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OPTIMAL CONTROL OF THE SPREAD OF MENINGITIS: IN THE PRESENCE OF BEHAVIOUR CHANGE OF THE SOCIETY AND INFORMATION DEPENDENT VACCINATION

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Abstract. A mathematical model of meningococcal meningitis with incidence dependent self-protection measure and vaccination is formulated and analysed. The vaccination considered is assumed to be given to everyone and is information dependent. It is shown that the disease free equilibrium is globally asymptotically stable, which implies that if the disease reproduction number $\mathscr{R}_0$ can be reduced to a value less than unity then it is possible to eradicate the meningococcal meningitis. In addition, the behaviour modification parameters are found to have significant impact on the dynamics of the disease. Moreover, an optimal control theory is applied to propose the optimal combination of efforts in controlling the disease. It is shown that the optimal use of controls, such as preventive education, vaccination and treatment reduces the incidence of the disease. It is also indicated that, incidence dependent self-protection measure and vaccination are important in controlling meningitis, and the cost-effectiveness analysis reveals that combining education with vaccination is the most cost-effective strategy in the setting of the model.

Keywords: epidemiological model; meningitis; information dependent vaccination; behaviour change; optimal control; cost-effectiveness analysis.

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1. INTRODUCTION

Meningitis is an inflammation of the meninges, the membranes that cover the central nervous system which is composed of the brain and spinal cord [5]. It is most often caused by infection (bacterial, viral, or fungal), but can also be produced by chemical irritation, subarachnoid haemorrhage, cancer and other conditions. Bacterial meningitis is rare but serious and most cases are usually caused by the bacteria Haemophilus influenzae, Neisseria meningitidis, and Streptococcus pneumoniae. Bacterial meningitis caused by Neisseria meningitidis is a very serious form of meningococcal meningitis and it has different types. There are 13 distinct serogroups of Neisseria meningitidis, those mostly causing disease being A, B, C, W-135, X and Y[9]. The serogroup A is the most commonly isolated pathogen in the African meningitis belt [40, 43]. Although the highest burden of disease is currently in Africa, epidemics can occur in any part of the world [30, 31, 34, 35, 38, 39]. The most common serogroup responsible for the disease in many of the industrialized countries is serogroup B, followed by C [41]. The meningococcal meningitis is a preventable disease if the population can effectively participate in using protective mechanisms. The best ways to prevent the disease are usually personal practices such as wearing a face mask, wash frequently, keep the environment clean, get vaccinated [9].

Mathematical modelling and numerical simulations give promising ways to understand the dynamics of the disease and have a significant effect on how to identify the effective preventive and control strategies. There are a number of mathematical models of meningococcal disease, with many of them investigating the bacterial level of the infection process and some of them modelling the spread and control of the infection [2, 4, 7, 12, 21, 35, 40, 44].

When an individual is infected by meningococcal meningitis the individual becomes asymptomatically infectious. This stage is called the carrier stage of the diseases. The carrier individuals are infected and infectious but show no signs of the disease. This makes the disease hard to monitor in the population, and yet these individuals play a large part in the spread of the infection. Tuckewell *et al* [41] did not include a separate carriage state in their model, but assumed that at steady states the carriers and non-carriers are at equilibrium. However, in dynamic models of the evolution of a disease in a population it is often important to distinguish between carriers and non carriers [10].

Based on the susceptible-carrier-infectious-removed (SCIR) structure, Simpson & Roberts [35] proposed a framework for a mathematical model and showed how the model could be used to calculate the predicted yearly incidence of infection in the absence of vaccination. Blyuss [4] used a mathematical model to identify crucial factors that determine the meningitis dynamics. The same paper further suggested that temporary population immunity plays a very important role and has to be taken into account during disease monitoring and when measuring the efficiency of vaccines being deployed. Asamoah *et al* [2] presented a mathematical framework of vaccination and treatment on *SCIRS* bacterial meningitis model. They applied optimal control to determine optimal (best) way of controlling the transmission of meningitis in the sub-Sahara African settings. They also concluded that it is advisable to encourage susceptible individuals to be vaccinated against meningitis and work on early detection and immediate treatment to better control the epidemic. Tartof *et al* [36] developed an age-structured mathematical model of serogroup A Neisseria meningitidis transmission, colonization, and disease in the African meningitis belt, and used this model to explore the impact of various vaccination strategies. But they did not report for the use of optimal control tools to find the best strategy. Recently, Agusto & Leite [1] formulated and analysed a mathematical model for meningitis with vaccination and applied optimal control analysis to propose the best strategy for the use of vaccination and wearing a face mask as control mechanisms. However, none of the studies in the above described references incorporate the effect of behaviour change in the host population. They all intrinsically assumed that people remain passive during the outbreak of the disease and vaccination is administered at a constant rate.

It is widely understood among the public health experts that an outbreak of meningococcal meningitis can be controlled more quickly if the public understands the available preventive mechanisms and respond in an appropriate way for the actions requested by the public health authorities. Health education is crucial to ensure the participation of the community. Informing the public is an integral and important part of meningococcal meningitis control strategies: the population must be informed of the outbreak and of the measures to be taken, including the importance of early case identification.

To the knowledge of the authors, all the mathematical models formulated to study the dynamics of meningococcal meningitis ignored the effect of preventive education and the participation of the population in reducing their risky behaviour. Usually people do not adopt the preventive mechanisms unless they perceived risk of acquiring the disease. Therefore, participation of the public in preventive actions depends on the information and the behaviour of individuals [6, 8, 17]. Thus, it appears to be necessary to include in the meningitis model the effect of behaviour change to participate in self protective actions and in getting vaccinated to understand the actual dynamics of the disease.

The effect of changing behaviour of individuals is important in epidemic outcomes, and now such effects are being included increasingly in mathematical models [22]. For example, the models considered in Manfredi and D'Onofrio [29] study the change in behaviour of the human population corresponding to the response to vaccination. Moreover, change of behaviour in a human population in adopting any preventive actions is studied by Kassa and Ouhinou [24] using the diffusion of innovation within a society. But these modeling schemes were used for disease types that are different from meningitis.

Human spontaneous behavioural responses, the changes in human behavioural patterns that involve personal decisions based on the available information about the disease or on individual's beliefs and attitudes, to an epidemic can significantly affect the spread of infectious diseases [32]. While behavioural responses to the spread of a disease have been accepted to play a key role, it has not been used in any of the studies for the dynamics of the meningococcal meningitis so far. We believe that introducing such behavioural change attributes in the model and analysing its effects in the dynamics can significantly improve the control strategies.

In this paper, we introduce the behaviour change function into the meningitis model and mathematically analyse it. The formulated model contains one more compartment S_e, which represents individuals who decide to change their behaviour due to an effect of a successful awareness campaign and the perceived threat to acquire the disease by the population. In addition, we develop an optimal control model taking the combination of different intervention mechanisms: awareness creation (which will be simply called '*education*' in this manuscript), vaccination and treatment as control measures and study the mathematical control analysis of

the model to propose an optimal control strategy for public health planning. Here, unlike to the usual way of defining the cost function as in most of the epidemiological models (either all quadratic or all linear forms), we apply a mixed control mechanism that combines both linear and nonlinear cost functions corresponding to different structure of the controls. However, this requires a careful analysis of the optimality condition at singularity points, which we managed to get the required conditions and formulations.

The paper is organized as follows: Section 2 gives the description and mathematical analysis of the model. In Section 3, the control problem for the model is formulated and the mathematical control analysis is applied to find the necessary conditions for the optimal controls. Some numerical simulations are performed in Section 4 to illustrate the trajectory of the sub population in the dynamics when various combination of controls are applied. Section 5 contains the cost-effectiveness analysis of different strategies taken. The paper is concluded with conclusive remarks in Section 6.

2. Mathematical Model and Its Analysis

2.1. Model Description. The mathematical model of meningococcal meningitis is studied by different researchers. We consider a basic mathematical model as formulated in literature [2, 4, 21, 44] to study the dynamics of this disease. Extending these models, we divide the overall population into five distinct compartments: the susceptible individuals who are separated into two parts: individuals who are fully susceptible to the infection (S), and individuals who are aware the risk (educated) as well as possible self protective measures and became less susceptible due to their action to change their behaviour, (S_e). The remaining population is further divided into carriers (C) who are carrying the infection and are infectious, but show no signs of the invasive disease, infected (I) who show the symptoms of the invasive disease and are still infectious and recovered (R) individuals, so that the total population is $N = S + S_e + C + I + R$.

We assume that individuals are recruited into the population at a constant rate of Λ. The susceptible individuals S and '*educated*' susceptible individuals S_e are vaccinated at constant vaccination rates of $\phi_1\nu$ and $\phi_2\nu$, respectively, where $0 < \phi_1 \leq \phi_2 < 1$ and ν is the information dependent vaccination ratio as in d'Onofrio *et al* [14]. The susceptible group, S progress to susceptible '*educated*' group S_e at a rate of αe, where $0 < \alpha < 1$ and e is the behavioural

change function as in Kassa & Ouhinou [25]. The individuals in the S_e group, as it is mentioned earlier, are susceptible individuals who are aware of the risk of meningitis and are involved in self protective action against the infection by applying any of the existing self protective mechanisms. However, in practice when the disease is endemic for longer period of time in the population, some individuals may become negligent and go back to practising some of the risky behaviour attributes. In the current model, we assume that the proportion of risk ignorant and negligent population from among the class S_e move back to the S class at a constant rate of m. Moreover, it is clear that individuals in the '*educated*' class are exposed to the infection with a rate smaller than other susceptible individuals.

The susceptible and '*educated*'-susceptible populations are also decreased due to the infection of the disease and transfer to the carriers C and infected groups I. We assume that the level of infectiousness of individuals in C is grater, say by a factor $\varepsilon > 1$ than the individuals in the I class. This could be attributed to the fact that carriers are asymptomatic and can freely mix with the community while those in the I class can possibly be sick and their possibility to mix with the public is limited.

The force of infection for each group is represented by the functional parameter $\lambda = c\beta\dfrac{\varepsilon C + I}{N}$ where c and β represent the contact and transmission rates, respectively. That is, c is the contact rate of individuals (which measures the frequency of contact by individuals and how many person per unit of time the infected person can possibly contact) and β represents the biological transmission probability per each contact. A portion κ of the newly infected individuals from the S class, $\kappa\lambda S$ go to the carriers group C, while the remaining $(1-\kappa)\lambda S$ progress directly to the infectious and symptomatic group I. A portion ι of the new infections that come from the S_e class, $\iota\gamma\lambda S_e$, join the C class and the remaining $(1-\iota)\gamma\lambda S_e$ join to the I group, where γ $(0 < \gamma < 1)$ is the overall average rate of non-effectiveness (failure) of existing self-preventive measures.

Carriers develop an invasive disease at a rate σ and recover at a rate δ without passing the I class, and individuals with invasive disease recover at a rate ρ. The model takes into account both the natural death rate μ and disease-induced mortality d. Recovery from the disease is not permanent. Due to the waning effect of vaccines and the behaviour of the disease, some portion

will join back to the susceptible group S, and the remaining to the susceptible '*educated*' group S_e, at a rate of, say $\eta\omega R$, and $(1-\eta)\omega R$, respectively, where $0<\eta<1$. Here ω is the rate of losing immunity and hence $\frac{1}{\omega}$ represents the average time that individuals remain in the recovered group R before they become susceptible.

In the model we include two types of information dependent values. The first one is the inclusion of '*educated*' compartment, S_e of the behaviour dependent model as in Kassa *et al* [25] and the second one is behaviour change for the adoption of vaccination as models in d'Onofrio *et al* [14] for information-related vaccinating behaviour.

One of the methods used to model the change in human behaviour during the outbreak of an epidemic is with the use of the mechanistic procedure in the "Diffusion of innovation" from marketing. This approach assumes the introduction of a protective mechanism from an infectious disease as an 'innovation' that is to be adopted by the population [3, 24, 25, 26]. In models of this approach so far, the level of prevalence of the disease is assumed to be the perceived treat for the population. However, for high fatality and short-lived diseases, like meningitis, the prevalence dependent awareness function may not reflect the reality. Therefore, we assumed in this study that awareness is driven by the magnitude of the incidence for meningitis.

The human learning behaviour does not keep on increasing for ever. After a sharp increase, the rate of increase in the awareness function e slows down and settles to some value asymptotically. Therefore, we define the awareness function $e(i)$ by the Hill-type function in terms of the incidence of the infection

$$e(i)=\frac{i^n}{i_*^n+i^n}, \tag{1}$$

where i is the incidence of the disease, which is given by

$$i(t)=\lambda(t)S(t)=c\beta S(t)\frac{\varepsilon C(t)+I(t)}{N(t)},$$

and i_* is the value of the incidence corresponding to the threshold infectivity in which half of the maximum behavioural change value is achieved.

Equivalently, using the force of infection λ, e can be given by

$$e=\frac{S^n\lambda^n}{i_*^n+S^n\lambda^n}=\frac{S^n(\varepsilon C+I)^n}{i_0^nN^n+S^n(\varepsilon C+I)^n},$$

where $i_0 = \dfrac{i_*}{c\beta}$.

Since $n = 2$ produces a slow increase in the function of e and resembles the graph described by Green & McAlister [19], we assumed $n = 2$ in the formula for e, and hence it is simplified as:

$$e(t) = \frac{S(t)^2\lambda(t)^2}{i_*^2 + S(t)^2\lambda(t)^2} = \frac{S(t)^2[\varepsilon C(t) + I(t)]^2}{i_0^2 N^2(t) + S^2[\varepsilon C(t) + I(t)]^2}. \tag{2}$$

We introduce this in the model as αeS in such a way that this portion of the individuals from the S class transfer to the S_e group. Here, α is the mean constant rate at which keen susceptible individuals get recruited into the educated class per unit of time, and αe will give the actual recruitment rate to the cohort of '*educated*' from the susceptible class. That is, αe is the behaviour change function for self-protective actions.

Because of its application, enforcement mechanism and cost, the use of vaccination can be considered differently as compared to the self-protective mechanisms described earlier. Even though public health authorities try to convince the population in the community to increase the uptake of vaccination at the time when the outbreak of the disease is detected, the response depends on various information the population is receiving. That means, the vaccine uptake function ν is information dependent and is assumed to be a function of information variable M as in d'Onofrio *et al* [14] and is defined as:

$$\nu(M) = \nu_0 + \nu_1(M), 0 < \nu_0 < 1.$$

The fixed or baseline value ν_0 models the fraction of individuals that are vaccinated independently on the available current and historical information irrespective of the incidence level of the disease in the population, whereas $\nu_1(M)$ models the fraction of individuals that are vaccinated depending on the social alarm caused by the disease. It is usually given by either

$$\nu_1(M) = \frac{a_1 M}{1 + b_1 M} -- \text{ Michaelis-Menten function, or}$$

$$\nu_1(M) = \frac{a_2 M^2}{1 + b_2 M^2} -- \text{ Holling type II function,}$$

where a_1, b_1, a_2, b_2 are some suitable positive constants. The variable M is an information variable governing the signal available to individuals as a function of prevalence or incidence of

infection. It is the information variable which summarizes information about the current state of the disease (i.e M depends only on current values of the state variables) or M summarizes information about past values of state variables as indicated by d'Onofrio *et al* [13]. The possible ways describing M as indicated in d'Onofrio *et al* [13] are:

1. $M = \alpha c\beta \dfrac{S(\varepsilon C + I)}{N}$: Information governing vaccinating behaviour depends on the current incidence (i.e. M is the currently reported absolute incidence), where $\alpha > 0$ is a reporting rate.
2. $M = k\dfrac{I}{N}$: Information governing vaccinating behaviour depends on the current prevalence (i.e M is a function of the current prevalence of the disease), where $k > 0$ is a parameter subsuming aspects such as pathogenicity.
3. $M = \dfrac{\alpha c\beta(\varepsilon C + I)}{\mu + \alpha\beta(\varepsilon C + I)}$: Information governing vaccinating behaviour is a saturating function of current incidence (i.e. M is a non-linear increasing function of standardized incidence which can be taken as a measure of the perceived risk of infection).
4. $M = \int_{-\infty}^{t} g(S(\tau), C(\tau), I(\tau))k(t-\tau)d\tau$: where M depends on past states with k representing memory decay function.

In our case we assume that M depends on the currently reported absolute incidence of the disease and is given by

$$M = \alpha c\beta S\frac{(\varepsilon C + I)}{N} = \alpha S\lambda. \tag{3}$$

Taking Holling type II form for vaccine coverage function:

$$\nu_1(M) = \frac{a_2 M^2}{1 + b_2 M^2} \tag{4}$$

we have,

$$\nu(t) = \nu_0 + \frac{a_2 M^2}{1 + b_2 M^2} = \nu_0 + \frac{a_2(\alpha c\beta)^2 S^2(\varepsilon C + I)^2}{N^2 + b_2(\alpha c\beta)^2 S^2(\varepsilon C + I)^2}. \tag{5}$$

For simplicity and mathematical tractability we assume that

$$\nu_0 = 0, a_2 = b_2,$$

and we get

$$\nu(t) = \frac{S(t)^2\lambda(t)^2}{i_*^2 + S(t)^2\lambda(t)^2} = \frac{S(t)^2[\varepsilon C(t) + I(t)]^2}{i_0^2 N^2(t) + S^2[\varepsilon C(t) + I(t)]^2}. \tag{6}$$

representing the information driven vaccination, where

$$i_0^2 = \frac{1}{a_2(\alpha c\beta)^2},$$

which has exactly similar structure as that of $e(t)$.

In addition, if we assume for mathematical simplicity that vaccine is perfect and the acquired immunity wanes at a rate equal to ω (which is the same as the rate of loosing immunity after recovery) the vaccinated group can be merged with the recovered group. Therefore, the system dynamics of the model becomes:

$$\dot{S} = \Lambda - \alpha eS + mS_e - \phi_1 \nu S + \eta\omega R - \lambda S - \mu S, \tag{7a}$$

$$\dot{S}_e = \alpha eS - mS_e - \phi_2 \nu S_e + (1-\eta)\omega R - \gamma\lambda S_e - \mu S_e, \tag{7b}$$

$$\dot{C} = \kappa\lambda S + \iota\gamma\lambda S_e - (\sigma + \delta + \mu)C, \tag{7c}$$

$$\dot{I} = (1-\kappa)\lambda S + (1-\iota)\gamma\lambda S_e + \sigma C - (\rho + d + \mu)I, \tag{7d}$$

$$\dot{R} = \phi_1 \nu S + \phi_2 \nu S_e + \delta C + \rho I - (\omega + \mu)R, \tag{7e}$$

where the upper dots denote the time derivatives and the total population N at any time t is given by

$$N(t) = S(t) + S_e(t) + C(t) + I(t) + R(t).$$

In addition, we also assume the following nonnegative initial conditions hold for the system (7),

$$S(0) = S_0 \geq 0,\ S_e(0) = S_{e0} \geq 0,\ C(0) = C_0 \geq 0,\ I(0) = I_0 \geq 0,\ R(0) = R_0 \geq 0.$$

The coefficient terms λ, e, ν are functions of the state variables and the other coefficients are assumed to be constant parameters and are summarized in Table 2.

Var. Coefficients	**Description**
λ	Force of infection.
e	Behaviour change function as in Kassa [24]
ν	Information dependent vaccination coverage function as in d'Onofrio [13]

TABLE 1. Description of variable parameters used in the model (7).

Remark 2.1.

(1) By taking $\nu_0 = 0$, we are assuming effectively that the vaccine uptake function $\nu(t)$ is fully incidence driven and hence it becomes zero when there is no disease incidence. Particularly, at the disease free equilibrium, both $e(t)$ and $\nu(t)$ become zero.

(2) The parameters in Table 1 are functions of the states and can be explained as follows.

- *force of infection:*

$$\lambda(t) = c\beta \frac{\varepsilon C(t) + I(t)}{N(t)}. \tag{8}$$

- *behaviour change functions:*

$$e(t) = \frac{S^2(t)[\varepsilon C(t) + I(t)]^2}{i_0^2 N^2(t) + S^2(t)[\varepsilon C(t) + I(t)]^2} = \nu(t), \tag{9}$$

where the constant i_0 represents the incidence producing half of the maximum behavioural change value.

2.2. Positivity and Boundedness of solutions. Since model (7) represents human populations, all parameters in the model are non-negative and one can show that the solutions of the system are non-negative, given non-negative initial values. The model system (7) will be analysed in a biologically feasible region, $\Omega \subset \mathbb{R}^5_+$ with

$$\Omega = \left\{ (S, S_e, C, I, R) \in \mathbb{R}^5_+ : S + S_e + C + I + R < \frac{\Lambda}{\mu} \right\}. \tag{10}$$

Now we need to show the boundedness of the solutions, that is the positive invariance of Ω (solutions in Ω remain in Ω for all $t > 0$). Adding all the five equations in the model system, we have the rate at which total population changes, which is given by

$$\dot{N} = \Lambda - \mu N - dI. \tag{11}$$

Since $dI(t) \geq 0$,

$$\dot{N}(t) \leq \Lambda - \mu N(t) \qquad for\, t \geq 0.$$

Solving this differential equation we get

$$N(t) \leq \frac{\Lambda}{\mu} + ce^{-\mu t}$$

where $c = N(0) - \frac{\Lambda}{\mu}$. Thus,

$$N(t) \leq \frac{\Lambda}{\mu} + (N(0) - \frac{\Lambda}{\mu})e^{-\mu t}. \tag{12}$$

For large t values, the limiting value of N is

$$N(t) \leq \frac{\Lambda}{\mu},$$

which implies that the trajectories of the model system are bounded.

In particular $N(0) \leq \frac{\Lambda}{\mu}$. Thus, the biologically feasible region Ω is positively invariant and attractive. Hence, the following theorem holds.

Theorem 2.2. *The region $\Omega \subset \mathbb{R}^5_+$ is positively-invariant for the model system* (7) *with non-negative initial conditions in $\mathbb{R}^5_+$.*

2.3. The Disease Free Equilibrium (DFE) and its stability. In the absence of infection (i.e $C = 0, I = 0$), the system of equations (7) has a disease free equilibrium E_0, given by

$$E_0 = \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0\right), \tag{13}$$

provided vaccination is given after the incidence of the disease.

At the disease free equilibrium E_0, the Jacobian matrix of the model system (7) will be J_{E_0}, where

$$J_{E_0} = \begin{bmatrix} -\mu & m & -c\beta\varepsilon & -c\beta & \eta\omega \\ 0 & -m-\mu & 0 & 0 & (1-\eta)\omega \\ 0 & 0 & \kappa c\beta\varepsilon - (\sigma+\delta+\mu)) & \kappa c\beta & 0 \\ 0 & 0 & (1-\kappa)c\beta\varepsilon + \sigma & (1-\kappa)c\beta - (\rho+\mu+d) & 0 \\ 0 & 0 & \delta & \rho & -(\omega+\mu) \end{bmatrix} \tag{14}$$

Letting $x_1(t) = S(t) - S_0, x_2(t) = S_e(t) - S_{e0}, x_3(t) = C(t) - C_0, x_4(t) = I(t) - I_0, x_5(t) = R(t) - R_0$, and linearising the system at E_0, we have the system

$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \\ x_3'(t) \\ x_4'(t) \\ x_5'(t) \end{bmatrix} = J_{\varepsilon_0} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix} \tag{15}$$

Hence, the characteristic polynomial will be

$$\begin{aligned} P(\chi) &= |\chi I - J_{E_0}| \\ &= \begin{vmatrix} \chi+\mu & -m & c\beta\varepsilon & c\beta & -\eta\omega \\ 0 & \chi+m+\mu & 0 & 0 & -(1-\eta)\omega \\ 0 & 0 & \chi-\kappa c\beta\varepsilon+(\sigma+\delta+\mu)) & -\kappa c\beta & 0 \\ 0 & 0 & -(1-\kappa)c\beta\varepsilon-\sigma & \chi-(1-\kappa)c\beta+(\rho+\mu+d) & 0 \\ 0 & 0 & -\delta & -\rho & \chi+(\omega+\mu) \end{vmatrix} \\ &= (\chi+\mu)(\chi+m+\mu)(\chi+(\omega+r+\mu))\{[\chi^2-[\kappa c\beta\varepsilon-(\sigma+\delta+\mu)+(1-\kappa)c\beta \\ &-(\rho+\mu+d)]\chi-\kappa c\beta\varepsilon(\rho+\mu+d)-(1-\kappa)c\beta(\sigma+\delta+\mu)-\kappa c\beta\sigma]\} \end{aligned}$$

which is a polynomial of degree five. The eigenvalues of the matrix J_{E_0} are the zeros of $P(\chi)$, which are

$$\chi_1 = -\mu, \quad \chi_2 = -(m+\mu), \quad \chi_3 = -(\omega+\mu),$$

and χ_4, χ_5 are roots of the quadratic equation

$$a\chi^2 + b\chi + c = 0, \tag{16}$$

where

$$a = 1,$$

$$b = -\kappa c\beta\varepsilon + (\sigma+\delta+\mu) - (1-\kappa)c\beta + (\rho+\mu+d)$$

$$c = -\kappa c\beta\varepsilon(\rho+\mu+d) - (1-\kappa)c\beta(\sigma+\delta+\mu) - \kappa c\beta\sigma + (\sigma+\delta+\mu)(\rho+\mu+d).$$

The three eigenvalues, χ_1, χ_2 and χ_3 are already found to be negative real numbers. However, for local stability of the disease free equilibrium, we must have all the eigenvalues to be with negative real parts. Now to show that χ_4 and χ_5 also have negative real parts, we first define the basic reproduction number, $\mathscr{R}_0$ (which can also be obtained through the next generation matrix method as in van den Driessche & Watmough [42]) for the model system (7) to be

$$\mathscr{R}_0 = \frac{\kappa c\beta\varepsilon}{(\sigma+\delta+\mu)} + \frac{(1-\kappa)c\beta}{(\rho+\mu+d)} + \frac{\kappa c\beta\sigma}{[(\sigma+\delta+\mu)+(\rho+\mu+d)]}. \tag{17}$$

Then we have,

$$\begin{aligned}
b =& -\kappa c\beta\varepsilon + (\sigma+\delta+\mu) - (1-\kappa)c\beta + (\rho+\mu+d)\\
=&[(\sigma+\delta+\mu)+(\rho+\mu+d)][1 - \frac{\kappa c\beta\varepsilon}{[(\sigma+\delta+\mu)+(\rho+\mu+d)]} - \frac{(1-\kappa)c\beta}{[(\sigma+\delta+\mu)+(\rho+\mu+d)]}]\\
>&[(\sigma+\delta+\mu)+(\rho+\mu+d)][1 - \frac{\kappa c\beta\varepsilon}{(\sigma+\delta+\mu)} - \frac{(1-\kappa)c\beta}{(\rho+\mu+d)}]\\
>&[(\sigma+\delta+\mu)+(\rho+\mu+d)][1 - \frac{\kappa c\beta\varepsilon}{(\sigma+\delta+\mu)} - \frac{(1-\kappa)c\beta}{(\rho+\mu+d)} - \frac{\kappa c\beta\sigma}{[(\sigma+\delta+\mu)+(\rho+\mu+d)]}]\\
=&[(\sigma+\delta+\mu)+(\rho+\mu+d)][1-\mathscr{R}_0].
\end{aligned}$$

Similarly,

$$\begin{aligned}
c =& -\kappa c\beta\varepsilon(\rho+\mu+d) - (1-\kappa)c\beta(\sigma+\delta+\mu) - \kappa c\beta\sigma + (\sigma+\delta+\mu)(\rho+\mu+d)\\
=&[(\sigma+\delta+\mu)(\rho+\mu+d)][1 - \frac{\kappa c\beta\varepsilon}{(\sigma+\delta+\mu)} - \frac{(1-\kappa)c\beta}{(\rho+\mu+d)} - \frac{\kappa c\beta\sigma}{[(\sigma+\delta+\mu)+(\rho+\mu+d)]}]\\
=&[(\sigma+\delta+\mu)+(\rho+\mu+d)][1-\mathscr{R}_0].
\end{aligned}$$

Thus, if $\mathscr{R}_0 < 1$, we have $a = 1, b > 0, c > 0$ and hence, by the Routh-Hurwitz Criterion for polynomials both the roots of the quadratic equation (16) have negative real parts. That means, all the five eigenvalues of the Jacobian matrix at E_0 have negative real parts for $\mathscr{R}_0 < 1$.

The linear stability of the disease free equilibrium E_0 is governed by the basic reproduction number $\mathscr{R}_0$. Hence the following theorem follows.

Theorem 2.3. *The system always has a unique disease free equilibrium E_0. If $\mathscr{R}_0 < 1$, then E_0 is locally asymptotically stable, and unstable otherwise.*

In epidemiological terms, Theorem 2.3 states that it is possible to control the epidemic if we can reduce the value of $\mathscr{R}_o < 1$ as long as the initial population is in the neighbourhood of the DFE point E_0. However, to ensure the independence of the possibility of elimination of the disease on the initial size of the population, we need to prove that the DFE point E_0 is globally asymptotically stable (GAS) for $\mathscr{R}_0 < 1$ as given in the following theorem.

Theorem 2.4. *If $\mathscr{R}_0 < 1$, then the disease free equilibrium E_0 is globally asymptotically stable in Ω.*

Proof. To prove this theorem, we use the settings of the Stability Theorem given by Kamgang-Sallet [23] (which we shall refer it hereafter as Kamgang-Sallet Theorem). Let $x_1 = (S, Se) \in \mathbb{R}^2$ and $x_2 = (C, I, R) \in \mathbb{R}^3$. Then the system (7) can be written as:

$$\dot{x}_1 = A_1(x)(x_1 - x_1^*) + A_{12}(x)x_2, \tag{18}$$

$$\dot{x}_2 = A_2(x)x_2, \tag{19}$$

where $x = (x_1, x_2) \in \mathbb{R}^2 \times \mathbb{R}^3$, $x_1^* = \left(\frac{\Lambda}{\nu_0+\mu}, \frac{\nu_0\Lambda}{\mu(\nu_0+\mu)}\right)$,

$$A_1(x) = \begin{pmatrix} -\mu & m \\ 0 & -(m+\mu) \end{pmatrix}, \; A_{12}(x,t) = \begin{pmatrix} -\frac{c\beta\varepsilon S}{N} & -\frac{c\beta S}{N} & \eta\omega \\ -\frac{\gamma c\beta\varepsilon Se}{N} & -\frac{\gamma c\beta Se}{N} & (1-\eta)\omega \end{pmatrix},$$

and

$$A_2(x) = \begin{pmatrix} \frac{c\beta\varepsilon(\kappa S+\iota\gamma Se)}{N} - (\sigma+\delta+\mu) & \frac{c\beta(\kappa S+\iota\gamma Se)}{N} & 0 \\ \frac{c\beta\varepsilon((1-\kappa)S+(1-\iota)\gamma Se)}{N} + \sigma & \frac{c\beta((1-\kappa)S+(1-\iota)\gamma Se)}{N} - (\rho+\mu+d) & 0 \\ \delta & \rho & -(\omega+\mu) \end{pmatrix}.$$

The five sufficient conditions of Kamgang-Sallet Theorem for GAS of the DFE can be checked to be satisfied as follows.

i. System (7) is a dynamical system on Ω as it is indicated in equation (10).

ii. The subsystem $\dot{x}_1 = A_1(x)(x_1 - x_1^*)$ is globally asymptotically stable at the equilibrium x_1^*. Here the coefficient matrix $A_1(x)$ is a constant matrix having negative eigenvalues. Therefore, the system is globally asymptotically stable at the point x_1^*. More precisely, it means that the solution of the subsystem converges to x_1^* for any initial condition.

iii. The matrix $A_2(x)$ is Metzler (*i.e.*, all the off-diagonal elements are non-negative) and irreducible for any given $x \in \Omega$, as can be seen from its expression.

iv. There exists an upper-bound matrix $\bar{A}_2$ for the set

$$\mathscr{M} = \{A_2(x) : x \in \Omega\}.$$

For instance,

$$\bar{A}_2 = A_2(x) = \begin{pmatrix} c\beta\varepsilon(\kappa+\iota\gamma) - (\sigma+\delta+\mu) & c\beta(\kappa+\iota\gamma) & 0 \\ c\beta\varepsilon((1-\kappa)+(1-\iota)\gamma)+\sigma & A_{22} & 0 \\ \delta & \rho & -(\omega+\mu) \end{pmatrix}.$$

is an upper-bound of $\mathscr{M}$, where $A_{22} = c\beta((1-\kappa)+(1-\iota)\gamma) - (\rho+\mu+d)$.

v. For $\mathscr{R}_0 \leq 1$ in (17)

$$\alpha(\bar{A}_2) = \max\left\{Re(\chi) : \chi \text{ eigenvalue of } \bar{A}_2\right\} \leq 0.$$

Hence, by the Kamgang-Sallet Theorem [23], the disease-free equilibrium is globally asymptotically stable for $\mathscr{R}_0 < 1$. □

3. Formulation of the Control Problem

In this section we try to incorporate different intervention mechanisms as control variables in the model system (7) and develop an optimal control problem. At the end of the section we will see the effects of the interventions on the spreading of the disease and the effect of these interventions on the incidence of the disease.

3.1. Model with Controls. The diseases like hepatitis, tuberculosis, meningitis, etc. are generally treated by both vaccination as well as treatment. In our system we use the following three possible interventions for control strategy of the meningitis disease:

- preventive education,
- vaccination of susceptible and educated-susceptible individuals, and
- treating carriers and infected individuals.

Hence, these interventions will serve as control parameters in the dynamics of the epidemic model. Further application of control may be either time independent (supplied at a constant

value) or dependent (supplied according to the demand of the time). In our case we assume the controls are time dependant.

(a) **Educating the population:** Increase the rate of self-initiated protective measures by the susceptible individuals to reduce the risk of contracting the disease. Precautions should be taken to avoid transfer of the meningococcal bacteria from the infected person to others, which is primarily by coughing and sneezing. Replace the term α in the system (7) by

$$\alpha_0 + u_1(t),$$

for some $\alpha_0 > 0$ with the control function $u_1(t)$, which measures the rate at which additional susceptible individuals are convinced to take part in behaviour modification. Because of practicality and economical limitations on the maximum rate of convincing individuals for behaviour modification, we assume that $\alpha_{max} > 0$ to be the maximum rate. That is,

$$0 \leq u_1 \leq \alpha_{max} - \alpha_0.$$

(b) **Vaccination:** Increase the rate of vaccination. The most effective way to protect bacterial meningitis is to get vaccinated. Vaccines for meningococcal disease are available but they are not free for most people. However, they may be funded by local district health boards during a disease outbreak. Currently available vaccines do not provide protection against all types of meningococcal disease, rather they are prepared only for strains A, C, Y, and W-135.

Let the current rate of vaccinations be $1 > \phi_{10} > 0$ and $1 > \phi_{20} > 0$ and $u_2(t)$ measures the rate at which additional susceptible are vaccinated. We will consider vaccination $u_2(t)$ as a continuous control and introduce it in the system dynamics as

$$(\phi_{10} + u_2(t))\nu S(t),\ (\phi_{20} + u_2(t))\nu S_e(t) \text{ with } 0 \leq u_2(t) \leq \phi_{\max},$$

where, $\phi_{\max} = \min\{1 - \phi_{10}, 1 - \phi_{20}\}$.

(c) **Treatment:** Increase the rate of recruitment for infected classes so that they receive treatment effectively and transfer to the recovered class. Once the bacteria that are causing the illness are identified, the person can be treated with the correct antibiotics.

The antibiotic will most likely be given directly into the blood stream through a drip (intravenously). Very high antibiotic doses may be required to fight the disease. Additional antibiotics may be required to prevent and/or combat secondary infections. The control functions $u_3(t)$ and $u_4(t)$ measure the rate at which additional infectious (both carriers, Infected-symptomatic) individuals transform to the "Recovered" class at any time t, where the current rate is at δ_0 and ρ_0 respectively. These controls will be seen in the dynamics as $(\delta_0 + u_3(t))C(t)$ and $(\rho_0 + u_4(t))I(t)$ by replacing $\delta C(t)$ and $\rho I(t)$ in the system respectively. Letting the constants $\delta_{max} > 0, \rho_{max} > 0$ to represent the maximum rates, we have

$$0 \leq u_3 \leq \delta_{max} - \delta_0 \text{ and } 0 \leq u_4 \leq \rho_{max} - \rho_0.$$

Now, using the above control parameters, the system of the disease dynamics (7) can be rewritten as:

$$\dot{S} = \Lambda - (\alpha_0 + u_1)eS + mS_e - (\phi_{10} + u_2)\nu S + \eta\omega R - \lambda S - \mu S, \tag{20a}$$

$$\dot{S}_e = (\alpha_0 + u_1)eS - (\phi_{20} + u_2)\nu S_e - mS_e + (1-\eta)\omega R - \gamma\lambda S_e - \mu S_e, \tag{20b}$$

$$\dot{C} = \kappa\lambda S + \iota\gamma\lambda S_e - ((\delta_0 + u_3) + \sigma + \mu)C, \tag{20c}$$

$$\dot{I} = (1-\kappa)\lambda S + (1-\iota)\gamma\lambda S_e + \sigma C - ((\rho_0 + u_4) + d + \mu)I, \tag{20d}$$

$$\dot{R} = (\phi_{10} + u_2)\nu S + (\phi_{20} + u_2)\nu S_e + (\delta_0 + u_3)C + (\rho_0 + u_4)I - (\omega + \mu)R, \tag{20e}$$

with given initial conditions on the sates,

$$S(0), S_e(0), C(0), I(0), R(0), \tag{21}$$

and bounded controls as indicated above.

3.2. Objective Function of the Optimal Control Problem. Having developed the system dynamics with controls, now we try to formulate the objective function to be minimized so that the optimal interventions can be obtained.

The main goal here is to minimize the total number of new infections (carriers and symptomatically infectious groups) in the planning period, while also minimizing the total cost of controlling the disease dynamics. That means, by constructing optimal values of Lebesgue

integrable, bounded control functions $u_i(t), i = 1,2,3,4$, we seek the best strategy that can control the dynamics of the epidemics modelled in (20). For controls which are implemented at individual level, such as giving treatment, etc., it is better to take the cost as linear as $B_3u_3(t)C(t), B_4u_4(t)I(t)$, where $B_i, i = 3,4$, represent a unit cost and u_3C, u_4I represent the total number of the C and I classes who are treated at time t. If the control is applied at a community level (mass campaign), the control for preventive education and vaccination can be considered as quadratic u_1^2, u_2^2 in the objective function. Since implementation of any community level public health intervention has increasing costs with reaching higher fraction of the population, we usually take a non-linear cost function, like the quadratic, to represent such costs. That is, we minimize the objective functional:

$$J = \int_0^{t_f} \left[(A_1 + B_3u_3(t))C(t) + (A_2 + B_4u_4(t))I(t) + \frac{B_1}{2}u_1^2(t) + \frac{B_2}{2}u_2^2(t) \right] dt, \tag{22}$$

where u_1, u_2, u_3, u_4 are Lebesgue measurable bounded functions on $[0, t_f]$.

The constants A_1, A_2, B_1 and B_2 could be considered as values that will balance the units of measurement and also may indicate the importance of one type of intervention over the other, at implementation level to the general public.

So, we seek to find optimal controls $u_1^*, u_2^*, u_3^*, u_4^*$ such that

$$J(u_1^*, u_2^*, u_3^*, u_4^*) = \min_{u_1, u_2, u_3, u_4} J(u_1, u_2, u_3, u_4) \tag{23}$$

where

$$\mathbf{u} = (u_1, u_2, u_3, u_4) \tag{24}$$

is the vector of Lebesgue integrable functions, which are all bounded controls.

Considering the objective function in (22), with system dynamics in (20) with initial conditions on the states and controls we have the optimal control problem:

$$\min_{u_1, u_2, u_3, u_4} J = \int_0^{t_f} \left[(A_1 + B_3u_3(t))C(t) + (A_2 + B_4u_4(t))I(t) + \frac{B_1}{2}u_1^2(t) + \frac{B_2}{2}u_2^2(t) \right] dt$$

Subject to

$$
\begin{aligned}
\dot{S} &= \Lambda - (\alpha_0 + u_1)eS + mS_e - (\phi_{10} + u_2)\nu S + \eta\omega R - \lambda S - \mu S, \\
\dot{S}_e &= (\alpha_0 + u_1)eS - (\phi_{20} + u_2)\nu S_e - mS_e + (1-\eta)\omega R - \gamma\lambda S_e - \mu S_e, \\
\dot{C} &= \kappa\lambda S + \iota\gamma\lambda S_e - ((\delta_0 + u_3) + \sigma + \mu)C, \\
\dot{I} &= (1-\kappa)\lambda S + (1-\iota)\gamma\lambda S_e + \sigma C - ((\rho_0 + u_4) + d + \mu)I, \\
\dot{R} &= (\phi_{10} + u_2)\nu S + (\phi_{20} + u_2)\nu S_e + (\delta_0 + u_3)C + (\rho_0 + u_4)I - (\omega + \mu)R,
\end{aligned} \tag{25}
$$

with given initial conditions on the sates,

$$
S(0), S_e(0), C(0), I(0), R(0). \tag{26}
$$

The controls are bounded as

$$
\begin{aligned}
&u_1(t) \in [0, \alpha_{\max} - \alpha_0], u_2(t) \in [0, \phi_{\max}], u_3(t) \in [0, \delta_{\max} - \delta_0], u_4(t) \in [0, \rho_{max} - \rho_0] \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for all } t \in [0, t_f].
\end{aligned} \tag{27}
$$

3.3. Existence of Optimal Control Solutions. The following theorem shows the existence of the optimal control solution to the above optimal control problem [16].

Theorem 3.1. *There exists an optimal control quadruple* $\boldsymbol{u}^* = (u_1^*, u_2^*, u_3^*, u_4^*)$*, and corresponding solution vector* $\boldsymbol{x}^* = (S^*, S_e^*, C^*, I^*, R^*)$ *to the state initial value problem* (25) *that minimizes the objective functional* $J(u)$ *of* (22) *over the set of admissible controls* $\mho$*.*

Proof. Let the marginal cost function L, the state function f and the admissible control $\mho$ be

$$
\begin{aligned}
&L(\mathbf{x}, \mathbf{u}, t) = (A_1 + B_3 u_3(t))C(t) + (A_2 + B_4 u_4(t))I(t) + \frac{B_1}{2}u_1^2(t) + \frac{B_2}{2}u_2^2(t), \\
&f(\mathbf{x}, \mathbf{u}, t) = \frac{d\mathbf{x}}{dt}, \text{ where } \frac{d\mathbf{x}}{dt} \text{ represents system (20), and} \\
&\mho = \{(u_1(t), u_2(t), u_3(t), u_4(t)) \in L^1(0, t_f) | 0 \le u_1(t) \le \alpha_{max} - \alpha_0, 0 \le u_2(t) \le \phi_{max}, \\
&0 \le u_3(t) \le \delta_{max} - \delta_0, 0 \le u_4(t) \le \rho_{max} - \rho_0, \forall t \in [0, t_f]\}.
\end{aligned}
$$

Since all the involved functions in the model are continuously differentiable, we need to verify the following four conditions given in Filippov-Cesari Theorem (cf. Theorem 3.1 in Hartl *et al.* [20]).

1. There exists an admissible solution pair $(\mathbf{x}, \mathbf{u})$.

2. Roxin's condition holds, *i.e*,

$$\Gamma(\mathbf{x},t) = \{(L(\mathbf{x},\mathbf{u},t)+\xi, f(\mathbf{x},\mathbf{u},t)) : \xi \leq 0, \mathbf{u} \in \mho\} \subset \mathbb{R}^5$$

for all $(\mathbf{x},t) \in \mathbb{R}^5 \times [0,t_f]$.

3. There exist $\delta > 0$ such that $\|\mathbf{x}\| < \delta$ for all admissible $\{\mathbf{x},\mathbf{u}\}$ and t.
4. There exist $\delta_1 > 0$ such that $\|\mathbf{u}\| < \delta_1$ for all $\mathbf{u} \in \mho(\mathbf{x},t)$ with $\|\mathbf{x}\| < \delta$.

With regard to the first condition, the bound established for the non-controlled system (7) has the same form if we also incorporate the control function parameters, as they eventually add up to zero. Hence, for any $\mathbf{u} \in \mho$ and the state variables, we have

$$0 \leq N(t) \leq \frac{\Lambda}{\mu}. \tag{28}$$

Moreover, the state system is continuous and bounded for any admissible control $\mathbf{u} \in \mho$. Therefore, the state system (20) has a unique solution corresponding to every admissible control $\mathbf{u} \in \mho$. [see Theorem I.3.1 in Coddington & Levinson [11], Theorem 9.2.1 in Lukes [28].] The second condition (Roxin's condition) is satisfied as the state system (20) is linear with respect to the control variables and $\mho$ is compact. Conditions 3. and 4. follow from (28) and the definition of the control set $\mho$. Therefore, by Filippov-Cesari Theorem, there exists an optimal control pair $\{\mathbf{x}^*,\mathbf{u}^*\}$ with $\mathbf{u}^*$ measurable, that solves the optimal control problem (25). □

3.4. Characterization of Optimal Control Solution. To formulate the necessary conditions for optimality, we need to define the Hamiltonian function of the optimal control problem (25). The Hamiltonian equation with marginal cost function, state variables and adjoint variables is given by:

$$\begin{aligned} H(\mathbf{x},\mathbf{u},\xi,t) = \; & [(A_1+B_3u_3(t))C(t)+(A_2+B_4u_4(t))I(t)+\frac{B_1}{2}u_1^2(t)+\frac{B_2}{2}u_2^2(t)] \\ & +\xi_1[\Lambda-(\alpha_0+u_1)eS+mS_e-(\phi_{10}+u_2)\nu S+\eta\omega R-\lambda S-\mu S] \\ & +\xi_2[(\alpha_0+u_1)eS-(\phi_{20}+u_2)\nu S_e-mS_e+(1-\eta)\omega R-\gamma\lambda S_e-\mu S_e] \\ & +\xi_3[\kappa\lambda S+\iota\gamma\lambda S_e-((\delta_0+u_3)+\sigma+\mu)C] \\ & +\xi_4[(1-\kappa)\lambda S+(1-\iota)\gamma\lambda S_e+\sigma C-((\rho_0+u_4)+d+\mu)I] \\ & +\xi_5[(\phi_{10}+u_2)\nu S+(\phi_{20}+u_2)\nu S_e+(\delta_0+u_3)C+(\rho_0+u_4)I-(\omega+\mu)R], \end{aligned} \tag{29}$$

where $\mathbf{x} = (S, S_e, C, I, R)$, $\mathbf{u} = (u_1, u_2, u_3, u_4)$, $\xi = (\xi_1, \xi_2, \xi_3, \xi_4, \xi_5)$, with

$$e(t) = \frac{S^2[\varepsilon C(t) + I(t)]^2}{i_0^2 N^2(t) + S^2[\varepsilon C(t) + I(t)]^2} = \nu(t), \text{ and } \lambda(t) = c\beta \frac{\varepsilon I(t) + I(t)}{N(t)}.$$

By using Pontryagin's Maximum Principle [27, 33], we derive necessary conditions for the optimal controls and corresponding states. That is, it is now possible to determine the optimal control variables, $(u_1^*, u_2^*, u_3^*, u_4^*)$ from the necessary conditions.

Now, the necessary conditions are given as follows.

1. **The adjoint equations:**

$$\begin{aligned}
\xi_1'(t) &= -\frac{\partial H}{\partial S}(t) \\
&= (\alpha_0 + u_1)(\varepsilon C + I)^2 \frac{3S^2[i_0^2 N^2 + S^2(\varepsilon C + I)^2] - S^3[2i_0^2 N + 2S(\varepsilon C + I)^2]}{i_0^2 N^2(t) + S^2[\varepsilon C(t) + I(t)]^2}(\xi_1 - \xi_2) \\
&+ (\phi_{10} + u_2)(\varepsilon C(t) + I(t))^2 \frac{3S^2[i_0^2 N^2 + S^2(\varepsilon C + I)^2] - S^3[2i_0^2 N + 2S(\varepsilon C + I)^2]}{i_0^2 N^2(t) + S^2[\varepsilon C(t) + I(t)]^2}(\xi_1 - \xi_5) \\
&+ (\phi_{20} + u_2)(\varepsilon C + I) S_e \frac{2S[i_0^2 N^2 + S^2(\varepsilon C + I)^2] - S^2[2i_0^2 N + 2S(\varepsilon C + I)^2]}{i_0^2 N^2(t) + S^2[\varepsilon C(t) + I(t)]^2}(\xi_2 - \xi_5) \\
&+ c\beta(\varepsilon C + I)\frac{(N - S)}{N^2}(\xi_1 - \kappa\xi_3 - (1 - \kappa)\xi_4) \\
&- \gamma c\beta(\varepsilon C + I)\frac{S_e}{N^2}(\xi_2 - \iota\xi_3 - (1 - \iota)\xi_4) \\
&+ \mu\xi_1
\end{aligned} \tag{30}$$

$$\begin{aligned}
\xi_2'(t) = &\ -\frac{\partial H}{\partial S_e}(t) \\
&= -(\alpha_0 + u_1)(\varepsilon C + I)^2 S^3 \frac{2i_0^2 N}{[i_0^2 N^2(t) + S^2[\varepsilon C(t) + I(t)]^2]^2}(\xi_1 - \xi_2) \\
&- (\phi_{10} + u_2)(\varepsilon C(t) + I(t))^2 S^3 \frac{2i_0^2 N}{[i_0^2 N^2(t) + S^2[\varepsilon C(t) + I(t)]^2]^2}(\xi_1 - \xi_5) \\
&+ (\phi_{20} + u_2)(\varepsilon C + I)^2 S^2 \frac{[i_0^2 N^2 + S^2(\varepsilon C + I)^2] - S_e[2i_0^2 N]}{[i_0^2 N^2(t) + S^2[\varepsilon C(t) + I(t)]^2]^2}(\xi_2 - \xi_5) \\
&- c\beta(\varepsilon C + I)\frac{S}{N^2}(\xi_1 - \kappa\xi_3 - (1 - \kappa)\xi_4) \\
&+ \gamma c\beta(\varepsilon C + I)\frac{(N - S_e)}{N^2}(\xi_2 - \iota\xi_3 - (1 - \iota)\xi_4) \\
&- m(\xi_1 - \xi_2) \\
&+ \mu\xi_2
\end{aligned} \tag{31}$$

$$
\begin{aligned}
\xi_3'(t) &= -\frac{\partial H}{\partial C}(t) \\
&= -(A_1 + B_3 u_3) \\
&+(\alpha_0 + u_1)S^3 \frac{2\varepsilon(\varepsilon C + I)[i_0^2 N^2(t)] - [2i_0^2 N[\varepsilon C(t) + I(t)]^2}{[i_0^2 N^2(t) + S^2[\varepsilon C(t) + I(t)]^2]^2}(\xi_1 - \xi_2) \\
&+(\phi_{10} + u_2)S^3 \frac{2\varepsilon(\varepsilon C + I)[i_0^2 N^2(t)] - [2i_0^2 N[\varepsilon C(t) + I(t)]^2}{[i_0^2 N^2(t) + S^2[\varepsilon C(t) + I(t)]^2]^2}(\xi_1 - \xi_5) \\
&+(\phi_{20} + u_2)S^2 S_e \frac{2\varepsilon(\varepsilon C + I)[i_0^2 N^2(t)] - [2i_0^2 N[\varepsilon C(t) + I(t)]^2}{[i_0^2 N^2(t) + S^2[\varepsilon C(t) + I(t)]^2]^2}(\xi_2 - \xi_5) \\
&+c\beta S \frac{\varepsilon N - (\varepsilon C + I)}{N^2}(\xi_1 - \kappa\xi_3 - (1-\kappa)\xi_4) \\
&+c\beta\gamma S_e \frac{\varepsilon N - (\varepsilon C + I)}{N^2}(\xi_2 - \iota\xi_3 - (1-\iota)\xi_4) \\
&+(\delta_0 + u_3)(\xi_3 - \xi_5) \\
&+\sigma(\xi_3 - \xi_4) \\
&+\mu\xi_3
\end{aligned} \tag{32}
$$

$$
\begin{aligned}
\xi_4'(t) &= -\frac{\partial H}{\partial I}(t) \\
&= -(A_2 + B_4 u_4) \\
&+(\alpha_0 + u_1)S^3 \frac{2\varepsilon(\varepsilon C + I)[i_0^2 N^2(t)] - [2i_0^2 N[\varepsilon C(t) + I(t)]^2}{[i_0^2 N^2(t) + S^2[\varepsilon C(t) + I(t)]^2]^2}(\xi_1 - \xi_2) \\
&+(\phi_{10} + u_2)S^3 \frac{2\varepsilon(\varepsilon C + I)[i_0^2 N^2(t)] - [2i_0^2 N[\varepsilon C(t) + I(t)]^2}{[i_0^2 N^2(t) + S^2[\varepsilon C(t) + I(t)]^2]^2}(\xi_1 - \xi_5) \\
&+(\phi_{20} + u_2)S^2 S_e \frac{2\varepsilon(\varepsilon C + I)[i_0^2 N^2(t)] - [2i_0^2 N[\varepsilon C(t) + I(t)]^2}{[i_0^2 N^2(t) + S^2[\varepsilon C(t) + I(t)]^2]^2}(\xi_2 - \xi_5) \\
&+c\beta S \frac{N - (\varepsilon C + I)}{N^2}(\xi_1 - \kappa\xi_3 - (1-\kappa)\xi_4) \\
&+c\beta\gamma S_e \frac{N - (\varepsilon C + I)}{N^2}(\xi_2 - \iota\xi_3 - (1-\iota)\xi_4) \\
&+(\rho_0 + u_4)(\xi_4 - \xi_5) \\
&+(d + \mu)\xi_4
\end{aligned} \tag{33}
$$

$$
\begin{aligned}
\xi_5'(t) &= -\frac{\partial H}{\partial R}(t) \\
&= -(\alpha_0+u_1)S^3(\varepsilon C+I)^2\frac{(2i_0^2N)}{[i_0^2N^2(t)+S^2[\varepsilon C(t)+I(t)]^2]^2}(\xi_1-\xi_2) \\
&-(\phi_{10}+u_2)S^3(\varepsilon C+I)^2\frac{(2i_0^2N)}{[i_0^2N^2(t)+S^2[\varepsilon C(t)+I(t)]^2]^2}(\xi_1-\xi_5) \\
&-(\phi_{20}+u_2)S^2S_e(\varepsilon C+I)^2\frac{(2i_0^2N)}{[i_0^2N^2(t)+S^2[\varepsilon C(t)+I(t)]^2]^2}(\xi_2-\xi_5) \\
&-c\beta S\frac{(\varepsilon C+I)}{N^2}(\xi_1-\kappa\xi_3-(1-\kappa)\xi_4) \\
&-c\beta\gamma S_e\frac{(\varepsilon C+I)}{N^2}(\xi_2-\iota\xi_3-(1-\iota)\xi_4) \\
&-\omega(\eta\xi_1+(1-\eta)\xi_2-\xi_5) \\
&+\mu\xi_5
\end{aligned} \tag{34}
$$

2. **The transversality conditions**

$$\xi_1(t_f)=\xi_2(t_f)=\xi_3(t_f)=\xi_4(t_f)=\xi_5(t_f)=0 \tag{35}$$

3. **The Minimum Conditions:**

When the minimum occurs in the interior of the control region

$$\frac{\partial H}{\partial u_i}=0, i=1,2,3,4. \tag{36}$$

Thus,

(37)

$$
\begin{aligned}
&B_1u_1-\frac{S^3[\varepsilon C(t)+I(t)]^2}{i_0^2N^2(t)+S^2[\varepsilon C(t)+I(t)]^2}(\xi_1-\xi_2)=0 \\
&B_2u_2-\frac{S^3[\varepsilon C(t)+I(t)]^2}{i_0^2N^2(t)+S^2[\varepsilon C(t)+I(t)]^2}(\xi_1-\xi_5)-\frac{S^2S_e[\varepsilon C(t)+I(t)]^2}{i_0^2N^2(t)+S^2[\varepsilon C(t)+I(t)]^2}(\xi_2-\xi_5)=0, \\
&B_3C-C(\xi_3-\xi_5)=0, \\
&B_4I-I(\xi_4-\xi_5)=0
\end{aligned}
$$

Since the first two equations of (37) involve controls u_1 and u_2 we have

$$
\begin{aligned}
u_1(t) &= \frac{1}{B_1}\frac{S^3(t)[\varepsilon C(t)+I(t)]^2}{i_0^2N^2(t)+S^2(t)[\varepsilon C(t)+I(t)]^2}(\xi_1(t)-\xi_2(t)) \\
u_2(t) &= \frac{1}{B_2}\frac{S^2(t)[\varepsilon C(t)+I(t)]^2}{i_0^2N^2(t)+S^2(t)[\varepsilon C(t)+I(t)]^2}[(\xi_1(t)-\xi_5(t))S(t)+(\xi_2(t)-\xi_5(t))S_e(t)].
\end{aligned} \tag{38}
$$

As these controls $, u_i's, i = 1,2$ are bounded, we arrive at

(39a)
$$u_1^* = \min\{\alpha_{\max} - \alpha_0, \max\{0, \frac{1}{B_1}\frac{[\varepsilon C(t) + I(t)]^2}{i_0^2 N^2(t) + S^2(t)[\varepsilon C(t) + I(t)]^2} S^3(t)(\xi_1 - \xi_2)\}\}$$

(39b)
$$u_2^* = \min\{\phi_{\max}, \max\{10, \frac{1}{B_2}\frac{S^2(t)[\varepsilon C(t) + I(t)]^2}{i_0^2 N^2(t) + S^2(t)[\varepsilon C(t) + I(t)]^2}[(\xi_1 - \xi_5)S(t) + (\xi_2 - \xi_5)S_e(t)]\}\}$$

The Hamiltonian with respect to the controls u_3 and u_4 are linear. Taking partial derivatives of the Hamiltonian with respect to these controls, we get

(40a)
$$\frac{\partial H}{\partial u_3} = B_3 C - C(\xi_3 - \xi_5),$$

(40b)
$$\frac{\partial H}{\partial u_4} = B_4 I - I(\xi_4 - \xi_5).$$

In these expressions, we have no information about the controls that minimize the system. So we have either Bang-Bang controls or singular controls. Now define the switching functions

(41a)
$$\psi_3(t) = B_3 C(t) - C(t)(\xi_3(t) - \xi_5(t)), \text{ and}$$

(41b)
$$\psi_4(t) = B_4 I(t) - I(t)(\xi_4(t) - \xi_5(t)).$$

In this case the characterization of $u_i, i = 3,4$ is given

(42)
$$u_i^*(t) = \begin{cases} 0 & \text{if } \psi_i(t) > 0, \\ ? & \text{if } \psi_i(t) = 0, \\ \delta_{max} - \delta_0, \rho_{max} - \rho_0 & \text{if } \psi_i(t) < 0, \end{cases} \text{ for } i = 3,4.$$

a. If $\psi_i(t) = 0$ cannot be sustained over an interval of time, but occurs only at finitely many points, then the control $u_i^*, i = 3,4$ is referred as **bang-bang.**
b. If $\psi_i(t) = 0$ for some interval of time, we say control $u_i^*, i = 3,4$ is **singular** on that interval.

Differentiating (41a) with respect to t twice and substituting values of C, ξ_3, ξ_5, and their corresponding derivatives gives an expression involving u_3. Solving for u_3 we have

$$u_3(t) = \frac{f_1(t)}{f_2(t) + f_3(t)}, \tag{43}$$

where f_1, f_2 and f_3 are functions of states and adjoint functions as given in Appendix A.

Differentiating the fourth equation of (41b) with respect to t twice and substituting values of I, ξ_4, ξ_5, and their corresponding derivatives gives an expression involving u_4. Solving for u_4 we have

$$u_4(t) = \frac{g_1(t)}{g_2(t) + g_3(t)}, \tag{44}$$

where g_1, g_2 and g_3 are functions of states and adjoint functions as given in Appendix B.

Now let

$$temp3 = u_3 \text{ and } temp4 = u_4$$

from equations (43) and (44). Then, the characterizations of the two controls u_3 and u_4 are given by

$$u_3^*(t) = \begin{cases} 0 & \text{if } \psi_3(t) > 0, \\ temp3 & \text{if } \psi_3(t) = 0, \\ \delta_{max} - \delta_0 & \text{if } \psi_3(t) < 0, \end{cases} \tag{45a}$$

$$u_4^*(t) = \begin{cases} 0 & \text{if } \psi_4(t) > 0, \\ temp4 & \text{if } \psi_4(t) = 0, \\ \rho_{max} - \rho_0 & \text{if } \psi_4(t) < 0, \end{cases} \tag{45b}$$

Since the model functions are convex with respect to the control variables, and due to a priori boundedness of the state and adjoint (or co-state) functions the optimal solution so obtained is unique for small time t_f (cf. Fister et al. 1998 [15]; Gaff and Schaefer 2009[18]).

4. NUMERICAL RESULTS AND SIMULATIONS

In this section, first we want to see the effects of some of the parameters without applying any control interventions. Then we see the effect of the control functions on the disease dynamics by performing numerical simulations. For the first case we solve the model system (7) with initial population using the Runge-Kutta method of fourth-order. For the second case we solve the optimality system which is the state and adjoint systems coupled with the optimal control characterization numerically using the forward-backward sweep method [27]. Using the initial values of the population and the set of parameter values described in Table 2, we integrated the model system (20) numerically applying the classical Runge-Kutta method of fourth-order. Starting with an initial guess for the control, the state system is solved forward in time. Using those new state values, the adjoint system is solved backward in time. The control is updated using a convex combination of the old control values and the new control values from the characterization. The linear controls are determined whether they are bang-bang or singular controls. The iterative method is repeated until convergence. All our numerical simulations are performed using the MATLAB software.

We use the parameter values given in Table 2.

For numerical simulations we use as initial conditions $S(0) = 800, S_e(0) = 300, C(0) = 50, I(0) = 40$, and $R(0) = 10$ and the associated costs $A_1 = 3, A_2 = 2, B_1 = 500, B_2 = 4, B_3 = 80, B_4 = 40$. In addition, the maximum proportion of the population to be convinced to take part in permanent self protective actions against the disease is taken to be $\alpha_{max} = 95\%$ (of the susceptible population), the maximum rate of vaccine coverage is assumed to be $\phi_{max} = 95\%$ (of both the susceptible and susceptible educated individuals) and the maximum rate of recovery of the carriers $\delta_{max} = 80\%$ while the maximum rate of recovery of the infected groups is assumed to be $\rho_{max} = 95\%$.

4.1. Effect of parameters without controls. To see how the change in a parameter affects the system dynamics it is possible to take two different values and simulate it. For example, we want to see the effect of the contact rate, c, which is involved in the expression of $\mathscr{R}_0$. Taking two different values of c, the smaller the contact rate gives the smaller expansion of the disease as given in Figure 1. In Figure 1a with contact rate $c = 1.5$ gives $\mathscr{R}_0 = 0.9079 < 1$, and is

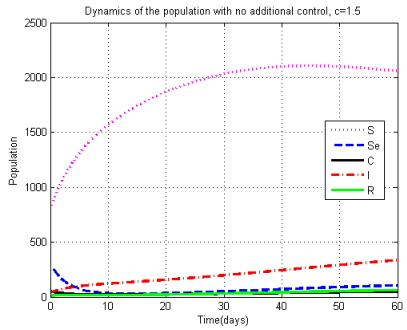
Parameter	Description	Values	Sources
Λ	The inflow rate of susceptible individuals	100	[2]
α_0	Current rate of dissemination of information about the disease	0.18	[25]
m	Rate of negligence or memory decay	0.3	Assumed
ϕ_{10}	Current rate of vaccine for susceptible	0.3	[1, 2]
ϕ_{20}	Current rate of vaccine for educated susceptible	0.4	[1, 2]
ω	Rate of immunity loss	0.63178	[12]
η	Proportion of recovered group to become susceptible.	0.4	Assumed
κ	Proportion of infected susceptibles to become carriers	0.5	Assumed
ι	Proportion of infected educated susceptibles to become carriers	0.6	Assumed
c	Average number of contacts per person per unit time	3.5	Assumed
β	Transmission rate (or rate of infection)	0.12	Assumed
σ	Rate of progressing from carrier to symptomatic class	0.37384	[12]
γ	Average rate of failure of existing self-preventive measures	0.3	Assumed
δ_0	Current recovery rate of carriers	0.29966	[12]
ρ_0	Current rate of recovery of the symptomatic individuals	0.0010959	[12]
μ	Per-capita natural mortality rate for all classes	0.027397	[12]
d	Additional death rate due to the disease	0.1	[2]
ε	Rate of infectiousness of a carrier individual	1.3	Assumed

TABLE 2. Description of constant parameters used in the model (20).

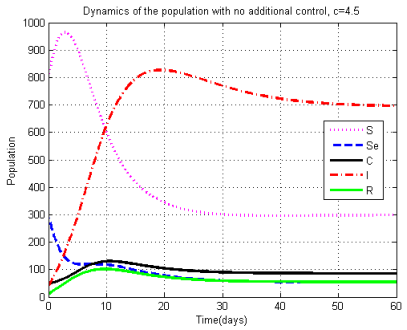
expected to lead to a possible eradication of the disease. Figure 1b is for states with contact rate $c = 4.5$ and gives $\mathscr{R}_0 = 2.7238 > 1$ and the disease becomes endemic.

Figure 2 shows the prevalence, I/N and incidence, λS of the disease with two different values of the contact rates $c = 1.5$ and $c = 4.5$. This shows for a smaller value of c a better reduction of the prevalence and the incidence of the disease.

Figure 3 shows the prevalence and incidence with two different values of the vaccination rates $\phi_{10} = 0.1, \phi_{20} = 0.1$ and $\phi_{10} = 0.5, \phi_{20} = 0.7$. This indicates that increasing the rate of vaccination reduces the prevalence and the incidence of the disease, as expected.

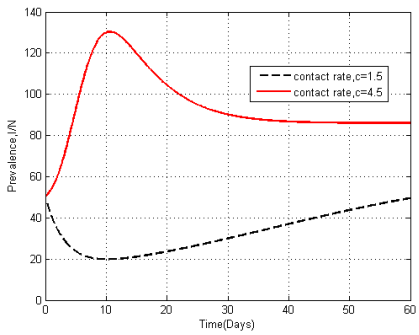


(A) states with small contact rate c=1.5

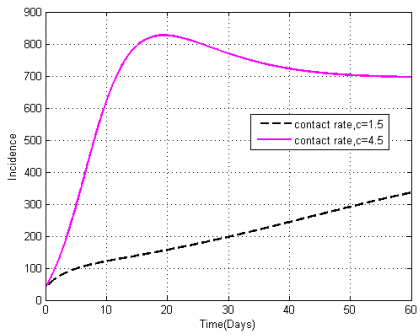


(B) states with big contact rate c=4.5

FIGURE 1. Time profile of the individuals as predicted by model system (7) with small and big contact rates, c, and the remaining parameter values are as given in Table 2. Figure 1a is for states with contact rate $c = 1.5$ and gives $\mathscr{R}_0 = 0.9079 < 1$. Figure 1b is for states with contact rate $c = 4.5$ and gives $\mathscr{R}_0 = 2.7238 > 1$.



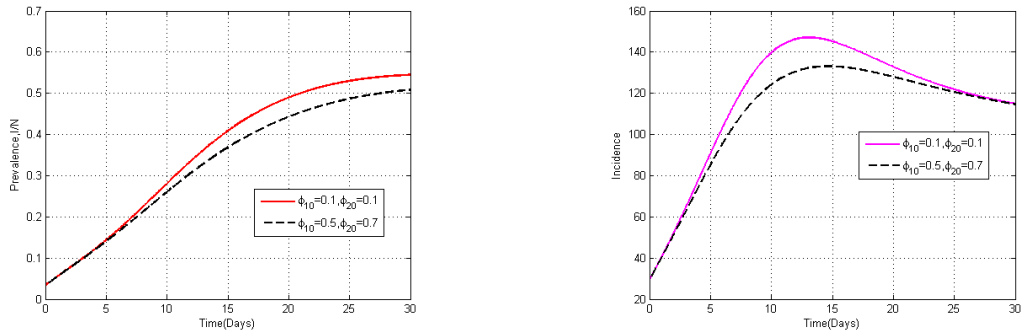
(A) Prevalence with big and small contacts



(B) Incidence with big and small contacts

FIGURE 2. The graphs of prevalence and incidence with big and small contact rates, c, and other parameter values are as given in Table 2. Figure 2a is for the prevalence with contact rates $c = 1.5$ and $c = 4.5$ Figure 2b is for the incidence with contact rates $c = 1.5$ and $c = 4.5$

At last we want to see the effect of treating infectious individuals both carriers and symptomatic groups. In addition to helping these groups recover from meningitis, treatment reduces their mean infectiousness period. It can be seen from Figure 4 that increasing the rate of treatment to a higher proportion of the infectious individuals decreases the prevalence of meningitis.



(A) Prevalence with big and small vaccination rates (B) Incidence with big and small vaccination rates

FIGURE 3. The graph of prevalence and incidence with big and small vaccination rates, where the remaining parameter values are as given in Table 2. Figure 3a is for the prevalence with vaccination rates $\phi_{10} = 0.1, \phi_{20} = 0.1$ and $\phi_{10} = 0.5, \phi_{20} = 0.7$ Figure 3b is for the incidence with vaccination rates $\phi_{10} = 0.1, \phi_{20} = 0.1$ and $\phi_{10} = 0.5, \phi_{20} = 0.7$

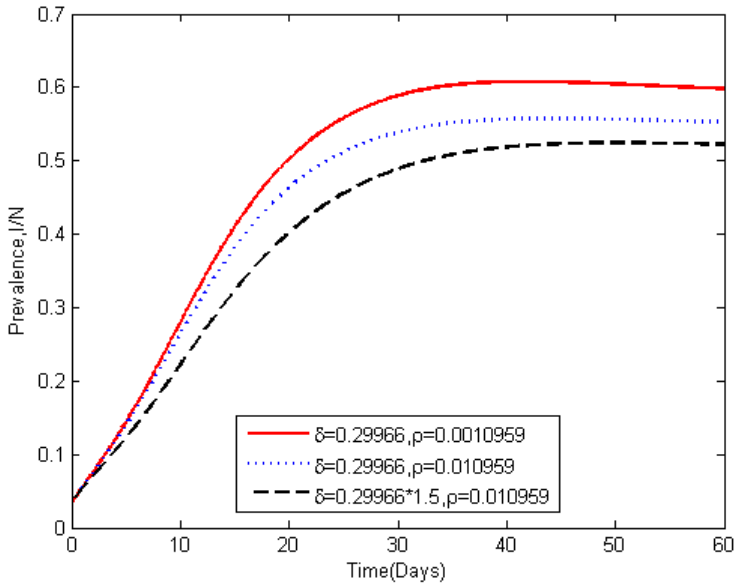
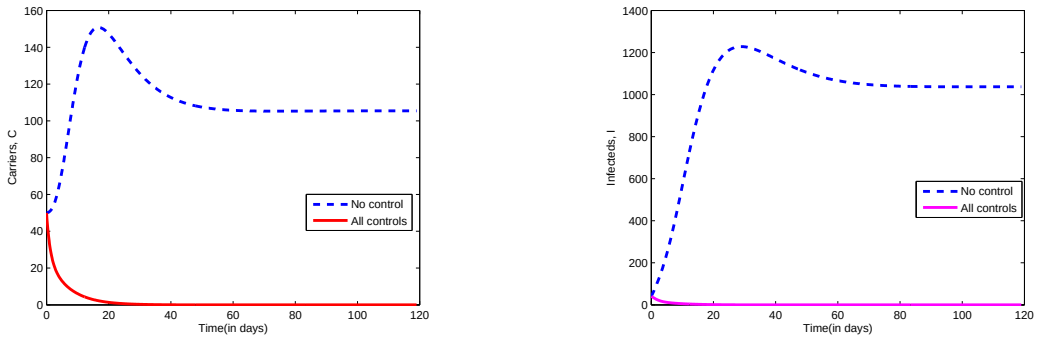


FIGURE 4. The graph of the prevalence of the the disease with different treatment rates. The remaining parameter values are as given in Table 2.

4.2. Effect of controls on the disease transmission. For a better control of the disease and to reduce its prevalence we have already introduced the intervention mechanisms called controls.

To see the influence of these intervention mechanisms on the system dynamics we simulate using the parameter values in Table 2 and initial conditions for states and the adjustment coefficients. We first used all controls (u_1, u_2, u_3, u_4) simultaneously to optimize the objective function J, second, we set all controls to zero and to optimize J. As it is given in Figure 5 one can see how much the infectious groups decrease with addition of controls.

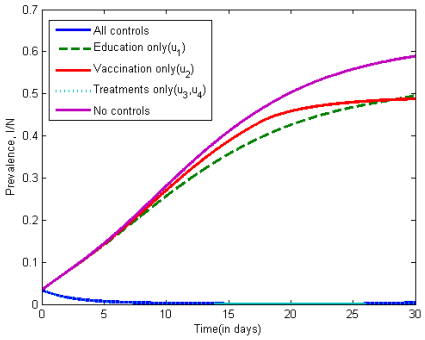


(A) Carriers (B) Infecteds

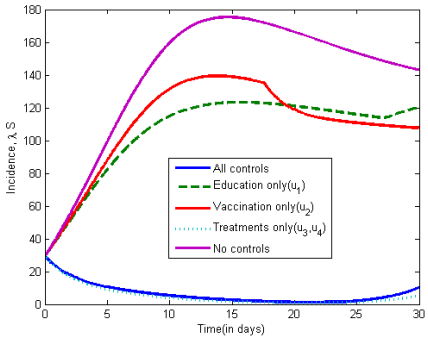
FIGURE 5. The graphs of prevalence (5a) and incidence (5b) with all controls (solid lines) and without controls (dashed lines) in various cases of the controls. The parameter values are as given in Table 2.

Next, we want to see the effects of the different combination of control mechanisms independently. In addition to the all and no addition of control, we set the controls u_2, u_3, u_4 to zero and optimize the objective function J over the control u_1, we set the controls u_1, u_3, u_4 to zero and optimize the objective function J over the control u_2 and taking treatment controls u_3, u_4 setting u_1, u_2 to zero and optimize the objective function J over the control u_3, u_4. We also estimate the cost of managing the disease with these different cases. This will help us to compare the benefit of applying the control measures in terms of reducing both the disease and economic burden on the population because of the epidemic.

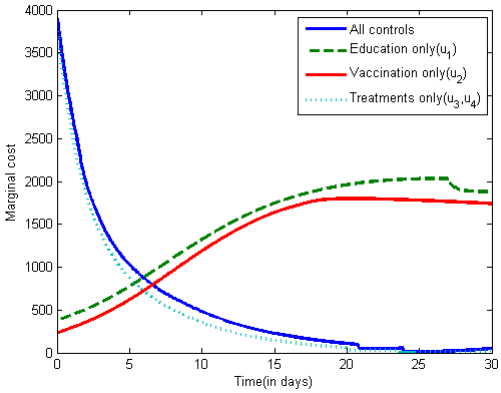
Figure 6 shows the effects of the controls Education (u_1), Vaccination (u_2) and treatments (u_3, u_4) applying independently on the prevalence 6a, incidence 6b of the diseases and the corresponding cost 6c. We can observe that the prevalence and incidence of the disease increases and stays very high if there is no additional control measure employed. In addition, if the strategy



(A) The Prevalence



(B) Incidence

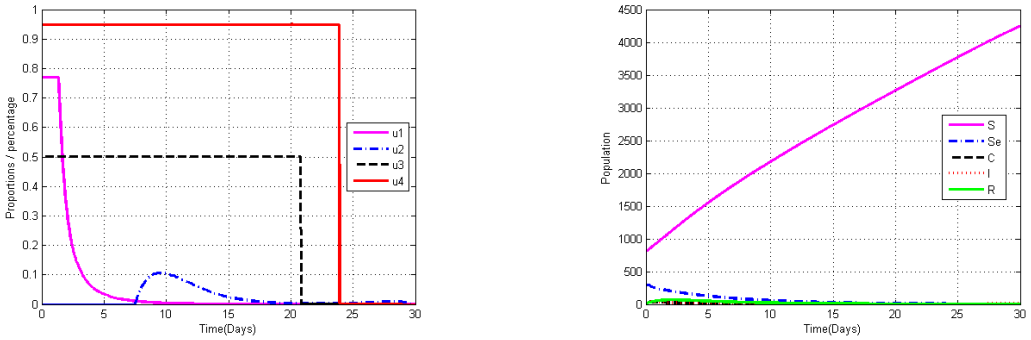


(C) Marginal cost

FIGURE 6. The variation of the prevalence of the disease, the incidence and the graph of the marginal cost of the interventions (per day) in various cases of the controls with parameter values as given in Table 2.

focusses only on prevention education with no additional effort is made to treat more infectious people, the result is better than the no-control strategy but has slightly less effect on the prevalence than the vaccination strategy. We can also see that treatment is the best preventive control of meningitis as compared to vaccination and education. The corresponding values of the marginal costs are drawn in Figure 6c.

When all the control efforts are put together, the resulting optimal control values and the corresponding states are shown in Figure 7. Figure 7a gives the graphs of optimal controls and Figure 7b is the variation of sizes of the five classes of the population when all controls are employed simultaneously. When we apply all interventions simultaneously and optimally,



(A) Optimal Controls (B) States with all controls

FIGURE 7. The graphs of optimal control values and the variation of sizes of the states when all controls are employed simultaneously with parameter values as given in Table 2.

the count for carriers and infected individuals falls rapidly with an increase of the susceptible groups as shown in Figure 7b. For example, addition of all controls gives around 98% decrease in the cumulative carrier groups within 119 days as seen in Figure 5a. In this case the optimal controls suggest that educating people and convincing them to participate in self protective schemes and apply treatment controls starting with their maximal possible values will result in best health and economic benefit to the community. However, the time profile for vaccination control starts with minimum intensity and rises and at last comes to its minimum value. There is around a 40% decrease in disease management cost using all optimal controls than applying no control within 30 days.

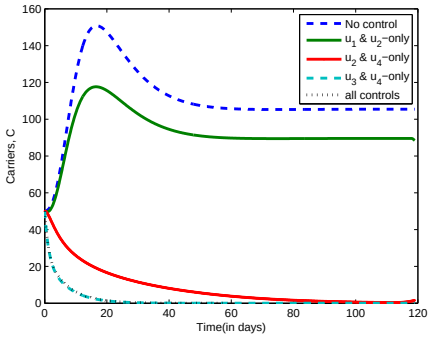
5. COST-EFFECTIVENESS ANALYSIS

In this section, we want to identify a strategy which is cost-effective compared to other strategies in applying the different controls. Since different combinations of the controls results in different outcome in the disease dynamics, as can be seen in Figure 8, we need to investigate the cost-effectiveness of each combination. To this effect, we use the incremental cost-effectiveness ratio (ICER) as given in [37], which is done by dividing the difference of costs between two strategies to the difference of the total number of their infections averted. The total cost of each strategy can be taken as the corresponding total cost, J values and to estimate the total number of

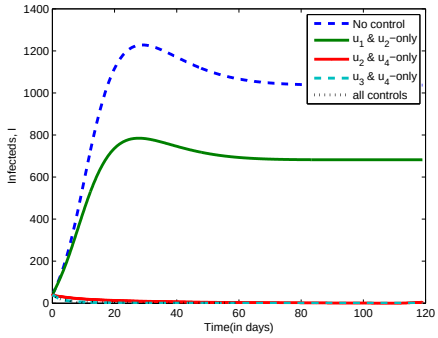
the infections averted we take the corresponding total of carriers C and infected (symptomatic) groups I. That is, as it is indicated in [1, 37], we consider the formula

$$\text{ICER} = \frac{\text{Difference in costs between strategies}}{\text{Difference in health effects between strategies}},$$

which helps to rank the suggested strategies in terms of their cost and health benefits.



(A) Count of Carrier individuals



(B) Count of Infected individuals

FIGURE 8. The graphs of carriers (8a) and infected groups (8b) for different strategies. The parameter values are as given in Table 2 with $A_1 = 3; A_2 = 2; B1 = 50; B2 = 1; B3 = 80; B4 = 60$.

The following are the proposed strategies.

- Strategy (a): Treating both carriers (C) and infected (symptomatic) (I) groups.
- Strategy (b): Vaccination and treating only the infected (symptomatic) (I) groups.
- Strategy (c): Education and vaccination.
- Strategy (d): Education, vaccination and treating both carriers (C) and infected (symptomatic) (I) groups.

Table 3 gives the number of infections averted and total costs of each strategy.

To determine the cost effective strategy according to the ICER, we first perform pairwise comparison and order them according to their decreasing ICER values. To this end, first we compare the cost-effectiveness of strategies (a) and (b):

$$\text{ICER(a)} = \frac{7789700}{3476600} = 2.24060864062589,$$

$$\text{ICER(b)} = \frac{7789700 - 1.7725}{3476600 - 0.7912} = 2.24060864070.$$

Strategy	Description	Total infections averted	Total cost
(a)	Treating both carriers (C) and infected (I) groups.	3476600	7789700
(b)	Vaccination and treating only the infected (I) groups.	0.7912	1.7725
(c)	Education and vaccination.	91.7007	193.9752
(d)	Education,vaccination and treating all.	3667300	8218500

TABLE 3. Number of infections averted and total costs of each strategy.

This shows that strategy (b) is slightly cheaper than strategy (a) by saving 2.24060864070. Which means strategy (b) is better than strategy (a). Excluding strategy (a) and comparing strategies (b) and (c), we have the following.

$$\text{ICER(b)} = \frac{1.7725}{0.7912} = 2.240267947421638,$$

$$\text{ICER(c)} = \frac{1.7725 - 193.9752}{0.7912 - 91.7007} = 2.1142201860.$$

This gives strategy (c) is better than strategy (b). Excluding strategy (b) and comparing strategies (c) and (d), we have the following.

$$\text{ICER(c)} = \frac{193.9752}{91.7007} = 2.11530773483735,$$

$$\text{ICER(d)} = \frac{193.9752 - 8218500}{91.7007 - 3667300} = 2.241025148849.$$

Which means that strategy (c) is still cheaper than strategy (d).

Therefore, from the pairwise comparisons we can conclude that strategy (c), education and vaccination is the most cost-effective strategy from all the four strategies suggested.

6. CONCLUDING REMARKS

In this paper, we formulated and presented behaviour dependent mathematical model of meningococcal meningitis. The self protection action is considered to be incident dependent and vaccination is also considered as information driven. We have seen that decreasing the

contact rate, treating the infectious groups and increasing the rate of vaccination reduces the prevalence of the disease.

Moreover, we applied optimal control method to the mathematical model of meningitis taking education, vaccination and treatment as control factors. This is used in making decisions about optimally allocating efforts to slow down the epidemic with an information educational campaign, vaccination and treatment. We found that taking all controls better reduces the prevalence and incidence of the disease than without controls. Education campaigns, vaccination and treatment are important options for the disease management. Our simulations indicate that treatment is the best strategy to decrease the transmission of the disease than vaccination and education. In general, we recommend to the health policy makers

i. to teach people to take vaccination of meningitis and to decrease the contact rate with those who are infectious as it decreases the prevalence of the disease.
ii. to consider and use all the intervention mechanisms optimally to decrease the prevalence and incidence of meningitis.
iii. in an area with limited resources, it is advisable to give emphasis on the treatment control method for a short gain in public health.
iv. to educate and vaccinate the population as it is the best cost-effective strategy to control and prevent meningitis.

ACKNOWLEDGEMENTS

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APPENDIX A. BANG-BANG CONTROL u_3

$$u_3(t) = \frac{f_1(t)}{f_2(t) + f_3(t)}, \tag{46}$$

where

$$\begin{aligned}
f_1 =&(\xi_5 + B_3)[\kappa c\beta\varepsilon\frac{CS}{N} + \iota\gamma c\beta\frac{CS_e}{N}] - B_3(\delta_0 + \sigma + \mu)C \\
&+ [(\xi_3 - \xi_5) - B_3][\kappa c\beta\frac{\varepsilon IS}{N} + \iota\gamma c\beta\frac{\varepsilon IS_e}{N}] + (\xi_4 - \xi_5)\sigma C \\
&+ A_1 C \\
&- \{\frac{2\varepsilon(\varepsilon C + I)i^2N^2}{[i^2N^2 + S^2(\varepsilon C + I)^2]^2}C\} \\
&[(\alpha_0 + u_1)S^3(\xi_1 - \xi_2) + (\phi_{10} + u_2)S^3(\xi_1 - \xi_5) + (\phi_{20} + u_2)S^2S_e(\xi_2 - \xi_5)] \\
&- \{\frac{2\varepsilon^2 i^2N^2[i^2N^2 - 3S^2(\varepsilon C + I)^2]}{[i^2N^2 + S^2(\varepsilon C + I)^2]^3}\} \\
&[(\alpha_0 + u_1)S^3C^2(\xi_1 - \xi_2) + (\phi_{10} + u_2)S^3C^2(\xi_1 - \xi_5) + (\phi_{20} + u_2)S^2S_eC^2(\xi_2 - \xi_5)] \\
&- [c\beta\varepsilon\frac{S}{N}(\xi_1 - (1-\kappa)\xi_4) + \gamma c\beta\varepsilon\frac{S_e}{N}(\xi_2 - (1-\iota)\xi_4) + \omega(\eta\xi_1 + (1-\eta)\xi_2 - \xi_5)]C,
\end{aligned}$$

$$\begin{aligned}
f_2 \;=&\; [(B_3 + \xi_5)\kappa c\beta\varepsilon\tfrac{S}{N}[\kappa c\beta\tfrac{(\varepsilon C+I)S}{N} + \iota\gamma c\beta\tfrac{(\varepsilon C+I)S_e}{N} - (\delta_0 + \sigma + \mu)C] \\
&+(B_3 + \xi_5)\kappa c\beta\tfrac{(I'S + (\varepsilon C+I)S')N - (\varepsilon C+I)S(\Lambda - \mu N - dI)}{N^2} \\
&[(B_3 + \xi_5)\iota\gamma c\beta\varepsilon\tfrac{S_e}{N}[\kappa c\beta\tfrac{(\varepsilon C+I)S}{N} + \iota\gamma c\beta\tfrac{(\varepsilon C+I)S_e}{N} - (\delta_0 + \sigma + \mu)C] \\
&+(B_3 + \xi_5)\iota\gamma c\beta\tfrac{(I'S_e + (\varepsilon C+I)S_e')N - (\varepsilon C+I)S_e(\Lambda - \mu N - dI)}{N^2} \\
&-B_3(\delta_0 + \sigma + \mu)[\kappa c\beta\tfrac{(\varepsilon C+I)S}{N} + \iota\gamma c\beta\tfrac{(\varepsilon C+I)S_e}{N} - (\delta_0 + \sigma + \mu)C] \\
&-[\kappa c\beta\tfrac{IS}{N} + \iota\gamma c\beta\tfrac{IS_e}{N}] \\
&\{ \\
&-A_1 + [\frac{2\varepsilon(\varepsilon C + I)i_0^2N^2 - 2i_0^2N[\varepsilon C + I]^2}{[i_0^2N^2 + S^2[\varepsilon C + I]^2]^2}] \\
&[(\alpha_0 + u_1)S^3(\xi_1 - \xi_2) + (\phi_{10} + u_2)S^3(\xi_1 - \xi_5) + (\phi_{20} + u_2)S^2S_e(\xi_2 - \xi_5)] \\
&+[c\beta S\tfrac{\varepsilon N - (\varepsilon C+I)}{N^2}][\xi_1 - \kappa\xi_3 - (1-\kappa)\xi_4] \\
&+[c\beta\gamma S_e\tfrac{\varepsilon N - (\varepsilon C+I)}{N^2}][\xi_2 - \iota\xi_3 - (1-\iota)\xi_4] \\
&+\delta_0(\xi_3 - \xi_5) + \sigma(\xi_3 - \xi_4) + \mu\xi_3\}
\end{aligned}$$

$$
\begin{aligned}
f_3 \;&= -\xi_3[\kappa c\beta\tfrac{(I'S+IS')N-IS(\Lambda-\mu N-dI))}{N^2}+\iota\gamma c\beta\tfrac{(I'S_e+IS'_e)N-IS_e(\Lambda-\mu N-dI))}{N^2}]\\
&+\xi'_5[\kappa c\beta\tfrac{\varepsilon C+I}{N}S+\iota\gamma c\beta\tfrac{\varepsilon C+I}{N}S_e-\sigma C]\\
&+(\xi_4-\xi_5)\sigma[\kappa c\beta\tfrac{(\varepsilon C+I)S}{N}+\iota\gamma c\beta\tfrac{(\varepsilon C+I)S_e}{N}-(\delta_0+\sigma+\mu)C]\\
&+[\kappa c\beta\tfrac{(\varepsilon C+I)S}{N}+\iota\gamma c\beta\tfrac{(\varepsilon C+I)S_e}{N}-(\delta_0+\sigma+\mu)C]\\
&\{A_1-\tfrac{2\varepsilon(\varepsilon C+I)i^2N^2C}{[i^2N^2+S^2(\varepsilon C+I)^2]^2}\\
&[(\alpha_0+u_1)S^3(\xi_1-\xi_2)+(\phi_{10}+u_2)S^3(\xi_1-\xi_5)+(\phi_{20}+u_2)S^2S_e(\xi_2-\xi_5)]\}\\
&-\tfrac{2\varepsilon(\varepsilon C+I)i^2N^2C}{[i^2N^2+S^2(\varepsilon C+I)^2]^2}\{(\alpha_0+u_1)[3S^2(\xi_1-\xi_2)S'+S^3(\xi'_1-\xi'_2)]\\
&+(\phi_{10}+u_2)[3S^2(\xi_1-\xi_5)S'+S^3(\xi'_1-\xi'_5)]\\
&+(\phi_{20}+u_2)[2SS_e(\xi_2-\xi_5)S'+S^2S'_e(\xi_2-\xi_5)+S^2S_e(\xi'_2-\xi'_5)]\}\\
&-[(\alpha_0+u_1)S^3C(\xi_1-\xi_2)+(\phi_{10}+u_2)S^3C(\xi_1-\xi_5)+(\phi_{20}+u_2)S^2S_eC(\xi_2-\xi_5)]\\
&\left[\frac{\begin{array}{c}[2\varepsilon i^2N^2I'+4\varepsilon(\varepsilon C+I)i^2N(\Lambda-\mu N-dI)[i^2N^2+S^2(\varepsilon C+I)^2]\\ -[2i^2N(\Lambda-\mu N-dI)+2SS'(\varepsilon C+I)^2+2S^2(\varepsilon C+I)I']4\varepsilon(\varepsilon C+I)i^2N^2]\end{array}}{\left[i^2N^2+S^2(\varepsilon C+I)^2\right]^3}\right]\\
&-[(\alpha_0+u_1)S^3C(\xi_1-\xi_2)+(\phi_{10}+u_2)S^3C(\xi_1-\xi_5)+(\phi_{20}+u_2)S^2S_eC(\xi_2-\xi_5)]\\
&\{\tfrac{2\varepsilon^2i^2N^2[i^2N^2+S^2(\varepsilon C+I)^2]-8\varepsilon^2i^2N^2S^2(\varepsilon C+I)^2}{[i^2N^2+S^2(\varepsilon C+I)^2]^3}]\}[\kappa c\beta\tfrac{(\varepsilon C+I)S}{N}+\iota\gamma c\beta\tfrac{(\varepsilon C+I)S_e}{N}\\
&-(\delta_0+\sigma+\mu)C]\\
&-c\beta\varepsilon\tfrac{S'CN-SC(\Lambda-\mu N-dI)}{N^2}[\xi_1-(1-\kappa)\xi_4]\\
&-c\beta\varepsilon\tfrac{SC}{N}[\xi'_1-(1-\kappa)\xi'_4]\\
&-c\beta\gamma\varepsilon\tfrac{S'_eCN-S_eC(\Lambda-\mu N-dI)}{N^2}[\xi_2-(1-\iota)\xi_4]\\
&-c\beta\gamma\varepsilon\tfrac{S_eC}{N}[\xi'_2-(1-\iota)\xi'_4]\\
&+\sigma\xi'_4C\\
&-\omega(\eta\xi'_1+(1-\eta)\xi'_2-\xi'_5)C\\
&-\{c\beta\varepsilon\tfrac{S}{N}[\xi_1-(1-\kappa)\xi_4]+c\beta\gamma\varepsilon\tfrac{S_e}{N}[\xi_2-(1-\iota)\xi_4)+\omega(\eta\xi_1+(1-\eta)\xi_2-\xi_5)\}\\
&\{\kappa c\beta\tfrac{(\varepsilon C+I)S}{N}+\iota\gamma c\beta\tfrac{(\varepsilon C+I)S_e}{N}-(\delta_0+\sigma+\mu)C\}.
\end{aligned}
$$

APPENDIX B. BANG-BANG CONTROL u_4

$$u_4(t) = \frac{g_1(t)}{g_2(t) + g_3(t)}, \tag{47}$$

where

$$\begin{aligned}
g_1 =&(\xi_5 + B_4)[(1-\kappa)c\beta\frac{IS}{N} + (1-\iota)\gamma c\beta\frac{IS_e}{N}] - B_4(\rho_0 + d + \mu)I \\
&+ [(\xi_4 - \xi_5) - B_4][(1-\kappa)c\beta\frac{\varepsilon CS}{N} + (1-\iota)\gamma c\beta\frac{\varepsilon CS_e}{N} + \sigma C] \\
&+ A_2 I \\
&- \{\frac{[2(\varepsilon C + I)i^2N^2}{[i^2N^2 + S^2(\varepsilon C + I)^2]^2}\} \\
&[(\alpha_0 + u_1)S^3I(\xi_1 - \xi_2) + (\phi_{10} + u_2)S^3I(\xi_1 - \xi_5) + (\phi_{20} + u_2)S^3I(\xi_2 - \xi_5)] \\
&- \{\frac{2i^2N^2(i^2N^2 - 3S^2(\varepsilon C + I)^2)I}{[i^2N^2 + S^2(\varepsilon C + I)^2]^3}\} \\
&[(\alpha_0 + u_1)S^3I(\xi_1 - \xi_2) + (\phi_{10} + u_2)S^3I(\xi_1 - \xi_5) + (\phi_{20} + u_2)S^3I(\xi_2 - \xi_5)] \\
&- [c\beta\frac{S}{N}(\xi_1 - \kappa\xi_3) + \gamma c\beta\frac{S_e}{N}(\xi_2 - \iota\xi_3) + \omega(\eta\xi_1 + (1-\eta)\xi_2 - \xi_5]I,
\end{aligned}$$

$$\begin{aligned}
g_2 = \;& (B_4 + \xi_5)(1-\kappa)c\beta\tfrac{S}{N}[(1-\kappa)c\beta\tfrac{(\varepsilon C+I)S}{N} + (1-\iota)\gamma c\beta\tfrac{(\varepsilon C+I)S_e}{N} + \sigma C - (\rho_0 + d + \mu)I] \\
&+(B_4 + \xi_5)(1-\kappa)c\beta\tfrac{(\varepsilon C'S + (\varepsilon C+I)S')N - (\varepsilon C+I)S(\Lambda - \mu N - dI)}{N^2} \\
&+(B_4 + \xi_5)(1-\iota)\gamma c\beta\tfrac{S_e}{N}[(1-\kappa)c\beta\tfrac{(\varepsilon C+I)S}{N} + (1-\iota)\gamma c\beta\tfrac{(\varepsilon C+I)S_e}{N} + \sigma C - (\rho_0 + d + \mu)I] \\
&+(B_4 + \xi_5)(1-\iota)\gamma c\beta\tfrac{(\varepsilon C'S_e + (\varepsilon C+I)S_e')N - (\varepsilon C+I)S_e(\Lambda - \mu N - dI)}{N^2} \\
&+(B_4 + \xi_5)\sigma C' \\
&-B_4(\rho_0 + d + \mu)[(1-\kappa)c\beta\tfrac{(\varepsilon C+I)S}{N} + (1-\iota)\gamma c\beta\tfrac{(\varepsilon C+I)S_e}{N} + \sigma C - (\rho_0 + d + \mu)I] \\
&-[(1-\kappa)c\beta\varepsilon\tfrac{CS}{N} + (1-\iota)\gamma c\beta\varepsilon\tfrac{CS_e}{N} + \sigma C] \\
&\{ \\
&-A_2 + [\frac{2(\varepsilon C + I)i_0^2N^2 - 2i_0^2N[\varepsilon C + I]^2}{[i_0^2N^2 + S^2[\varepsilon C + I]^2]^2}] \\
&[(\alpha_0 + u_1)S^3(\xi_1 - \xi_2) + (\phi_{10} + u_2)S^3(\xi_1 - \xi_5) + (\phi_{20} + u_2)S^2S_e(\xi_2 - \xi_5)] \\
&+[c\beta\tfrac{N - (\varepsilon C+I)}{N^2}][\xi_1 S - \kappa\xi_3 S - (1-\kappa)\xi_4 S + \gamma\xi_2 S_e - \iota\gamma\xi_3 S_e - (1-\iota)\gamma\xi_4 S_e] \\
&+\rho_0(\xi_4 - \xi_5) + (d + \mu)\xi_4\}
\end{aligned}$$

$$
\begin{aligned}
&g_3 = \\
&-\xi_4[(1-\kappa)c\beta\varepsilon\tfrac{(C'S+CS')N-CS(\Lambda-\mu N-dI))}{N^2}+(1-\iota)\gamma c\beta\tfrac{(C'S_e+CS'_e)N-CS_e(\Lambda-\mu N-dI))}{N^2}+\sigma C'] \\
&+\xi'_5[(1-\kappa)c\beta\tfrac{\varepsilon C+I}{N}S+(1-\iota)\gamma c\beta\tfrac{\varepsilon C+I}{N}S_e+\sigma C] \\
&\\
&+[(1-\kappa)c\beta\tfrac{(\varepsilon C+I)S}{N}+(1-\iota)\gamma c\beta\tfrac{(\varepsilon C+I)S_e}{N}+\sigma C-(\rho_0+d+\mu)I] \\
&\{\, A_2-\tfrac{2(\varepsilon C+I)i^2N^2}{[i^2N^2+S^2(\varepsilon C+I)^2]^2} \\
&[(\alpha_0+u_1)S^3(\xi_1-\xi_2)+(\phi_{10}+u_2)S^3(\xi_1-\xi_5)+(\phi_{20}+u_2)S^2S_e(\xi_2-\xi_5)]\,\} \\
&-\tfrac{2(\varepsilon C+I)i^2N^2}{[i^2N^2+S^2(\varepsilon C+I)^2]^2} \\
&\{\,(\alpha_0+u_1)[3S^2(\xi_1-\xi_2)IS'+S^3I(\xi'_1-\xi'_2)] \\
&+(\phi_{10}+u_2)[3S^2(\xi_1-\xi_5)IS'+S^3I(\xi'_1-\xi'_5)] \\
&+(\phi_{20}+u_2)[2SS_e(\xi_2-\xi_5)IS'+S^2S'_eI(\xi_2-\xi_5)+S^2S_eI(\xi'_2-\xi'_5)]\} \\
&\\
&-[(\alpha_0+u_1)S^3I(\xi_1-\xi_2)+(\phi_{10}+u_2)S^3I(\xi_1-\xi_5)+(\phi_{20}+u_2)S^2S_eI(\xi_2-\xi_5)] \\
&\left[\begin{array}{r}[2\varepsilon i^2N^2C'+4(\varepsilon C+I)i^2N(\Lambda-\mu N-dI)][i^2N^2+S^2(\varepsilon C+I)^2]-8(\varepsilon C+I)i^2N^2i^2N \\ \hline (\Lambda-\mu N-dI)-8(\varepsilon C+I)^3i^2N^2SS'-8(\varepsilon C+I)^2i^2N^2S^2(\varepsilon C') \\ \hline [i^2N^2+S^2(\varepsilon C+I)^2]^3\end{array}\right] \\
&-[(\alpha_0+u_1)S^3I(\xi_1-\xi_2)+(\phi_{10}+u_2)S^3I(\xi_1-\xi_5)+(\phi_{20}+u_2)S^2S_eI(\xi_2-\xi_5)] \\
&[\tfrac{2i^2N^2(i^2N^2-3S^2(\varepsilon C+I)^2)}{[i^2N^2+S^2(\varepsilon C+I)^2]^3}][(1-\kappa)c\beta\tfrac{(\varepsilon C+I)S}{N}+(1-\iota)\gamma c\beta\tfrac{(\varepsilon C+I)S_e}{N}+\sigma C-(\rho_0+d+\mu)I] \\
&-c\beta\tfrac{S'IN-SI(\Lambda-\mu N-dI)}{N^2}[\xi_1-\kappa\xi_3] \\
&-c\beta\tfrac{SI}{N}[\xi'_1-\kappa\xi'_3] \\
&-c\beta\gamma\tfrac{S'_eIN-S_eI(\Lambda-\mu N-dI)}{N^2}[\xi_2-\iota\xi_3] \\
&-c\beta\gamma\tfrac{S_eI}{N}[\xi'_2-\iota\xi'_3] \\
&-\omega(\eta\xi'_1+(1-\eta)\xi'_2-\xi'_5)I \\
&-\{c\beta\tfrac{S}{N}[\xi_1-\kappa\xi_3]+c\beta\gamma\tfrac{S_e}{N}[\xi_2-\iota\xi_3)+\omega(\eta\xi_1+(1-\eta)\xi_2-\xi_5\} \\
&\{(1-\kappa)c\beta\tfrac{(\varepsilon C+I)S}{N}+(1-\iota)\gamma c\beta\tfrac{(\varepsilon C+I)S_e}{N}+\sigma C-(\rho_0+d+\mu)I\}.
\end{aligned}
$$

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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