Available online at http://scik.org
Commun. Math. Biol. Neurosci. 2020, 2020:91
https://doi.org/10.28919/cmbn/5029
ISSN: 2052-2541

MATHEMATICAL MODELING OF CANCER AND HEPATITIS CO-DYNAMICS WITH NON-LOCAL AND NON-SINGULAR KERNEL

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Abstract. Cancer and hepatitis are increasingly becoming a global threat and reducing the total workforce in the world. Atangana-Baleanu in Caputo sense (ABC) is employed to examine the co-infection model of cancer and hepatitis dynamics. The Banach space theory is used to establish the existence and uniqueness solutions of the co-infected model. The stability analysis and reproductive number is investigated. Numerical simulation based on Adams-Moulton rule is made use to obtain qualitative information on the co-infected model. The numerical solution results suggest that the fractional order values and the chosen parameter values have influence on the dynamics of cancer and hepatitis cells in the human body.

Keywords: Mitta-Leffler; Atangana-Baleanu; hepatitis; existence and uniqueness; cancer; dynamics.

2010 AMS Subject Classification: 26A33, 37N25, 93D20.

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Received September 16, 2020

1. INTRODUCTION

Cancer has become very dangerous to human race across the entire world. This disease may be caused by human and environmental factors. Hepatitis of all types have been identified as one of the main causes of cancer in the world. It can be stated that large number of people get infected with cancer related problem through several factors. It is estimated that 90.5 million people had cancer related disease over the globe leading to 8.8 million human death each year [1–3]. Studies have shown that about 22% of cancer death may be related to hepatitis disease of any type [1, 4] . The hepatitis family cause inflammation of the liver tissue which end up being chronic or acute [3]. Hepatitis A and E are mostly contracted through contaminated food [5, 6]. Hepatitis B can be obtained through sexual intercourse or though childbirth [1, 6]. Hepatitis C is obtained from contaminated blood. For someone to be infected with hepatitis D, the person must have been then infected with hepatitis B [1, 7].

Quality information about etiology of the co-infection would go a long way to help health practitioner to make quality decision making [1]. There have been countless number of mathematical models studies on cancer and hepatitis in the quest for providing qualitative information in the context of integer. In recent times, non-integer models have gained serious attention to researchers due to its quality predictions. Several fractional derivatives have been developed and applied in real life problem and Atangana-Beleanu in Caputo sense based on non-local and non-singular kernel [8–11]. This operator has the ability to shift from one function to one another when stretched making it more accurate for predictions purposes [11] .

Baleanu et al. [12] used fractional optimal control to examine the best strategy for tumor-immune surveillance with non-singular operator. The authors in [13] constructed a system of partial differential equation to explore the dynamics of cancer tumor. Farman et al. [14] developed a fractional derivative model with sole emphasis on vaccine as a strategy of controlling cancer spread in the cells. Makhlouf et al. [15] investigated the role of CD4+T cells in tumor-immune interactions. Shi et al. [16] explored the dynamics of hepatitis B virus with holling II function response. Ahmed and El-Saka [5] developed a mathematical model on fractional dynamics of hepatitis C in order to obtain some qualitative information about the disease. Alzahrani and Khan [6] developed a hepatitis E mathematical model with optimal control for

the purpose of establishing the best strategy in minimizing the spread of the disease. Saad et al. [7] constructed a fractional mathematical model with emphasis on chronic hepatitis C virus infection model with immune response. Saidalieva and Hidirova [17] presented a hepatitis D virus mathematical model hinged on co-infection and super-infection with the aim of obtaining some qualitative information on the disease.

Mtisi et al. [18] constructed a co-infection model of malaria and tuberculosis to investigate the dynamics of the two diseases. Mukandavire et al. [19] developed a mathematical analysis of a model for HIV-malaria co-infection and obtain some dynamical qualitative information. Bonyah and Asiedu [20] used fractional derivative to study the dynamics of filariasis-schistosomiasis co-infection and presented uniqueness and existence of solutions of the co-infection model. Sanga et al. [21] developed mathematical model that examined the co-dynamics of cervical cancer and HIV diseases and obtained reproduction number and numerical results that presented some useful qualitative information. Moualeu et al. [22] constructed a co-infection model of HIV and hepatitis and examined the dynamics of these disease and explored numerical dynamical behavior of these diseases.

The aim of this work is to examine the uniqueness and existence of solution of co-infection of cancer and hepatitis diseases in the context of Mittag-Leffler function and to present numerical solution for qualitative information about the co-infection.

2. Mathematical Model Formulation

Mathematical modeling as a concept has been widely studied by many researchers [23–25]. The model is a modified version by Abiodun et al. [1] in which the total cells population $N(t)$ is partitioned into the following sub-populations of susceptible cells $(X(t))$, infected cancer cells $(Y(t))$, infected cells with hepatitis virus $(Z(t))$, recovered cells from cancer $(R_Y(t))$ and recovered cells from hepatitis virus $(R_Z(t))$. The respective cells growth rate is denoted by r_i and K is the carrying capacity. α and ρ are disease induced death rate for cancer and hepatitis virus correspondingly. γ and σ are the fractions of recovered cells from cancer and hepatitis virus respectively that become susceptible cells. ω and ϕ are proportion of cells that recovered from cancer and hepatitis virus in that order. The b and d parameters are the fraction of cells that are infected with cancer and hepatitis virus correspondingly. The following nonlinear differential

equation represents the interactions among the various compartments.

$$
\begin{aligned}
\frac{dX}{dt} &= r_1 X\left(1-\tfrac{X}{K}\right) - bXY - dXZ + \sigma R_Z + \gamma R_Y - \mu X,\\
\frac{dY}{dt} &= r_2 Y\left(1-\tfrac{Y}{K}\right) + bXY - (\alpha+\omega+\mu)Y,\\
\frac{dZ}{dt} &= r_3 Z\left(1-\tfrac{Z}{K}\right) + dXZ - (\rho+\phi+\mu)Z,\\
\frac{dR_Y}{dt} &= \omega Y - (\gamma+\mu)R_Y,\\
\frac{dR_Z}{dt} &= \phi Z - (\sigma+\mu)R_Z.
\end{aligned}
\tag{1}
$$

with the following initial conditions

$X_0 = X(0), Y_0 = Y(0), Z_0 = Z(0), R_Y 0 = R_Y(0), R_Z 0 = R_Z(0)$.

Definition 1. [8] The integral operator under the Atangana Baleanu-Caputo Sense is defined by the following expression:

$$
{}^{\mathrm{ABC}}\mathbb{D}^{\chi}_{0,t}[f(t)] = \frac{1-\chi}{\mathbf{B}(\chi)} f(t) + \frac{\chi}{\mathbf{B}(\chi)\Gamma(\chi)} \int_0^t f(s)(t-s)^{\chi-1} ds. \tag{2}
$$

Governing equations of the model. Taking into account the equation (2) to system (1) and applying the ABC operator, we obtain the following:

$$
\begin{cases}
{}^{\mathrm{ABC}}\mathbb{D}^{\chi}_{0,t}[X(t)] = r_1^{\chi} X\left(1-\dfrac{X}{K}\right) - b^{\chi} XY - d^{\chi} XZ + \sigma^{\chi} R_Z + \gamma^{\chi} R_Y - \mu^{\chi} X,\\
{}^{\mathrm{ABC}}\mathbb{D}^{\chi}_{0,t}[Y(t)] = r_2^{\chi} Y\left(1-\dfrac{Y}{K}\right) + b^{\chi} XY - (\alpha^{\chi}+\omega^{\chi}+\mu^{\chi})Y,\\
{}^{\mathrm{ABC}}\mathbb{D}^{\chi}_{0,t}[Z(t)] = r_3^{\chi} Z\left(1-\dfrac{Z}{K}\right) + d^{\chi} XZ - (\rho^{\chi}+\phi^{\chi}+\mu^{\chi})Z,\\
{}^{\mathrm{ABC}}\mathbb{D}^{\chi}_{0,t}[R_Y(t)] = \omega^{\chi} Y - (\gamma^{\chi}+\mu^{\chi})R_Y,\\
{}^{\mathrm{ABC}}\mathbb{D}^{\chi}_{0,t}[R_Z(t)] = \phi^{\chi} Z - (\sigma^{\chi}+\mu^{\chi})R_Z.
\end{cases}
\tag{3}
$$

with

$$
X(t), Y(t), Z(t), R_Y(t), R_Z(t) \geq 0.
$$

2.1. Invariant Region. Let the total population at time t is represented by $N(t)$, which satisfied,

$$N(t) = X(t) + Y(t) + Z(t) + R_Y(t) + R_Z(t).$$

The equation (3) gives

$$\frac{dN}{dt} = r_1^{\chi} X\left(1 - \frac{X}{K}\right) + r_2^{\chi} Y\left(1 - \frac{Y}{K}\right) + r_3^{\chi} Z\left(1 - \frac{Z}{K}\right) - \alpha^{\chi} Y - \rho^{\chi} Z - \mu^{\chi} N(t).$$

Now, assuming that there is no disease induced death rate, i.e., $\alpha^{\chi} = 0$ and $\rho^{\chi} = 0$,
so that

$$\begin{aligned}\frac{dN}{dt} &= r_1^{\chi} X\left(1 - \frac{X}{K}\right) + r_2^{\chi} Y\left(1 - \frac{Y}{K}\right) + r_3^{\chi} Z\left(1 - \frac{Z}{K}\right) - \mu^{\chi} N(t), \\ &\leq r_1^{\chi} X + r_2^{\chi} Y + r_3^{\chi} Z - \mu^{\chi} N(t)\end{aligned}$$

Now as $t \to \infty$, we obtain $N \leq \frac{r_1^{\chi} X + r_2^{\chi} Y + r_3^{\chi} Z}{\mu^{\chi}}$. We study (3) in the following closed set

$$\Delta = \left\{(X, Y, Z, R_Y, R_Z) \in R_+^5 : 0 < X, Y, Z, R_Y, R_Z \leq \frac{r_1^{\chi} X + r_2^{\chi} Y + r_3^{\chi} Z}{\mu^{\chi}}\right\}.$$

3. Existence and Uniqueness Criteria

In this part, the existence of solutions of the cancer and hepatitis model (3) is considered using fixed point techniques. In this regard, applying the ABC fractional integral operator on Eq. (3), we obtain

$$X(t) - X(0) = \frac{1-\chi}{B(\chi)} K_1(\chi, t, X(t)) + \frac{\chi}{B(\chi)\Gamma(\chi)} \times \int_0^t (t-\vartheta)^{\chi-1} K_1(\chi, \vartheta, X(\vartheta)) d\vartheta,$$

$$Y(t) - Y(0) = \frac{1-\chi}{B(\chi)} K_2(\chi, t, Y(t)) + \frac{\chi}{B(\chi)\Gamma(\chi)} \times \int_0^t (t-\vartheta)^{\chi-1} K_2(\chi, \vartheta, Y(\vartheta)) d\vartheta, \tag{4}$$

$$Z(t) - Z(0) = \frac{1-\chi}{B(\chi)} K_3(\chi, t, Z(t)) + \frac{\chi}{B(\chi)\Gamma(\chi)} \times \int_0^t (t-\vartheta)^{\chi-1} K_3(\chi, \vartheta, Z(\vartheta)) d\vartheta,$$

$$R_Y(t)-R_Y(0)=\frac{1-\chi}{B(\chi)}K_4(\chi,t,R_Y(t))+\frac{\chi}{B(\chi)\Gamma(\chi)}\times\int_0^t(t-\vartheta)^{\chi-1}K_4(\chi,\vartheta,R_Y(\vartheta))d\vartheta,$$

$$R_Z(t)-R_Z(0)=\frac{1-\chi}{B(\chi)}K_5(\chi,t,R_Z(t))+\frac{\chi}{B(\chi)\Gamma(\chi)}\times\int_0^t(t-\vartheta)^{\chi-1}K_5(\chi,\vartheta,R_Z(\vartheta))d\vartheta,$$

For simplicity, we define functions K_i, and some constants η_i, $i\in\mathbf{N}_1^5$, below:

$$\begin{aligned}
K_1(\chi,t,S(t)) &= r_1^\chi X\left(1-\tfrac{X}{K}\right)-b^\chi XY-d^\chi XZ+\sigma^\chi R_Z+\gamma^\chi R_Y-\mu^\chi X,\\
K_2(\chi,t,Y(t)) &= r_2^\chi Y\left(1-\tfrac{Y}{K}\right)+b^\chi XY-(\alpha^\chi+\omega^\chi+\mu^\chi)Y,\\
K_3(\chi,t,Z(t)) &= r_3^\chi Z\left(1-\tfrac{Z}{K}\right)+d^\chi XZ-(\rho^\chi+\phi^\chi+\mu^\chi)Z,\\
K_4(\chi,t,R_Y(t)) &= \omega^\chi Y-(\gamma^\chi+\mu^\chi)R_Y,\\
K_5(\chi,t,R_Z(t)) &= \phi^\chi Z-(\sigma^\chi+\mu^\chi)R_Z,
\end{aligned}\tag{5}$$

and

$$\eta_1=\left\|\left(r_1^\chi\left(1-\frac{X}{K}\right)+b^\chi Y+d^\chi Z+\mu^\chi\right)\right\|$$

$$\eta_2=\left\|\left(r_2^\chi\left(1-\frac{Y}{K}\right)+b^\chi X-(\alpha^\chi+\omega^\chi+\mu^\chi)\right)\right\|$$

$$\eta_3=\left\|\left(r_3^\chi\left(1-\frac{Z}{K}\right)+d^\chi X-(\rho^\chi+\phi^\chi+\mu^\chi)\right)\right\|$$

$$\eta_4=\|((\gamma^\chi+\mu^\chi))\|$$

$$\eta_5=\|((\sigma^\chi+\mu^\chi))\|.$$

For proving our results, we assume the following assumption: (B) For the following continuous functions $X, X^*, Y, Y^*, Z, Z^*, R_Y, R_Y^*, R_Z, R_Z^* \in L[0,1]$, such that $\|X(t)\| \leq \mathscr{L}_1, \|Y(t)\| \leq \mathscr{L}_2, \|Z(t)\| \leq \mathscr{L}_3, \|R_Y(t)\| \leq \mathscr{L}_4, \|R_Y(t)\| \leq \mathscr{L}_5$.

Theorem 1. *The kernels $k_i, i \in \mathbf{N}^5$ are satisfying the Lipschitz condition if the assumption B is true and are contractions provided that $\eta_i < 1$ for every $i \in \mathbf{N}^5$.*

Proof. First, we prove that $K_1(\chi, t, S(t))$ satisfies Lipschitz condition. For $X(t)$ and $X^*(t)$ using the first Eq. (5), we have

$$\begin{aligned}
\|K_1(\chi,t,X(t)) - K_1(\chi,t,X^*(t))\| &= \|r_1^\chi X\left(1-\frac{X}{K}\right) - b^\chi XY - d^\chi XZ + \sigma^\chi R_Z + \gamma^\chi R_Y - \mu^\chi X \\
&\quad - \left(r_1^\chi X^*\left(1-\frac{X^*}{K}\right) - b^\chi X^*Y - d^\chi X^*Z + \sigma^\chi R_Z + \gamma^\chi R_Y - \mu^\chi X^*\right)\| \\
&\leq \left[r_1^\chi\left(1-\frac{X}{K}\right) + b^\chi Y + d^\chi Z + \mu^\chi\right]\|X - X^*\| \\
&= \eta_1\|X - X^*\|.
\end{aligned}$$

Second, we prove that $K_2(\chi, t, Y(t))$ satisfies Lipschitz condition. For $Y(t)$ and $Y^*(t)$ using the second Eq. (5), we have

$$\begin{aligned}
\|K_2(\chi,t,Y(t)) - K_2(\chi,t,Y^*(t))\| &= \|r_2^\chi Y\left(1-\frac{Y}{K}\right) + b^\chi XY - (\alpha^\chi + \omega^\chi + \mu^\chi)Y \\
&\quad - \left(r_2^\chi Y\left(1-\frac{Y}{K}\right) + b^\chi XY^* - (\alpha^\chi + \omega^\chi + \mu^\chi)Y^*\right)\| \\
&\leq \left[r_2^\chi\left(1-\frac{Y}{K}\right) + b^\chi X + (\alpha^\chi + \omega^\chi + \mu^\chi)\right]\|Y - Y^*\| \\
&= \eta_2\|Y - Y^*\|.
\end{aligned} \tag{6}$$

Third, we prove that $K_3(\chi, t, Z(t))$ satisfies Lipschitz condition. For $Z(t)$ and $Z^*(t)$ using the third Eq. (5), we have

$$
\begin{aligned}
\|K_3(\chi,t,Z(t)) - K_3(\chi,t,Z^*(t))\| &= \|r_3^\chi Z\left(1-\frac{Z}{K}\right) + d^\chi XZ - (\rho^\chi+\phi^\chi+\mu^\chi)Z \\
&\quad - \left(r_3^\chi Z^*\left(1-\frac{Z}{K}\right) + d^\chi XZ^* - (\rho^\chi+\phi^\chi+\mu^\chi)Z^*\right)\| \\
&\leq \left[r_3^\chi\left(1-\frac{Z}{K}\right) + d^\chi X + (\rho^\chi+\phi^\chi+\mu^\chi)\right]\|Z-Z^*\| \\
&= \eta_3\|Z-Z^*\|
\end{aligned} \tag{7}
$$

Fourth, we prove that $K_4(\chi,t,R_Y(t))$ satisfies Lipschitz condition. For $R_Y(t)$ and $R_Y^*(t)$ using the fourth Eq. (5), we have

$$
\begin{aligned}
\|K_4(\chi,t,R_Y(t)) - K_4(\chi,t,R_Y^*(t))\| &= \|\omega^\chi Y - (\gamma^\chi+\mu^\chi)R_Y \\
&\quad - (\omega^\chi Y - (\gamma^\chi+\mu^\chi)R_Y^*)\| \\
&\leq [(\gamma^\chi+\mu^\chi)]\|R_Y - R_Y^*\| \\
&= \eta_4\|R_Y - R_Y^*\|
\end{aligned} \tag{8}
$$

Fifth, we prove that $K_5(\chi,t,R_Z(t))$ satisfies Lipschitz condition. For $R_Z(t)$ and $R_Z^*(t)$ using the fifth Eq. (5), we have

$$
\begin{aligned}
\|K_5(\chi,t,R_Z(t)) - K_5(\chi,t,Z^*(t))\| &= \|\phi^\chi Z - (\sigma^\chi+\mu^\chi)R_Z \\
&\quad - (\phi^\chi Z - (\sigma^\chi+\mu^\chi)R_Z^*)\| \\
&\leq [(\sigma^\chi+\mu^\chi)]\|R_Z - R_Z^*\| \\
&= \eta_5\|R_Z - R_Z^*\|
\end{aligned} \tag{9}
$$

Hence, all the kernels K_i, $i \in \mathbf{N}^5$ are satisfying Lipschitz condition, and they are contractions with $\eta_i < 1, i \in \mathbf{N}^5$. This completes the proof.

Going in a recursive manner, the expressions in (4) yields

(10)

$$
\begin{aligned}
X_n(t)-X(0) &= \frac{1-\chi}{B(\chi)}K_1(\chi,t,X_{n-1}(t)) + \frac{\chi}{B(\chi)\Gamma(\chi)}\times\int_0^t (t-\vartheta)^{\chi-1}K_1(\chi,\vartheta,X_{n-1}(\vartheta))d\vartheta,\\
Y_n(t)-Y(0) &= \frac{1-\chi}{B(\chi)}K_2(\chi,t,Y_{n-1}(t)) + \frac{\chi}{B(\chi)\Gamma(\chi)}\times\int_0^t (t-\vartheta)^{\chi-1}K_2(\chi,\vartheta,Y_{n-1}(\vartheta))d\vartheta,\\
Z_n(t)-Z(0) &= \frac{1-\chi}{B(\chi)}K_3(\chi,t,Z_{n-1}(t)) + \frac{\chi}{B(\chi)\Gamma(\chi)}\times\int_0^t (t-\vartheta)^{\chi-1}K_3(\chi,\vartheta,Z_{n-1}(\vartheta))d\vartheta,\\
R_{Y_n}(t)-R_Y(0) &= \frac{1-\chi}{B(\chi)}K_4(\chi,t,R_{Y_{n-1}}(t)) + \frac{\chi}{B(\chi)\Gamma(\chi)}\times\int_0^t (t-\vartheta)^{\chi-1}K_4(\chi,\vartheta,R_{Y_{n-1}}(\vartheta))d\vartheta,\\
R_{Z_n}(t)-R_Z(0) &= \frac{1-\chi}{B(\chi)}K_5(\chi,t,R_{Z_{n-1}}(t)) + \frac{\chi}{B(\chi)\Gamma(\chi)}\times\int_0^t (t-\vartheta)^{\chi-1}K_5(\chi,\vartheta,R_{Z_{n-1}}(\vartheta))d\vartheta,
\end{aligned}
$$

together with $X_0(t)=X(0), Y_0(t)=Y(0), Z_0(t)=Z(0), R_{Y_0}(t)=R_Y(0)$ and $R_{Z_0}(t)=R_Z(0)$. When the successive terms difference is taken, we get

(11)

$$
\begin{aligned}
\Xi_{X,n}(t) =& X_n(t)-X_{n-1}(t) = \frac{1-\chi}{B(\chi)}(K_1(\chi,t,X_{n-1}(t))-K_1(\chi,t,X_{n-2}(t)))\\
&+\frac{\chi}{B(\chi)\Gamma(\chi)}\int_0^t (t-\vartheta)^{\chi-1}\left(K_1\left(\chi,\vartheta,X_{n-1}(\vartheta)\right)-K_1\left(\chi,\vartheta,X_{n-2}(\vartheta)\right)\right)d\vartheta\\
\Xi_{Y,n}(t) =& Y_n(t)-Y_{n-1}(t) = \frac{1-\chi}{B(\chi)}(K_2(\chi,t,Y_{n-1}(t))-K_2(\chi,t,Y_{n-2}(t)))\\
&+\frac{\chi}{B(\chi)\Gamma(\chi)}\int_0^l (t-\vartheta)^{\chi-1}\left(K_2\left(\chi,\vartheta,Y_{n-1}(\vartheta)\right)-K_2\left(\chi,\vartheta,Y_{n-2}(\vartheta)\right)\right)d\vartheta\\
\Xi_{Z,n}(t) =& Z_n(t)-Z_{n-1}(t) = \frac{1-\chi}{B(\chi)}(K_3(\chi,t,Z_{n-1}(t))-K_3(\chi,t,Z_{n-2}(t)))\\
&+\frac{\chi}{B(\chi)\Gamma(\chi)}\int_0^t (t-\vartheta)^{\chi-1}\left(K_3\left(\chi,\vartheta,Z_{n-1}(\vartheta)\right)-K_3\left(\chi,\vartheta,Z_{n-2}(\vartheta)\right)\right)d\vartheta\\
\Xi_{R_Y,n}(t) =& R_{Y_n}(t)-R_{Y_{n-1}}(t) = \frac{1-\chi}{B(\chi)}(K_4(\chi,t,R_{Y_{n-1}}(t))-K_4(\chi,t,R_{Y_{n-2}}(t)))\\
&+\frac{\chi}{B(\chi)\Gamma(\chi)}\int_0^t (t-\vartheta)^{\chi-1}\left(K_4\left(\chi,\vartheta,R_{Y_{n-1}}(\vartheta)\right)-K_4\left(\chi,\vartheta,R_{Y_{n-2}}(\vartheta)\right)\right)d\vartheta\\
\Xi_{R_Z,n}(t) =& F_n(t)-R_{Z_{n-1}}(t) = \frac{1-\chi}{B(\chi)}(K_5(\chi,t,R_{Z_{n-1}}(t))-K_5(\chi,t,R_{Z_{n-2}}(t)))\\
&+\frac{\chi}{B(\chi)\Gamma(\chi)}\int_0^t (t-\vartheta)^{\chi-1}\left(K_5\left(\chi,\vartheta,R_{n-1}(\vartheta)\right)-K_5\left(\chi,\vartheta,R_{Z_{n-2}}(\vartheta)\right)\right)d\vartheta
\end{aligned}
$$

It is vital to observe that $X_n(t)=\sum_{i=0}^n \Xi_{X,i}(t)$, $Y_n(t)=\sum_{i=0}^n \Xi_{Y,i}(t)$, $Z_n(t)=\sum_{i=0}^n \Xi_{Z,i}(t)$, $R_{Y_n}(t)=\sum_{i=0}^n \Xi_{R_Y,i}(t)$, $R_{Z_n}(t)=\sum_{i=0}^n \Xi_{R_Z,i}(t)$. Additionally, by using Eqs. (10)-(11) and considering that $\Xi_{X,n-1}(t)=X_{n-1}(t)-X_{n-2}(t)$, $\Xi_{Y,n-1}(t)=Y_{n-1}(t)-Y_{n-2}(t)$, $\Xi_{Z,n-1}(t)=$

$Z_{n-1}(t)-Z_{n-2}(t)$, $\Xi_{R_Y,n-1}(t)=R_{Y_{n-1}}(t)-R_{Y_{n-2}}(t)$, $\Xi_{R_Z,n-1}(t)=R_{Z_{n-1}}(t)-R_{Z_{n-2}}(t)$, we reach

$$\|\Xi_{X,n}(t)\| \leq \frac{1-\chi}{B(\chi)}\eta_1 \left\|\Xi_{X,n-1}(t)\right\| \frac{\chi}{B(\chi)\Gamma(\chi)}\eta_1 \times \int_0^t (t-\vartheta)^{\chi-1}\left\|\Xi_{X,n-1}(\vartheta)\right\| d\vartheta$$

$$\|\Xi_{Y,n}(t)\| \leq \frac{1-\chi}{B(\chi)}\eta_2 \left\|\Xi_{Y,n-1}(t)\right\| \frac{\chi}{B(\chi)\Gamma(\chi)}\eta_2 \times \int_0^t (t-\vartheta)^{\chi-1}\left\|\Xi_{Y,n-1}(\vartheta)\right\| d\vartheta$$

$$\|\Xi_{Z,n}(t)\| \leq \frac{1-\chi}{B(\chi)}\eta_3 \left\|\Xi_{Z,n-1}(t)\right\| \frac{\chi}{B(\chi)\Gamma(\chi)}\eta_3 \times \int_0^t (t-\vartheta)^{\chi-1}\left\|\Xi_{Z,n-1}(\vartheta)\right\| d\vartheta \tag{12}$$

$$\|\Xi_{R_Y,n}(t)\| \leq \frac{1-\chi}{B(\chi)}\eta_4 \left\|\Xi_{R_Y,n-1}(t)\right\| \frac{\chi}{B(\chi)\Gamma(\chi)}\eta_4 \times \int_0^t (t-\vartheta)^{\chi-1}\left\|\Xi_{R_Y,n-1}(\vartheta)\right\| d\vartheta$$

$$\|\Xi_{R_Z,n}(t)\| \leq \frac{1-\chi}{B(\chi)}\eta_5 \left\|\Xi_{R_Z,n-1}(t)\right\| \frac{\chi}{B(\chi)\Gamma(\chi)}\eta_5 \times \int_0^t (t-\vartheta)^{\chi-1}\left\|\Xi_{R_Z,n-1}(\vartheta)\right\| d\vartheta$$

Now, the following theorem will be proved.

Theorem 2. *Surmising that the following condition holds*

$$\frac{1-\chi}{B(\chi)}\eta_i + \frac{\chi}{B(\chi)\Gamma(\chi)}b^{\chi}\eta_i < 1, i = 1,2,\ldots,5. \tag{13}$$

Then, (3) has a unique solution for $t \in [0,b]$.

Proof. It is shown $X(t), Y(t), Z(t), R_Y(t)$ and $R_Z(t)$ are bounded functions. In addition, the symbols K_1, K_2, K_3, K_4 and K_5 hold for Lipchitz condition. Therefore, utilizing Eq. (12) together with a recursive hypothesis, we arrive at

$$\|\Xi_{X,n}(t)\| \leq \|X_0(t)\| \left(\frac{1-\chi}{B(\chi)}\eta_1 + \frac{\chi b^{\chi}}{B(\chi)\Gamma(\chi)}\eta_1\right)^n$$

$$\|\Xi_{Y,n}(t)\| \leq \|Y_0(t)\| \left(\frac{1-\chi}{B(\chi)}\eta_3 + \frac{\chi b^{\chi}}{B(\chi)\Gamma(\chi)}\eta_2\right)^n$$

$$\|\Xi_{Z,n}(t)\| \leq \|Z_0(t)\| \left(\frac{1-\chi}{B(\chi)}\eta_3 + \frac{\chi b^{\chi}}{B(\chi)\Gamma(\chi)}\eta_3\right)^n \tag{14}$$

$$\|\Xi_{R_Y,n}(t)\| \leq \|R_{Y_0}(t)\| \left(\frac{1-\chi}{B(\chi)}\eta_4 + \frac{\chi b^{\chi}}{B(\chi)\Gamma(\chi)}\eta_4\right)^n$$

$$\|\Xi_{R_Z,n}(t)\| \leq \|R_{Z_0}(t)\| \left(\frac{1-\chi}{B(\chi)}\eta_5 + \frac{\chi b^{\chi}}{B(\chi)\Gamma(\chi)}\eta_5\right)^n$$

Thus, one can see that sequences satisfy and exist

$\|\Xi_{X,n}(t)\| \to 0, \quad \|\Xi_{Y,n}(t)\| \to 0, \quad \|\Xi_{Z,n}(t)\| \to 0, \quad \|\Xi_{R_Y,n}(t)\| \to 0, \quad \|\Xi_{R_Z,n}(t)\| \to 0$

as $n \to \infty$. Moreover, from Eq. (14) and imposing the triangle inequality, for any k, we have

$$
\begin{aligned}
&\|X_{n+k}(t) - X_n(t)\| \leq \sum_{j=n+1}^{n+k} Z_1^j = \tfrac{Z_1^{n+1} - Z_1^{n+k+1}}{1-Z_1} \\
&\|Y_{n+k}(t) - Y_n(t)\| \leq \sum_{j=n+1}^{n+k} Z_2^j = \tfrac{Z_2^{n+1} - Z_2^{n+k+1}}{1-Z_2} \\
&\|Z_{n+k}(t) - Z_n(t)\| \leq \sum_{j=n+1}^{n+k} Z_3^j = \tfrac{Z_3^{n+1} - Z_3^{n+k+1}}{1-Z_3} \\
&\left\|R_{Y_{n+k}}(t) - R_{Y_n}(t)\right\| \leq \sum_{j=n+1}^{n+k} Z_4^j = \tfrac{Z_4^{n+1} - Z_4^{n+k+1}}{1-Z_4} \\
&\left\|R_{Z_{n+k}}(t) - R_{Z_n}(t)\right\| \leq \sum_{i=n+1}^{n+k} Z_5^j = \tfrac{Z_5^{n+1} - Z_5^{n+k+1}}{1-Z_5},
\end{aligned} \tag{15}
$$

with $Z_i = \frac{1-\chi}{B(\chi)}\eta_i + \frac{\chi}{B(\chi)\Gamma(\chi)} b^{\chi}\eta_i < 1$ by hypothesis. Therefore, X_n, Y_n, Z_n, R_{Y_n} and R_{Z_n} can be seen as a Cauchy sequences in the Banach space B. This has demonstrated that they are uniformly convergent [26]. Imposing the limit theorem in Eq. (11) as $n \to \infty$ affirms that the limit of these sequences is the unique solution of (3). This establishes the existence of a unique solution for Eq. (3) under the condition (13).

3.1. Basic reproductive number R_0. The disease free equilibrium point of system (3) is denoted by E^0, i.e

$$
E^0 = (X^0, Y^0, Z^0, R_Y^0, R_Z^0) = \left(\frac{K(r_1^{\chi} - \mu^{\chi})}{r_1^{\chi}}, 0, 0, 0, 0\right). \tag{16}
$$

The spread of diseases are simply associated with the basic reproduction number (R_0). This quantity presents an indication of the nature of the spread. If this threshold quantity is $R_0 < 1$, the disease will die out of the population. But if the threshold quantity is $R_0 > 1$ then the disease will persist in the population in the long run. To compute the basic reproduction number for

this model, let (Y,Z) be the infected compartment, then it follows from system (3):

$$\begin{cases} {}^{\mathrm{ABC}}\mathbb{D}^{\chi}_{0,t}[Y(t)] = r_2^{\chi} Y\left(1-\dfrac{Y}{K}\right) + b^{\chi} XY - (\alpha^{\chi}+\omega^{\chi}+\mu^{\chi})Y, \\ {}^{\mathrm{ABC}}\mathbb{D}^{\chi}_{0,t}[Z(t)] = r_3^{\chi} Z\left(1-\dfrac{Z}{K}\right) + d^{\chi} XZ - (\rho^{\chi}+\phi^{\chi}+\mu^{\chi})Z. \end{cases} \tag{17}$$

Using the next Generation Matrix Approach, the Jacobian matrix J for the above system at the disease free equilibrium point E^0 is given by

$$J = \begin{pmatrix} r_2^{\chi} + b^{\chi}X^0 - (\alpha^{\chi}+\omega^{\chi}+\mu^{\chi}) & 0 \\ 0 & r_3^{\chi} + d^{\chi}X^0 - (\rho^{\chi}+\phi^{\chi}+\mu^{\chi}) \end{pmatrix}.$$

Now decomposing the matrix J in the term of F and V i.e $J = F - V$ we get

$$F = \begin{pmatrix} r_2^{\chi} + b^{\chi}X^0 & 0 \\ 0 & r_3^{\chi} + d^{\chi}X^0 \end{pmatrix},$$

and

$$V = \begin{pmatrix} (\alpha^{\chi}+\omega^{\chi}+\mu^{\chi}) & 0 \\ 0 & (\rho^{\chi}+\phi^{\chi}+\mu^{\chi}) \end{pmatrix}.$$

The basic reproduction number (R_0) is the spectral radius of the of the matrix (FV^{-1}) and for the present model it is given by

$$R_{0_{Can}} = \frac{r_1^{\chi} r_2^{\chi} + K(r_1^{\chi} - \mu^{\chi})b^{\chi}}{r_1^{\chi}(\alpha^{\chi}+\omega^{\chi}+\mu^{\chi})},$$

and

$$R_{0_{Hep}} = \frac{r_1^{\chi} r_3^{\chi} + K(r_1^{\chi} - \mu^{\chi})d^{\chi}}{r_1^{\chi}(\rho^{\chi}+\phi^{\chi}+\mu^{\chi})}.$$

Also, the reproduction number cancer-hepatitis co-infection model denoted by $R_{0_{CH}}$, is given by

$$R_{0_{CH}} = max\{R_{0_{Can}}, R_{0_{Hep}}\},$$

where $R_{0_{Can}}$ and $R_{0_{Hep}}$ are the two spectral radii indicating the R_0 for cancer and hepatitis disease in that order. Practically, $R_{0_{Cen}}$ and $R_{0_{Hep}}$ are the average rates of new infections caused by one infected individual in a given period.

3.2. Endemic equilibrium point. The endemic equilibrium point for the system (3) is denoted by $E^* = (X^*, Y^*, Z^*, R_Y^*, R_Z^*)$ for which the disease is endemic in the population (i.e. at least one of Y^* and Z^* is nonzero). The system (3) are rearranged to get X^*, Y^*, Z^*, R_Y^* and R_Z^*. This gives

$$
\begin{cases}
X^* = \dfrac{(-L+\Pi)Kr_2^{\chi}r_2^{\chi}}{2(\gamma^{\chi}r_2^{\chi}r_3^{\chi}+b^{2\chi}K^2r_3^{\chi}+d^{2\chi}K^2r_2^{\chi})}), \\
Y^* = \dfrac{K(r_2^{\chi}+d^{\chi}X^*-(\alpha^{\chi}+\omega^{\chi}+\mu^{\chi})}{r_2^{\chi}}, \\
Z^* = \dfrac{K(r_3^{\chi}+d^{\chi}X^*-(\rho^{\chi}+\phi^{\chi}+\mu^{\chi})}{r_3^{\chi}}, \\
R_Y^* = \dfrac{\omega^{\chi}K(r_2^{\chi}+d^{\chi}X^*-(\alpha^{\chi}+\omega^{\chi}+\mu^{\chi})}{r_2^{\chi}(\gamma^{\chi}+\mu^{\chi})}, \\
R_Z^* = \dfrac{\phi K(r_3^{\chi}+d^{\chi}X^*-(\rho^{\chi}+\phi^{\chi}+\mu^{\chi})}{r_3^{\chi}(\sigma^{\chi}+\mu^{\chi})},
\end{cases}
\tag{18}
$$

where

$$
L = \frac{bK(r_2^{\chi}-(\alpha^{\chi}+\omega^{\chi}+\mu^{\chi})}{r_2^{\chi}} + \frac{dK(r_3^{\chi}-(\rho^{\chi}+\phi^{\chi}+\mu^{\chi})}{r_3^{\chi}} + \mu^{\chi} - \frac{\sigma^{\chi}\phi^{\chi}d^{\chi}K}{r_3^{\chi}(\sigma^{\chi}+\mu^{\chi})} - \frac{\omega^{\chi}\gamma^{\chi}b^{\chi}K}{r_2^{\chi}(\gamma^{\chi}+\mu^{\chi})}
$$

and

$$
\Pi = \sqrt{A}
$$

$$
A = L^2 + 4\left(\frac{\gamma^{\chi}r_2^{\chi}r_3^{\chi}+b^{2\chi}K^2r_3^{\chi}+d^{2\chi}K^2r_2^{\chi}}{Kr_2^{\chi}r_2^{\chi}}\right)\left(\frac{\sigma^{\chi}\phi^{\chi}K(r_3^{\chi}-(\rho^{\chi}+\phi^{\chi}+\mu^{\chi}}{r_3^{\chi}(\sigma^{\chi}+\mu^{\chi})} + \frac{\omega^{\chi}\gamma^{\chi}K(r_2^{\chi}-(\alpha^{\chi}+\omega^{\chi}+\mu^{\chi})}{r_2^{\chi}(\gamma^{\chi}+\mu^{\chi})}\right).
$$

For

$$
\begin{aligned}
b^{\chi}Kr_2^{\chi}+d^{\chi}Kr_3^{\chi}+r_2^{\chi}r_3^{\chi}\mu^{\chi} < \frac{r_2^{\chi}\sigma^{\chi}\phi^{\chi}d^{\chi}K}{(\sigma^{\chi}+\mu^{\chi})} + \frac{r_3^{\chi}\omega^{\chi}\gamma^{\chi}b^{\chi}K}{(\gamma^{\chi}+\mu^{\chi})} \\
+Kb^{\chi}(\alpha^{\chi}+\omega^{\chi}+\mu^{\chi}) + Kd^{\chi}(\rho^{\chi}+\pi^{\chi}+\mu^{\chi})
\end{aligned}
$$

the positivity of the above equilibrium point (18) is assured.

4. Local Stability

We prove the local stability of the system (3) of the disease free equilibrium state E^0.

4.1. Disease free equilibrium local stability.

Theorem 3. *The disease free equilibrium (DFE) point E^0 of the system (3) is locally asymptotically stable if $R_0 < 1$.*

proof. Jacobian matrix of the system (3) at E^0 is

$$(19)\ J(E^0) = \begin{pmatrix} r_1^\chi - \frac{2X^0}{K} - \mu^\chi & -b^\chi X^0 & -d^\chi X^0 & \gamma^\chi & \sigma^\chi \\ 0 & a_{22} & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 \\ 0 & \omega^\chi & 0 & -(\gamma^\chi + \mu^\chi) & 0 \\ 0 & 0 & \phi^\chi & 0 & -(\sigma^\chi + \mu^\chi) \end{pmatrix}.$$

where $a_{22} = r_2^\chi - (\alpha^\chi + \omega^\chi + \mu^\chi) + bX^0, a_{33} = r_3^\chi + dX^0 - (\rho^\chi + \phi^\chi + \mu^\chi)$.

The characteristic equation of $J(E^0)$ takes the following form:

$$[\lambda_1 + (r_1^\chi - \frac{2X^0}{K} - \mu^\chi)][\lambda_2 + (\sigma^\chi + \mu^\chi)][\lambda_3 + (\gamma^\chi + \mu^\chi)][\lambda_4 + (r_2^\chi - (\alpha^\chi + \omega^\chi + \mu^\chi) + bX^0)]$$
$$[\lambda_5 + (r_3^\chi + dX^0 - (\rho^\chi + \phi^\chi + \mu^\chi))] = 0,$$

Since the three eigenvalues of the characteristic equation of $J(E^0)$ is negative.
i.e $\lambda_1 = (r_1^\chi - \frac{2(r_1^\chi - \mu^\chi)}{r_1^\chi} - \mu^\chi), \quad \lambda_2 = -(\sigma^\chi + \mu^\chi)$ and $\lambda_3 = -(\gamma^\chi + \mu^\chi)$.
Similarly $\lambda_4 = (r_2^\chi - (\alpha^\chi + \omega^\chi + \mu^\chi) + bX^0)$ is negative if
$r_2^\chi - (\alpha^\chi + \omega^\chi + \mu^\chi) + b\frac{K(r_1^\chi - \mu^\chi)}{r_1^\chi} < 0$, implies that

$$r_2^\chi r_1^\chi + bK(r_1^\chi - \mu^\chi) < r_1^\chi(\alpha^\chi + \omega^\chi + \mu^\chi),\ or\ \frac{r_1^\chi r_2^\chi + K(r_1^\chi - \mu^\chi)b}{r_1^\chi(\alpha^\chi + \omega^\chi + \mu^\chi)} < 1.$$

By definition of

$$R_{0_{Can}} = \frac{r_1^\chi r_2^\chi + K(r_1^\chi - \mu^\chi)b}{r_1^\chi(\alpha^\chi + \omega^\chi + \mu^\chi)} < 1.$$

Similarly $\lambda_5 = (r_3^\chi + dX^0 - (\rho^\chi + \phi^\chi + \mu^\chi))$ is negative
if $(r_3^\chi + d\frac{K(r_1^\chi - \mu^\chi)}{r_1^\chi} - (\rho^\chi + \phi^\chi + \mu^\chi)) < 0$, implies that

$$(r_3^\chi r_1^\chi + dK(r_1^\chi - \mu^\chi)) < r_1^\chi(\rho^\chi + \phi^\chi + \mu^\chi),\ or\ \frac{r_1^\chi r_3^\chi + K(r_1^\chi - \mu^\chi)d}{r_1^\chi(\rho^\chi + \phi^\chi + \mu^\chi)} < 1.$$

By definition

$$R_{0_{Hep}} = \frac{r_1^{\chi} r_3^{\chi} + K(r_1^{\chi} - \mu^{\chi})d}{r_1^{\chi}(\rho^{\chi} + \phi^{\chi} + \mu^{\chi})} < 1.$$

So $R_0 = max(R_{0_{Can}}, R_{Hep})$ implies that $R_0 < 1$.

This shows that the DFE point asymptotically stable if $R_0 < 1$.

4.2. Numerical Scheme for Cancer and Hepatitis Model. The numerical scheme for the model (2) based on the Adams-Moulton rule is given by [27]

$$(20) \qquad {}_0^{AB}\mathscr{I}_t^{\alpha}[f(t_{x+1})] = \frac{1-\alpha}{B(\alpha)}\left[\frac{f(t_{x+1}) - f(t_x)}{2}\right] + \frac{\alpha}{\Gamma(\alpha)}\sum_{z=0}^{\infty}\left[\frac{f(t_{z+1}) - f(t_z)}{2}\right]\xi_z^{\alpha}$$

where $\xi_z^{\alpha} = (z+1)^{1-\alpha} - (z)^{1-\alpha}$.

Using the above numerical scheme, we have

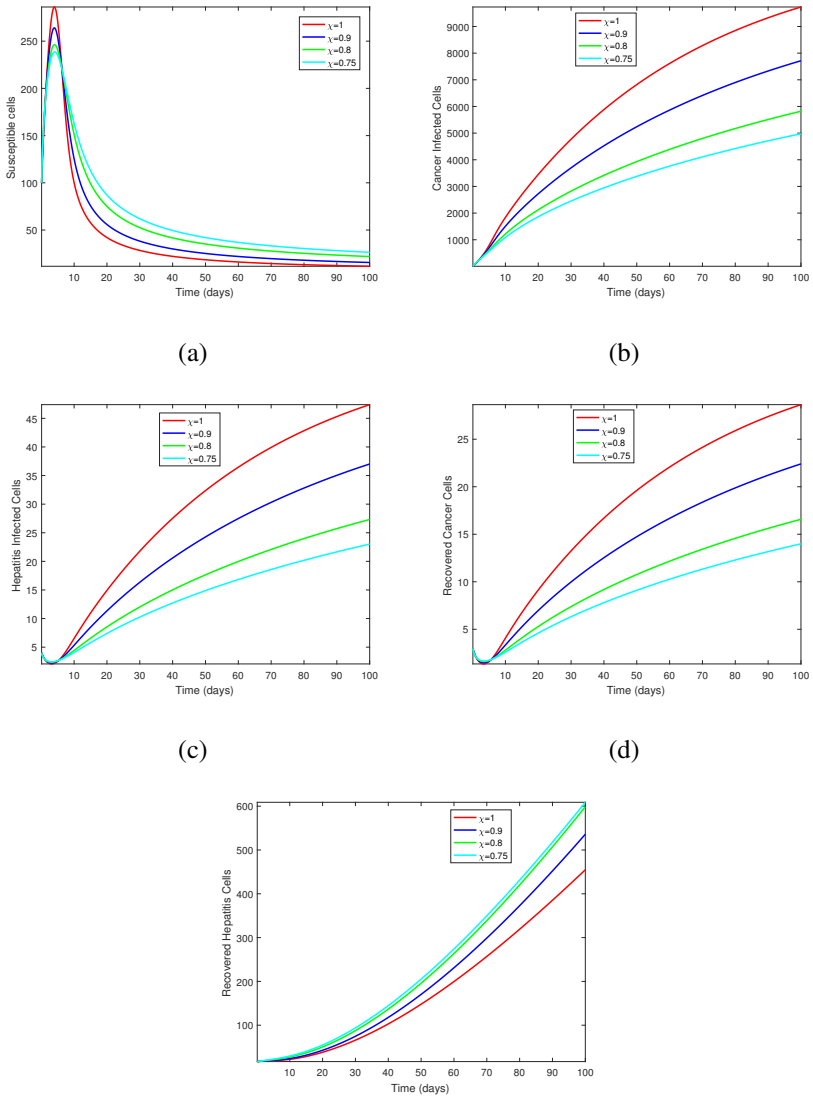
$$\begin{aligned}
X_{(x+1)}(t) - X_{(x)}(t) &= X(0)^n(t) + [\frac{1-\chi}{B(\chi)}[r_1^{\chi}\{\frac{X_{(x+1)}(t) - X_{(x)}(t)}{2}\}\{1 - \frac{X_{(x+1)}(t) - X_{(x)}(t)}{2K}\} \\
&- b^{\chi}\{\{\frac{X_{(x+1)}(t) - X_{(x)}(t)}{2}\}\{\frac{Y_{(x+1)}(t) - Y_{(x)}(t)}{2}\}\} - d^{\chi}\{\{\frac{X_{(x+1)}(t) - X_{(x)}(t)}{2}\}\{\frac{Z_{(x+1)}(t) - Z_{(x)}(t)}{2}\}\} \\
&+ \sigma^{\chi}\{\{\frac{R_{Z(x+1)}(t) - R_{Z(x)}(t)}{2}\}\} + \gamma^{\chi}\{(\frac{R_{Y(x+1)}(t) - R_{Y(x)}(t)}{2}\}\} - \mu^{\chi}\{\{\frac{X_{(x+1)}(t) - X_{(x)}(t)}{2}\}\}]] \\
&+ \frac{\chi}{B(\chi)}\sum_{z=0}^{\infty}\xi_z^{\chi}[r_1^{\chi}\{\frac{X_{(z+1)}(t) - X_{(z)}(t)}{2}\}\{1 - \frac{X_{(z+1)}(t) - X_{(z)}(t)}{2K}\} \\
&- b^{\chi}\{\{\frac{X_{(z+1)}(t) - X_{(z)}(t)}{2}\}\{\frac{Y_{(z+1)}(t) - Y_{(z)}(t)}{2}\}\} - d^{\chi}\{\{\frac{X_{(z+1)}(t) - X_{(z)}(t)}{2}\}\{\frac{Z_{(z+1)}(t) - Z_{(z)}(t)}{2}\}\} \\
&+ \sigma^{\chi}\{\{\frac{R_{Z(z+1)}(t) - R_{Z(z)}(t)}{2}\} + \gamma\{\frac{R_{Y(z+1)}(t) - R_{Y(z)}(t)}{2}\}\} - \mu^{\chi}\{\{\frac{X_{(z+1)}(t) - X_{(z)}(t)}{2}\}\}].
\end{aligned}$$

$$\vdots$$

$$R_{Z(x+1)}(t) - R_{Z(x)}(t) = R_Z(0)^n(t) + [\frac{1-\chi}{B(\chi)}[\phi^{\chi}\{\frac{Z_{(x+1)}(t) - Z_{(x)}(t)}{2}\} - (\sigma^{\chi} + \mu^{\chi})\{\frac{R_{Z(x+1)}(t) - R_{Z(x)}(t)}{2}\}]$$

$$+ \frac{\chi}{B(\chi)}\sum_{z=0}^{\infty}\xi_z^{\chi}\{\phi^{\chi}\{\frac{Z_{(z+1)}(t) - Z_{(z)}(t)}{2}\} - (\sigma^{\chi} + \mu^{\chi})\{\frac{R_{Z(z+1)}(t) - R_{Z(z)}(t)}{2}\}\}]$$

4.3. Numerical results and discussions. This section examines the numerical simulation results based on numerical scheme as Section 4.2 for cancer and hepatitis cells model (3). The numerical method employed on equation (3) hinged on Adams-Moulton Rule. The numerical simulation is undertaken making use of following parameter values as in [1], $r_1 = 0.4, r_2 = 0.3, r_3 = 0.00004, b = 0.01, d = 0.885, \sigma = 0.03, \mu = 0.0002, \alpha = 0.003, \rho = 0.0141, \omega = 0.2, \phi = 0.001$.



(a) (b) (c) (d) (e)

FIGURE 1. Numerical simulation for cancer and hepatitis cells co-infection model (3) at $\chi = 1, 0.9, 0.80, 0.75$

Figure 1 represents the numerical simulation results based on Mittag-Leffler Generalized Function which is characterized by crossover property when stretched from one operator to one another. The operator has a statistical representation making it more viable. Figure 1(a) is the susceptible cells and as the fractional order χ derivative increases the number of susceptible cells reduces. In Figure 1(b), the number of infected cancer cells increase as the fractional order χ value increase. Figure 1(c) depicts hepatitis infected cells and the number of infected cells increase as the fractional order derivative values. Figure 1(d) is the recovered cancer cells and as the fractional order values increase more infected cancer cells are recovered. Figure 1(e) is the recovered hepatitis cells and generally the number of recovered cells increase, however, as the fractional order increase the number of recovered cells from hepatitis decrease. It is can be seen that the fractional order derivative has influence on the dynamics of the spread of cancer and hepatitis cells in the human body.

5. Conclusion

This paper explored the dynamics of cancer and hepatitis cells co-infection model in the context of Mittag-Leffler Function. The uniqueness and existence of solutions were proved. The stability analysis and reproductive number of the co-infection model were carried out. Numerical solution based on parameter values in [1] employed to obtain qualitative information about the co-infection. The graphical results indicated that the order of fractional derivative and the chosen parameter have effect on the dynamics of the disease. The numerical results obtained does not affect the artificial singularities as in Caputo operator. The crossover property of Mittag-Leffler kernel enable the numerical results to be predicted accurately. The operator can be employed to investigate other complex biological models.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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