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OPTIMAL PREVENTION STRATEGY OF THE TYPE *SIR* COVID-19 SPREAD MODEL IN INDONESIA

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Abstract. This paper studied the optimal control for the spread of the type *SIR* COVID-19 in Indonesia. Optimal control provided by prevention of COVID-19 with Large-Scale Social Restrictions in the Framework of Accelerated Handling COVID-19 (u_1^*) and COVID-19 treatment efforts (u_2^*). Data obtained from COVID-19.go.id from July 1-September 30, 2020. The increase in COVID-19 cases can be seen from the basic reproduction number ($\boldsymbol{R_0}$) greater than one, which means that the number of active cases of COVID-19 in Indonesia continues to increase. The Indonesian government has made efforts to overcome the spread of COVID-19 but the number of active cases of COVID-19 is still increasing. Optimal control u_1^* and u_2^* can significantly reduce the number of active COVID-19 individuals compared without control. Optimal control u_2^* significantly can increase the number of individuals who recovered COVID-19 compared without control.

Keywords: COVID-19; *SIR type* model; optimal control; prevention; treatment.

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1. INTRODUCTION

In 1918, a significance proportion of the deaths were from pneumonia followed by influenza disease. The COVID-19 pandemic brushed across the world, initially starting in Wuhan China at the end of 2019. Nearly 2020 it began to spread to other countries including Indonesia starting on March 2, 2020 [1].

Mathematics plays a role in predicting, analyzing, evaluating, and monitoring the spread of COVID-19 cases. Several researchers have reviewed the model of the spread of COVID-19, among others [2, 3, 4, 5, 6, 7]. Dynamics model of the COVID-19 outbreak and efficacy of Government interferences is found in [2]. Modeling analysis and simulation of COVID-19 with morbidity data in Anhui, China has been studied in [3]. The COVID-19 epidemic control model in terms of the latent subpopulation was established in [4, 5]. The spreading of the COVID-19 in Wuhan with the reflection of individual interactive and governmental behavior, e.g., travel constraint, and hospital was reported in [6]. Analysis of the control of COVID-19 with the contact of policy interventions and meteorological determinants has been discussed in [7]. The authors in [7] have been planning detailed vaccination ideas for COVID-19 in dissimilar countermeasures form and show the efficacy of vaccination.

The optimal prevention strategy of avian influenza pandemic has been devoted in [8] for the control of the epidemic influenza to analysis prevention control, which is connected with relieve and quarantine policy, including its execution cost. Really, cost affects the optimal of countermeasures principles and the control of the virus transmission. This paper reviewed the optimal control of the COVID-19 spread model in Indonesia.

The paper is consists of: Section 2 describes the COVID-19 transmission model with control terms. Application of the COVID-19 model in Section 3. The analysis of optimal controls and includes some numerical studies of the optimal controls is given in Section 4 and discuss our results.

2. SPREAD MODEL OF THE *SIR*-TYPE COVID-19 IN INDONESIA

Epidemic spread models are studied in this paper by using a logistic differential equation form. The number of initial cases of infection, spread and recovered were obtained from COVID-19 data go.id. Suppose the initial number of infected cases in a population is P_0, and the rate of spread of infected cases is $r > 0$. Then number of epidemic growth at time t can be expressed as

$$\frac{dP}{dt} = rP,$$

Solution of exponential growth, for time $t = 0$ is

$$P(t) = P_0 e^{rt}$$

where $P(0) = P_0$. The epidemic growth dynamics as [9] is given by

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{M}\right), \tag{1}$$

with P is the cumulative number infected, and $M > 0$ maximum population size that the environment could carry, while $\frac{dP}{dt}$ is the growth rate of infected cases, r and M are constants. Logistic growth increases starting from the initial time, but decreases at the end time. When infections reach a certain proportion, epidemic growth shows an exponential trend, after being given epidemic prevention, the epidemic situation gradually slows down the rate of spread. If $P(0) = P_0 > 0$, represents the initial number of infected cases, then solving the equation (1) is

$$P = \frac{M}{1+Ce^{-rt}},$$

with $C = \frac{K-P_0}{P_0}$.

The COVID-19 model is grouped into three sub-populations, namely the suspected, actively infected, and recovered sub-populations. The model COVID-19 is based on the Kermack McKendrick model [10]. Suspect individual (S) is a person who has symptoms of a cold cough, fever or sore throat who has a chronicle of travel to the area of the spread of COVID-19 or have a history of contact with people with COVID-19. Suspect phase (S) can change status to become infected, if you have direct contact with a person who is positive for the corona virus, but if you have been infected with COVID-19 is not included in the subpopulation of suspects. In the infectious phase (I), the medical team works hard to treat the infected individuals so that the rate of recovery is optimal. The Government of the Republic of Indonesia has provided a special hospital to handle corona virus patients so that the virus does not spread to other patients. A quarantine building was also built to accommodate and treat the symptomatic and infected to reduce the rate of contact between those who were still symptomatic and infected with people who were still healthy. Even with these efforts, there were still many victims who died, with a fairly high mortality rate. If an infected individual becomes cured of COVID-19 then that individual is not included in the infected subpopulation. The third phase, namely recovered population (R). In this phase, people who are declared cured, after recovering, need to be recovered so they do not get infected with the coronavirus again.

The model assumption is given as follows.

(i) The population is closed because the time is short.

(ii) Pay attention to deaths from COVID-19.

(iii) Individual S who has swab tests, confirmed positive and actively enters compartment I.

(iv) Individuals who recover or die from COVID-19 are not included in the infected sub-population I.

(v) Individuals recovering from COVID-19 enter compartment R.

The schematic diagram of the model studied is shown in Figure 1:

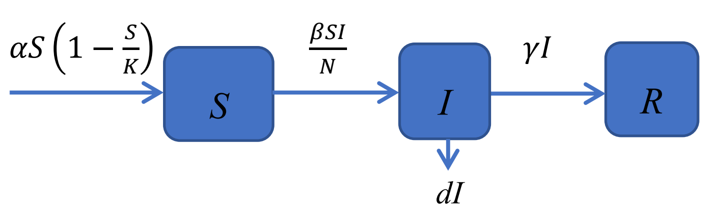

Figure 1. Schematic diagram of the COVID-19 type *SIR*

The model studied is a *SIR*-type epidemic spread model as follows :

$$\frac{dS}{dt} = \alpha S\left(1 - \frac{S}{K}\right) - \frac{\beta SI}{N} \tag{2}$$

$$\frac{dI}{dt} = \frac{\beta SI}{N} - (\gamma + d)I \tag{3}$$

$$\frac{dR}{dt} = \gamma I, \tag{4}$$

with $N(t) = S(t) + I(t) + R(t)$, parameters α is the growth rate of the individual suspect compartments, parameters β is the transmission rate from the individual suspected compartment to active COVID-19, γ is transfer rate from actively infected to recovered individual compartments, and d is death rate due to COVID-19. K is suspected carrying capacity. Carrying capacity K, parameters α, β, γ and d, are estimated from COVID-19.go.id data by using the Ordinary Least Square method (OLS) with minimal error.

3. APPLICATION OF THE SIR-TYPE COVID-19 SPREAD MODEL IN INDONESIA

The COVID-19 data used is July 1- September 30, 2020 in Indonesia. The reason of the data was taken started Juli 1, 2020 because the data is complete for the subpopulation of suspected and active COVID-19 in Indonesia. Based on COVID-19.go.id data [1], the initial population is $S(0) = 58488$, $I(0) = 29241$, $R(0) = 25595$, with each carrying capacity $K = 20.000.000$, parameters $\alpha = 0{,}047$, $\beta = 0{,}154$, $\gamma = 0{,}036$, and $d = 0{,}002$. The comparison of the spread of the suspected COVID-19 based on COVID-19 go.id data and the model can be seen in Figure 2.

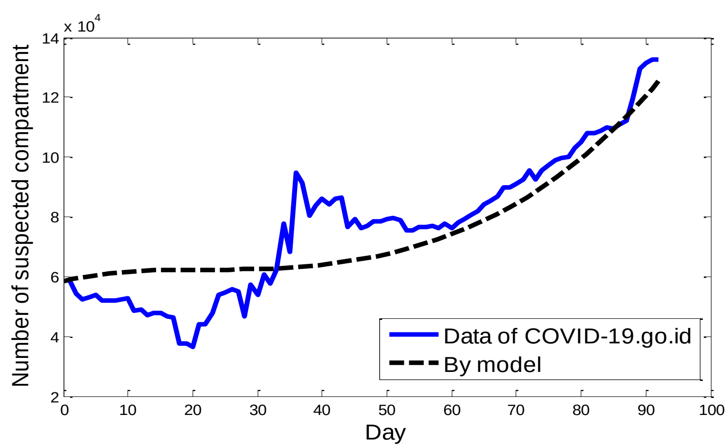

Figure 2. Number of suspected COVID-19, Juli 1- September 30, 2020 in Indonesia from COVID-19.go.id data and the model

Based on Figure 2, from early July, 2nd week of August and end of September 2020, the number of COVID-19 suspects from the COVID-19.go.id data and models is still almost the same, while at other times the number of suspected COVID-19 from COVID-19.go.id data with the model is rather different.

The dynamic of active cases of COVID-19, 1 June-31 August 2020 in Indonesia based on COVID-19.go.id data and the model studied can be seen as in Figure 3 below:

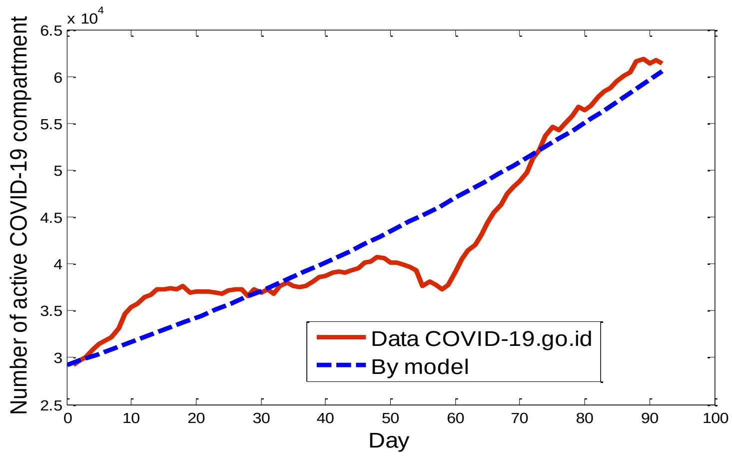

Figure 3. Number of active COVID-19, July 1-September 30, 2020 in Indonesia from COVID-19.go.id data with model studied

Based on Figure 3, early July, the fourth week of July, and the end of September 2020, the number of active COVID-19 cases in Indonesia from COVID-19.go.id data with the model is almost the same, while the other time the active number of COVID-19 was from the COVID-19.go.id data with the model is rather different.

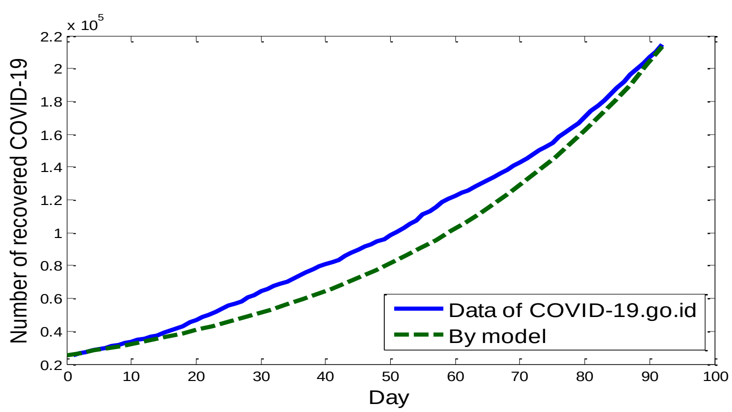

Figure 4. The number of recovered from COVID-19, July 1- September 30, 2020 in Indonesia from COVID-19.go.id data with the model studied

The spread of recovery from COVID-19, 1 July 1- September 30, 2020, in Indonesia based on COVID-19.go.id data and the model can be seen as in Figure 4. Based on Figure 4, the week to July 1-2 and at the end of September 2020 cases recovered from COVID-19 in Indonesia based on COVID-19.go.id data and almost the similar with the model, whereas July 13 - September 28, 2020, the number of recovered from COVID-19 in Indonesia was based on COVID-19.go.id data and models studied is rather different.

The Basic Reproduction Number

The standard parameters for determining whether the disease is spreading or not i.e. the basic reproduction number ($\boldsymbol{R_0}$). If $\boldsymbol{R_0} > 1$, then the disease spreads or the number of individuals with new infections is greater than those who are cured, whereas if $\boldsymbol{R_0} \leq 1$, then the disease decreases or the number of new infections is less than cured. We adopt the method in [11] to determine the basic reproduction number by using the next generation matrix. Hence, we obtain:

$$\boldsymbol{R_0} = \frac{\beta}{\gamma + d} \tag{5}$$

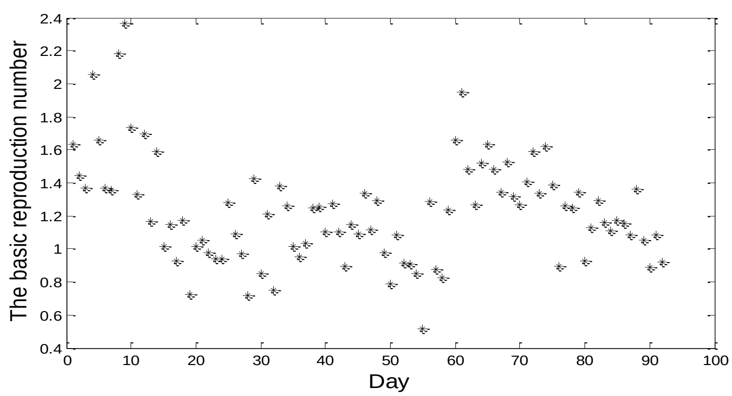

Figure 5. The basic reproduction number of COVID-19, Juli 1-September 30, 2020 in Indonesia

In Figure 5, it can be seen from July 1-September 30, 2020 the average value of the basic reproduction number ($\boldsymbol{R_0}$) > 1, meaning that the spread of COVID-19 is increasing. Meanwhile, July 19, 22-24, 27-28, 30 and August 1, 5, 12, 18-19, 21-24, 26-27, and September 14, 18, 28, 30, 2020 $\boldsymbol{R_0} \leq 1$, this means that the spread of COVID-19 in Indonesia has decreased.

4. Optimal Prevention Control on COVID-19 in Indonesia

The efforts have been made by the government to protect the spread of COVID-19 by providing control include: (i) Control with Government Supervision no. 21 of 2020 about Large-Scale Social Restraints (LSSR) in the Context of Quickening the Handling of COVID-19 ($u_1(t)$). LSSR is boundary on certain activities of population in an area suspect of being infected with the COVID-19 to stop its possible spread. Determination LSSR, each area must satisfy the following standards: The number of positive cases and the number of deaths from disease increases and spread really and rapidly to several areas. There are epidemiological connections with identical incidents in other areas or nations. (ii) Control treatment by providing a supply of vitamins C, D and E in an effort to accelerate healing of COVID-19 infection ($u_2(t)$). The system of differential equations after given control is as follows:

$$\frac{dS}{dt} = \alpha S\left(1 - \frac{S}{K}\right) - \beta(1 - u_1)\frac{SI}{N} \tag{6}$$

$$\frac{dI}{dt} = \beta(1 - u_1)\frac{SI}{N} - \gamma(1 + u_2)I - dI \tag{7}$$

$$\frac{dR}{dt} = \gamma(1 + u_2)I. \tag{8}$$

Optimal control objective functional which is studied as follows:

$$J(u_1, u_2) = \int_0^{t_f}\left(AI(t) + B_1u_1^2(t) + B_2u_2^2(t)\right)dt, \tag{9}$$

with coefficient A, B_1, and B_2 are the weight balance of the number of active COVID-19 compartments, weights that correspond to the control $u_1(t)$, and weights that correspond to the control $u_2(t)$, respectively. While, t_f is period end time. Suppose u_1^* and u_2^* are optimal control of the system (6)-(8) and (9), such that it satisfy:

$$J(u_1^*, u_2^*) = \min_{\kappa} J(u_1, u_2) \tag{10}$$

with $\kappa = \{(u_1, u_2) \epsilon L^1(0, T) |\ 0 \le u_i \le 1, i = 1, 2\}$.

Hamiltonian equation of the system (6)-(8) and (9) are as follows [4, 12, 13, 14]:

$$H = AI + B_1u_1^2 + B_2u_2^2 + \lambda_1\frac{dS}{dt} + \lambda_2\frac{dI}{dt} + \lambda_3\frac{dR}{dt}. \tag{11}$$

Theorem 1

Let $S^*(t), I^*(t), R^*(t)$ *be optimal state solutions with associated optimal control variables* $u_1^*(t)$ *and* $u_2^*(t)$ *for the optimal control problem* (6)-(8) and (11). *Then there exist adjoin* *variables λ_i, i = 1, 2, 3 satisfing*

$$\frac{d\lambda_1}{dt} = \lambda_1\left(\frac{2S}{K} - \alpha\right) + (\lambda_1 - \lambda_2)\beta(1 - u_1)\frac{I}{N}) \tag{12}$$

$$\frac{d\lambda_2}{dt} = -A + \beta(\lambda_1 - \lambda_2)(1 - u_1)\frac{S}{N} + \gamma(\lambda_2 - \lambda_3)(1 + u_2) + \lambda_2 d \tag{13}$$

$$\frac{d\lambda_3}{dt} = 0. \tag{14}$$

The transversality conditions of are given by $\lambda_i(t_f) = 0, i = 1, 2$. Finally, from the optimality condition, we obtain the following optimal controls:

$$u_1^* = \min\left\{max\left\{0, \frac{\beta(\lambda_2 - \lambda_1)SI}{2B_1 N}\right\}, 1\right\}$$

$$u_2^* = \min\left\{max\left\{0, \frac{\gamma(\lambda_2 - \lambda_3)I}{2B_2}\right\}, 1\right\}.$$

Proof: We use Pontrygain's Maximum Principle [15] on our model system (11), and the Hamiltonian is given by,

$$H = AI + B_1 u_1^2 + B_2 u_2^2 + \lambda_1(\alpha S(1 - \frac{S}{K}) - \beta(1 - u_1)\frac{SI}{N}) + \lambda_2(\beta(1 - u_1)\frac{SI}{N} -$$
$$\gamma(1 + u_2)I - dI + \lambda_3(\gamma(1 + u_2)I$$

$$\frac{d\lambda_1}{dt} = -\frac{\partial H}{\partial S}$$
$$= \lambda_1\left(\frac{2S}{K} - \alpha\right) + (\lambda_1 - \lambda_2)\beta(1 - u_1)\frac{I}{N})$$
$$\frac{d\lambda_2}{dt} = -\frac{\partial H}{\partial I}$$
$$= -A + \beta(\lambda_1 - \lambda_2)(1 - u_1)\frac{S}{N} + \gamma(\lambda_2 - \lambda_3)(1 + u_2) + \lambda_2 d$$
$$\frac{d\lambda_3}{dt} = -\frac{\partial H}{\partial R} = 0,$$

and must satisfy transversality conditions $\lambda(t_f) = 0$ for values i = 1, 2, 3. There exist unique optimal controls $u_1^*(t)$ and $u_2^*(t)$ which minimize *J* over *U*: The optimality necessary conditions that $\frac{\partial H}{\partial u_1} = 0$ and $\frac{\partial H}{\partial u_2} = 0$, then, by the bounds on the controls, it is easy to obtain and in the form $u_1^*(t) = \frac{\beta(\lambda_2 - \lambda_1)SI}{2B_1 N}$ and $u_2^*(t) = \frac{\gamma(\lambda_2 - \lambda_3)I}{2B_2}$. The optimal control of disease, the reproduction numbers declared to be as follows:

$$R_0^* = \frac{\beta(1-u_1)}{\gamma(1+u_2)+d}.$$

To demonstrate the numerical simulation, we assume the weight parameters into account A = 10, B_1 = 20, and B_2 = 30, so that control variables can be said to minimize objective functionality. We used the fourth order Runge-Kutta method to solve the numerical of the optimal control problem [16]. The numerical solution for the COVID-19 in Indonesia for the active infected subpopulation is given as follows.

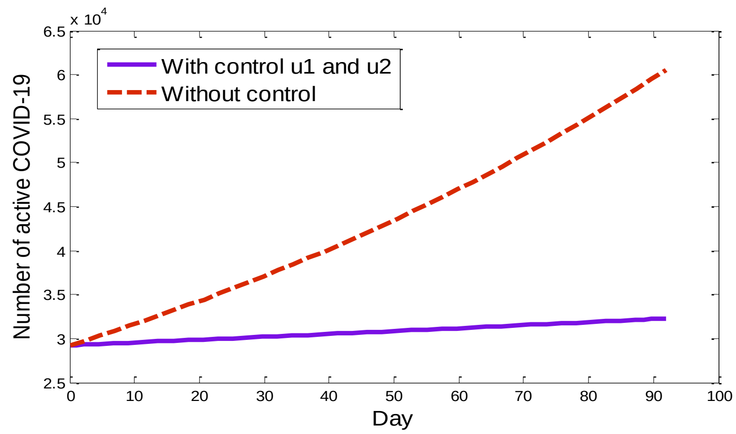

Figure 6. The dynamics of I with u_1^* and u_2^* controls

Figure 6 shows the optimal control on COVID-19 using the prevention (u_1^*) and the treatment (u_2^*) to optimize the objective function J. The results in Figure 6 show that there is a significant difference in the population I with optimal control strategy u_1^* and u_2^* compared to without control. The number of active COVID-19 by using the controls more decreased compared to without control.

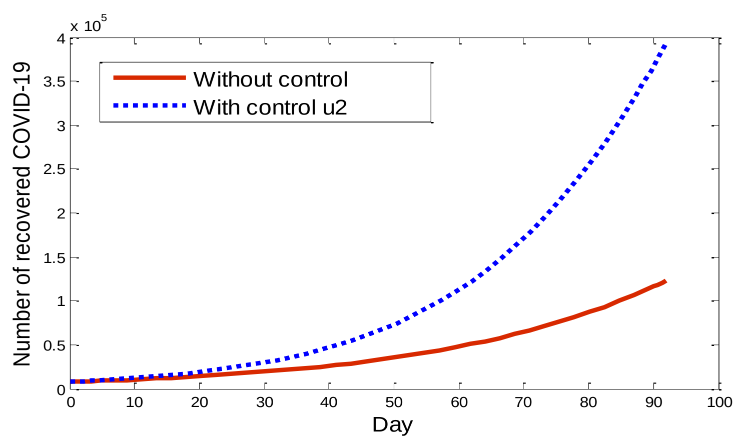

Figure 7. The dynamics of R with u_2^* control

Figure 7 display the recovered population (R) with and without control. The results in Figure 7 show a significant difference in the population R with optimal control strategy u_2^* compared to without control. We can see in Figure 7 that the number of recovered population more decrease compared to without control. The profile of the optimal prevention control u_1^* and optimal treatment control u_2^* is depicted in Figure 8.

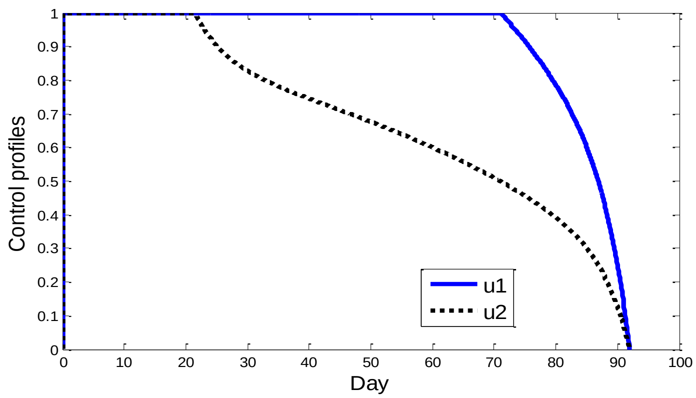

Figure 8. The profile of the optimal controls u_1^* and u_2^*

5. Discussion and Conclusion

To date, various efforts have been made by medical personnel in each country, and WHO has been trying to find a cure and a vaccine for COVID-19, while the spread of COVID-19 in the world has not been controlled. The model studied in this paper discusses the type *SIR* COVID-19 model in Indonesia. Based on COVID-19.go.id-data, the number of positive cases of COVID-19 continues to increase, which can be seen from $\boldsymbol{R_0} > 1$. Based on the model studied compared to COVID-19.go.id data, there is a difference in the number of active COVID-19 (I) and who are recovered (R), from mid-July-September 2020, meanwhile, at the beginning and end of July-September 2020, the model studied can predict the number of positive cases of COVID-19 (I) and who are recovered (R) corresponding to COVID-19.go.id data. Optimal control of COVID-19 in the form prevention (u_1^*) and COVID-19 treatment efforts (u_2^*) can reduce the number of individuals who are active COVID-19 compared with without control. The application of optimal control (u_2^*) significantly can increase the number of individuals who recovered COVID-19 compared to without control.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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