms. Then, in section five the methods will be used to solve the models given in equations (1), (3) and (5) with the given initial conditions.

Let us introduce the following nonlinear differential equation:

$$(7) \quad L_i(S_i(t)) + N_i(S_1(t), S_2(t), S_3(t), S_4(t)) = 0, i = 1,2,3,4.$$

with the initial conditions

$$(8) \quad S_i(0) = r_i, i = 1,2,3,4.$$

where t represents the independent variable, S_i the unknown function, L_i are the linear operators, N_i are the nonlinear operators. Let us start by introducing the basic ideas of the following five iterative methods.

3.1. The basic concept of the DJM

We can re-write the Eq. (7) as:

$$(9) \quad L_i \left(\sum_{n=0}^{\infty} S_{i,n}(t) \right) + N_i \left(\sum_{n=0}^{\infty} S_{1,n}(t), \sum_{n=0}^{\infty} S_{2,n}(t), \sum_n S_{3,n}(t), \sum_n S_{4,n}(t) \right) = 0,$$

By integrating both sides of Eq. (9) and applying the given conditions of the problem and introducing the inverse operator given in the following form [33]:

$$(10) \quad L_i^{-1}(\cdot) = \int_0^t (\cdot) d\omega, i = 1,2,3,4.$$

we get,

$$(11) \quad \sum_{n=0}^{\infty} S_{i,n}(t) = S_{i,0}(t) + L_i^{-1} \left(N_i \left(\sum_{n=0}^{\infty} S_{1,n}(t), \sum_{n=0}^{\infty} S_{2,n}(t), \sum_n S_{3,n}(t), \sum_n S_{4,n}(t) \right) \right),$$

$$i = 1,2,3,4.$$

Let us consider

$$(12) \quad \tilde{S}_{i,1}(t) = L_i^{-1} \left(N_i(\tilde{S}_{1,0}, \tilde{S}_{2,0}, \tilde{S}_{3,0}, \tilde{S}_{4,0}) \right), i = 1,2,3,4.$$

Hence,

$$(13) \quad S_{i,0}(t) = \tilde{S}_{i,0} = r_i, \quad i = 1,2,3,4.$$

and,

$$(14) \quad \tilde{S}_{i,2}(t) = L_i^{-1} \left(N_i(\tilde{S}_{1,0}, \tilde{S}_{2,0}, \tilde{S}_{3,0}, \tilde{S}_{4,0}) + (\tilde{S}_{1,1}, \tilde{S}_{2,1}, \tilde{S}_{3,1}, \tilde{S}_{4,1}) \right) - L_i^{-1} \left(N_i(\tilde{S}_{1,0}, \tilde{S}_{2,0}, \tilde{S}_{3,0}, \tilde{S}_{4,0}) \right),$$

In general,

$$(15) \quad \tilde{S}_{i,m+1}(t) = L_i^{-1} \left(N_i \left((\tilde{S}_{1,0}, \tilde{S}_{2,0}, \tilde{S}_{3,0}, \tilde{S}_{4,0})(t) + (\tilde{S}_{1,1}, \tilde{S}_{2,1}, \tilde{S}_{3,1}, \tilde{S}_{4,1})(t) + (\tilde{S}_{1,2}, \tilde{S}_{2,2}, \tilde{S}_{3,2}, \tilde{S}_{4,2})(t) \dots + (\tilde{S}_{1,m}, \tilde{S}_{2,m}, \tilde{S}_{3,m}, \tilde{S}_{4,m})(t) \right) \right) - L_i^{-1} \left(N_i \left((\tilde{S}_{1,0}, \tilde{S}_{2,0}, \tilde{S}_{3,0}, \tilde{S}_{4,0})(t) + (\tilde{S}_{1,1}, \tilde{S}_{2,1}, \tilde{S}_{3,1}, \tilde{S}_{4,1})(t) + (\tilde{S}_{1,2}, \tilde{S}_{2,2}, \tilde{S}_{3,2}, \tilde{S}_{4,2})(t) \dots + (\tilde{S}_{1,m-1}, \tilde{S}_{2,m-1}, \tilde{S}_{3,m-1}, \tilde{S}_{4,m-1})(t) \right) \right), m \geq 1$$

The m th-term approximate solution of Eq. (7) is given by

$$(16) \quad S_{i,m} = \sum_{n=0}^m \tilde{S}_{i,n}(t), i = 1,2,3,4.$$

Finally, the solution S_1, S_2, S_3 and S_4 will be presented in this form

$$(17) \quad S_i(t) = \sum_{n=0}^{\infty} \tilde{S}_{i,n}(t), i = 1,2,3,4.$$

3.2. The basic concept of the TAM

We first begin by solving the following initial value problem

$$(18) \quad L_i(S_{i,0}(t)) = 0 \text{ and } S_{i,0}(0) = r_i, i = 1,2,3,4.$$

and,

$$(19) \quad L_i(S_{i,1}(t)) + N_i(S_{1,0}, S_{2,0}, S_{3,0}, S_{4,0}) = 0 \text{ and } S_{i,1}(0) = r_i, i = 1,2,3,4.$$

Thus, in general we have to solve:

$$(20) \quad L_i(S_{i,n+1}(t)) + N_i(S_{1,n}(t), S_{2,n}(t), S_{3,n}(t), S_{4,n}(t)) = 0,$$

and $S_{i,n+1}(0) = r_i, i = 1,2,3,4, n = 1,2,..$

$S_{i,n}$, represents an approximate solution to Eq. (7). Then, the solution to the problem can be found by [34].

$$(21) \quad S_i(t) = \lim_{n \rightarrow \infty} S_{i,n}(t), i = 1,2,3,4.$$

3.3. The basic concept of the BCM

Let us first consider the Eq. (7), and using the inverse operator given in Eq. (10), we have [25]:

$$(22) \quad S_i(t) = L_i^{-1}(N_i(S_1(t), S_2(t), S_3(t), S_4(t))), i = 1,2,3,4.$$

Define successive approximations as:

$$(23) \quad S_{i,0}(t) = r_i, i = 1,2,3,4.$$

$$(24) \quad S_{i,1}(t) = S_{i,0}(t) + L_i^{-1} \left(N_i \left(S_{1,0}(t), S_{2,0}(t), S_{3,0}(t), S_{4,0}(t) \right) \right), i = 1,2,3,4.$$

⋮

$$(25) \quad S_{i,n}(t) = S_{i,0}(t) + L_i^{-1} \left(N_i \left(S_{1,n-1}(t), S_{2,n-1}(t), S_{3,n-1}(t), S_{4,n-1}(t) \right) \right),$$

$i = 1,2,3,4. n = 1,2,..$

BCM is based on the Banach fixed theorem. the solution of Eq. (7) is given by

$$(26) \quad S_i(t) = \lim_{n \rightarrow \infty} S_{i,n}(t), i = 1,2,3,4.$$

3.4. The basic concept of the ADM

The ADM introduces for Eq. (7) in the following form [35]:

$$(27) \quad S_{i,0}(t) = r_i,$$

$$(28) \quad S_{i,n+1}(t) = L_i^{-1} \left(\sum_{n=0}^{\infty} S_{i,n}(t) \right) + L_i^{-1} \left(N_i \left(\sum_{n=0}^{\infty} S_1(t), \sum_{n=0}^{\infty} S_2(t), \sum_{n=0}^{\infty} S_3(t), \sum_{n=0}^{\infty} S_4(t) \right) \right), n \geq 0$$

For the nonlinear solution $N_i(t)$ the infinite series of polynomials becomes:

$$(29) \quad \begin{aligned} & N_i \left(\sum_{n=0}^{\infty} S_1(t), \sum_{n=0}^{\infty} S_2(t), \sum_{n=0}^{\infty} S_3(t), \sum_{n=0}^{\infty} S_4(t) \right) \\ &= \sum_{n=0}^{\infty} A_{i,n} (S_{1,0}, \dots, S_{1,n}, S_{2,0}, \dots, S_{2,n}, \dots, S_{3,0}, \dots, S_{3,n}, S_{4,0}, \dots, S_{4,n}), \quad n \geq 0 \end{aligned}$$

The Adomian polynomial $A_{i,n}$ are obtained from the following formula [35].

$$(30) \quad A_{i,n} = \frac{1}{n!} \frac{d^n}{d\lambda^n} N_i \left[\left(\sum_{j=0}^n \lambda^j S_{1,j}, \sum_{j=0}^n \lambda^j S_{2,j}, \sum_{j=0}^n \lambda^j S_{3,j}, \sum_{j=0}^n \lambda^j S_{4,j} \right), \lambda = 0 \right], i = 1,2,3,4.$$

$$n = 1,2, \dots$$

The formulas of the first several Adomian polynomials from $A_{i,0}$ to $A_{i,3}$ for $S_i(t)$ have been listed below as given in [35]:

$$(31) \quad \begin{aligned} A_{i,0} &= N_i(S_{1,0}, S_{2,0}, S_{3,0}, S_{4,0}), \\ A_{i,1} &= S_{i,1} N_i'(S_{1,0}, S_{2,0}, S_{3,0}, S_{4,0}), \\ A_{i,2} &= S_{i,2} N_i'(S_{1,0}, S_{2,0}, S_{3,0}, S_{4,0}) + \frac{1}{2!} S_{i,1}^2 N_i''(S_{1,0}, S_{2,0}, S_{3,0}, S_{4,0}), \\ A_{i,3} &= S_{i,3} N_i'(S_{1,0}, S_{2,0}, S_{3,0}, S_{4,0}) + S_{i,1} S_{i,2} N_i''(S_{1,0}, S_{2,0}, S_{3,0}, S_{4,0}) + \\ & \quad \frac{1}{3!} S_{i,1}^3 N_i'''(S_{1,0}, S_{2,0}, S_{3,0}, S_{4,0}), \\ & \quad : \end{aligned}$$

and so on.

We approximate the solution S_i by the truncated series:

$$(32) \quad \varphi_{i,m}(S_i, t) = S_{i,m} = \sum_{n=0}^{m-1} S_{i,n}.$$

3.5. The basic concept of the VIM

The VIM can be presented for Eq. (7) in the following form [36].

$$\begin{aligned}
 (33) \quad & S_{i,n+1}(t) \\
 & = S_{i,n}(t) \\
 & + \int_0^t \lambda_i(\omega) \left(L_i(S_{i,n}(\omega)) + N_i(\widetilde{S}_{1,n}(\omega), \widetilde{S}_{2,n}(\omega), \widetilde{S}_{3,n}(\omega), \widetilde{S}_{4,n}(\omega)) \right) d\omega, \\
 & i = 1,2,3,4.
 \end{aligned}$$

Where λ_i , is a general Lagrange multiplier that can all be optimal way identified via the Variation Theory, and $\widetilde{S}_{i,n}$, $i = 1,2,3,4$ as a restricted variation. The Lagrange multiplier λ_i , may be a value or a function, and given the general formula [37].

$$(34) \quad \lambda_i = (-1)^n \frac{1}{(n-1)!} (\omega - t)^{n-1}, n \geq 1.$$

For the first order $\lambda_i = -1$.

However, for fast convergence, the function $S_{i,0}(t)$ should be selected by using the initial condition as follow:

$$(35) \quad S_{i,0}(t) = S_i(0).$$

the solution is given by

$$(36) \quad S_i(t) = \lim_{n \rightarrow \infty} S_{i,n}(t), i = 1,2,3,4.$$

4. CONVERGENCE OF THE PROPOSED METHODS

In this section, we introduce the convergence principles of the proposed methods by using some theorems [38]. For the DJM and ADM the convergence can be directly proved. However, to demonstrate the convergence for the TAM, VIM and BCM, we must follow the following procedure:

Let,

$$\begin{aligned}
 v_0 & = S_0(t), \\
 v_1 & = F[v_0], \\
 v_2 & = F[v_0 + v_1],
 \end{aligned}$$

:

$$(37) \quad v_{n+1} = F[v_0 + v_1 + \cdots + v_n].$$

where, F is the operator which can be defined by

$$(38) \quad F(v_K) = W_K(t) - \sum_{j=0}^{K-1} v_j(t), \quad K = 1, 2, \dots$$

where, W_K is the solution for the problem in the following form,

For the TAM:

$$(39) \quad L(v_K(t)) + N\left(\sum_{j=0}^{K-1} v_j(t)\right) = 0, \quad K = 1, 2, \dots$$

For the BCM or VIM:

$$(40) \quad v_K(t) = v_0(t) + N\left(\sum_{j=0}^{K-1} v_j(t)\right), \quad K = 1, 2, \dots$$

By using the same conditions which will be used for the approximate iterative method. We get,

$$S(t) = \lim_{n \rightarrow \infty} S_n = \sum_{n=0}^{\infty} v_n$$

Therefore, by using (37) and (38), one can get the solution by

$$(41) \quad S(t) = \sum_{j=0}^{\infty} v_j.$$

In the recursive algorithm of DJM, TAM, BCM, ADM and VIM, the following theorems [38] will provide the sufficient condition for achieving the convergence of our proposed methods.

Theorem 4.1 [38]. "Let F introduce in Eq. (38), be an operator from Hilbert space

H to H . $S_n(t) = \sum_{j=0}^n v_j$ is converges if $\exists 0 < \alpha < 1$ if $\|F[v_0 + v_1 + \cdots + v_{j+1}]\| \leq \alpha \|F[v_0 + v_1 + \cdots + v_j]\|$ (where $\|v_{j+1}\| \leq \alpha \|v_j\|$) $\forall j = 0, 1, 2, \dots$."

This theorem is a special case of a fixed point theorem which is a sufficient condition for the study of convergence.

Theorem 4.2[38]. "If the series $S(t) = \sum_{j=0}^{\infty} v_j$ converges, then this series represents the exact solution S ".

In another meaning, for each rank j , if the parameters are defined $\exists 0 < \alpha < 1$ such that

$$\|F[v_0 + v_1 + \dots + v_{j+1}]\| \leq \alpha \|F[v_0 + v_1 + \dots + v_j]\| \quad (\text{that is } \|v_{j+1}\| \leq \alpha \|v_j\|) \quad \forall j = 0, 1,$$

2,....., if the parameters are used for each iteration j

$$(42) \quad \vartheta_j^i = \begin{cases} \frac{\|v_{j+1}\|}{\|v_j\|}, & \|v_j\| \neq 0 \\ 0, & \|v_j\| = 0 \end{cases}$$

Then $\sum_{j=0}^{\infty} v_j$ converges to the exact solution $S(t)$, when $0 \leq \vartheta_j^i < 1, \forall j = 0, 1, 2, \dots, i = 1, 2, 3, 4$. We evaluate the ϑ_j^i .

5. SOLVING MATHEMATICAL MODELS OF COVID-19 BY THE PROPOSED METHODS

In this section, the five iterative methods introduced in section three will be used to solve the three phases mathematical models of the COVID-19.

5.1. Solving phase I for mathematical model of COVID-19

5.1.1. Solving phase I by using the DJM:

In current work, we consider the following values for the parameters: $r_1 = r_2 = r_3 = 1$ and $r_4 = 0$, where $M = S + E + I + R$ and $\beta = 0.4029, q = 0.41, k = 0.78, \gamma = 0.182$, see [1].

By using the steps given in subsection 3.1, and using the Eqs. (12), (13) and (14), we get

$$\tilde{S}_0(t) = \tilde{E}_0(t) = \tilde{I}_0(t) = 1, \quad \tilde{R}_0(t) = 0,$$

and,

$$(43) \quad \tilde{S}_1(t) = -0.239054t, \quad \tilde{E}_1(t) = -0.1709461t, \quad \tilde{I}_1(t) = 0.228t, \quad \text{and } \tilde{R}_1(t) = 0.182t.$$

Then,

$$(44) \quad \begin{aligned} \tilde{S}_2(t) &= 2.7755610^{-17}t + 0.0222168t^2 + 0.00101304t^3, \\ \tilde{E}_2(t) &= 0.0128271t^2 + 0.00101304t^3, \quad \tilde{I}_2(t) = -0.0557919t^2, \\ \tilde{R}_2(t) &= 0.020748t^2, \end{aligned}$$

Hence, we get the following approximate solutions,

$$(45) \quad \begin{cases} S_1(t) = \tilde{S}_0 + \tilde{S}_1 = 1 - 0.239054t, \\ E_1(t) = \tilde{E}_0 + \tilde{E}_1 = 1 - 0.170946t, \\ I_1(t) = \tilde{I}_0 + \tilde{I}_1 = 1 + 0.228t, \\ R_1(t) = \tilde{R}_0 + \tilde{R}_1 = 0.182t. \end{cases}$$

and,

$$(46) \quad \begin{cases} S_2(t) = \tilde{S}_0 + \tilde{S}_1 + \tilde{S}_2 = 1. - 0.239054t + 0.0222168t^2 + 0.00101304t^3, \\ E_2(t) = \tilde{E}_0 + \tilde{E}_1 + \tilde{E}_2 = 1. - 0.170946t + 0.0128271t^2 - 0.00101304t^3, \\ I_2(t) = \tilde{I}_0 + \tilde{I}_1 + \tilde{I}_2 = 1. + 0.228t - 0.0557919t^2, \\ R_2(t) = \tilde{R}_0 + \tilde{R}_1 + \tilde{R}_2 = 0.182t + 0.020748t^2. \end{cases}$$

:

The obtained approximated solution for six iterations by the DJM will be:

$$(47) \quad \begin{cases} S_6(t) = \tilde{S}_0 + \tilde{S}_1 + \dots + \tilde{S}_6 = 1. - 0.239054t + 0.0222168t^2 + 0.00129242t^3 \\ \quad \quad \quad - 0.000608091t^4 + 0.0000662304t^5 - 1.3985210^{-7}t^6, \\ E_6(t) = \tilde{E}_0 + \tilde{E}_1 + \dots + \tilde{E}_6 = 1. - 0.170946t + 0.0128271t^2 - 0.00304546t^3 \\ \quad \quad \quad + 0.00092025t^4 - 0.000141691t^5 + 9.8220710^{-6}t^6, \\ I_6(t) = \tilde{I}_0 + \tilde{I}_1 + \dots + \tilde{I}_6 = 1. + 0.228t - 0.0557919t^2 + 0.00513775t^3 \\ \quad \quad \quad - 0.000545927t^4 + 0.0000953322t^5 - 0.000012574t^6, \\ R_6(t) = \tilde{R}_0 + \tilde{R}_1 + \dots + \tilde{R}_6 = 0.182t + 0.020748t^2 - 0.00338471042t^3 \\ \quad \quad \quad + 0.000233767t^4 - 0.0000198717t^5 + 2.8917410^{-6}t^6, \end{cases}$$

5.1.2. Solving phase I by using the TAM:

In order to apply the TAM to solve the model of phase I, we follow similar steps given in subsection 3.2, the we get the following approximate solution for six iterations:

$$(48) \quad \begin{cases} S_6(t) = 1. - 0.239054t + 0.0222168t^2 + 0.00129242t^3 - 0.000608091t^4 \\ \quad \quad \quad + 0.0000662304t^5 - 1.3985210^{-7}t^6, \\ E_6(t) = 1. - 0.170946t + 0.0128271t^2 - 0.00304546t^3 + 0.00092025t^4 \\ \quad \quad \quad - 0.000141691t^5 + 9.8220710^{-6}t^6, \\ I_6(t) = 1. + 0.228t - 0.0557919t^2 + 0.00513775t^3 - 0.000545927t^4 \\ \quad \quad \quad + 0.0000953322t^5 - 0.000012574t^6, \\ R_6(t) = 0.182t + 0.020748t^2 - 0.00338471042t^3 + 0.000233767t^4 \\ \quad \quad \quad - 0.0000198717t^5 + 2.8917410^{-6}t^6, \end{cases}$$

5.1.3. Solving phase I by using the BCM:

To solve the phase I by using the BCM, we follow similar steps given in subsection 3.3, and the approximate solution will be the same to the TAM given in Eq. (48).

5.1.4. Solving phase I by using the ADM:

Let us consider the Eq. (1) with the given initial conditions the Eq. (2). Integrating both sides of Eq. (1) from 0 to t and using the given initial conditions, we have

$$(49) \quad \begin{cases} S(t) = 1 + L_1^{-1} \left(- \left(\frac{\beta k}{M} A_n + \frac{\beta}{M} B_n \right) \right), \\ E(t) = 1 + L_2^{-1} \left(\left(\frac{\beta k}{M} A_n + \frac{\beta}{M} B_n \right) - qE_n \right), \\ I(t) = 1 + L_3^{-1} (qE_n - \gamma I_n), \\ R(t) = 0 + L_4^{-1} (\gamma I_n), \end{cases}$$

where A_n, B_n are the Adomian polynomials, which evaluated from the nonlinear terms SE and SI , as follows:

$$\begin{aligned} A_0 &= S_0 E_0, \\ A_1 &= S_1 E_0 + E_1 S_0, \\ A_2 &= S_2 E_0 + S_1 E_1 + S_0 E_2, \\ &\vdots \\ B_0 &= S_0 I_0, \\ B_1 &= S_1 I_0 + I_1 S_0, \\ B_2 &= S_2 I_0 + S_1 I_1 + S_0 I_2, \\ &\vdots \end{aligned}$$

By applying the ADM, we obtain

$$S_0(t) = 1, E_0(t) = 1, I_0(t) = 1 \text{ and } R_0(t) = 0.$$

Also,

$$S_1(t) = -0.239054 t, E_1(t) = -0.170946 t, I_1(t) = 0.228t \text{ and } R_1(t) = 0.182t.$$

$$S_2(t) = 0.0222168 t^2, E_2(t) = 0.0128271 t^2, I_2(t) = -0.0557919 t^2$$

$$\text{and } R_2(t) = 0.020748 t^2.$$

Then, we get approximate solutions

$$\begin{cases} \varphi_1(S, t) = S_0 + S_1 = 1 - 0.239054 t, \\ \varphi_1(E, t) = E_0 + E_1 = 1 - 0.170946 t, \\ \varphi_1(I, t) = I_0 + I_1 = 1 + 0.228 t, \\ \varphi_1(R, t) = R_0 + R_1 = 0.182 t, \end{cases}$$

$$\text{and, } \begin{cases} \varphi_2(S, t) = S_0 + S_1 + S_2 = 1. - 0.239054 t + 0.0222168 t^2, \\ \varphi_2(E, t) = E_0 + E_1 + E_2 = 1. - 0.170946 t + 0.0128271 t^2, \\ \varphi_2(I, t) = I_0 + I_1 + I_2 = 1 + 0.228 t - 0.0557919 t^2, \\ \varphi_2(R, t) = R_0 + R_1 + R_2 = 0.182 t + 0.020748 t^2, \end{cases}$$

So, the obtained approximated solution for six iterations by the ADM will be:

$$\begin{cases} \varphi_6(S, t) = S_0 + S_1 + \dots + S_6 = 1. - 0.239054 t + 0.0222168 t^2 + 0.00129242 t^3 \\ \quad - 0.000608091 t^4 + 0.0000662304 t^5 - 1.3985210^{-7} t^6, \\ \varphi_6(E, t) = E_0 + E_1 + \dots + E_6 = 1. - 0.170946 t + 0.0128271 t^2 - 0.00304546 t^3 \\ \quad + 0.00092025 t^4 - 0.000141691 t^5 + 9.8220710^{-6} t^6, \\ \varphi_6(I, t) = I_0 + I_1 + \dots + I_6 = 1. + 0.228 t - 0.0557919 t^2 + 0.00513775 t^3 \\ \quad - 0.000545927 t^4 + 0.0000953322 t^5 - 0.000012574 t^6, \\ \varphi_6(R, t) = R_0 + R_1 + \dots + R_6 = 0.182 t + 0.020748 t^2 - 0.00338471042 t^3 \\ \quad + 0.000233767 t^4 - 0.0000198717 t^5 + 2.8917410^{-6} t^6, \end{cases}$$

5.1.5. Solving phase I by using VIM:

To solve the phase I by using the VIM, we follow similar steps given in subsection 3.4, and the approximate solution will be the same to the TAM given in Eq. (48).

5.2. Solving phase II for mathematical model of COVID-19

Let us consider the phase II:

$$(50) \quad \begin{cases} \frac{dS}{dt} = \frac{-\beta_1 SE}{M}, \\ \frac{dE}{dt} = \frac{\beta_1 SE}{M} - qE, \\ \frac{dQ}{dt} = qE - \sigma Q, \\ \frac{dR}{dt} = \sigma Q. \end{cases}$$

The initial conditions are

$$(51) \quad S(0) = 4, E(0) = 9, Q(0) = 4 \text{ and } R(0) = 2.$$

In current work, we consider the following values: $r_1 = 4, r_2 = 9, r_3 = 4, r_4 = 2, M = S + E + Q + R, \beta_1 = 0.208, q = 0.24, \sigma = 0.04$, see [1].

In this phase, the results for all the DJM, TAM, BCM and VIM are the same. Therefore, we selected the TAM to solve this phase.

5.2.1 Solving phase II by using TAM:

In order to apply the TAM to solve the model of phase II, we follow similar steps given in subsection 3.2.

From the Eqs. (50) and (51). We get:

$$\begin{cases} S_1 = 4 - 0.394105 t, \\ E_1 = 9 - 1.76589 t, \\ Q_1 = 4 + 2. t, \\ R_1 = 2 + 0.16 t. \end{cases}$$

So, we have

$$\begin{cases} S_2 = 4 - 0.394105 t + 0.0580787 t^2 - 0.0025396 t^3, \\ E_2 = 9 - 1.76589 t + 0.153829 t^2 + 0.0025396 t^3, \\ Q_2 = 4 + 2. t - 0.251907 t^2, \\ R_2 = 2 + 0.16 t + 0.04 t^2. \end{cases}$$

:

and so on. By continuing in this way, we get the following approximate solution for six iterations:

$$\begin{cases} S_6 = 4 - 0.394105 t + 0.0580787 t^2 - 0.00669239 t^3 + 0.000672915 t^4 - 0.0000605972 t^5 \\ \quad + 4.8347810^{-6} t^6, \\ E_6 = 9 - 1.76589 t + 0.153829 t^2 - 0.00561391 t^3 - 0.00033608 t^4 + 0.000076729 t^5 \\ \quad - 7.9039410^{-6} t^6, \\ Q_6 = 4 + 2. t - 0.251907 t^2 + 0.0156651 t^3 - 0.000493485 t^4 - 0.000012184 t^5 \\ \quad + 3.1503910^{-6} t^6, \\ R_6 = 2 + 0.16 t + 0.04 t^2 - 0.00335876 t^3 + 0.000156651 t^4 - 3.9478810^{-6} t^5 \\ \quad - 8.1226410^{-8} t^6, \end{cases}$$

5.2.2. Solving phase II by using ADM:

Let us consider the Eq. (50) with the given initial conditions given in the Eq. (51). Integrating both sides of Eq. (50) from 0 to t and using the given initial conditions, we have

$$(52) \quad \begin{cases} S(t) = 4 + L_1^{-1} \left(- \left(\frac{\beta_1}{M} A_n \right) \right), \\ E(t) = 9 + L_2^{-1} \left(\frac{\beta_1}{M} A_n - q E_n \right), \\ Q(t) = 4 + L_3^{-1} (q E_n - \sigma Q_n), \\ R(t) = 2 + L_4^{-1} (\sigma Q_n), \end{cases}$$

where A_n , is the Adomian polynomial, which evaluated from the nonlinear terms SE , as follows:

$$A_0 = S_0 E_0,$$

$$A_1 = S_1 E_0 + E_1 S_0,$$

$$(53) \quad A_2 = S_2 E_0 + S_1 E_1 + S_0 E_2,$$

:

By applying the ADM, we obtain

$$S_0(t) = 4, E_0(t) = 9, Q_0(t) = 4 \text{ and } R_0(t) = 2.$$

Also,

$$S_1(t) = -0.394105 t, E_1(t) = -1.76589 t, Q_1(t) = 2. t \text{ and } R_1(t) = 0.16 t.$$

$$S_2(t) = 0.0580787 t^2, E_2(t) = 0.153829 t^2, Q_2(t) = -0.251907 t^2 \text{ and } R_2(t) = 0.04 t^2.$$

Then, we get approximate solutions

$$\begin{cases} \varphi_1(S, t) = S_0 + S_1 = 4 - 0.394105 t, \\ \varphi_1(E, t) = E_0 + E_1 = 9 - 1.76589 t, \\ \varphi_1(Q, t) = Q_0 + Q_1 = 4 + 2. t, \\ \varphi_1(R, t) = R_0 + R_1 = 2 + 0.16 t, \end{cases}$$

and,

$$\begin{cases} \varphi_2(S, t) = S_0 + S_1 + S_2 = 4 - 0.394105 t + 0.0580787 t^2, \\ \varphi_2(E, t) = E_0 + E_1 + E_2 = 9 - 1.76589 t + 0.153829 t^2, \\ \varphi_2(Q, t) = Q_0 + Q_1 + Q_2 = 4 + 2. t - 0.251907 t^2, \\ \varphi_2(R, t) = R_0 + R_1 + R_2 = 2 + 0.16 t + 0.04 t^2, \end{cases}$$

So, the obtained approximated solution for six iterations by the ADM will be:

$$\begin{cases} \varphi_6(S, t) = S_0 + S_1 + \dots + S_6 = 4. - 0.394105 t + 0.0580787 t^2 - 0.00669239 t^3 + 0.000672915 t^4 \\ \quad - 0.0000605972 t^5 + 4.8347810^{-6} t^6, \\ \varphi_6(E, t) = E_0 + E_1 + \dots + E_6 = 9. - 1.76589 t + 0.153829 t^2 - 0.00561391 t^3 - 0.00033608 t^4 \\ \quad + 0.000076729 t^5 - 7.9039410^{-6} t^6, \\ \varphi_6(Q, t) = Q_0 + Q_1 + \dots + Q_6 = 4. + 2. t - 0.251907 t^2 + 0.0156651 t^3 - 0.000493485 t^4 \\ \quad - 0.000012184 t^5 + 3.1503910^{-6} t^6, \\ \varphi_6(R, t) = R_0 + R_1 + \dots + R_6 = 2. + 0.16 t + 0.04 t^2 - 0.00335876 t^3 + 0.000156651 t^4 \\ \quad - 3.9478810^{-6} t^5 - 8.1226410^{-8} t^6, \end{cases}$$

5.3. Solving phase III for mathematical model of COVID-19

The phase III is the same phase II, except replacing β_1 by β_2 .

$$(54) \quad \begin{cases} \frac{dS}{dt} = \frac{-\beta_2 SE}{M}, \\ \frac{dE}{dt} = \frac{\beta_2 SE}{M} - qE, \\ \frac{dQ}{dt} = qE - \sigma Q, \\ \frac{dR}{dt} = \sigma Q. \end{cases}$$

with the initial conditions

$$(55) \quad S(0) = 3, E(0) = 4, Q(0) = 6 \text{ and } R(0) = 4.$$

In current work, we consider the following values: $r_1 = 3, r_2 = 4, r_3 = 6, r_4 = 4, M = S + E + Q + R, \beta_2 = 0.1311, q = 0.229, \sigma = 0.051$, see[1].

Once again it was observed that the obtained series approximate solutions via the four methods proposed are the same. So, let solve this phase by using the BCM.

5.3.1 Solving phase III by using BCM:

To solve the phase III by using the BCM, we follow similar steps given in subsection 3.3, by implementing the inverse operator given in (10) and using the given initial conditions, we get:

$$\begin{aligned} & \left. \begin{aligned} S_1 &= 3 - 0.0925412 t, \\ E_1 &= 4 - 0.823459 t, \\ Q_1 &= 6 + 0.61 t, \\ R_1 &= 4 + 0.306 t, \end{aligned} \right\} \\ \text{and, } & \left\{ \begin{aligned} S_2 &= 3 - 0.0925412 t + 0.0109528 t^2 - 0.000195889 t^3, \\ E_2 &= 4 - 0.823459 t + 0.0833332 t^2 + 0.000195889 t^3, \\ Q_2 &= 6 + 0.61 t - 0.109841 t^2, \\ R_2 &= 4 + 0.306 t + 0.015555 t^2, \end{aligned} \right. \\ & : \end{aligned}$$

and so on. We obtain the following approximate solution for six iterations:

$$\left\{ \begin{aligned} S_6 &= 3 - 0.0925412 t + 0.0109528 t^2 - 0.000951156 t^3 + 0.0000708815 t^4 - 4.9303710^{-6} t^5 \\ & \quad + 3.2997910^{-7} t^6, \\ E_6 &= 4 - 0.823459 t + 0.0833332 t^2 - 0.00540995 t^3 + 0.000238838 t^4 - 6.0084210^{-6} t^5 \\ & \quad - 1.0065810^{-7} t^6, \\ Q_6 &= 6 + 0.61 t - 0.109841 t^2 + 0.0082284 t^3 - 0.000414632 t^4 + 0.000015168 t^5 \\ & \quad - 3.5824910^{-7} t^6, \\ R_6 &= 4 + 0.306 t + 0.015555 t^2 - 0.0018673 t^3 + 0.000104912 t^4 - 4.2292410^{-6} t^5 + 1.2892810^{-7} t^6, \end{aligned} \right.$$

5.3.2 Solving phase III by using ADM:

Let us consider the Eq. (54) with the initial conditions given in the Eq. (55). Integrating both sides of Eq. (54) from 0 to t and using the given initial conditions, we have

$$(56) \quad \begin{cases} S(t) = 3 + L_1^{-1} \left(- \left(\frac{\beta_2}{M} A_n \right) \right), \\ E(t) = 4 + L_2^{-1} \left(\frac{\beta_2}{M} A_n - q E_n \right), \\ Q(t) = 6 + L_3^{-1} (q E_n - \sigma Q_n), \\ R(t) = 4 + L_4^{-1} (\sigma Q_n), \end{cases}$$

where A_n , is the Adomian polynomial, which evaluated from the nonlinear terms SE , is the same in Eq. (53).

By applying the ADM, we obtain $S_0(t) = 3, E_0(t) = 4, Q_0(t) = 6$ and $R_0(t) = 4$. Also,

$$S_1(t) = -0.0925412 t, E_1(t) = -0.823459 t, Q_1(t) = 0.61 t \text{ and } R_1(t) = 0.306 t.$$

$$S_2(t) = 0.0109528 t^2, E_2(t) = 0.0833332 t^2, Q_2(t) = -0.109841 t^2 \text{ and } R_2(t) = 0.015555 t^2.$$

$$\text{Then, we get approximate solutions } \begin{cases} \varphi_1(S, t) = S_0 + S_1 = 3 - 0.0925412 t, \\ \varphi_1(E, t) = E_0 + E_1 = 4 - 0.823459 t, \\ \varphi_1(Q, t) = Q_0 + Q_1 = 6 + 0.61 t, \\ \varphi_1(R, t) = R_0 + R_1 = 4 + 0.306 t, \end{cases}$$

and,

$$\begin{cases} \varphi_2(S, t) = S_0 + S_1 + S_2 = 3 - 0.0925412 t + 0.0109528 t^2, \\ \varphi_2(E, t) = E_0 + E_1 + E_2 = 4 - 0.823459 t + 0.0833332 t^2, \\ \varphi_2(Q, t) = Q_0 + Q_1 + Q_2 = 6 + 0.61 t - 0.109841 t^2, \\ \varphi_2(R, t) = R_0 + R_1 + R_2 = 4 + 0.306 t + 0.015555 t^2, \end{cases}$$

So, the obtained approximated solution by the ADM for six iterations will be:

$$\begin{cases} \varphi_6(S, t) = S_0 + S_1 + \dots + S_6 = 3 - 0.0925412 t + 0.0109528 t^2 - 0.000951156 t^3 + 0.0000708815 t^4 \\ \quad - 4.9303710^{-6} t^5 + 3.2997910^{-7} t^6, \\ \varphi_6(E, t) = E_0 + E_1 + \dots + E_6 = 4 - 0.823459 t + 0.0833332 t^2 - 0.00540995 t^3 + 0.000238838 t^4 \\ \quad - 6.0084210^{-6} t^5 - 1.0065810^{-7} t^6, \\ \varphi_6(Q, t) = Q_0 + Q_1 + \dots + Q_6 = 6 + 0.61 t - 0.109841 t^2 + 0.0082284 t^3 - 0.000414632 t^4 \\ \quad + 0.000015168 t^5 - 3.5824910^{-7} t^6, \\ \varphi_6(R, t) = R_0 + R_1 + \dots + R_6 = 4 + 0.306 t + 0.015555 t^2 - 0.0018673 t^3 + 0.000104912 t^4 \\ \quad - 4.2292410^{-6} t^5 + 1.2892810^{-7} t^6, \end{cases}$$

6. NUMERICAL RESULTS AND CONVERGENCE

6.1. Numerical results

In this section, the accuracy of the approximate solutions obtained is investigated for the proposed methods for the three phases. the maximum remaining error MER_{in} will be calculated [18].

For Phase I: The error remainder of the Phase I can be defined as

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$$(57) \quad \begin{cases} ER_{1n} = S'_n - \left(\frac{-\beta S_n(kE_n + I_n)}{M} \right), \\ ER_{2n} = E'_n - \left(\frac{\beta S_n(kE_n + I_n)}{M} - qE_n \right), \\ ER_{3n} = I'_n - (qE_n - \gamma I_n), \\ ER_{4n} = R'_n - (\gamma I_n). \end{cases}$$

and the MER_{in} is:

$$(58) \quad MER_{in} = \max_{0 \leq t \leq 1} |ER_{in}(t)|, i = 1, 2, 3, 4.$$

Figs. 1-a, 1-b, 1-c and 1-d, show the comparison between the obtained results and the classical fourth order Runge-Kutta (RK4) method, a good agreement can be noticed for $S(t)$, $E(t)$, $I(t)$ and $R(t)$.

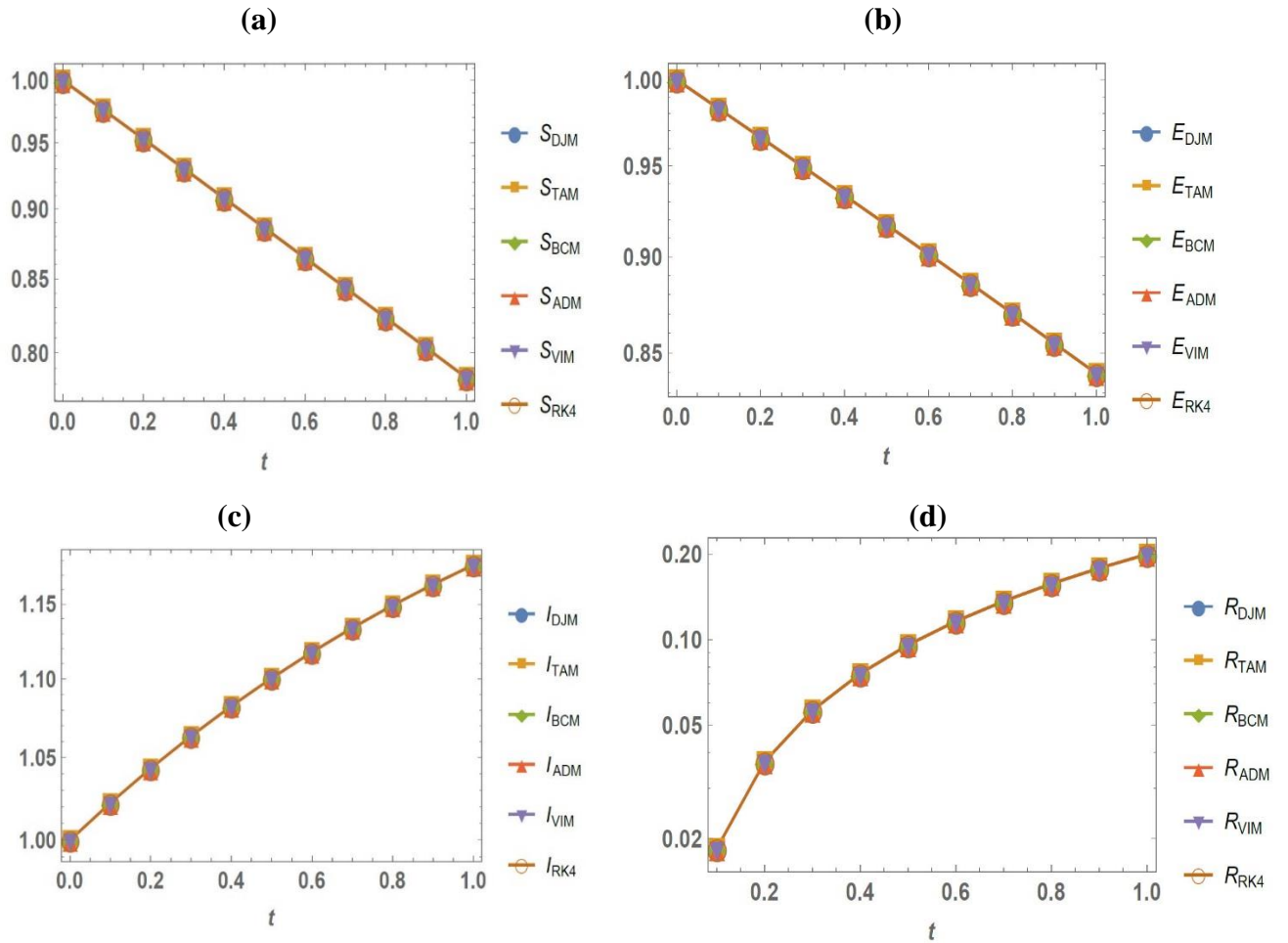


Fig. 1: (a) Comparison of the RK4 method with the proposed methods solution for $S(t)$.

(b) Comparison of the RK4 method with the proposed methods solution for $E(t)$.

(c) Comparison of the RK4 method with the proposed methods solution for $I(t)$.

(d) Comparison of the RK4 method with the proposed methods solution for $R(t)$.

The values of MER_{1n} , MER_{2n} , MER_{3n} and MER_{4n} for phase I which are obtained by proposed four methods : DJM, TAM, BCM and VIM compared with the resulted by the ADM in the table 1, where the values of n is increasing from 1 to 6. It can be seen that the errors of the DJM, TAM, BCM and VIM are less than the ADM.

Table 1:The maximal error remainder: MER_{in} by the (DJM, TAM, BCM and VIM) and ADM for phase I.

n	MER_{1n} for $S(t)$		MER_{2n} for $E(t)$	
	The proposed four methods	ADM	The proposed four methods	ADM
1	0.0474728	0.0474728	0.022615	0.022615
2	0.000943276	0.00226145	0.00390048	0.00752056
3	0.000366545	0.00222427	0.000972352	0.00347291
4	0.0000444335	0.000339584	0.00015223	0.000716887
5	3.5806810^{-6}	6.2623910^{-6}	0.0000166732	0.0000643557
6	2.7568910^{-7}	4.8716610^{-6}	1.4537710^{-6}	8.4461810^{-7}
n	MER_{3n} for $I(t)$		MER_{4n} for $R(t)$	
	The proposed four methods	ADM	The proposed four methods	ADM
1	0.111584	0.111584	0.041496	0.041496
2	0.0149979	0.0154132	0.0101541	0.0101541
3	0.00152198	0.00218371	0.000916171	0.00093507
4	0.000179234	0.000476661	0.000071437	0.0000993587
5	0.0000198693	0.0000754437	6.7767910^{-6}	0.0000173505
6	1.7995410^{-6}	6.3155110^{-6}	6.2145910^{-7}	2.2884610^{-6}

For Phase II: To test the accuracy of the approximate solution, the remaining error function is defined as:

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$$(59) \quad \begin{cases} ER_{1n} = S'_n - \left(\frac{-\beta_1 S_n E_n}{M}\right), \\ ER_{2n} = E'_n - \left(\frac{\beta_1 S_n E_n}{M} - qE_n\right), \\ ER_{3n} = Q'_n - (qE_n - \sigma Q_n), \\ ER_{4n} = R'_n - (\sigma Q_n). \end{cases}$$

and the MER_{in} is:

$$(60) \quad MER_{in} = \max_{0 \leq t \leq 1} |ER_{in}(t)|, i = 1, 2, 3, 4.$$

Moreover, the numerical comparison between the obtained approximate solutions and the RK4 for phase II are presented in Figs. 2-a, 2-b, 2-c and 2-d, and a good agreement can be clearly seen.

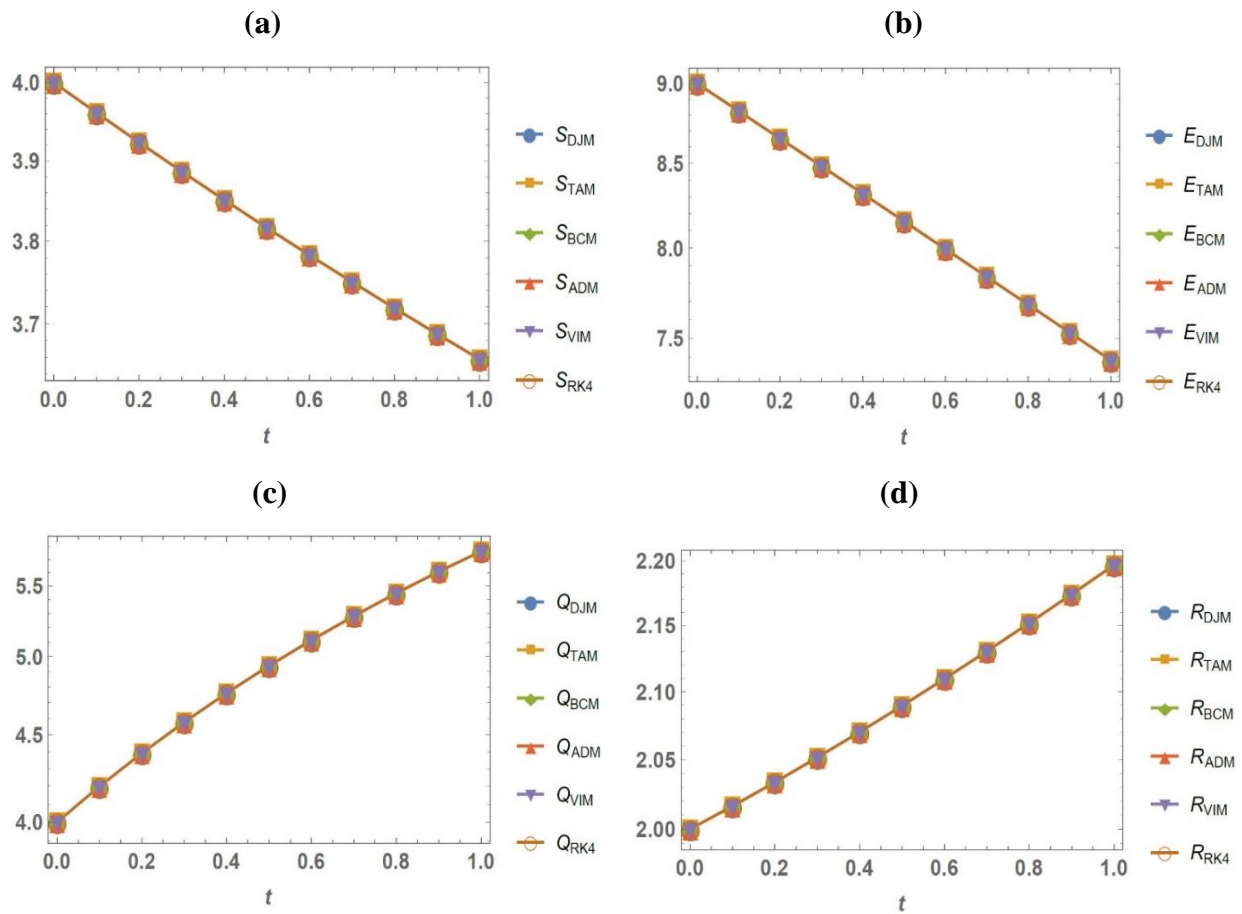


Fig. 2: (a) Comparison of the RK4 method with the proposed methods solution for $S(t)$.

(b) Comparison of the RK4 method with the proposed methods solution for $E(t)$.

(c) Comparison of the RK4 method with the proposed methods solution for $Q(t)$.

(d) Comparison of the RK4 method with the proposed methods solution for $R(t)$.

Also, the values of MER_{1n} , MER_{2n} , MER_{3n} and MER_{4n} for phase II which are obtained by the DJM, TAM, BCM and VIM are compared with the ADM in the table 2. It can be also seen the errors of the proposed methods are decreasing when increasing the iterations. Moreover, the errors of the DJM, TAM, BCM and VIM are less than the error of the ADM.

Table 2: The maximal error remainder: MER_{in} by the (DJM, TAM, BCM, VIM) and the ADM for phase II.

n	MER_{1n} for $S(t)$		MER_{2n} for $E(t)$	
	The proposed four methods	ADM	The proposed four methods	ADM
1	0.108539	0.108539	0.315276	0.315276
2	0.0106661	0.0183885	0.0268623	0.0185304
3	0.000649997	0.00245468	0.00145253	0.00110735
4	0.0000276022	0.000277899	0.0000566017	0.000358559
5	8.8845710^{-7}	0.0000269111	1.7018510^{-6}	0.0000453261
6	2.2780410^{-8}	2.1718110^{-6}	4.1168610^{-8}	4.0687610^{-6}
n	MER_{3n} for $Q(t)$		MER_{4n} for $R(t)$	
	The proposed four methods	ADM	The proposed four methods	ADM
1	0.503815	0.503815	0.08	0.08
2	0.0476047	0.0469952	0.0100763	0.0100763
3	0.00273522	0.00197394	0.000632697	0.000626602
4	0.000111261	0.0000609198	0.0000270571	0.0000197394
5	3.4627310^{-6}	0.0000189023	8.7241710^{-7}	4.8735810^{-7}
6	8.6332110^{-8}	2.0229610^{-6}	2.238310^{-8}	1.2601510^{-7}

For phase III: To examine the accuracy for the approximate solution for phase III, the error remainder function is same as Eq. (59) and Eq.(60), with the replacing of β_1 by β_2 .

The numerical comparison between the obtained solutions by the proposed methods, and the RK4 for phase III, are shown in Figs.3-a, 3-b, 3-c and 3-d, once again a good agreement between the solution can be noticed.

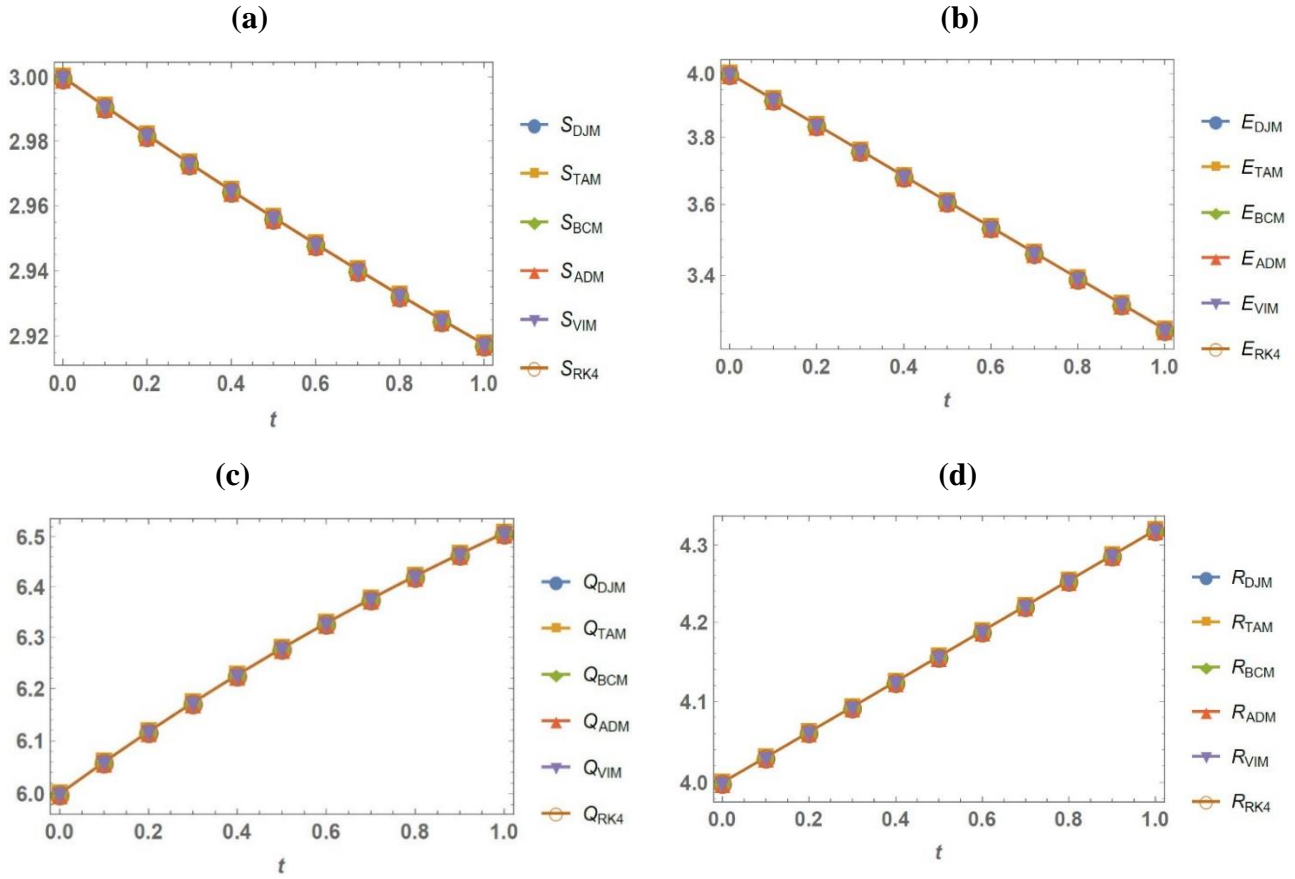


Fig. 3: (a) Comparison of the RK4 method with the proposed methods solution for $S(t)$.
 (b) Comparison of the RK4 method with the proposed methods solution for $E(t)$.
 (c) Comparison of the RK4 method with the proposed methods solution for $Q(t)$.
 (d) Comparison of the RK4 method with the proposed methods solution for $R(t)$.

Finally, the values of MER_{1n} , MER_{2n} , MER_{3n} and MER_{4n} for phase III which are obtained by the DJM, TAM, BCM and VIM are compared with the ADM in the table 3, less errors are obtained in all methods than the ADM.

Table 3: The maximal error remainder: MER_{in} by the (DJM, TAM, BCM, VIM) and ADM for phase III.

n	MER_{1n} for $S(t)$		MER_{2n} for $E(t)$	
	The proposed four methods	ADM	The proposed four methods	ADM
1	0.0213179	0.0213179	0.167254	0.167254
2	0.0021433	0.00273148	0.0169849	0.0163518
3	0.000145284	0.000267615	0.00114813	0.000971263
4	7.3598710^{-6}	0.0000230639	0.000058182	0.00003163
5	2.9820910^{-7}	1.8468810^{-6}	2.3581810^{-6}	4.7095710^{-7}
6	1.0068310^{-8}	1.3900610^{-7}	7.9638610^{-8}	1.6205610^{-7}
n	MER_{3n} for $Q(t)$		MER_{4n} for $R(t)$	
	The proposed four methods	ADM	The proposed four methods	ADM
1	0.219682	0.219682	0.03111	0.03111
2	0.0247301	0.0246852	0.00560189	0.00560189
3	0.00171363	0.00165853	0.00042022	0.000419648
4	0.0000873652	0.0000758401	0.0000218233	0.0000211462
5	3.5460310^{-6}	2.149510^{-6}	8.8964410^{-7}	7.7356910^{-7}
6	1.1978710^{-7}	4.7798510^{-8}	3.0080510^{-8}	1.8270710^{-8}

6.2. The convergence of the proposed methods

For phase I: The solutions obtained from the DJM, TAM, BCM, VIM are same. So we will choose the DJM to prove the convergence analysis, let choose the same components obtained by the DJM. By using the terms given by the series $\sum_{j=0}^{\infty} v_j$ in Eq.(41) to satisfy the convergent conditions by evaluating the values of ϑ_j^i , we get

For the $S(t)$, we have

$$\vartheta_0^1 = \frac{\|v_1\|}{\|v_0\|} = 0.239054 < 1, \quad \vartheta_1^1 = \frac{\|v_2\|}{\|v_1\|} = 0.0971742 < 1,$$

$$\vartheta_2^1 = \frac{\|v_3\|}{\|v_2\|} = 0.00738787 < 1, \quad \vartheta_3^1 = \frac{\|v_4\|}{\|v_3\|} = 0.584993 < 1,$$

$$\vartheta_4^1 = \frac{\|v_5\|}{\|v_4\|} = 0.0924299 < 1, \quad \vartheta_5^1 = \frac{\|v_6\|}{\|v_5\|} = 0.0666509 < 1.$$

Also, for the $E(t)$, we have

$$\vartheta_0^2 = \frac{\|v_1\|}{\|v_0\|} = 0.170946 < 1, \quad \vartheta_1^2 = \frac{\|v_2\|}{\|v_1\|} = 0.0691098 < 1,$$

$$\vartheta_2^2 = \frac{\|v_3\|}{\|v_2\|} = 0.12507 < 1, \quad \vartheta_3^2 = \frac{\|v_4\|}{\|v_3\|} = 0.177939 < 1,$$

$$\vartheta_4^2 = \frac{\|v_5\|}{\|v_4\|} = 0.121456 < 1, \quad \vartheta_5^2 = \frac{\|v_6\|}{\|v_5\|} = 0.0899812 < 1.$$

So, for the $I(t)$, we have

$$\vartheta_0^3 = \frac{\|v_1\|}{\|v_0\|} = 0.228 < 1, \quad \vartheta_1^3 = \frac{\|v_2\|}{\|v_1\|} = 0.244701 < 1,$$

$$\vartheta_2^3 = \frac{\|v_3\|}{\|v_2\|} = 0.0902265 < 1, \quad \vartheta_3^3 = \frac{\|v_4\|}{\|v_3\|} = 0.0779734 < 1,$$

$$\vartheta_4^3 = \frac{\|v_5\|}{\|v_4\|} = 0.0948639 < 1, \quad \vartheta_5^3 = \frac{\|v_6\|}{\|v_5\|} = 0.0917041 < 1.$$

For the $R(t)$, we have

$$\vartheta_0^4 = \frac{\|v_1\|}{\|v_0\|} = 0, \quad \vartheta_1^4 = \frac{\|v_2\|}{\|v_1\|} = 0.114 < 1,$$

$$\vartheta_2^4 = \frac{\|v_3\|}{\|v_2\|} = 0.163134 < 1, \quad \vartheta_3^4 = \frac{\|v_4\|}{\|v_3\|} = 0.067949 < 1,$$

$$\vartheta_4^4 = \frac{\|v_5\|}{\|v_4\|} = 0.0634021 < 1, \quad \vartheta_5^4 = \frac{\|v_6\|}{\|v_5\|} = 0.0795336 < 1.$$

In order to prove the convergence analysis for the ADM, let choose the same components obtained by the ADM.

By using these iterations for computing the values of ϑ_j^i , we get

For the $S(t)$, we have

$$\vartheta_0^1 = \frac{\|v_1\|}{\|v_0\|} = 0.239054 < 1, \quad \vartheta_1^1 = \frac{\|v_2\|}{\|v_1\|} = 0.0929365 < 1,$$

$$\vartheta_2^1 = \frac{\|v_3\|}{\|v_2\|} = 0.058173 < 1, \quad \vartheta_3^1 = \frac{\|v_4\|}{\|v_3\|} = 0.470505 < 1,$$

$$\vartheta_4^1 = \frac{\|v_5\|}{\|v_4\|} = 0.108915 < 1, \quad \vartheta_5^1 = \frac{\|v_6\|}{\|v_5\|} = 0.00211159 < 1.$$

Also, for the $E(t)$, we have

$$\begin{aligned} \vartheta_0^2 &= \frac{\|v_1\|}{\|v_0\|} = 0.170946 < 1, & \vartheta_1^2 &= \frac{\|v_2\|}{\|v_1\|} = 0.0750359 < 1, \\ \vartheta_2^2 &= \frac{\|v_3\|}{\|v_2\|} = 0.237424 < 1, & \vartheta_3^2 &= \frac{\|v_4\|}{\|v_3\|} = 0.302171 < 1, \\ \vartheta_4^2 &= \frac{\|v_5\|}{\|v_4\|} = 0.15397 < 1, & \vartheta_5^2 &= \frac{\|v_6\|}{\|v_5\|} = 0.0693204 < 1. \end{aligned}$$

So, for the $I(t)$, we have

$$\begin{aligned} \vartheta_0^3 &= \frac{\|v_1\|}{\|v_0\|} = 0.228 < 1, & \vartheta_1^3 &= \frac{\|v_2\|}{\|v_1\|} = 0.244701 < 1, \\ \vartheta_2^3 &= \frac{\|v_3\|}{\|v_2\|} = 0.0920876 < 1, & \vartheta_3^3 &= \frac{\|v_4\|}{\|v_3\|} = 0.106258 < 1, \\ \vartheta_4^3 &= \frac{\|v_5\|}{\|v_4\|} = 0.174625 < 1, & \vartheta_5^3 &= \frac{\|v_6\|}{\|v_5\|} = 0.131896 < 1. \end{aligned}$$

For the $R(t)$, we have

$$\begin{aligned} \vartheta_0^4 &= \frac{\|v_1\|}{\|v_0\|} = 0, & \vartheta_1^4 &= \frac{\|v_2\|}{\|v_1\|} = 0.114 < 1, \\ \vartheta_2^4 &= \frac{\|v_3\|}{\|v_2\|} = 0.163134 < 1, & \vartheta_3^4 &= \frac{\|v_4\|}{\|v_3\|} = 0.0690657 < 1, \\ \vartheta_4^4 &= \frac{\|v_5\|}{\|v_4\|} = 0.0850064 < 1, & \vartheta_5^4 &= \frac{\|v_6\|}{\|v_5\|} = 0.14552 < 1. \end{aligned}$$

The ϑ_j^i values are less than one, for $j \geq 0$, $0 < t \leq 1$. Hence, the DJM and ADM are approach satisfied the convergence condition.

For phase II: In order to prove the convergence analysis for the four proposed methods which provided the same results, we choose the TAM, we have applied the process as presented in section 4. The obtained ϑ_j^i values are

For the $S(t)$, we have

$$\begin{aligned}\vartheta_0^1 &= \frac{\|v_1\|}{\|v_0\|} = 0.0985263 < 1, & \vartheta_1^1 &= \frac{\|v_2\|}{\|v_1\|} = 0.140924 < 1, \\ \vartheta_2^1 &= \frac{\|v_3\|}{\|v_2\|} = 0.0665865 < 1, & \vartheta_3^1 &= \frac{\|v_4\|}{\|v_3\|} = 0.0453493 < 1, \\ \vartheta_4^1 &= \frac{\|v_5\|}{\|v_4\|} = 0.0336642 < 1, & \vartheta_5^1 &= \frac{\|v_6\|}{\|v_5\|} = 0.0265429 < 1.\end{aligned}$$

Also, for the $E(t)$, we have

$$\begin{aligned}\vartheta_0^2 &= \frac{\|v_1\|}{\|v_0\|} = 0.196211 < 1, & \vartheta_1^2 &= \frac{\|v_2\|}{\|v_1\|} = 0.088549 < 1, \\ \vartheta_2^2 &= \frac{\|v_3\|}{\|v_2\|} = 0.0560249 < 1, & \vartheta_3^2 &= \frac{\|v_4\|}{\|v_3\|} = 0.0400489 < 1, \\ \vartheta_4^2 &= \frac{\|v_5\|}{\|v_4\|} = 0.0307624 < 1, & \vartheta_5^2 &= \frac{\|v_6\|}{\|v_5\|} = 0.0246878 < 1.\end{aligned}$$

Moreover, for the $Q(t)$, we have

$$\begin{aligned}\vartheta_0^3 &= \frac{\|v_1\|}{\|v_0\|} = 0.5 < 1, & \vartheta_1^3 &= \frac{\|v_2\|}{\|v_1\|} = 0.125954 < 1, \\ \vartheta_2^3 &= \frac{\|v_3\|}{\|v_2\|} = 0.0627907 < 1, & \vartheta_3^3 &= \frac{\|v_4\|}{\|v_3\|} = 0.0427647 < 1, \\ \vartheta_4^3 &= \frac{\|v_5\|}{\|v_4\|} = 0.0322435 < 1, & \vartheta_5^3 &= \frac{\|v_6\|}{\|v_5\|} = 0.0256563 < 1.\end{aligned}$$

In addition, for the $R(t)$, we have

$$\begin{aligned}\vartheta_0^4 &= \frac{\|v_1\|}{\|v_0\|} = 0.08 < 1, & \vartheta_1^4 &= \frac{\|v_2\|}{\|v_1\|} = 0.25 < 1, \\ \vartheta_2^4 &= \frac{\|v_3\|}{\|v_2\|} = 0.0839691 < 1, & \vartheta_3^4 &= \frac{\|v_4\|}{\|v_3\|} = 0.0470023 < 1, \\ \vartheta_4^4 &= \frac{\|v_5\|}{\|v_4\|} = 0.034026 < 1, & \vartheta_5^4 &= \frac{\|v_6\|}{\|v_5\|} = 0.0266705 < 1.\end{aligned}$$

To prove the convergence analysis for the ADM, we evaluated the values of ϑ_j^i for the components obtained by the ADM. Therefore, we get:

For the $S(t)$,

$$\begin{aligned}\vartheta_0^1 &= \frac{\|v_1\|}{\|v_0\|} = 0.0985263 < 1, & \vartheta_1^1 &= \frac{\|v_2\|}{\|v_1\|} = 0.147368 < 1, \\ \vartheta_2^1 &= \frac{\|v_3\|}{\|v_2\|} = 0.11523 < 1, & \vartheta_3^1 &= \frac{\|v_4\|}{\|v_3\|} = 0.100549 < 1, \\ \vartheta_4^1 &= \frac{\|v_5\|}{\|v_4\|} = 0.0900518 < 1, & \vartheta_5^1 &= \frac{\|v_6\|}{\|v_5\|} = 0.0797856 < 1.\end{aligned}$$

Also, for the $E(t)$, we have

$$\begin{aligned}\vartheta_0^2 &= \frac{\|v_1\|}{\|v_0\|} = 0.196211 < 1, & \vartheta_1^2 &= \frac{\|v_2\|}{\|v_1\|} = 0.0871109 < 1, \\ \vartheta_2^2 &= \frac{\|v_3\|}{\|v_2\|} = 0.0364946 < 1, & \vartheta_3^2 &= \frac{\|v_4\|}{\|v_3\|} = 0.0598656 < 1, \\ \vartheta_4^2 &= \frac{\|v_5\|}{\|v_4\|} = 0.228306 < 1, & \vartheta_5^2 &= \frac{\|v_6\|}{\|v_5\|} = 0.103011 < 1.\end{aligned}$$

Moreover, for the $Q(t)$, we have

$$\begin{aligned}\vartheta_0^3 &= \frac{\|v_1\|}{\|v_0\|} = 0.5 < 1, & \vartheta_1^3 &= \frac{\|v_2\|}{\|v_1\|} = 0.125954 < 1, \\ \vartheta_2^3 &= \frac{\|v_3\|}{\|v_2\|} = 0.0621858 < 1, & \vartheta_3^3 &= \frac{\|v_4\|}{\|v_3\|} = 0.0315023 < 1, \\ \vartheta_4^3 &= \frac{\|v_5\|}{\|v_4\|} = 0.0246896 < 1, & \vartheta_5^3 &= \frac{\|v_6\|}{\|v_5\|} = 0.258568 < 1.\end{aligned}$$

Finally, for the $R(t)$, we have

$$\begin{aligned}\vartheta_0^4 &= \frac{\|v_1\|}{\|v_0\|} = 0.08 < 1, & \vartheta_1^4 &= \frac{\|v_2\|}{\|v_1\|} = 0.25 < 1, \\ \vartheta_2^4 &= \frac{\|v_3\|}{\|v_2\|} = 0.0839691 < 1, & \vartheta_3^4 &= \frac{\|v_4\|}{\|v_3\|} = 0.0466393 < 1, \\ \vartheta_4^4 &= \frac{\|v_5\|}{\|v_4\|} = 0.0252018 < 1, & \vartheta_5^4 &= \frac{\|v_6\|}{\|v_5\|} = 0.0205747 < 1.\end{aligned}$$

The ϑ_j^i values are less than one, for $j \geq 0$, $0 < t \leq 1$. Hence, the TAM and ADM are satisfied the convergence condition.

For phase III: To prove the convergence analysis for four proposed methods which provided the same results, we choose the BCM, we have implemented the process as in section 4. The obtained ϑ_j^i values are

For the $S(t)$, we have

$$\begin{aligned}\vartheta_0^1 &= \frac{\|v_1\|}{\|v_0\|} = 0.0308471 < 1, & \vartheta_1^1 &= \frac{\|v_2\|}{\|v_1\|} = 0.116239 < 1, \\ \vartheta_2^1 &= \frac{\|v_3\|}{\|v_2\|} = 0.0673282 < 1, & \vartheta_3^1 &= \frac{\|v_4\|}{\|v_3\|} = 0.0506346 < 1, \\ \vartheta_4^1 &= \frac{\|v_5\|}{\|v_4\|} = 0.0404352 < 1, & \vartheta_5^1 &= \frac{\|v_6\|}{\|v_5\|} = 0.0337091 < 1.\end{aligned}$$

Also, for the $E(t)$, we have

$$\begin{aligned}\vartheta_0^2 &= \frac{\|v_1\|}{\|v_0\|} = 0.205865 < 1, & \vartheta_1^2 &= \frac{\|v_2\|}{\|v_1\|} = 0.101437 < 1, \\ \vartheta_2^2 &= \frac{\|v_3\|}{\|v_2\|} = 0.067618 < 1, & \vartheta_3^2 &= \frac{\|v_4\|}{\|v_3\|} = 0.0506737 < 1, \\ \vartheta_4^2 &= \frac{\|v_5\|}{\|v_4\|} = 0.0405296 < 1, & \vartheta_5^2 &= \frac{\|v_6\|}{\|v_5\|} = 0.0337703 < 1.\end{aligned}$$

Moreover, for the $Q(t)$, we have

$$\begin{aligned}\vartheta_0^3 &= \frac{\|v_1\|}{\|v_0\|} = 0.101667 < 1, & \vartheta_1^3 &= \frac{\|v_2\|}{\|v_1\|} = 0.180067 < 1, \\ \vartheta_2^3 &= \frac{\|v_3\|}{\|v_2\|} = 0.075014 < 1, & \vartheta_3^3 &= \frac{\|v_4\|}{\|v_3\|} = 0.0519329 < 1, \\ \vartheta_4^3 &= \frac{\|v_5\|}{\|v_4\|} = 0.0407658 < 1, & \vartheta_5^3 &= \frac{\|v_6\|}{\|v_5\|} = 0.0338118 < 1.\end{aligned}$$

In addition, for the $R(t)$, we have

$$\begin{aligned}\vartheta_0^4 &= \frac{\|v_1\|}{\|v_0\|} = 0.0765 < 1, & \vartheta_1^4 &= \frac{\|v_2\|}{\|v_1\|} = 0.0508333 < 1, \\ \vartheta_2^4 &= \frac{\|v_3\|}{\|v_2\|} = 0.120045 < 1, & \vartheta_3^4 &= \frac{\|v_4\|}{\|v_3\|} = 0.0562452 < 1, \\ \vartheta_4^4 &= \frac{\|v_5\|}{\|v_4\|} = 0.041525 < 1, & \vartheta_5^4 &= \frac{\|v_6\|}{\|v_5\|} = 0.0339573 < 1.\end{aligned}$$

To prove the convergence analysis for the ADM, we will computed the values of ϑ_j^i , we have:

For the $S(t)$,

$$\begin{aligned}\vartheta_0^1 &= \frac{\|v_1\|}{\|v_0\|} = 0.0308471 < 1, & \vartheta_1^1 &= \frac{\|v_2\|}{\|v_1\|} = 0.118356 < 1, \\ \vartheta_2^1 &= \frac{\|v_3\|}{\|v_2\|} = 0.0868414 < 1, & \vartheta_3^1 &= \frac{\|v_4\|}{\|v_3\|} = 0.0745215 < 1, \\ \vartheta_4^1 &= \frac{\|v_5\|}{\|v_4\|} = 0.0695578 < 1, & \vartheta_5^1 &= \frac{\|v_6\|}{\|v_5\|} = 0.0669278 < 1.\end{aligned}$$

Also, for the $E(t)$, we have

$$\begin{aligned}\vartheta_0^2 &= \frac{\|v_1\|}{\|v_0\|} = 0.205865 < 1, & \vartheta_1^2 &= \frac{\|v_2\|}{\|v_1\|} = 0.101199 < 1, \\ \vartheta_2^2 &= \frac{\|v_3\|}{\|v_2\|} = 0.0649195 < 1, & \vartheta_3^2 &= \frac{\|v_4\|}{\|v_3\|} = 0.0441479 < 1, \\ \vartheta_4^2 &= \frac{\|v_5\|}{\|v_4\|} = 0.0251569 < 1, & \vartheta_5^2 &= \frac{\|v_6\|}{\|v_5\|} = 0.0167528 < 1.\end{aligned}$$

and for the $Q(t)$, we have

$$\begin{aligned}\vartheta_0^3 &= \frac{\|v_1\|}{\|v_0\|} = 0.101667 < 1, & \vartheta_1^3 &= \frac{\|v_2\|}{\|v_1\|} = 0.180067 < 1, \\ \vartheta_2^3 &= \frac{\|v_3\|}{\|v_2\|} = 0.0749119 < 1, & \vartheta_3^3 &= \frac{\|v_4\|}{\|v_3\|} = 0.0503903 < 1, \\ \vartheta_4^3 &= \frac{\|v_5\|}{\|v_4\|} = 0.0365819 < 1, & \vartheta_5^3 &= \frac{\|v_6\|}{\|v_5\|} = 0.0236187 < 1.\end{aligned}$$

Finally, for the $R(t)$, we have

$$\begin{aligned}\vartheta_0^4 &= \frac{\|v_1\|}{\|v_0\|} = 0.0765 < 1, & \vartheta_1^4 &= \frac{\|v_2\|}{\|v_1\|} = 0.0508333 < 1, \\ \vartheta_2^4 &= \frac{\|v_3\|}{\|v_2\|} = 0.120045 < 1, & \vartheta_3^4 &= \frac{\|v_4\|}{\|v_3\|} = 0.0561839 < 1, \\ \vartheta_4^4 &= \frac{\|v_5\|}{\|v_4\|} = 0.040312 < 1, & \vartheta_5^4 &= \frac{\|v_6\|}{\|v_5\|} = 0.0304849 < 1.\end{aligned}$$

The ϑ_j^i values are less than one, for $j \geq 0$, $0 < t \leq 1$. Hence, the BCM and ADM are satisfied the convergence condition.

7. CONCLUSION

This paper implemented five iterative methods: the DJM, TAM, BCM, VIM and ADM to solve the mathematical models that represented the coronavirus pandemic COVID-19 in three phases. The obtained approximate solutions were presented in a series terms. Moreover, the maximum errors remainder were calculated to verify the convergence of the obtained solutions and it appeared the errors for the DJM, TAM, BCM and VIM are the same and less than the ADM. Furthermore, the convergence of the proposed methods was demonstrated based on used the Banach fixed point theorem. In addition, the obtained numerical results were compared with the fourth order Runge-Kutta (RK4) method and good matches were achieved.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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