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A SPATIOTEMPORAL MODEL WITH OPTIMAL CONTROL FOR THE NOVEL CORONAVIRUS EPIDEMIC IN WUHAN, CHINA

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Abstract. We propose a spatiotemporal model with optimal control to investigate the current of the coronavirus epidemic in Wuhan. Our model formulated as a system of parabolic partial differential equations. Immunity is forced through vaccine distribution considered a control variable. Our objective is to prove the existence of solutions to the state system and also the existence of optimal control.

Keywords: COVID-19; spread of epidemic; compartmental model; optimal control.

2010 AMS Subject Classification: 93A30.

1. Introduction

A severe outbreak of respiratory illness started in Wuhan, a city of 11 million people in central China, in December 2019. The virus is believed to have a zoonotic origin. This is the third zoonotic human coronavirus emerging in the current century, after the severe acute respiratory

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syndrome coronavirus (SARS-CoV) in 2002 that spread to 37 countries and the Middle East respiratory syndrome coronavirus (MERS-CoV) in 2012 that spread to 27 countries.

To give a full account on mathematical study of diseases would require a book in itself. However, an interesting overview on the use of mathematical models in epidemi- ology, can be found in Baily et al. [1], Anderson et al. [2], Hethcote [3], Brauer and Castillo-Chavez [4], Keeling and Rohani [5] and Huppert and Katriel [6].

The COVID-19 outbreak is currently on-going and the number of infections has been fast growing since the onset of the epidemic. As a bold effort to contain the epidemic, the Chinese government ordered to lock down Wuhan on January 23 and at least 15 other cities in the following days, effectively restricting the movement of more than 50 million people in central China, which is considered as the largest quarantine in human history.

A number of modeling studies have already been performed for the COVID-19 epidemic. Chayu Yang and Jin Wang [7] presented a new mathematical model for COVID-19 that incorporates multiple transmission pathways, including both the environment-to-human and human-to-human routes. In particular, they introduce an environmental compartment that represents the pathogen concentration in the environmental reservoir. A susceptible individual may contract the disease through the interaction with the contaminated environment, with an infectious but asymptomatic individual, or with an infectious and symptomatic individual. Meanwhile, the transmission rates in this model depend on the epidemiological status and environmental conditions which change with time. In particular, when the infection level is high, people would be motivated to take necessary action to reduce the contact with the infected individuals and contaminated environment so as to protect themselves and their families, leading to a reduction of the average transmission rates.

In the present paper, we propose another extention of the model presented by Chayu Yang and Jin Wang, in which we incorporate the spatial behavior of populations and a term of control, we assume that the direct, human-to-human transmission rates are constants. An interesting overview on the use of mathematical models in epidemiology, are incorporate the spatial behavior of populations and a term of control(see [8-11]).

Recently, in modern population dynamics, the use of reaction-diffusion is the simplest mechanism used to model the spread of the population. Reaction-diffusion model is a typical spatially extended model. It involves time, space and consists of several interaction species which can diffuse within the spatial domain [12]. Examples of the use of reaction-diffusion models can be seen in many ecologi- cal and epidemiological contexts. For example, in ecology, Guin and Mandal [12,13] proposed an interesting analysis on a diffusive predator-prey model. In mathematical epidemiology, a few extensions of the basic SIR model that involve reaction-diffusion mechanism have been formulated and mathematically analyzed. Web [14] analyzed a one dimensional SIR model which included constant diffusive movement of all individuals as well as no-flux boundary conditions. Milner and Zhao [15] proposed and analyzed an SIR model based on hyperbolic partial differential equations, in which susceptible individuals move away from foci of infection, and all individuals move away from overcrowded regions.

The remaining parts of this paper are organized as follows: Section 2 is devoted to the mathematical model of the novel coronavirus and the associated optimal control problem. In Section 3, we prove the existence of a global strong solution for our system. In Section 4, we prove the existence of an optimal solution. Finally, we conclude the paper in Section 5.

2. Mathematical Model

A mathematical model of Coronavirus transmission is based on the model for the novel coronavirus epidemic in Wuhan, it categorizes each individual into one of four compartments: the susceptible (denoted by S), the exposed (denoted by E), the infected (denoted by I), and the recovered (denoted by R). Individuals in the exposed class are in the incubation period; they do not show symptoms but are still capable of infecting others. Thus, another interpretation of the E and I compartment in our model is that they contain asymptomatic infected and symptomatic infected individuals, respectively. The model takes the following form:

$$
\begin{aligned}
\frac{\partial S}{\partial t} &= \Lambda - \beta_E SE - \beta_I SI - \beta_V SV - \mu S \\
\frac{\partial E}{\partial t} &= \beta_E SE + \beta_I SI + \beta_V SV - (\alpha + \mu) E \\
\frac{\partial I}{\partial t} &= \alpha E - (w + \gamma + \mu) I \\
\frac{\partial R}{\partial t} &= \gamma I - \mu R \\
\frac{\partial V}{\partial t} &= \xi_1 E + \xi_2 I - \sigma V
\end{aligned} \tag{1}
$$

Where V is the concentration of the coronavirus in the environmental reservoir, the parameter Λ represents the population influx, μ is the natural death rate of human hosts, α^{-1} is the incubation period between the infection and the onset of symptoms, w is the disease-induced death rate, γ is the rate of recovery from infection, ξ_1 and ξ_2 are the respective rates of the exposed and infected individuals contributing the coronavirus to the environmental reservoir, and σ is the removal rate of the virus from the environment. β_E and β_I represent the direct, human-to-human transmission rates between the exposed and susceptible individuals, and between the infected and susceptible individuals respectively, and β_V represents the indirect, environment-to- human transmission rate. Given that higher values of E, I and V would motivate stronger control measures that could reduce the transmission rates. Specifically, we make the following assumption: β_E, β_I and β_V are all positive

We propose another extension of this model, in which we incorporate the spatial behavior of the populations and a term of control representing a vaccination program. The main motivation is to study the effect of a vaccination campaign on the spread of infectious diseases in the context of a more realistic model that takes into account the spatial diffusion. We chose the vaccination as strategy of control because it still remains among the powerful tool that prevent and control the spread of infection. We Assume that the population habitat is a spatially heterogeneous environment, the populations tend to move to regions and their densities will depend on space. The subpopulation in all three compartments are thus tracked not only on time t but also on the spatial location x, leading to the notations $S(t,x), E(t,x), I(t,x)$,and $R(t,x)$ which represent the densities of the four populations at the time t and the spatial position x. In addition, we assume that the spatial diffusion is through space with $\lambda_1, \lambda_2, \lambda_3and\lambda_4$ are the self-diffusion coefficients for each class. With the assumptions explained above in mind, we get the following system of reaction-diffusion equations as a model for the spatial spread of the novel coronavirus:

$$
\begin{aligned}
\frac{\partial S}{\partial t} &= \lambda_1 \Delta S + \Lambda - \beta_E SE - \beta_I SI - \beta_V SV - \mu S \\
\frac{\partial E}{\partial t} &= \lambda_2 \Delta E + \beta_E SE + \beta_I SI + \beta_V SV - (\alpha + \mu) E \\
\frac{\partial I}{\partial t} &= \lambda_3 \Delta I + \alpha E - (w + \gamma + \mu) \\
\frac{\partial R}{\partial t} &= \lambda_4 \Delta R + \gamma I - \mu R \\
\frac{\partial V}{\partial t} &= \xi_1 E + \xi_2 I - \sigma V
\end{aligned} \tag{2}
$$

$$I(t,x) \in Q = [0,T] \times \Omega.$$

With the homogeneous Neumann boundary conditions

$$\frac{\partial S}{\partial \eta} = \frac{\partial E}{\partial \eta} = \frac{\partial I}{\partial \eta} = \frac{\partial R}{\partial \eta} = \frac{\partial V}{\partial \eta} = 0, (t,x) \in \Sigma = [0,T] \times \partial\Omega$$

$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ represents the usual Laplacian operator, Ω is fixed and bounded domain in $\mathbb{R}^2$with smooth boundary $\partial\Omega$ is the outward unit normal vector onthe boundary, the time t belongs to a finite interval $[0,T]$, while x varies in Ω. Here the homogeneous Neumann boundary condition implies that the abovesystem is self-contained and there is no emigration across the boundary. The initial distribution of the foor populations is supposed to be

$S(0,x) = S_0 > 0, E(0,x) = E_0 > 0, I(0,x) = I_0 > 0, R(0;x) = R_0 > 0$ and $V(0;x) = V_0 > 0$

Strategy of control: We chose a vaccination program, so into the model (2) we include a control u that represents the density of susceptible individuals being vaccinated per timeunit and space. We assume that all susceptible vaccinates are transferred directly to the removed class.

The dynamics of the controlled system is given by:

$$
\begin{aligned}
\frac{\partial S}{\partial t} &= \lambda_1 \Delta S + \Lambda - \beta_E SE - \beta_I SI - \beta_V SV - \mu S - uS \\
\frac{\partial E}{\partial t} &= \lambda_2 \Delta E + \beta_E SE + \beta_I SI + \beta_V SV - (\alpha + \mu) E \\
\frac{\partial I}{\partial t} &= \lambda_3 \Delta I + \alpha E - (w + \gamma + \mu) I \\
\frac{\partial R}{\partial t} &= \lambda_4 \Delta R + \gamma I - \mu R + uS \\
\frac{\partial V}{\partial t} &= \xi_1 E + \xi_2 I - \sigma V
\end{aligned} \tag{3}
$$

$(t,x) \in Q = [0,T] \times \Omega$ with the homogeneous Neumann boundary conditions

$$\frac{\partial S}{\partial \eta} = \frac{\partial E}{\partial \eta} = \frac{\partial I}{\partial \eta} = \frac{\partial R}{\partial \eta} = \frac{\partial V}{\partial \eta} = 0, (t,x) \in \Sigma = [0,T] \times \partial\Omega \tag{4}$$

and for $x \in \Omega$

$$S(0,x) = S_0, E(0,x) = E_0, I(0,x) = I_0, R(0,x) = R_0, V(0,x) = V_0$$

Our goal is to minimize the density of infected individuals and the cost of vaccination program. Mathematically, it can be interpreted by optimization of the objective functional

$$J(S,E,I,R,V,u) = \|I\|^2_{L^2(Q)} + \|I(T,.)\|^2_{L^2(\Omega)} + \theta \|u\|^2_{L^2(Q)} \tag{5}$$

Where u belongs to the set U_{ad} of admissible controls

$$U_{ad} = \left\{ u \in L^\infty(Q) : \|u\|_{L^\infty(Q)} < 1 \, and \, u > 0 \right\} \tag{6}$$

3. Existence of Global Solution

For $y = (y_1, y_2, y_3, y_4, y_5)$ and $y^0 = (y_1^0, y_2^0, y_3^0, y_4^0, y_5^0)$

We can put $y = (S,E,I,R,V)$ and $y^0 = (S_0, E_0, I_0, R_0, V_0)$

$H(\Omega) = (L^2(\Omega))^5$ and A the linear operator defined as follow

$A : D(A) \subset H(\Omega) \to H(\Omega)$

$$
\begin{aligned}
Ay &= (\lambda_1 \Delta y_1, \lambda_2 \Delta y_2, \lambda_3 \Delta y_3, \lambda_4 \Delta y_4, 0) \in D(A) \\
\forall y &= (y_1, y_2, y_3, y_4, y_5) \in D(A)
\end{aligned} \tag{7}
$$

(8)
$$D(A)=\left\{y=(y_1,y_2,y_3,y_4,y_5)\in(H^2(\Omega)^5,\frac{\partial y_1}{\partial\eta}=\frac{\partial y_2}{\partial\eta}=\frac{\partial y_3}{\partial\eta}=\frac{\partial y_4}{\partial\eta}=\frac{\partial y_5}{\partial\eta}=0,a.ex\in\partial\Omega\quad\right\}$$

If we consider the function

$$f(y(t)) \;=\; (f_1(y(t)),f_2(y(t)),f_3(y(t)),f_4(y(t)),f_5(y(t))$$

With

$$\begin{aligned} f_1(y(t)) &= \Lambda-\beta_E yy_2-\beta_I y_1y_3-\beta_V y_1y_5-\mu y_1-uy_1 \\ f_2(y(t)) &= \beta_E y_1y_2+\beta_I y_1y_3+\beta_V y_1y_5-(\alpha+\mu)y_2 \\ f_3(y(t)) &= \alpha y_2-(w+\gamma+\mu)y_3 \\ f_4(y(t)) &= \gamma y_3-\mu y_4+uy_1 \\ f_5(y(t)) &= \xi_1 y_2+\xi_2 y_3-\sigma y_5 \end{aligned} \tag{9}$$

Then problem $(3)-(5)$ can be rewritten in the space $H(\Omega)$ under the form

$$\begin{aligned} \frac{\partial y}{\partial t} &= Ay+f(y(t)) \\ y(0) &= y^0 \end{aligned} \tag{10}$$

$$t\in[0,T]$$

We denote $L(T,\Omega)=L^2(0,T;H^2(\Omega))\bigcap L^\infty(0,T;H^1(\Omega))$

Theorem 1: Let Ω be a bounded domain from $\mathbb{R}^2$, with the boundary of class $C^{2+\theta},\theta>0$ If $\beta_E,\beta_I,\beta_V,\mu,\alpha,w,\gamma,\xi_1,\xi_2,\sigma>0$ $u\in U_{ad},y\in D(A)$ and$y_i^0\geq 0$ on Ω (for $i=1,2,3,4,5$) The problem (3)-(5) has a unique (global) strong solution $y\in W^{1,2}(0,T;H(\Omega))$

such that $y_1,y_2,y_3,y_4,y_5\in L(T,\Omega)\bigcap L^\infty(Q)$ and $y_i\geq 0$ on Q for$i=1,2,3,4,5$ In addition, there exists$C>0$ independent of u (and of the corresponding solution y) such that for a $t\in[0,T]$ for $i=1,2,3,4,5$

$$\|\frac{\partial y_i}{\partial t}\|_{L^2(Q)}+\|y_i\|_{L^2(0,T,H^2(\Omega)}+\|y_i\|_{H^1(\Omega)}+\|y_i\|_{L^\infty(Q)}\leq C \tag{11}$$

Proof. As$|y_i|\leq N$ for$i=1,2,3,4,5$ thus function $f=(f_1,f_2,f_3,f_4,f_5)$ becomes Lipschiz continuous in $y=(y_1,y_2,y_3,y_4,y_5)$ uniformly with respect to$t\in[0,T]$, (See [16-18]), Eq.(11)

admits a unique strong solution $y = (y_1, y_2, y_3, y_4, y_5) \in W^{1,2}(0,T;H(\Omega))$ with $y_1, y_2, y_3, y_4, y_5 \in L(T,\Omega)$ let's prove the boundedness of y on Q.

If we denote: $M = max\|f_1\|_{L^\infty(Q)}, \|y_1^0\|_{L^\infty(\Omega)}$ and $S(t), t \geq 0$ in the C_0-semi-group generated by the operator $B : D(A) \subset L^2(\Omega) \to L^2(\Omega)$ where $By = \lambda_1 \Delta y$ and $D(B) = y_1 \in H^2(\Omega), \partial y/\partial\eta = 0, a.e\partial\Omega$ The function $Y_1(t,x) = y_1 - Mt - \|y_1^0\|_{L^\infty(\Omega)}$ satisfies the Cauchy problem

$$\begin{aligned} \frac{\partial Y_1}{\partial t} &= \lambda_1 \Delta Y_1 + f_1(y(t)) - M \\ Y_1(0,x) &= y_1^0 - \|y_1^0\|_{L^\infty(\Omega)} \end{aligned} \tag{12}$$

$t \in [0,T]$

The corresponding strong solution is:

$$Y_1 t) = S(t)(y_1^0 - \|y_1^0\|_{L^\infty(\Omega)}) + \int_0^t S(t-s)(f_1(y(t)) - M)ds$$

Since $y_1^0 - \|y_1^0\|_{L^\infty(\Omega)} \leq 0$ and $f_1(y(t)) - M \leq 0$ it follows that $Y_1(t,x) \leq 0, \forall (t,x) \in Q$

And the function: $W_1(t,x) = y_1 + Mt + \|y_1^0\|_{L^\infty(\Omega)}$ satisfies the Cauchy problem

$$\begin{aligned} \partial W_1/\partial t &= \lambda_1 \Delta Y_1 + f_1(y(t)) + M \\ W_1(0,x) &= y_1^0 + \|y_1^0\|_{L^\infty(\Omega)} \end{aligned} \tag{13}$$

The corresponding strong solution is

$$W_1(t) = S(t)(y_1^0 + \|y_1^0\|_{L^\infty(\Omega)}) + \int_0^t S(t-s)(f_1(y(t)) + M)ds$$

Since$y_1^0 + \|y_1^0\|_{L^\infty(\Omega)} \geq 0$ and $f_1(y(t)) + M \geq 0$ it follows that $W_1(t,x) \geq 0, \forall (t,x) \in Q$

Then $|y_1(t,x)| \leq Mt + \|y_1^0\|_{L^\infty(\Omega)}$ $\forall (t,x) \in Q$

And analogously

$|y_i(t,x)| \leq Mt + \|y_i^0\|_{L^\infty(\Omega)}$ $\forall (t,x) \in Q$ for $i = 2,3,4,5$ So we have proved that $y_i \in L^\infty(Q)$ $\forall (t,x) \in Q$ for $i = 1,2,3,4,5$

Thus we have proved that $y_i \in L^\infty(Q)$ $(\forall (t,x) \in Q)$ for $i = 1,2,3,4,5$

By the first equation of (2) one obtains

$$\int_0^t \int_\Omega \left| \frac{\partial y_1}{\partial s} \right|^2 dsdx + \lambda_1^2 \int_0^t \int_\Omega |\triangle y_1|^2 dsdx - 2\lambda_1 \int_0^t \int_\Omega \frac{\partial y_1}{\partial s} \triangle y_1 dsdx$$

$$= \int_0^t \int_\Omega (\Lambda - \beta_E y_1 y_2 - \beta_I y_1 y_3 - \beta_V y_1 y_5 - \mu y_1 - u y_1)^2 \, ds dx$$

Using the regularity of $y_1$and the Green's formula, we have

$$\int_0^t \int_\Omega \frac{\partial y_1}{\partial s} \triangle y_1 ds dx \quad = \quad \int_\Omega \left(-|\nabla y_1|^2 + \left|\nabla y_1^0\right|^2 \right) dx$$

then

$$\int_0^t \int_\Omega \left| \frac{\partial y_1}{\partial s} \right|^2 ds dx + \lambda_1^2 \int_0^t \int_\Omega |\triangle y_1|^2 ds dx + 2\lambda_1 \int_\Omega |\nabla y_1|^2 dx - 2\lambda_1 \int_\Omega \left|\nabla y_1^0\right|^2 dx$$

$$= \int_0^t \int_\Omega (\Lambda - \beta_E y_1 y_2 - \beta_I y_1 y_3 - \beta_V y_1 y_5 - \mu y_1 - u y_1)^2 \, ds dx$$

Since $\|y_i\|_{L^\infty(Q)}$ for $i = 1,2,3,4,5$ are bounded independently of u and $y_1^0 \in H^2(\Omega)$ we deduce that:

$$y_1 \in L^\infty\left(0,T;H^1(\Omega)\right)$$

and the inequality in (11) holds for $i = 1$. similalry for y_2, y_3, y_4, y_5.In order to show the positiveness of y_i for $i = 1,2,3,4,5$ we write system (2) in the form:

$$\begin{aligned}
\frac{\partial y_1}{\partial t} &= \lambda_1 \Delta y_1 + H_1(y_1, y_2, y_3, y_4, y_5) \\
\frac{\partial y_2}{\partial t} &= \lambda_2 \Delta y_2 + H_2(y_1, y_2, y_3, y_4, y_5) \\
\frac{\partial y_3}{\partial t} &= \lambda_3 \Delta y_3 + H_3(y_1, y_2, y_3, y_4, y_5) \\
\frac{\partial y_4}{\partial t} &= \lambda_4 \Delta y_4 + H_4(y_1, y_2, y_3, y_4, y_5) \\
\frac{\partial y_5}{\partial t} &= H_5(y_1, y_2, y_3, y_4, y_5)
\end{aligned}$$

It is easy to see that the functions $H_i(y_1, y_2, y_3, y_4, y_5)$ for $i = 1,2,3,4,5$ are continuously differentiable satisfying

$$\begin{aligned}
H_1(0, y_2, y_3, y_4, y_5) &= \Lambda \geq 0 \\
H_2(y_1, 0, y_3, y_4, y_5) &= \beta_I y_1 y_3 + \beta_V y_1 y_5 \geq 0 \\
H_3(y_1, y_2, 0, y_4, y_5) &= \alpha y_2 \geq 0 \\
H_4(y_1, y_2, y_3, 0, y_5) &= \gamma y_3 + u y_1 \geq 0 \\
H_5(y_1, y_2, y_3, y_4, 0) &= \xi_1 y_2 + \xi_2 y_3 \geq 0
\end{aligned}$$

For all $y_1, y_2, y_3, y_4, y_5 \geq 0$ (See [19]). This completes the proof.

4. Existence of the Optimal Solution

This section is devoted to the existence of an optimal solution. The main result of this section is following

Theorem 2: If $\Lambda, \beta_E, \beta_I, \beta_V, \mu, \alpha, w, \gamma, \xi_1, \xi_2, \sigma > 0$ and $y^0 \in D(A)$, $y_i^0 \geq 0$ on Ω, for $i = 1,2,3,4,5$, then the optimal problem (2-6) admits an optimal solution (y^*, u^*)

proof. Let $J^* = inf\{J(y,u)\}$Where $u \in U_{ad}$ and y is the corresponding solution of (3)-(5). so J^* is finite. Therefore there exist a sequence (y^n, u^n) with $u^n \in U_{ad}$, $y^n = \left(y_1^n, y_2^n, y_3^n, y_4^n, y_5^n\right) \in W^{1,2}(0,T;H(\Omega))$ such that

$$(14) \qquad \begin{aligned}
\frac{\partial y_1^n}{\partial t} &= \lambda_1 \Delta y_1^n + \Lambda - \beta_E y_1^n y_2^n - \beta_I y_1^n y_3^n - \beta_V y_1^n y_5^n - \mu y_1^n - u y_1^n \\
\frac{\partial y_2^n}{\partial t} &= \lambda_2 \Delta y_2^n + \beta_E y_1^n y_2^n + \beta_I y_1^n y_3^n + \beta_V y_1^n y_5^n - (\alpha + \mu) y_2^n \\
\frac{\partial y_3^n}{\partial t} &= \lambda_3 \Delta y_3^n + \alpha y_2^n - (w + \gamma + \mu) y_3^n \\
\frac{\partial y_4^n}{\partial t} &= \lambda_4 \Delta y_4^n + \gamma y_3^n - \mu y_4^n + u y_1^n \\
\frac{\partial y_5^n}{\partial t} &= \xi_1 y_2^n + \xi_2 y_3^n - \sigma y_5^n
\end{aligned}$$

with the homogeneuos Neumann boundary conditions

$$(15) \qquad \frac{\partial y_1^n}{\partial \eta} = \frac{\partial y_2^n}{\partial \eta} = \frac{\partial y_3^n}{\partial \eta} = \frac{\partial y_4^n}{\partial \eta} = \frac{\partial y_5^n}{\partial \eta} = 0$$

$(t,x) \in \Sigma$

$$(16) \qquad y_i^n(0,x) = y_i^0,$$

for $i = 1,2,3,4,5$ with $x \in \Omega$

and

$$(17) \qquad J^* \leq J(y^n, u^n) \leq J^* + \frac{1}{n}$$

$(\forall n \geq 1)$

since $H^1(\Omega)$ is compactly embedded in $L^2(\Omega)$, we infer that $y_1^n(t)$ is compact in $L^2(\Omega)$ show that $\left\{y_1^n(t), n \geq 1\right\}$ is equiacontinuous in $C\left([0,T] : L^2(\Omega)\right)$.

By the first equation from (14) we have

$$\frac{\partial y_1^n}{\partial t} y_1^n = \lambda_1 \Delta y_1^n y_1^n + \Lambda y_1^n - \beta_E (y_1^n)^2 y_2^n - \beta_I (y_1^n)^2 y_3^n - \beta_V (y_1^n)^2 y_5^n - \mu (y_1^n)^2 - u (y_1^n)^2 \tag{18}$$

Then $\forall t \in [0,T]$

(19)

$$\int_\Omega (y_1^n)^2 (t,x)\, dx = \int_\Omega \left(y_1^0\right)^2 (x) dx$$
$$+2 \int_0^t \int_\Omega \left[\lambda_1 \Delta y_1^n y_1^n + \Lambda y_1^n - \beta_E (y_1^n)^2 y_2^n - \beta_I (y_1^n)^2 y_3^n - \beta_V (y_1^n)^2 y_5^n - \mu (y_1^n)^2 - u (y_1^n)^2\right] dx d\zeta, \forall t \in [0,T]$$

By theorem (1) there exists a constant $C > 0$ independent of n such that for all $n \geq 1$, $t \in [0,T]$

$$\left\| \frac{\partial y_i^n}{\partial t} \right\|_{L^2(Q)} \leq C, \|y_i^n\|_{L^2(0,T;H^2(\Omega))}$$
$$\leq C, \|y_i^n\|_{H^2(\Omega)} \leq C, fori = 1,2,3,4,5 \tag{20}$$

For all $n \geq 1, t \in [0,T]$, the sequence y_i^n is bounded in $C\left([0,T] : L^2(\Omega)\right); \triangle y_i^n, u_i^n$ and $\frac{\partial y_i^n}{\partial t}$ are bounded in $L^2(Q)$ for $i = 1,2,3,4,5$. This implies that for all $s,t \in [0,T]$

$$\left| \int_\Omega (y_1^n)^2 (t,x)\, dx - \int_\Omega (y_1^n)^2 (s,x)\, dx \right| \leq K |t-s| \tag{21}$$

The Ascoli-Arzela Theorem (See [20]) implies that y_1^n is compact in $C\left([0,T] : L^2(\Omega)\right)$. Hence, selecting further sequences, if necessary, we have$y_1^n \to y_1^*$ in $L^2(\Omega)$ uniformly with respect to t.

and analogously

$y_i^n \to y_i^*$ in $L^2(\Omega)$ uniformly with respect to t, for $i = 2,3,4,5$.

then $y_2^n(T) \to y_2^*(T)$ in $L^2(\Omega)$

The boundedness of $\triangle y_i^n$ in $L^2(\Omega)$, implies its weak convergence, namely $\triangle y_i^n \rightharpoonup \triangle y_i^*$ in $L^2(\Omega)$ $i = 1,2,3,4,5$. Here and everywhere below the sign $\rightharpoonup$ denotes the weak convergence in the specified space. Estimates (20) lead to

$$\frac{\partial y_i^n}{\partial t} \rightharpoonup \frac{\partial y_i^*}{\partial t} inL^2(Q), i = 1 = 1,2,3,4,5$$

$$y_i^n \rightharpoonup y_i^* inL^2\left(0,T; H^2(\Omega)\right), i = 1,2,3,4,5$$

$$y_i^n \rightharpoonup y_i^* inL^\infty\left(0,T; H^1(\Omega)\right), i = 1,2,3,4,5$$

Writing $y_1^n y_2^n - y_1^* y_2^* = \left(y_1^n - y_1^*\right) y_2^n + y_1^n \left(y_2^n - y_2^*\right)$ and making use of the convergences $y_i^n \longrightarrow y_i^*$in $L^2(Q)$, $i = 1,2,3,4,5$ and of boundedness of y_1^n, y_2^n in $L^\infty(Q)$, one arrives at $y_1^n y_2^n \mapsto y_1^* y_2^*$ in $L^2(Q)$. We also have $u^n \rightharpoonup u^*$ in $L^2(Q)$ on a subsequence denoted again u^n. since U_{ad} is closed and convex set in $L^2(Q)$, it is weakly closed, so $u^* \in U_{ad}$ and as above $u^n y_1^n \longrightarrow u^* y_1^*$ in $L^2(Q)$. Now we may pass to the limit in $L^2(Q)$ as $n \longrightarrow \infty$ in (14-17) to deduce that $(y^*, u^*)$is an optimal solution. The proof is complete.

5. CONCLUSION

The work in this paper contributes to a growing literature on modeling the spatial spread of a novel coronavirus epidemic in Wuhan, China. We present an application of optimal control theory to spatiotemporal epidemic models described by a system of partial differential equations. The control variable is the spatial and temporal distribution. We have based our mathematical work on the use of semigroup theory and optimal control to show the existence of solutions for our state system, as well as prove the existence of an optimal control.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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