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MATHEMATICAL MODELING OF COVID-19 IN MOROCCO AND THE IMPACT OF CONTROLLING MEASURES

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Abstract. The novel coronavirus disease (COVID-19) may be introduced into a population through arrival passengers outside the host population. The development of mathematical models is used to forecast, evaluate, and attempt to control the spread of diseases. In this paper, a new epidemiological model with seven compartments that include passengers is constructed to describe the COVID-19 dynamic in Morocco. The most common parameter is the reproduction number R_0 which determines whether a disease will die out or expand. The sensitivity analysis was applied to the model to discover which parameter has a high impact on R_0 . The final size relation for the epidemic models was derived to find its relationship with the reproduction number. Some parameter values are estimated by fitting the COVID-19 data in Morocco, which is considered from 2^{nd} March to 19^{th} July 2020. Finally, we verified the importance of testing and awareness of the community to break the spread of Coronavirus disease.

Keywords: mathematical model; COVID-19; basic reproduction number; final size; sensitivity analysis.

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1. Introduction

COVID-19 pandemic (coronavirus pandemic) is an ongoing disease that emerged by the end of December 2019 in Wuhan city in China. This infectious disease which directly targets the human respiratory system, was spread outside Chinese borders rapidly. Around the world, more than $155,297,800$ confirmed cases and around $3,243,963$ deaths up to May 6, 2021 [1]. The COVID-19 was recognized as a pandemic by the World Health Organization (WHO) on 11^{ed} March 2020. In Morocco, the first confirmed imported COVID-19 case was on March 2, 2020; it involved a Moroccan expatriate coming from Italy, the first case related to the local transmission was registered on March 13, 2020. The total number of infected increased to 844 up to April 4, 2020, of which 83.1% are indigenous cases. The number of infected cases continued to increase gradually, which lead the government to implement control measures, including closing the land, air, and maritime borders starting from March 15, 2020. Then all educational institutions were closed, including schools, universities, and others. They were also closing mosques since March 16, 2020. Also, the authorities rushed to stop the spread of the disease by imposing a ban inside the country and implementing a system to track cases and monitor those in contact with them. By the end of July, there were $6,311$ active cases with 535 deaths.

Mathematical modeling of infectious disease performs a key role in efforts that focus on forecasting, evaluating, controlling, and implementing strategies to deal with the outbreak of the disease. Several models were developed about COVID-19. Qiangying et al.[2],developed an SEIR model that focuses on the transmission and trends of the COVID-19 outbreak. Lhous et al.[3] developed a SEIRQD model to study the importance of optimal control for the second stage lockdown of COVID-19 in Morocco. In [4], Djilali et al. investigated through a SEIUH model of the COVID-19 impact unreported cases in Morocco, Algeria, and Egypt. Takasar et al. [5] designed a SEIQR model to predict the COVID-19 epidemic. Zhang et al. [6] estimated R0 about 2.28 for the COVID-19 model in the early stage of the outbreak. Giordano et al. [7] developed a mathematical model, SIDARTHE, which suggested tools to assess the consequences of possible strategies against the COVID-19 pandemic. Boutayeb et al.[8], proposed a multi-region SIR model of COVID-19 for different zones in Morocco to study the spread of the disease. Bentout et al.[9], presented a COVID-19 model to forecast the size of the epidemic in the United Arab Emirates, Algeria, and the United States of America.

In this study, a mathematical model with seven compartments, Susceptible, Passengers, Exposed, Asymptomatic, Infectious isolated, Infectious free, Treated and Recovered, is formulated to describe the spread of COVID-19 in the population. The goal for this study is forecasting COVID-19 transmission and the essential keys to control the spread of the diseases to enhance the authorities' efforts against this pandemic in Morocco. This paper consists of seven sections as follows: section 2, the model is proposed to describe the transmission of COVID-19. In section 3, the study case when the country lockdown the borders. In section 4, sensitivity analysis was presented for the original model when easing the COVID-19 re- restrictions. Section 5, illustrated the final size relation, while section 6, presented the estimation and the prediction for the spread of COVID-19. Finally, section 7 showed the conclusion and recommendation.

2. Mathematical Model

The following Ordinary Differential Equations (ODEs) are used to describe the transmission of covid-19 in Morocco:

$$
\left\{
\begin{array}{lcl}
\dfrac{dS(t)}{dt} & = & p_S\Lambda - \lambda(t)S(t) \\[2ex]
\dfrac{dE(t)}{dt} & = & p_E\Lambda + \lambda(t)S(t) - \sigma E(t) \\[2ex]
\dfrac{dA(t)}{dt} & = & p_A\Lambda + \sigma\rho E(t) - \alpha A(t) \\[2ex]
\dfrac{dI(t)}{dt} & = & p_I\Lambda + \sigma(1-\rho)E(t) + \alpha A(t) - (\tau + \theta_I + \delta_I)I(t) \\[2ex]
\dfrac{dT(t)}{dt} & = & \tau I(t) - (\theta_T + \delta_T)T(t) \\[2ex]
\dfrac{dR(t)}{dt} & = & \theta_I I(t) + \theta_T T(t)
\end{array}
\right.
\tag{1}
$$

$$\lambda(t) = \beta_E \frac{E(t)}{N} + \beta_A \frac{A(t)}{N} + \beta_I \frac{I(t)}{N} \quad \text{And} \quad \sum p_i = 1,\ i \in \{S, E, A, I\}$$

The total population is given by $N(t) = S(t) + E(t) + A(t) + I(t) + T(t) + R(t)$ and $\dot{N} = \Lambda - \delta_I I(t) - \delta_T T(t)$. According to disease state the total population N was divided into six compartments: general susceptible individuals $S(t)$, exposed and pre-asymptomatic $E(t)$, asymptomatic $A(t)$, infected (symptomatic infection) $I(t)$, treatment (hospitalized) $T(t)$ and recovered $R(t)$. In this model, if Λ is the number of passengers from oversea countries arriving to Morocco per day and p_i is the proportion of passenger individuals in compartment i, where $i \in \{S, E, A, I\}$.

All the parameters of the model are defined in table (1) and assumed to be positive and the initial condition of the system (1) are given by: $S(0) = S_0 > 0, E(0) = E_0 \geq 0, A(0) = A_0 \geq 0, I(0) = I_0 \geq 0, T(0) = T_0 \geq 0, R(0) = R_0 \geq 0$.

The variables of the system are positive and bounded (Standard method).

TABLE 1. Description of the parameters of the system (1)

Parameter	Description
Λ	The number of passenger arriving to Morocco per day
p_i	the proportion of passenger individuals in compartment i, $i \in \{S, E, A, I\}$.
β_E	Transmission coefficient from Exposed individuals
β_A	Transmission coefficient from asymptomatic individuals
β_I	Transmission coefficient from infected isolated individuals
σ	Rate at which exposed individuals progresses to infectious compartments A and I
ρ, $(1-\rho)$	The proportion of asymptomatic infected and infected (symptomatic infection) individuals
α	Rate at which asymptomatic individuals progresses to infected compartment
$\lambda(t)$	Infection rate for Moroccan population
τ	Treatment rate of infectious
θ_I, θ_T	Rate at which infected and treated individuals progresses to recovered compartments respectively
δ_I, δ_T	The disease induced death rates due to infected and treated individuals respectively

The flowchart of the model is presented in figure (1)

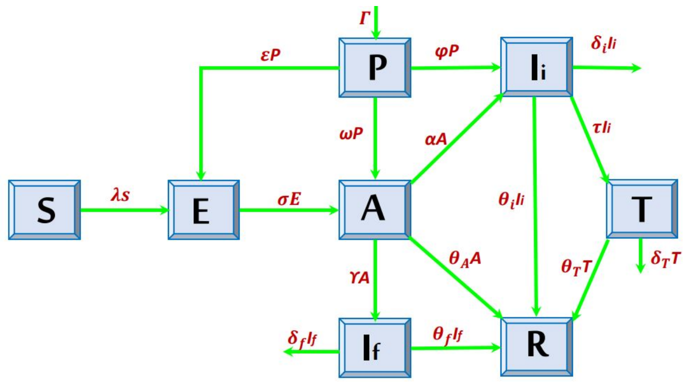

FIGURE 1. Flow chart of compartments of COVID-19 model in Morocco.

3. LOCKDOWNS AND EXTREME RESTRICTIONS TERM

In this section, we will study the case when the country have implemented restrictions on population movement and lockdowns the borders to slow the spread of the disease. In this case, travelers to the country have been stopped, the model (1) will be reduced to (2) :

$$(2)\qquad \begin{cases} \dfrac{dS(t)}{dt} &= -\lambda S(t) \\ \dfrac{dE(t)}{dt} &= \lambda S(t)-\sigma E(t) \\ \dfrac{dA(t)}{dt} &= \sigma E(t)-(\alpha+\gamma+\theta_A)A(t) \\ \dfrac{dI_i(t)}{dt} &= \alpha A(t)-(\delta_i+\tau+\theta_i)I_i(t) \\ \dfrac{dI_f(t)}{dt} &= \gamma A(t)-(\delta_f+\theta_f)I_f(t) \\ \dfrac{dT(t)}{dt} &= \tau I_i(t)-(\delta_T+\theta_T)T(t) \\ \dfrac{dR(t)}{dt} &= \theta_A A(t)+\theta_i I_i(t)+\theta_f I_f(t)+\theta_T T(t) \end{cases}$$

with $\lambda = \dfrac{\beta_E E(t)+\beta_A A(t)+\beta_i I_i(t)+\beta_f I_f(t)}{N}$.

3.1. The Basic Reproduction Number. The basic reproduction number $\mathscr{R}_0$ is the average number of individuals that one infected with the COVID 19 is likely to transmit in a susceptible population and it is calculated using the next generation matrix [10]. It is also called the threshold quantity which characterizes the spread of the disease through the population. The new passengers are assumed to be infected, Therefore, the disease compartments are ,E,A,I_i and I_f. The disease free equilibrium point of 2 is $e_0 = (S_0, 0, 0, 0, 0, 0)$, $S_0 = N_0$ which represents the susceptible population in the absence of disease.

$$\mathscr{F} = \begin{pmatrix} \lambda S^*(t) \\ 0 \\ 0 \\ 0 \end{pmatrix}, \mathscr{V} = \mathscr{V}^- - \mathscr{V}^+ = \begin{pmatrix} \sigma E^*(t) \\ c_2 A^*(t) - \sigma E^*(t) \\ c_3 I_i^*(t) - \alpha A^*(t) \\ c_4 I_f^*(t) - \gamma A^*(t) \end{pmatrix}$$

$$c_1 = (\omega + \varphi + \varepsilon), \quad c_2 = (\alpha + \gamma + \theta_A), \quad c_3 = (\delta_i + \tau + \theta_i), \quad c_4 = (\delta_f + \theta_f), \quad c_5 = (\delta_T + \theta_T)$$

The matrices F and V are:

$$F = \frac{\partial \mathscr{F}(e_0)}{\partial X_j} = \begin{pmatrix} \beta_E & \beta_A & \beta_i & \beta_f \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, V = \frac{\partial \mathscr{V}(e_0)}{\partial X_j} = \begin{pmatrix} \sigma & 0 & 0 & 0 \\ -\sigma & c_2 & 0 & 0 \\ 0 & -\alpha & c_3 & 0 \\ 0 & -\gamma & 0 & c_4 \end{pmatrix}$$

$$V^{-1} = \begin{pmatrix} \frac{1}{\sigma} & 0 & 0 & 0 \\ \frac{1}{c_2} & \frac{1}{c_2} & 0 & 0 \\ \frac{\alpha}{c_2 c_3} & \frac{\alpha}{c_2 c_3} & \frac{1}{c_3} & 0 \\ \frac{\gamma}{c_2 c_4} & \frac{\gamma}{c_2 c_4} & 0 & \frac{1}{c_4} \end{pmatrix}$$

Therefore

$$\mathscr{R}_0 = \rho(FV^{-1}) = \frac{\beta_E}{\sigma} + \frac{\beta_A}{c_2} + \frac{\alpha\beta_i}{c_2 c_3} + \frac{\beta_f \gamma}{c_2 c_4} \tag{3}$$

$$c_1 = (\omega + \varphi + \varepsilon), c_2 = (\alpha + \gamma + \theta_A), c_3 = (\delta_i + \tau + \theta_i), c_4 = (\delta_f + \theta_f)$$

From the model, the parameter γ represent the rate at which asymptomatic individuals progresses to infectious free compartment. To explore the impact of γ on the basic reproduction number $\mathscr{R}_0$, we consider the following function

$$\mathscr{R}_0(\gamma) = \frac{a\gamma + b}{\sigma\gamma + c} \tag{4}$$

$$a = \sigma(\frac{\beta_f}{\delta_f + \theta_f}) + \beta_E \ , \ b = \sigma(\beta_A + \frac{\alpha\beta_i}{(\delta_i + \tau + \theta_i)}) + \beta_E(\alpha + \theta_A) \ , \ c = \sigma(\alpha + \theta_A)$$

Note that all parameters are non-negative, the γ-intercepts corresponding to $\mathscr{R}_0(\gamma) = 1$ is given by $\gamma^* = \dfrac{c-b}{a-\sigma}$ when $\gamma \neq \frac{-c}{\sigma}$, also $\mathscr{R}_0(0) = \frac{b}{c}$. The function $\mathscr{R}_0(\gamma)$ decrease when $ac - \sigma b < 0$ and the horizontal asymptotes are $\mathscr{R}_0(\gamma) = \frac{a}{\sigma}$. Therefore, in the first quadrant the following was found:

(1) If $\frac{a}{\sigma} < \frac{b}{c} < 1$ then $\mathscr{R}_0(\gamma) < 1$, then the system is locally asymptotically stable for any γ.
(2) If $\frac{a}{\sigma} < 1 < \frac{b}{c}$ then the system is unstable and it can be stable for $\gamma > \gamma^*$.
(3) If $1 < \frac{a}{\sigma} < \frac{b}{c}$ then the system is unstable.

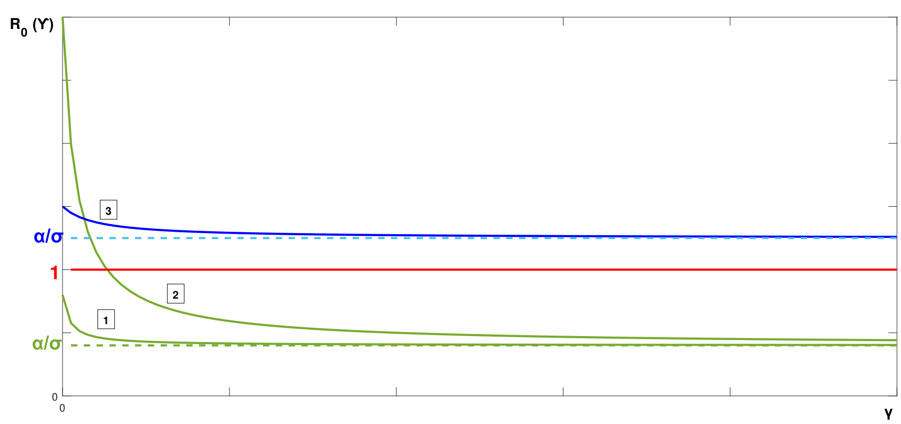

FIGURE 2. The function $\mathscr{R}_0(\gamma)$ at decreasing state

4. EASING COVID-19 RESTRICTIONS

In this section, we will consider the original model where we assume that the country open up their borders and we analyze the effect of travelers coming from countries with high risk infection the COVID-19. The equation P in the model 1 reflect the travelers that could carry the infection either as asymptomatic or mild infection. The goal is to understand how the relaxing the travel restriction would affect the dynamic of the model. In fact , we will perform the sensitivity analysis of the model based on this new assumption. It is clear that the form $\mathscr{R}_0$ does not change with this new variable, it is also important to see the effect of the variable to other parameters and variables

4.1. Sensitivity Analysis. In order to apply the model, the parameters has been estimated using pandemic official data from The Ministry of Health in Morocco. Since each parameter has its own significance, so that all the parameters were defined to be non negative and bounded.

The sensitivity index of each parameter that correlates with the basic reproductive numbers $\mathscr{R}_0(\gamma)$ has been calculated and presented in table (2), and the graphical bar-graph results has been obtained in figures (3) and (4). The sensitivity analysis for this epidemic threshold illustrate the important of each parameter to the covid-19 transmission in order to discover which parameter has a high impact on $\mathscr{R}_0(\gamma)$.

The normalized forward sensitivity index of $\mathscr{R}_0$ which is differentiable with respect to a given parameter ρ is defined by

$$\zeta_\rho^{\mathscr{R}_0} = \frac{\partial \mathscr{R}_0}{\partial \rho}\frac{\rho}{\mathscr{R}_0} \tag{5}$$

In case 5 the sensitivity index for $\mathscr{R}_0$ towards parameters β_A and γ is calculated as follow:

$$\begin{aligned}
\zeta_{\beta_A}^{\mathscr{R}_0} &= \frac{\partial \mathscr{R}_0}{\partial \beta_A}\frac{\beta_A}{\mathscr{R}_0} = \frac{\sigma}{(\sigma(\alpha+\theta_A)+\gamma\sigma)}\frac{\beta_A}{\mathscr{R}_0} = 0.4554 \\
\zeta_{\gamma}^{\mathscr{R}_0} &= \frac{\partial \mathscr{R}_0}{\partial \gamma}\frac{\gamma}{\mathscr{R}_0} \\
&= \beta_E + \frac{\beta_f\sigma}{(\delta_f+\theta_f)} - \frac{(\sigma(\sigma(\beta_A+\frac{(\alpha\beta_i)}{(\delta_i+\tau+\theta_i)})+\beta_E(\alpha+\theta_A)))}{(\sigma(\alpha+\theta_A)+\gamma\sigma)^2}\frac{\gamma}{\mathscr{R}_0} = -0.1157
\end{aligned}$$

TABLE 2. The sensitivity index of each parameter that correlates with $\mathscr{R}_0(\gamma)$

Cases	Case 1		Case 2		Case 3		Case 4		Case 5		Case 6	
Parameters	Values	S. index	Values	S.index	Values	S. index	Values	S. index	Values	S. index	Values	S. index
β_E	0.015	0.1794	0.015	0.0846	0.015	0.1016	0.011	0.1803	0.035	0.5120	0.098	0.3427
β_A	0.035	0.8047	0.035	0.9098	0.035	0.8924	0.011	0.8012	0.035	0.4554	0.035	0.6508
β_i	0.015	0.0146	0.015	0.0047	0.015	0.0046	0.011	0.0162	0.015	0.0296	0.035	0.0042
β_f	0.025	0.0013	0.025	9.6055e-04	0.045	0.0014	0.02	0.0023	0.045	0.0030	0.015	0.0023
α	0.0012	-0.0194	0.0012	-0.0236	0.0012	-0.0227	0.002	-0.0214	0.015	-0.0203	0.045	-0.0083
γ	0.015	-0.4052	0.015	-0.4127	0.015	-0.4121	0.015	-0.4015	0.015	-0.1157	0.0015	-0.2220
τ	0.002	-2.96E-04	0.002	-9.5475e-05	0.002	-9.3655e-05	0.002	-3.2698e-04	0.002	-5.9742e-04	0.015	-8.5378e-05
θ_A	0.02	-0.3783	0.02	-0.4723	0.02	-0.4560	0.02	-0.3755	0.095	-0.3156	0.002	-0.4176
θ_i	0.092	-0.0136	0.092	-0.0044	0.092	-0.0043	0.092	-0.0150	0.092	-0.0275	0.05	-0.0039
θ_f	0.028	-9.79E-04	0.0280	-7.4710e-04	0.0280	-0.0011	0.026	-0.0018	0.0280	-0.0023	0.092	-0.0018
δ_i	0.005	-7.39E-04	0.005	-2.3869e-04	0.005	-2.3414e-04	0.005	-8.1745e-04	0.005	-0.0015	0.0280	-2.1344e-04
δ_f	0.008	-3.01E-04	0.008	-2.1346e-04	0.008	-3.0767e-04	0.008	-5.4830e-04	0.008	-6.6414e-04	0.005	-5.0494e-04
σ	0.098	-0.1777	0.098	0.0834	0.08	-0.1000	0.098	-0.1775	0.098	-0.5082	0.008	-0.3397

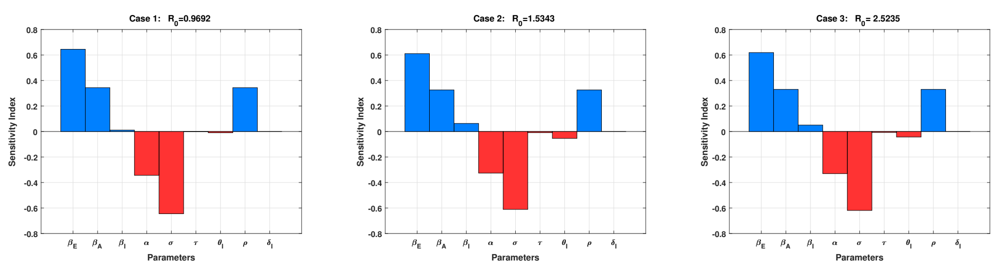

FIGURE 3. Sensitivity Cases of $R_0(\gamma)$ with respect to the parameters when $\frac{a}{\sigma} < \frac{b}{c}$

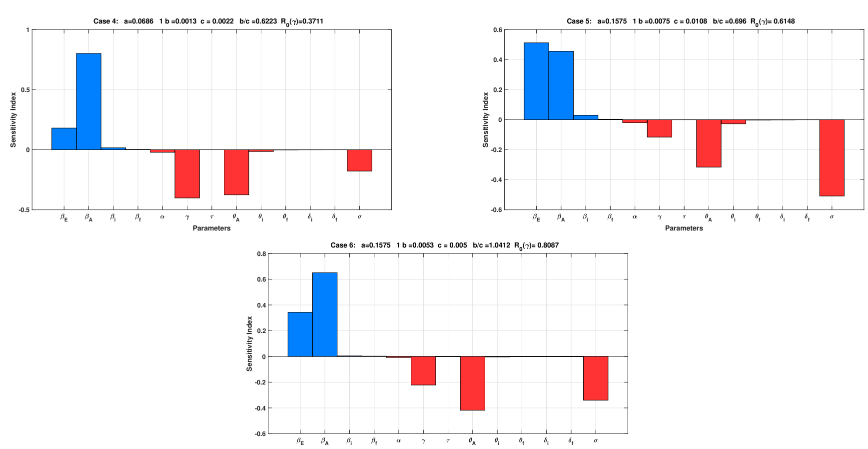

FIGURE 4. Sensitivity Cases of $R_0(\gamma)$ with respect to the parameters when $\frac{a}{\sigma} > \frac{b}{c}$

In general, all sensitivity cases show that some parameters are significant to the changes in the basic reproductive number $R_0(\gamma)$. Those parameters are β_E and β_A, the transmission coefficient from exposed and asymptomatic individuals respectively. α the rate at which asymptomatic individuals progresses to infectious isolated compartment, γ the rate at which asymptomatic individuals progresses to infectious free compartment and σ, the rate at which exposed individuals progresses to asymptomatic compartment. The sensitivity index illustrates that contact with asymptomatic individuals has the most chance of the COVID-19 transmission. Therefore, if $\zeta_{\beta_A}^{\mathscr{R}_0} = 0.4554$, this means if the parameter β_A increased (or decreased) by 45.54% then $R_0(\gamma)$ will increase (or decrease) by 45.54%. The same for $\zeta_{\gamma}^{\mathscr{R}_0} = -0.1157$ the decrease (or increase) of the parameter γ by 11.57% will decrease (or increase) $R_0(\gamma)$ by the same percentage.

5. The Final Size Relation

According to [11] the set of susceptible x and infected y compartments can be represented as:

$$\left\{ \begin{array}{rcl} \dot{x} & = & \prod D(y)\beta b x - V x \\ \\ \dot{y} & = & -\prod D(y)\beta b x \end{array} \right. \tag{6}$$

In terms of our notation, there is only one susceptible class, that is m=1, n=5 (the number of infected compartments), $D = m \times m = 1$ is the diagonal matrix with y as the main diagonal, y=S,

$$x = \begin{bmatrix} P \\ E \\ A \\ I_i \\ I_f \end{bmatrix} \quad , \quad \prod = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\beta b = \frac{1}{N} \begin{bmatrix} \beta_p & \beta_E & \beta_A & \beta_i & \beta_f \end{bmatrix}$$

The final size relation for S :

$$\dot{S} = \frac{-1}{N} \quad [\beta_p \quad \beta_E \quad \beta_A \quad \beta_i \quad \beta_f] \quad Sx$$

Both sides divided by S and taking the integration from $0 \to$ t, then

$$\int_0^t \frac{\dot{S}}{S} dS = \frac{-1}{N} \begin{bmatrix} \beta_p & \beta_E & \beta_A & \beta_i & \beta_f \end{bmatrix} \int_0^t x dS$$

$$\ln \frac{S(0)}{S(\infty)} = \frac{1}{N} \begin{bmatrix} \beta_p & \beta_E & \beta_A & \beta_i & \beta_f \end{bmatrix} \int_0^\infty x dS$$

But $\int_0^\infty x dS = V^{-1} \prod (y(0) - y(\infty)) + V^{-1} x(0)$

$$\ln \frac{S(0)}{S(\infty)} = \frac{1}{Nc_1} \left[\beta_p (P_0 + S(0)) + \frac{\beta_E}{\sigma} \Phi_1 + \frac{\beta_A}{c_2} (\Phi_2 + \Phi_3) + \frac{\alpha\beta_i}{c_2 c_3} (\Phi_2 + I_{i0} c_1 c_2 \alpha + \Phi_4) + \frac{\beta_f \gamma}{c_2 c_4} (\Phi_2 + I_{f0} c_1 c_2 \gamma + \Phi_3) - \Phi_8 S(\infty) \right]$$

- **Case 1:** $E(0) = I_i(0) = I_f(0) = 0$ while $P(0) = P_0 > 0$ and $A(0) = A_0 > 0$

$$\ln \frac{S(0)}{S(\infty)} = \frac{1}{Nc_1} \left[\beta_p (P_0 + S(0)) + \frac{\beta_E}{\sigma} \Phi_5 + \frac{\beta_A}{c_2} (c_1 A_0 + \Phi_3) + \frac{\alpha\beta_i}{c_2 c_3} (c_1 A_0 + \Phi_4) + \frac{\beta_f \gamma}{c_2 c_4} (c_1 A_0 + \Phi_3) - \Phi_8 S(\infty) \right]$$

- **Case 2:** $E(0) = I_i(0) = 0$ while $P(0) = P_0 > 0$, $A(0) = A_0 > 0$ and $I_f(0) = I_{f0} > 0$

$$\ln \frac{S(0)}{S(\infty)} = \frac{1}{Nc_1} \left[\beta_p (P_0 + S(0)) + \frac{\beta_E}{\sigma} \Phi_5 + \frac{\beta_A}{c_2} (c_1 A_0 + \Phi_3) + \frac{\alpha\beta_i}{c_2 c_3} (c_1 A_0 + \Phi_4) + \frac{\beta_f \gamma}{c_2 c_4} (c_1 A_0 + I_{f0} c_1 c_2 \gamma + \Phi_3) - \Phi_8 S(\infty) \right]$$

- **Case 3:** $E(0) = P(0) = 0$ while $A(0) = A_0 > 0$, $I_i(0) = I_{i0} > 0$, $I_f(0) = I_{f0} > 0$

$$\ln \frac{S(0)}{S(\infty)} = \frac{1}{Nc_1} \left[\beta_p S(0) + \frac{\beta_E}{\sigma} S(0)\varepsilon + \frac{\beta_A}{c_2} (c_1 A_0 + \Phi_6) + \frac{\alpha\beta_i}{c_2 c_3} c_1 A_0 + I_{i0} (c_1 c_2 \alpha + \Phi_7) + \frac{\beta_f \gamma}{c_2 c_4} (c_1 A_0 + I_{f0} c_1 c_2 \gamma + \Phi_6) - \Phi_8 S(\infty) \right]$$

$\Phi_1 = (E_0 c_1 + P_0 \varepsilon + S(0)\varepsilon)$ $\quad \Phi_2 = c_1 (A_0 + E_0)$

$\Phi_3 = (P_0 + S(0))(\omega + \varepsilon)$ $\quad \Phi_4 = (P_0 + S(0))(\varepsilon + c_2 \alpha \varphi + \omega)$

$\Phi_5 = \varepsilon (P_0 + S(0))$ $\quad \Phi_6 = S(0)(\omega + \varepsilon)$

$\Phi_7 = S(0)(\varepsilon + c_2 \alpha \varphi + \omega)$ $\quad \Phi_8 = \beta_p + \frac{\beta_E}{\sigma} \varepsilon + (\frac{\beta_A}{c_2} + \frac{\alpha\beta_i}{c_2 c_3} + \frac{\beta_f \gamma}{c_2 c_4})(\omega + \varepsilon) + \frac{\alpha\beta_i}{c_2 c_3} c_2 \alpha \varphi$

6. Fitting Model to Data and Illustrations

Figure 8 describes the model 7 fitting with COVID-19 data for the first 20 weeks which started from 2 March 2020, when the first imported case was confirmed in Morocco. In order to adjust the number of infected people with the real data some parameters are adjusted to get a good fitting.

$$
(7) \qquad \begin{cases}
\frac{dS(t)}{dt} &= -\lambda S(t) \\
\frac{dE(t)}{dt} &= \lambda S(t) - \sigma E(t) \\
\frac{dA(t)}{dt} &= \sigma E(t) - (\alpha + \gamma + \theta_A) A(t) \\
\frac{dI_i(t)}{dt} &= \alpha A(t) - (\delta_i + \tau + \theta_i) I_i(t) \\
\frac{dI_f(t)}{dt} &= \gamma A(t) - (\delta_f + \theta_f) I_f(t) \\
\frac{dT(t)}{dt} &= \tau I_i(t) - (\delta_T + \theta_T) T(t) \\
\frac{dR(t)}{dt} &= \theta_A A(t) + \theta_i I_i(t) + \theta_f I_f(t) + \theta_T T(t)
\end{cases}
$$

The total population of Morocco is 36,940,535. So that, we consider $N = 36 \times 10^6$ and the initial conditions as $(S_0, E_0, A_0, I_{i0}, I_{f0}, T_0, R_o) = (N - 208, 200, 5, 1, 2, 0, 0)$. We also set the parameter values as follows: $\delta_i = 0.02$, $\delta_f = 0.02$, $\delta_T = 0.016$, $\theta_A = 0.025$, $\theta_i = 0.025$, $\theta_f = 0.025$, $\tau = 0.12$ and $\gamma = 0.012$. The other parameters are adjusted in order to fit the data of the total infected individuals in different stages as illustrated in table 3 .

TABLE 3. The parameter values in all stages

Stage	Period	β_E	β_A	β_i	β_f	α	σ	R_0
1	2 March-10 April	0.115	0.115	0.115	0.115	0.075	0.079	2.797
2	11 April-10 May	0.00322	0.00322	0.00322	0.00322	0.075	0.079	0.0783
3	11-20 May	0.00322	0.00322	0.00322	0.00322	0.075	0.0395	0.1147
4	21 -30 May	0.00322	0.00322	0.00322	0.00322	0.005	0.0395	0.1373
5	31 May-9 June	0.092	0.092	0.0322	0.092	0.0012	0.079	3.2163
6	10-19 June	0.115	0.138	0.05175	0.115	0.005	0.079	4.3666
7	20 -29 June	0.15525	0.14145	0.1587	0.15525	0.087	0.1185	2.5167
8	30 June -9 July	0.023	0.023	0.0046	0.023	0.008	0.00395	4.7864
9	10-19 July	0.023	0.023	0.00322	0.023	0.005	0.00395	4.7094

Data on testing is important to know the number of confirmed cases, testing is the window onto this pandemic and how it is spreading.

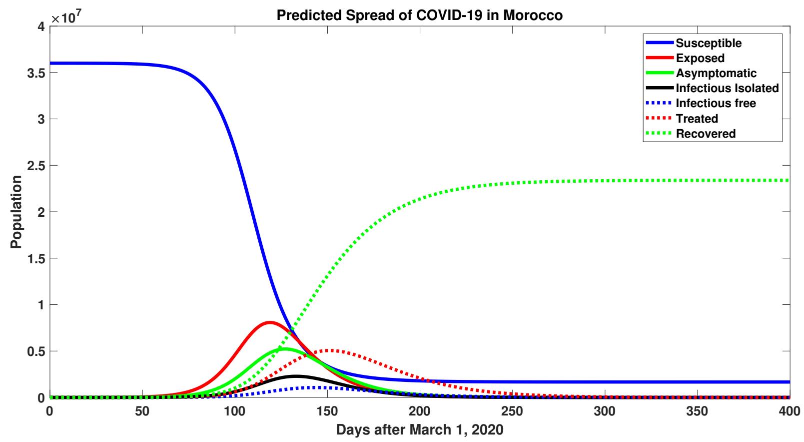

FIGURE 5. $E_0 = 600, A_0 = 70, I_{i0} = 20, I_{f0} = 15, T_0 = 2, R_0 = 0, \beta_E = \beta_A = \beta_i = \beta_f = 0.115, \sigma = 0.079, \alpha = 0.075, \gamma = 0.012, \tau = 0.12, \theta_A = \theta_i = \theta_f = \theta_T = 0.025, \delta_i = \delta_f = 0.02, \delta_T = 0.016$

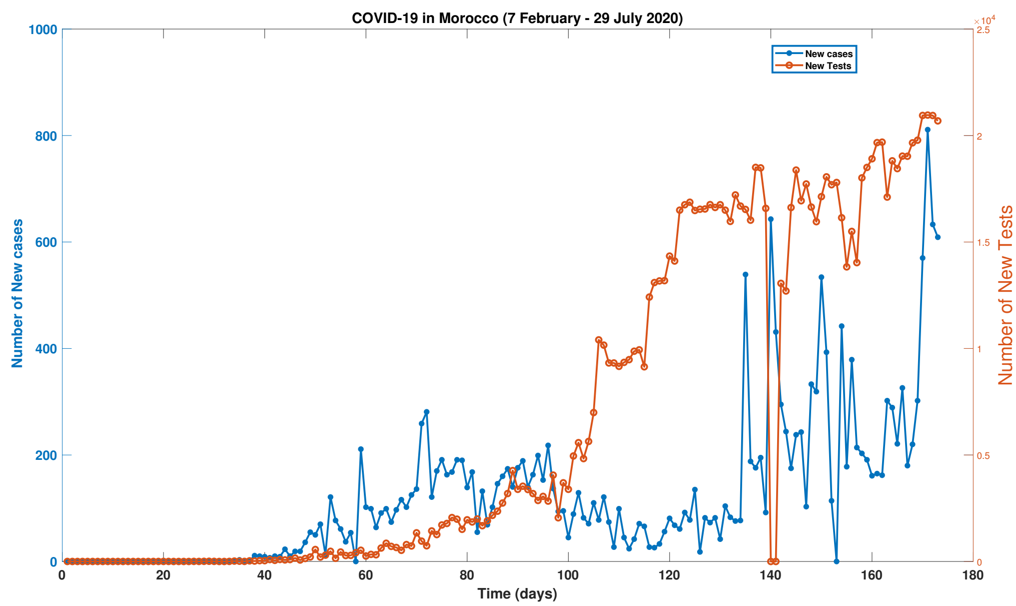

FIGURE 6. The number of Newly Cases with Number of New Tests.

Figure 6 illustrate the test of COVID-19 compared with the new cases, the curve in orange show the number of test is increasing over time while the number of new cases witnessed a fluctuation between high and low. The tests returning a positive result is known as the positive rate, the rate in the first month of the beginning of the pandemic was more than 20%. After that the rate decreases between 2% and 3% until the 120 days when the rate started to rose again and become 10%. WHO in May, published that a positive rate of less than 5% is one indicator that the pandemic is under control [1]. Figure 7 illustrates the cumulative number of infected individuals compared with the number of new tests. There are efforts to increase the number of tests, and in contrast the number of confirmed cases persists to rise. In fact, the actual data for infected people is likely to be much higher than the number of confirmed cases and this can be explored by intensifying testing.

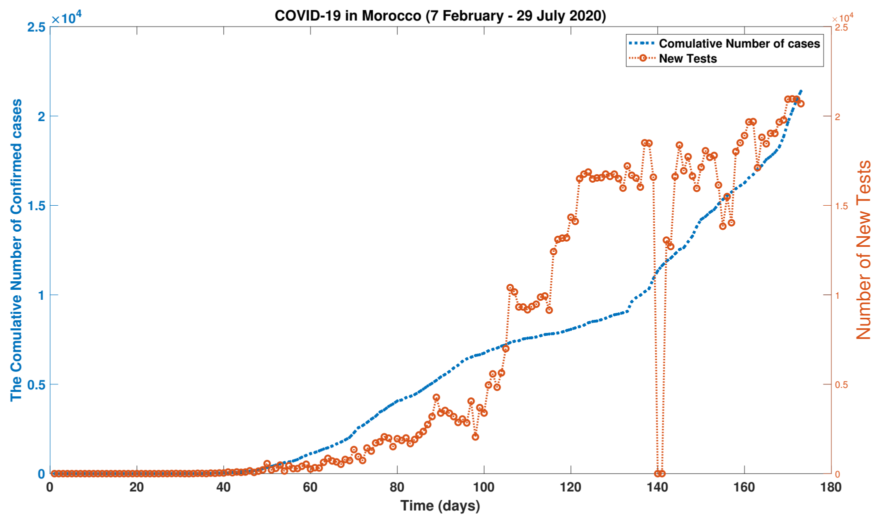

FIGURE 7. The cumulative Number of Confirmed Cases with Number of New Tests.

The infected cases are calculated by subtracting the death and recovered cases from the confirmed cases and the best model fit along with the best fit parameter values are shown in figure 8, The first day in the graph represent 2 March 2020. It can be observed from the graph that the COVID-19 mathematical model 7 produces a good adjustment to the real data and predicts that the pandemic will be controlled in Morocco due to a $R_0 = 0.1373$ during the first short wave until 6 June 2020. The second wave up to 19 July, which recorded the highest increases in the number of new cases which led R_0 to exceeding 4.5.

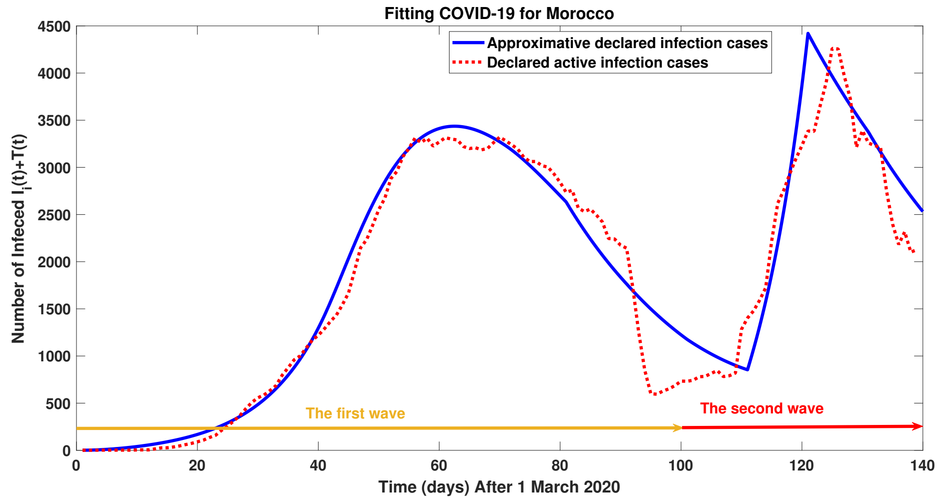

FIGURE 8. Fits of the Model of COVID-19 Pandemic in Morocco.

The spread of the disease depends on some influencing parameters that can be controlled. Therefore, the described stages in table 3 depend on parameters $\beta_E, \beta_A, \beta_i, \beta_f, \alpha$ and σ, but the impact of γ is knew previously in sensitivity section. It is very important to conclude the following:

- The first stage correspond to $\beta_E, \beta_A, \beta_i$ and β_f with a high values, and the reproduction number R_0 rose to around 2.8. That mean the contact rate between people at the beginning of the pandemic in Morocco was high. It can be seen from the graph the number of infected people increased rapidly.
- The next three stages 2, 3 and 4 the reproduction number reduced to less than one. The values of $\beta_E, \beta_A, \beta_i$ and β_f reduced by 97.2% and σ reduced by at most 50%. The major peak of the cumulative infected was 3351 on 30 April.
- In stage five, the cure decreased until the number become around 595 on 26 of May but the contact rate start to rise again, and that leads the value of R_0 to be greater than one.
- The last four stages which represent the second wave, the values of the parameters that represent the contact rate were between 0.0032 and 0.16. $R_0 = 2.5$ was recorded as the smallest value and exceeded 4.7 sometimes in this unstable wave. Therefore, the total number of infected people increased dramatically to the major peak of the pandemic to reach 4261 on 3 July.

- In conclusion, Morocco applied the medical emergency in a controlled manner in the first wave, which resulted in reducing the level of contact between people. In the second wave, the medical emergency was loosened with restrictions but people did not respect these restrictions, social distancing, wearing masks and not going outside unless necessary.The government also has stepped up testing and tracking of the hotspots.All these factors and others have led to more COVID-19 cases.

7. Conclusion and Recommendations

A mathematical model with seven compartments is proposed to study the spread of the COVID-19 in Morocco. The main feature of the model is to include the effect of the passengers that could enter the country if the authority to relax the travel restriction. In order to study the dynamic of our model, we calculated the basic reproduction number R_0. using the final size relation[11], we obtain a relation between the R_0 and the final size. The movement restrictions during the medical emergency is just an emergency measure; respecting the instructions and measures taken by the government and public awareness of responsibility are among the control measure that to reduce the spread of the disease. In addition, the massive testing help to understand the spread of the pandemic. Therefore, the country needs to do more testing to be able to quantify the efficacy of the control measures and track the spread of the virus properly.

Moreover, the authorities must increase awareness of the community, since the human behaviour are the main reason of continuous spread of the disease in any community. The gatherings without observing the precautionary and preventive measures all contributed to rising infection rates with the Coronavirus.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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