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NONLINEAR DYNAMICS OF COVID-19 SEIR INFECTION MODEL WITH OPTIMAL CONTROL ANALYSIS

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Abstract. In this study, we have presented a data-driven SEIR compartmental model for the 2019 coronavirus infections in Ghana. Using the fminsearch optimization routine in Matlab, and the reported cumulative infected cases of COVID-19 in Ghana from 13th March 2020 to 6th October 2020, we have estimated the basic reproduction number, $R_0 \approx 1.0413$. We have further developed a controlled SEIR dynamical model for COVID-19 disease with a personal protection control strategy. We have derived an optimality system from our proposed optimal control problem. Using the fourth Runge-Kutta iterative scheme with the forward-backward method, we have performed numerical simulations for the model problem. From the numerical results, we can argue that proper personal protection practices can help reduce the disease transmission in the susceptible human population.

Keywords: SEIR COVID-19 model; optimal control; Runge-Kutta fourth iterative scheme.

2010 AMS Subject Classification: 93A30.

1. Introduction

The recent global outbreak of COVID-19 disease is causing a lot of fear and panic among people in the world. This disease is highly infectious and has killed many people worldwide.

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The situation report of the World Health Organization (WHO) on the recent COVID-19 pandemic disease published on October 06, 2020, indicates that the global total cumulative infected and death cases were 35,347,404 and 1,039,406 respectively. Personal protection protocols such as regular washing of hands under running water with soap, social distancing, avoiding mass gatherings, wearing a face mask, and the use of hand sanitizer has become some of the highly recommended preventive measures against the spread of the 2019 coronavirus disease.

Epidemiological modeling has contributed immensely in understanding infectious diseases spread dynamics and control strategies [1–4]. Ever since the outbreak of this highly contagious disease, several authors have contributed to the literature of epidemiological modeling, see, e.g., [5–31]. In [32], the authors proposed and studied a generalized data-driven SEIR COVID-19 epidemic model. The authors in [33] developed and studied an SEIR COVID-19 transmission model that incorporates the lockdown effect and the transmission variability between symptomatic and asymptomatic individuals in India. In [34], projected figures regarding COVID-19 infections were forecast for several cities in China using Boltzmann function regression analysis. In [35], the authors constructed a new and novel age-structured COVID-19 dynamical model. Fang et al. [36], discussed a data-driven SEIR epidemic model to investigate the dynamical behavior of SARS-CoV-2. Based on the famous Karmack-McKendrick epidemic modeling framework, the authors in [37], explored the effect of non-pharmaceutical control strategies on the spreading dynamics of COVID-19. In the work of Li et al. [38], they applied Gaussian distribution theory to design a prediction and propagation-based model for COVID-19 disease. Chowell et al. [39] have developed and analyzed an SEIR-type COVID-19 compartmental model characterized by 11-nonlinear ordinary differential equations. In [40], an $SE_1E_2I_1I_2I_3R$ epidemiological mathematical model was proposed to evaluate the risk of COVID-19 pandemic beyond China. Their study shows that China's epidemic had reached its peak and therefore recommended that, the most effective control measure for nations with low connectivity with China to curb the outbreak in their cities is to cause further decline in their importation numbers by imposing travel restrictions and entry screening.

A SEIQR COVID-19 dynamical model that captures isolation class is studied in [41]. The authors performed numerical simulations for their mathematical model using Runge-Kutta fourth-order and nonstandard finite difference iterative schemes. Higazy [42], proposed and studied deterministic SIDARTHE Caputo fractional-order model and optimal control fractional SIDARTHE COVID-19 epidemiological model. The spatial dynamics of the highly infectious COVID-19 disease is investigated using Moran's I spatial statistic [43]. Rong [44], introduced and analyzed an SEIR-type mathematical modeling formulation to explore the spreading dynamics of COVID-19 infectious disease. Following the results of their simulations, the authors argued that reducing the waiting time for diagnosis and improving the proportion of timely diagnosis could significantly help in reducing the spread of the disease. The work done by the authors in [45], deals with mathematical modeling of COVID-19 disease using SEIR compartmental framework with discrete stochastic dynamics. The authors in [46] introduced and presented a data-driven compartmental model for COVID-19 disease. The work done by the authors in [47], explored the transmission dynamics of COVID-19 using a mathematical epidemic model with Monte Carlo simulations in Matlab. In [48], the authors presented and analyzed a novel data-driven prediction based stochastic model for COVID-19 infection in India.

The application of optimal control theory in constructing and analyzing nonlinear dynamical systems in infectious diseases modeling and control strategies is immense and has been explored by several authors see, e.g., [49–64]. Mallela [65], proposed a dynamical mathematical model with a control strategy to investigate the recent COVID-19 infectious disease. In a recent mathematical model proposed by Asamoah and co-workers [66], they used nonlinear dynamical systems both in an autonomous and non-autonomous sense to study the COVID-19 pandemic in Ghana. The work done by the authors in [67] deals with the application of Pontryagin's maximum principle to study some SIR and SEIR Ebola infection models. A deterministic COVID-19 model is proposed and studied by the authors in [68] to explore the spreading dynamics and possible mitigation strategies. Cui et al. [69] constructed and studied a data-driven SEIR type deterministic model for the 2019 coronavirus disease that captures quarantined individuals in susceptible, exposed, and infected compartmental classes. The authors in [70], have considered and analyzed a co-infection controlled dynamic model for schistosomiasis and Cholera. Another co-infection deterministic optimal control modeling approach is investigated by the authors in [71], in a new novel epidemiological model for the Zika virus and dengue fever vector-borne infectious diseases. The recent mathematical model introduced and analyzed by the authors in [72], studied the dynamical behavior of COVID-19 using nonlinear differential equations with time-dependent control functions. Their optimal control problem for the COVID-19 disease was an extension of the non-optimal control deterministic COVID-19 model formulated and numerically studied in [44]. A Susceptible-Exposed-Hospitalized infected-Quarantine-Recovered compartmental framework is applied in [73] to investigate the transmission dynamics of the 2019 coronavirus disease using both autonomous and non-autonomous differential equations models. A dynamical deterministic optimal control compartmental model for COVID-19 disease using quarantine as time-dependent control function is proposed and numerically studied in [74].

This study is concerned with a data-driven SEIR compartmental model for the 2019 coronavirus infections in Ghana. Our first objective in this work is to formulate a deterministic SEIR COVID-19 model and then perform data fitting to estimate the values of the model parameters based on the reported cumulative COVID infected cases in Ghana from 13th March 2020 to 6th October 2020. We will then compute the basic reproduction number, R_0, for the SEIR epidemic model based on the estimated model parameters. Our mathematical modeling formulation is motivated by works in [3, 75, 76] and data-driven Ebola epidemic models studied by authors in [77, 78]. Our second objective in this present study is inspired by the aforementioned literature on optimal control modeling of infectious diseases to construct a new optimal control nonlinear dynamical problem for the 2019 coronavirus disease using personal protection as a time-dependent control function.

The remainder of this research paper is outlined as follows. Section 2 is concerned with formulating an SEIR mathematical model for COVID-19 dynamics and estimating model parameters values using the reported cumulative infected cases of COVID-19 in Ghana from 13th

March 2020 to 6th October 2020 to compute the basic reproduction number, R_0. Using personal protection as a time-dependent control function, we will formulate and study a new deterministic optimal control model for the deadly COVID-19 disease in section 3. We will also derive an optimality system from our proposed mathematical model in the same section. In section 4, numerical simulations are carried out for the derived optimality system. We will finally conclude the work in section 5.

2. SEIR COVID-19 MATHEMATICAL MODEL

This section deals with constructing a nonlinear dynamical COVID-19 mathematical model using the classical SEIR deterministic modeling framework. Following the epidemiological compartmental modeling approach in [3, 75, 76], we partition the total population (N) into four sub-populations namely Susceptible, Exposed, Infected and Recovered represented by S_c, E_c, I_c and R_c respectively. With the assumption of constant population dynamics, the deterministic model describing the COVID-19 infection is given by;

$$
\begin{aligned}
\frac{dS_c(t)}{dt} &= -\frac{\beta S_c(t) I_c(t)}{N} & S_c(0) &= S_{c0} \geq 0,\\
\frac{dE_c(t)}{dt} &= \frac{\beta S_c(t) I_c(t)}{N} - dE_c(t) & E_c(0) &= E_{c0} \geq 0,\\
\frac{dI_c(t)}{dt} &= dE_c(t) - \gamma I_c(t) & I_c(0) &= I_{c0} \geq 0,\\
\frac{dR_c(t)}{dt} &= \gamma I_c(t) & R_c(0) &= R_{c0} \geq 0.
\end{aligned} \tag{1}
$$

with $S_c(t) + E_c(t) + I_c(t) + R_c(t) = N$

where β, $\frac{1}{\gamma}$ and $\frac{1}{d}$ represent disease transmission rate, average durations of infectiousness and incubation respectively. The basic reproduction number for this type of epidemiological model is given by $R_0 = \frac{\beta}{\gamma}$.

Our main objective in this section is to perform data fitting to estimate the model parameters β and γ using the the reported cumulative infected cases of COVID-19 in Ghana from 13th March 2020 to 6th October 2020. We will then compute the basic reproduction number, R_0, for the SEIR epidemic model based on the estimated model parameters. For this purpose, we include another differential equation that captures the dynamics of cumulative infected cases of this deadly disease in Ghana to the SEIR model (1). The modified SEIR COVID-19 model with cumulative infected cases (C) is given by

$$\begin{aligned}
\frac{dS_c(t)}{dt} &= -\frac{\beta S_c(t)I_c(t)}{N} \\
\frac{dE_c(t)}{dt} &= \frac{\beta S_c(t)I_c(t)}{N} - dE_c(t) \\
\frac{dI_c(t)}{dt} &= dE_c(t) - \gamma I_c(t) \\
\frac{dR_c(t)}{dt} &= \gamma I_c(t) \\
\frac{dC(t)}{dt} &= dE_c(t)
\end{aligned} \tag{2}$$

It is important to mention that the fifth differential equation in the COVID-19 model (2) has been used to capture the dynamics of cumulative infected cases of the Ebola disease outbreak by the authors in [77, 78].

For the data fitting we use the total population of $N = 30955202$ as projected by Ghana Statistical Service [79] and assume initial conditions: $S_{c_0} = 30954500, E_{c_0} = 700, I_{c_0} = 2, R_{c_0} = 0, C_0 = 2$. The incubation period, $\frac{1}{d} = 5.2$ days as reported in the work by Tian et al. [64] was used for the data fitting. Using the fminsearch optimization routine in Matlab and the reported cumulative infected COVID-19 cases in Ghana (from March 13 to October 06, 2020) as reported in ourworldindata.org [80], the estimated values for model parameters β and γ are 1.8980 and 1.8228 respectively. It follows from the estimated model parameters that the basic reproduction number, $R_0 \approx 1.0413$. The fminsearch Matlab routine which implements the Nelder-Mead optimization algorithm has recently been applied in the works of the authors in [81–86] to estimate their model parameters.

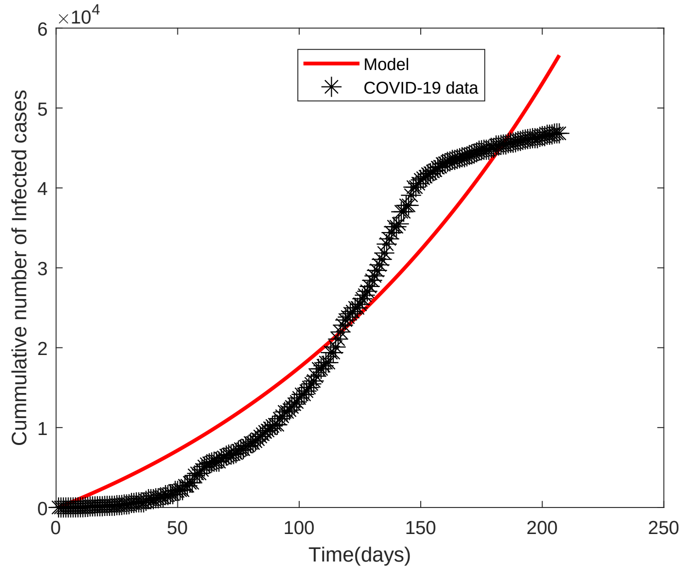

FIGURE 1. Plot of cumulative COVID-19 infected cases in Ghana and the mathematical model (2)

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It is clear from Figure 1 that the COVID-19 model 2 best fit the reported cumulative data of Ghana.

In the next section, we will construct a new COVID-19 optimal control problem. As recently presented in the works of the authors in [58, 59, 67], the optimal control problem that we will formulate and analyze in the next section will be based on the scaled COVID-19 SEIR model problem (3) given below.

By following the works of the authors in [3, 67] and ignoring the steps involved, the scaled COVID-19 SEIR deterministic model is given below.

$$
\begin{aligned}
\frac{ds_c(t)}{dt} &= -\beta s_c(t) i_c(t) & s_c(0) &= s_{c0} \geq 0, \\
\frac{de_c(t)}{dt} &= \beta s_t(t) i_c(t) - d e_c(t) & e_c(0) &= e_{c0} \geq 0, \\
\frac{di_c(t)}{dt} &= d e_c(t) - \gamma i_c(t) & i_c(0) &= i_{c0} \geq 0, \\
\frac{dr_c(t)}{dt} &= \gamma i_c(t) & r_c(0) &= r_{c0} \geq 0.
\end{aligned} \tag{3}
$$

with $s_c(t)+e_c(t)+i_c(t)+r_c(t)=1$,

where $s_c(t),\ e_c(t),\ i_c(t)$ and $r_c(t)$ represent new state variables in proportional sense.

3. Optimal Control SEIR COVID-19 Model

This section deals with using the scaled SEIR epidemiological model (3) formulated in the previous section to construct a new COVID-19 optimal control problem using personal protection as a time-dependent control function $(\vartheta(t))$. Our aim for constructing this new control strategy for this deadly pandemic is that we seek to minimize the number of exposed and infected individuals in the population and the cost of personal protection. For this purpose, we minimize the new quadratic objective functional $\mathscr{J}(\vartheta)$ given below

$$\mathscr{J}(\vartheta) := \int_0^T \left(A_1 e_c(t) + A_2 i_c(t) + \frac{1}{2} M \vartheta^2(t) \right) dt. \tag{4}$$

subject to

$$\begin{aligned}
\frac{ds_c(t)}{dt} &= -\Big(1-\vartheta(t)\Big)\beta s_c(t) i_c(t) & s_c(0) &= s_{c0} \geq 0,\\
\frac{de_c(t)}{dt} &= \Big(1-\vartheta(t)\Big)\beta s_t(t) i_c(t) - d e_c(t) & e_c(0) &= e_{c0} \geq 0,\\
\frac{di_c(t)}{dt} &= d e_c(t) - \gamma i_c(t) & i_c(0) &= i_{c0} \geq 0,\\
\frac{dr_c(t)}{dt} &= \gamma i_c(t) & r_c(0) &= r_{c0} \geq 0.
\end{aligned} \tag{5}$$

$$\mathscr{U} := \{\vartheta : \vartheta \text{ is Lebesgue measurable}, 0 \leq \vartheta(t) \leq 1, t \in [0,T].\} \tag{6}$$

where A_1 and A_2 are weight constants related to exposed and infected individuals and M is related to the controlled function, $\vartheta(t)$.

The Hamiltonian related to this optimal control problem is given by

$$
\begin{aligned}
\mathscr{H} &= A_1 e_c(t) + A_2 i_c(t) + \frac{1}{2} M \vartheta^2(t) \\
&+ \zeta_1 \Big[- \Big(1 - \vartheta(t)\Big) \beta s_c(t) i_c(t) \Big] \\
&+ \zeta_2 \Big[\Big(1 - \vartheta(t)\Big) \beta s_c(t) i_c(t) - d e_c(t) \Big] \\
&+ \zeta_3 \Big[(d e_c(t) - \gamma i_c(t) \Big] \\
&+ \zeta_4 \Big[\gamma i_c(t) \Big]
\end{aligned}
\tag{7}
$$

By using the well-known Maximum Principle introduced and studied by Pontryagin et al. [87], we can find an optimal solution for a given dynamical optimal control problem as follows

Given an optimal control problem with (ψ, ϑ) as its optimal solution, there exist a special vector function $\zeta = (\zeta_1,\ \zeta_2,\ \cdots, \zeta_n)$ consisting of adjoint variables which satisfies the system below.

$$
\begin{cases}
\frac{d\psi}{dt} = \frac{\partial H(t,\ \psi,\ \vartheta,\ \zeta)}{\partial \zeta}, \\
0 = \frac{\partial H(t,\ \psi,\ \vartheta,\ \zeta)}{\partial \vartheta}, \\
\frac{d\zeta}{dt} = -\frac{\partial H(t,\ \psi,\ \vartheta,\ \zeta)}{\partial \psi}.
\end{cases}
\tag{8}
$$

Following the special equation (8) and the constructed Hamiltonian function (7) from the optimal control problem, the adjoint or co-state equations and the optimal control characterization for the COVID-19 disease mathematical model are given below.

Theorem 1. *Suppose that ϑ^* is an optimal control and $({s_c}^*, {e_c}^*, {i_c}^*, {r_c}^*)$ as an optimal state solution for the optimal control problem (4)-(5) that minimize $\mathscr{J}(\vartheta)$ over $\mathscr{U}$, then there exist adjoint or co-state variables ζ_i, ζ_2, ζ_3, and ζ_4 satisfying the equations below;*

$$
\begin{aligned}
\frac{d\zeta_1}{dt} &= (\zeta_1-\zeta_2)(1-\vartheta^*(t))\beta i_c^*(t) \\
\frac{d\zeta_2}{dt} &= -A_1+(\zeta_2-\zeta_1)d \\
\frac{d\zeta_3}{dt} &= -A_2+(\zeta_1-\zeta_2)(1-\vartheta^*(t))\beta s_c^*(t)+(\zeta_3-\zeta_4)\gamma \\
\frac{d\zeta_4}{dt} &= 0
\end{aligned} \tag{9}
$$

with transversality conditions

$$\zeta_k(T)=0,\ k=1,2,3,4. \tag{10}$$

where $\vartheta^*(t)$ satisfies the optimality condition given by equation (11)

$$\vartheta^*(t)=\min\left\{\max\left\{0,\frac{(\zeta_2-\zeta_1)\beta s_c^*(t)i_c^*(t)}{M}\right\},1\right\} \tag{11}$$

Proof. To derive the adjoint or co-state system and its transversality conditions, we need to apply the well-known Pontryagin's maximum principle [87] and then differentiate the constructed Hamiltonian function (7) partially with respect to the state variables as follows:

$$
\begin{cases}
\frac{d\zeta_1}{dt} = -\frac{\partial\mathscr{H}}{\partial s_c}, \\
\frac{d\zeta_2}{dt} = -\frac{\partial\mathscr{H}}{\partial e_c}, \\
\frac{d\zeta_3}{dt} = -\frac{\partial\mathscr{H}}{\partial i_c}, \\
\frac{d\zeta_4}{dt} = -\frac{\partial\mathscr{H}}{\partial r_c}.
\end{cases} \tag{12}
$$

with

$$\zeta_k(T) = 0,\ k = 1,2,3,4. \tag{13}$$

Finally, knowing that on the interior of the control set $\mathscr{U}$, we have

$$\frac{\partial \mathscr{H}}{\partial \vartheta} = 0. \tag{14}$$

Solving equation (14) for ϑ^* yields the control characterization (11) □

4. NUMERICAL RESULTS AND DISCUSSION

This section of our study deals with computing numerical solutions for the constructed optimality system from our newly proposed COVID-19 optimal control problem. We have therefore in this study applied the widely used and reliable fourth Runge-Kutta iterative scheme with the forward-backward method for solving optimality systems in nonlinear dynamical optimal control problems to generate our numerical results. The details of this useful numerical scheme with some interesting applications in biology can be found in the optimal control modeling textbook written by Lenhart and Workman [88]. Many authors who apply indirect methods for their optimal control problems normally consider this efficient numerical scheme for solving their optimality systems see, e.g., [54–56, 61, 64, 71]. The numerical simulation is conducted by the use of the fitted parameter values given as $\beta = 1.8980$ and $\gamma = 1.8228$ with initial conditions as: $s_{c_0} = 0.95, e_{c_0} = 0.047, i_{c_0} = 0.003, r_{c_0} = 0$. We also assume weight constant values given as: $A_1 = 5, A_2 = 5, M = 50$. We adapt the incubation period, $\frac{1}{d} = 5.2$ days as reported in the study by Tian et al. [64]. In both figure 2 and figure 3, there is a substantial decrease in the exposed and infected individuals, respectively in the control model than the mathematical model without control. Figure 4 depicts the plot of the optimal control function, $\vartheta(t)$ over time.

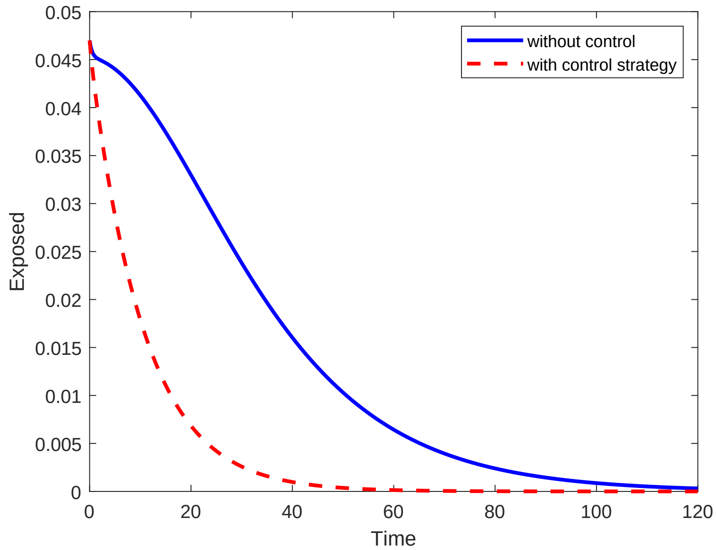

FIGURE 2. Solution trajectory for exposed individuals with and without control strategy

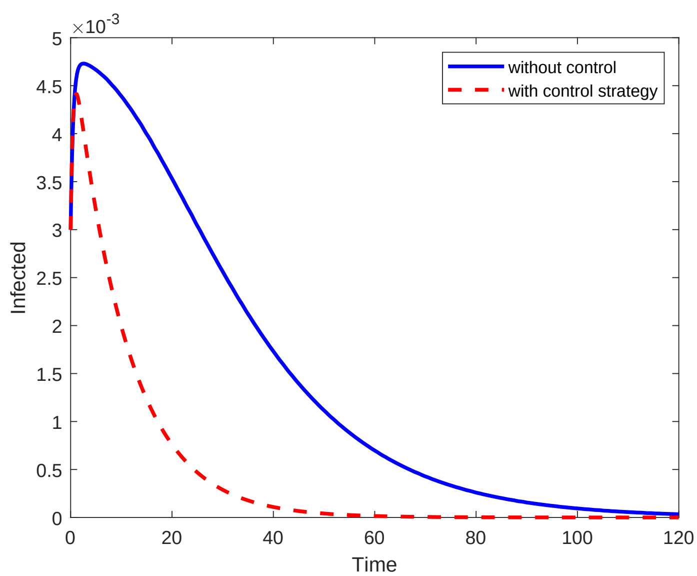

FIGURE 3. Solution trajectory for infected individuals with and without control strategy

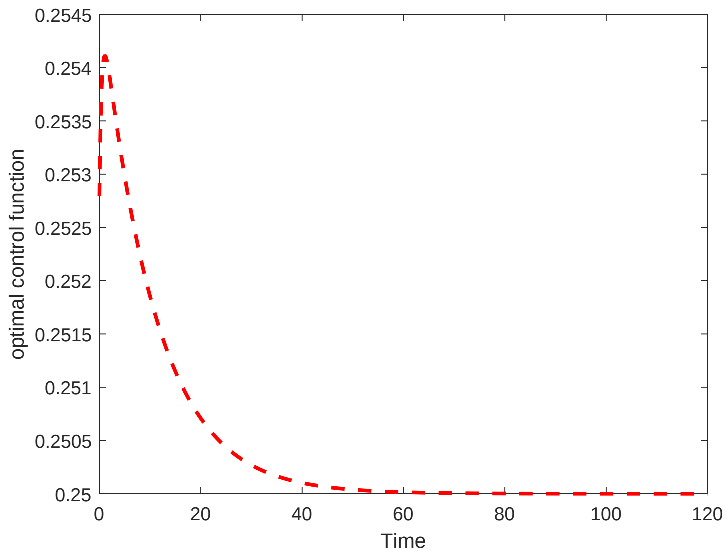

FIGURE 4. Optimal control function, $\vartheta(t)$

5. CONCLUSION

In this paper, we considered a simple data-driven SEIR compartmental model for the 2019 coronavirus infections in Ghana. We obtained the estimated model parameters using the fminsearch optimization routine in Matlab, and the reported cumulative infected cases of COVID-19 in Ghana from 13th March 2020 to 6th October 2020. We computed the basic reproduction number based on the fitted estimated values of the model parameters. In this work, we have also derived and analyzed a nonlinear dynamical optimal control SEIR COVID-19 model with personal protection as a time-dependent control function. We applied the well-known Maximum Principle in optimal control modeling of dynamical processes to also derive an optimality system for the model problem. We have further generated numerical results for the formulated optimality system using an efficient numerical scheme for optimal control problems detailed in [88]. From our numerical results, we can argue that proper personal protection practices can help reduce the disease transmission in the susceptible human population.

Data Availability

The authors confirm that the sources of data sets supporting the findings of this study are reported within the article.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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