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LOGISTIC GROWTH MODEL AND MODIFIED VERSIONS FOR THE CUMULATIVE NUMBER OF CONFIRMED CASES OF COVID-19 IN SAUDI ARABIA

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Abstract: In this study, mathematical modeling is applied to investigate the epidemiology of COVID-19 in Saudi Arabia. The aim was to estimate the cumulative number of infected cases using a logistic growth function and three modified models. The daily cumulative number of confirmed cases was collected from the online COVID-19 dashboard provided by the Ministry of Health. The period covered in this study began from 2nd March until the 20th August 2020. Data was fitted to the logistic growth function and three modified versions by using an online tool which implements the least squares estimate method. The results show all models fit significantly. The predictions from all these models are very similar and encouraging. According to the results the growth rate should decline from approximately 21st Jun 2020 onwards and the maximum number of cumulative confirmed infected cases in Saudi Arabia would be around 324,000 predicting an end to the pandemic by April 2021.

Keywords: confirmed cases estimation; COVID-19; data fitting; logistic growth; Saudi Arabia.

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1. INTRODUCTION

In December 2019, a sudden outbreak of severe pneumonia occurred in Wuhan, China. It was subsequently discovered that this disease was caused by a virus transmitted from bats, through still unknown intermediary animals, to humans. The virus was classified as 2019 Novel Corona Virus (2019-nCoV) or the severe acute respiratory syndrome corona virus 2 (SARS_CoV-2), and the disease was named COrona VIrus Disease 2019 (COVID-19) [1]. The effect of COVID-19 varies from slight, self-limiting respiratory tract disease to brutal progressive pneumonia, multiple organ dysfunction, and death [2]. The number of cases began to increase exponentially as the disease spread from Wuhan City to the whole Hubei Province and then to other provinces in China as well as other countries. As the virus continued to spread, modeling studies of the number of infected cases reported an epidemic doubling time of 7.4 days [3,4].

So far, the COVID-19 pandemic has been responsible for over twenty million reported cases worldwide, including over 700,000 deaths (as of 8th Aug 2020). In Saudi Arabia alone, the total number of reported cases is approaching 300,000, including over 3,000 deaths (as of 8th Aug 2020) [5]. The absence of an antiviral treatment and a preventative vaccine against COVID-19 has compelled the application of non-pharmaceutical interventions (NPI) to protect the wider population by reducing the spread of the pandemic.

In Saudi Arabia, the measures implemented by the Saudi government started before COVID-19 had spread to the Middle East. From the beginning of February, flights from and to China were stopped even before any confirmed infected cases were reported in the country. On the 2nd of March, the first case of COVID-19 was reported and further restrictions began to be implemented including the temporary suspension of Umrah (Islamic pilgrimage to Makkah that can be taken any time of the year), the closure of schools and universities all over the country, the suspension of international and then domestic flights, the suspension of all sporting events, the closure of all commercial complexes except for supermarkets and pharmacies, and the temporary suspension of attendance at workplaces in all government and then private sectors [6]. On the 23rd of March an 11-hour curfew was placed on the whole country for 21 days, this was then extended to a 15-hour curfew that continued till the 22nd of May, followed by a full lockdown from 23rd to 27th May, after which a curfew was set form 28th to 30th May. Thereafter, curfews were placed only on regions suffering the highest critical cases and, on the 21st of Jun, all curfews were lifted and all economic and commercial activates were allowed to resume, taking into account the strict application of all approved prevention protocols including social distancing, wearing masks and limiting gatherings to a maximum of 50 people [7]. The immediate precautionary measures and regulations that were implemented by the Saudi government have resulted in the reduction of the exponential growth rate of the pandemic and decreased both the death rate and the number of critical cases [8,9]. It has been reported that "The global death rate from COVID-19 is ten times higher than that of Saudi Arabia" [10].

Currently COVID-19 has attracted the attention of researchers due to its effect on people's health and on the economy. Mathematical modeling is a useful tool to understand the spread of diseases. Available data could be fitted to a certain model to help scientists predict the future and better prepare for this pandemic; it can also help them gain deeper understanding of this phenomenon. Already several papers on mathematical modeling of this outbreak have been published, for example [11-15] with much more ongoing research on the horizon.

The logistic growth function is commonly used to model epidemics. These epidemics follow an initially exponential growth, and then as more people get infected, the growth slows down approaching an upper limit, indicating that most of the population has been infected and thus have become immune to the disease. This function was used to model the outbreak of SARS in Singapore [16] and in Hong Kong [17]. This model could also be used to study COVID-19, as the disease's growth is similar to the logistic growth. Alboaneen *et al* fitted data from Saudi Arabia from 2/3/2020 to 15/5/2020 to a logistic function to predict the total number of cases and the final phase [14]. Elhassan and Gaafar studied two scenarios of the outbreak by using a logistic growth model to estimate the peak-time and final size [8]. Scenario 1 covered the period from 2/3/2020 to 28/5/2020 and scenario 2 was for the period from 29/5/2020 to 21/6/2020, through which lockdown was relaxed. Komies *et al* also used a logistic growth model for the data collected from Saudi Arabia and the UK from 31/1/2020 to 1/4/2020 [9]. They compared the results between the two countries and showed how the implemented measures by the Saudi government helped reduce the number of deaths and limit the spread of this disease. In this paper we are interested in studying this pandemic in Saudi Arabia by using the logistic growth function and modified versions, which were used by [13] to estimate cumulative number of deaths by COVID-19 in USA. We aim to apply this logistic growth function to predict the cumulative number of infected cases in Saudi Arabia. We will collect the daily cumulative number of confirmed cases from the online COVID-19 dashboard provided by the Ministry of Health in Saudi Arabia [18]. We will cover the period from 2/3/2020 (first confirmed case) to 20/8/2020. Then, we will fit this data to the logistic growth function and modified versions [13,19,20] by using an online tool which implements the least squares estimate method [21]. Each function estimates the inflection point (the point after which the growth rate starts to slow down) and the carrying capacity (maximum size). Afterwards, we will calculate the statistical measures: SSE (sum of squared error), MSE (mean square error), RMSE (root mean square error), and R^2 (the coefficient of determination) by using MATLAB to determine which function best fits the data. This will provide more information about this pandemic and its behavior and it will help the government to prepare and plan for the future. The paper is organized as follows: the data regarding the daily cumulative number of confirmed cases of COVID-19 in Saudi Arabia is given in the second section, the data fitting with logistic functions is in the third section, and the results are in the fourth section. Finally, the discussion and conclusion are presented in the fifth section.

2. DATA: DAILY CUMULATIVE NUMBER OF CASES IN SAUDI ARABIA

We collected the daily cumulative number of confirmed cases of COVID-19 in Saudi Arabia provided by the Ministry of Health [18]. The period covered was from the 2nd March 2020, which was the date of the first confirmed case in Saudi Arabia, until 20th August 2020. Table 1 shows the daily cumulative number of confirmed cases for this period. Also Figure 1 shows a plot of these numbers against time (days). The periods: from 2nd till 22nd March the Saudi government started applying precautions, as discussed in the introduction, (illustrated by light blue stars in Figure 1), from 23rd March till 22nd May comprised curfews ranging from 11-15 hours per day (illustrated by black stars in Figure 1), from 23rd to 27th May was a full lockdown (illustrated by green stars in Figure 1), form 28th to 30th May was a 16 hour curfew (illustrated by red stars in Figure 1), from 31st May till 20th Jun curfews were placed only on regions suffering the highest critical cases (illustrated by blue stars in Figure 1), and from 21st Jun onwards all curfews were lifted (illustrated by purple stars in Figure 1).

Table 1: The daily cumulative number of confirmed cases of COVID-19 in Saudi Arabia for the period: 2/3/2020-20/8/2020, which is plotted in Figure 1.

Date	**Cumul. cases**	**Date**	**Cumul. cases**	**Date**	**Cumul. cases**	**Date**	**Cumul. cases**	**Date**	**Cumul. cases**
02/03/20	1	06/04/20	2605	11/05/20	42925	15/06/20	136315	20/07/20	255825
03/03/20	1	07/04/20	2932	12/05/20	44830	16/06/20	141234	21/07/20	258156
04/03/20	2	08/04/20	3287	13/05/20	46869	17/06/20	145991	22/07/20	260394
05/03/20	5	09/04/20	3651	14/05/20	49176	18/06/20	150292	23/07/20	262772
06/03/20	5	10/04/20	4033	15/05/20	52016	19/06/20	154233	24/07/20	264973
07/03/20	7	11/04/20	4462	16/05/20	54752	20/06/20	157612	25/07/20	266941
08/03/20	11	12/04/20	4934	17/05/20	57345	21/06/20	161005	26/07/20	268934
09/03/20	15	13/04/20	5369	18/05/20	59854	22/06/20	164144	27/07/20	270831
10/03/20	20	14/04/20	5862	19/05/20	62545	23/06/20	167267	28/07/20	272590
11/03/20	21	15/04/20	6380	20/05/20	65077	24/06/20	170639	29/07/20	274451
12/03/20	45	16/04/20	7142	21/05/20	67719	25/06/20	174577	30/07/20	275905
13/03/20	86	17/04/20	8274	22/05/20	70161	26/06/20	178504	31/07/20	277478

14/03/20	103	18/04/20	9362	23/05/20	72560	27/06/20	182776	01/08/20	278835
15/03/20	118	19/04/20	10484	24/05/20	74795	28/06/20	186436	02/08/20	280093
16/03/20	133	20/04/20	11631	25/05/20	76726	29/06/20	190823	03/08/20	281435
17/03/20	171	21/04/20	12772	26/05/20	78541	30/06/20	194323	04/08/20	282824
18/03/20	238	22/04/20	13930	27/05/20	80185	01/07/20	197608	05/08/20	284226
19/03/20	274	23/04/20	15102	28/05/20	81766	02/07/20	201801	06/08/20	285793
20/03/20	344	24/04/20	16299	29/05/20	83384	03/07/20	205929	07/08/20	287262
21/03/20	392	25/04/20	17522	30/05/20	85261	04/07/20	209509	08/08/20	288690
22/03/20	511	26/04/20	18811	31/05/20	87142	05/07/20	213716	09/08/20	289947
23/03/20	562	27/04/20	20077	01/06/20	89011	06/07/20	217108	10/08/20	291468
24/03/20	767	28/04/20	21402	02/06/20	91182	07/07/20	220144	11/08/20	293037
25/03/20	900	29/04/20	22753	03/06/20	93157	08/07/20	223327	12/08/20	294519
26/03/20	1012	30/04/20	24097	04/06/20	95748	09/07/20	226500	13/08/20	295902
27/03/20	1104	01/05/20	25459	05/06/20	98869	10/07/20	229480	14/08/20	297315
28/03/20	1203	02/05/20	27011	06/06/20	101914	11/07/20	232259	15/08/20	298542
29/03/20	1299	03/05/20	28656	07/06/20	105283	12/07/20	235111	16/08/20	299914
30/03/20	1453	04/05/20	30251	08/06/20	108571	13/07/20	237803	17/08/20	301323
31/03/20	1563	05/05/20	31938	09/06/20	112288	14/07/20	240490	18/08/20	302686
01/04/20	1720	06/05/20	33731	10/06/20	116021	15/07/20	243238	19/08/20	303973
02/04/20	1885	07/05/20	35432	11/06/20	119942	16/07/20	245851	20/08/20	305186
03/04/20	2039	08/05/20	37136	12/06/20	123308	17/07/20	248416		
04/04/20	2221	09/05/20	39048	13/06/20	127541	18/07/20	250920		
05/04/20	2402	10/05/20	41014	14/06/20	132048	19/07/20	253349		

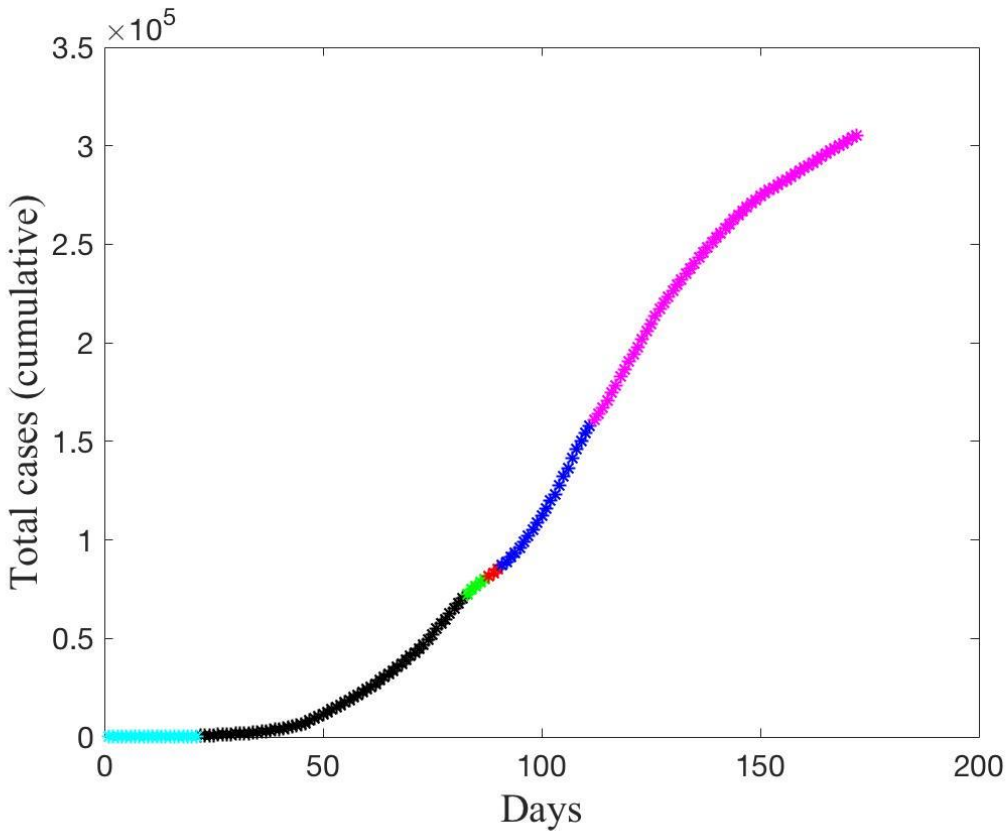

Figure 1: The plot of the daily cumulative number of confirmed cases of COVID-19 in Saudi Arabia for the period: 2/3/2020 (first day) until 20/8/2020 (172 days). The colored stars represent either lockdown, different types of curfews, or lifting the curfew (see the text for details).

Figure 1 shows that at the beginning, the growth rate was very slow. Then, after approximately one month, the growth accelerated rapidly even though lockdown was imposed. Afterwards, it steadily slowed down, as it appeared to reach a limit. Note that the lockdown was lifted on 21st Jun 2020 (day 112). The graph looks similar to an S-shape, this is called a *logistic growth*, and will be explained in the next section.

3. Method: Data Fitting with Logistic Functions

In this section we will introduce the logistic function and other similar functions, which we will use to fit the data. The logistic function is commonly used to model a population, which grows exponentially at the beginning, then, after a certain time, called the peak-time, the growth slows down as the population gets closer to a limit (carrying capacity). This happens for different reasons depending on the population, for example an environment can sustain a certain number of specie with limited recourses. The logistic function [22,23] is given by:

$$\textbf{Logistic function}: \ P(t) = \frac{K}{1+Ae^{-rt}},$$

where P is the population at time t, K is the carrying capacity, r is the logistic growth rate, P_0 is the initial population at $t = 0$, and $A = \frac{K-P_0}{P_0}$.

An infectious pathogen will spread fast among humans who have no immunity (exponential growth). As the number of susceptible individuals becomes less over time, the pandemic eventually levels off (logistic growth). The number of daily cumulative infected people with COVID-19 is expected to have a similar behavior, which is shown in Figure 1 for Saudi Arabia. Thus, the logistic function is a reasonable choice to fit the data regarding this pandemic as done by [24]. The data from Saudi Arabia were also fitted with a logistic function by [8,9,14]. However, in this paper we will consider a longer time period and also consider other modified versions of this function, which [13] used two of, to estimate the cumulative number of deaths in the United States. These three modified functions are used for comparison; and thus find which one best fits the data.

The modified models are:

$$\textbf{Model 1}\ [19]: \ P(t) = \frac{a(1-e^{-bt})}{1+\beta e^{-bt}},$$

$$\textbf{Model 2}\ [20]: \ P(t) = \frac{a}{1+d\frac{1+\beta}{\beta+e^{bt}}},$$

$$\textbf{Model 3}\ [13]: \ P(t) = c + \frac{a}{1+d\frac{1+\beta}{\beta+e^{bt}}}.$$

The parameters of the models are a, b, c, d, β. They will be determined by fitting the data from Table 1.

4. RESULTS

In this section we fit the data from Table 1 to the logistic function and the three models. We used and online tool to estimate the parameters by using the least squares estimate method [21]. Then, we used MATLAB to plot the figures (see Figures 2-5) and calculate the statistical measures: R^2, SSE, MSE, and RMSE. These statistical values along with the parameters' values are given in Table 2.

The value of R^2 (the coefficient of determination) in Table 2 shows that model 3 gives the best fit; and also, this is shown from the values of SSE, MSE, and RMSE. By using this model to fit the data, we get the carrying capacity, which is 323,761 (the maximum number of cumulative confirmed cases). It will take approximately seven months to reach this value. This means that the pandemic is expected to end approximately at the beginning of April 2021.

Table 2: The parameters' values and statistical measures of the logistic function and the three models.

Model	Parameters	Carrying capacity and inflection point	R^2	SSE	MSE	RMSE
Logistic	K=318164; A=209.601; r=0.0479439.	318,164 inflection point at day 111(20/6/2020)	0.9991	1934340871	11246168	3354
Model 1	a =320519; b =0.0467251; β =183.688.	320,519 inflection point at day 112 (21/6/2020)	0.9993	1506634345	8759502	2960
Model 2	a =320470; b =0.0467317; d =28110.2; β =-0.993429.	320,470 inflection point at day 112 (21/6/2020)	0.9993	1509201753	8774429	2962
Model 3	a =360363; b =0.0450135; c =-36601.6; d=9.74847; β =13.1632;	323,761 inflection point at day 111 (20/6/2020)	0.9994	1170937660	6807777	2609

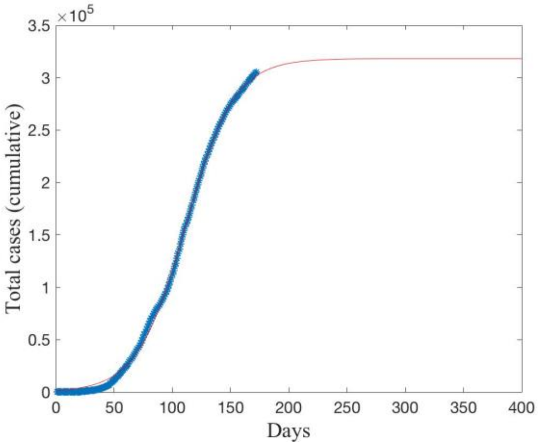

Figure 2: The plot of the cumulative number of confirmed cases (blue stars) and the logistic function (red curve) for the period: 2/3/2020 (first day) until 5/4/2021 (400 days).

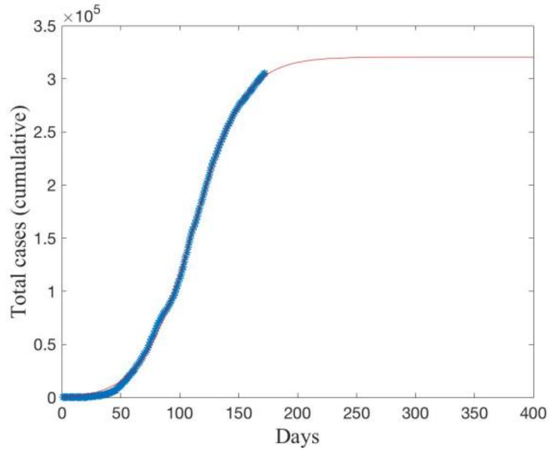

Figure 3: The plot of the cumulative number of confirmed cases (blue stars) and model 1 (red curve) for the period: 2/3/2020 (first day) until 5/4/2021 (400 days).

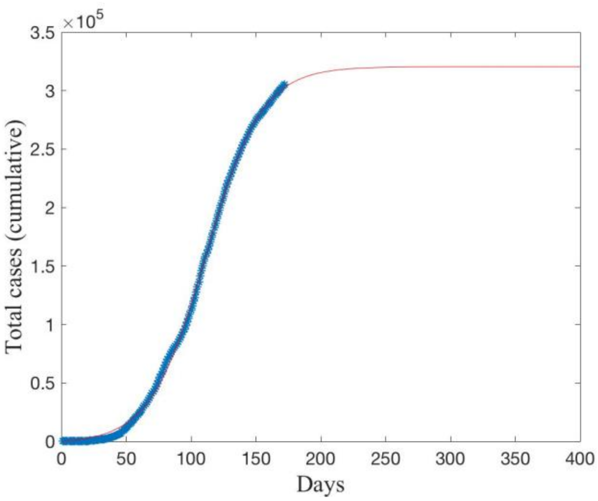

Figure 4: The plot of the cumulative number of confirmed cases (blue stars) and model 2 (red curve) for the period: 2/3/2020 (first day) until 5/4/2021 (400 days).

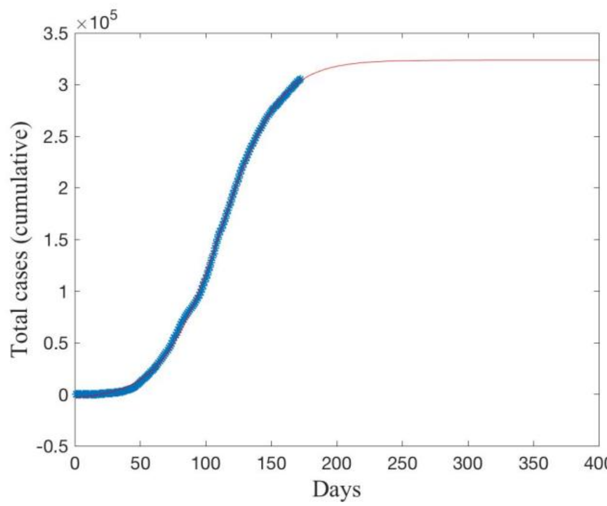

Figure 5: The plot of the cumulative number of confirmed cases (blue stars) and model 3 (red curve) for the period: 2/3/2020 (first day) until 5/4/2021 (400 days).

5. DISCUSSIONS AND CONCLUSION

Understanding the trend of COVID-19 is essential to determine the appropriate precautionary measures to limit the adverse effects of the pandemic and to avoid overwhelming healthcare facilities, especially intensive care units. Mathematical models in the field of infectious diseases can provide predictions of the number of infected cases during the pandemic, the duration of the pandemic, and the whole pandemic curve. This information assists governments and healthcare ministries or organizations in making decisions and strategically planning the control of pandemics.

The pandemic began in Saudi Arabia in early March with the number of confirmed cases approaching 300,000 by early August, of which 268,385 have recovered and 3300 have died [18]. The lower cases of mortality and the slower growth rate are attributed to the high alertness exhibited by the Saudi government in rapidly implementing exceptional precautionary measures.

In this study the logistic function and three modified models were applied to predict the number of cumulative confirmed infected cases in Saudi Arabia due to COVID-19. According to the experimental results, the logistic function and all modified models gave similar results however; the model proposed by Pham 2020 (model 3) had the best fitting. According to this model, the peak of the infection rate is day 111, which corresponds to the 20^{th} Jun 2020. Coincidently, this is in alignment with the government's ease of restrictions [7]. Model 3 predicts that the final number of cases is around 324,000 cases. The later date and the greater number of cases than the previous predictions by Alboaneen et al [14], Elhassan and Gaafar [8] and Komies *et al* [9] are attributed to the larger data we had available. According to our calculations it will take approximately seven months to reach this value. We thus predict that the pandemic will end around the beginning of April 2021. This date is much later than the prediction of Elhassan and Gaafar [8] and Komies *et al* [9]. We expect the predictions given in our study to be more reliable as the data covered a longer period than the former mentioned studies and more than one model was used. The predictions proposed by the models used in our study are based on the available data; errors in the data could influence the accuracy of the predictions. The availability of more data would improve future research. The mathematical models used in our study could assist in future predictions of the number of confirmed cases if the manner through which COVID-19 spreads does not change unexpectedly. This is a novel virus and still under investigation with lots more to be discovered. Actually, the mathematical models used in our study could be applied in the future to investigate the epidemiology of other infectious diseases.

In conclusion, the four models used in our study gave similar predictions of the spread of COVID-19 in Saudi Arabia, with model 3 giving the best fit. All models predicted a continues spread of the disease however the growth rate begins to slow down around day 111. The predicted final number of confirmed infected cases is around 324,000 cases, it will take approximately seven months to reach this value. We thus predict that the pandemic will end around the beginning of April 2021, by which time the disease would have continued spreading in Saudi Arabia for just over a year. All predictions must be interpreted with caution, as all models are simplifications of reality. However, the logistic models used in our study have a valuable advantage in terms of anticipating the pandemic trend, using these models can significantly improve estimates of the number of infected cases and help the government and the Ministry of Health make better plans and implement improved NPI. Nonetheless, further modifications might need to be added to these models in the case of the availability of a vaccine.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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